Heavy Ion Physics and the Quark-Gluon-Plasma

Thermodynamics of the Quark-Gluon Plasma

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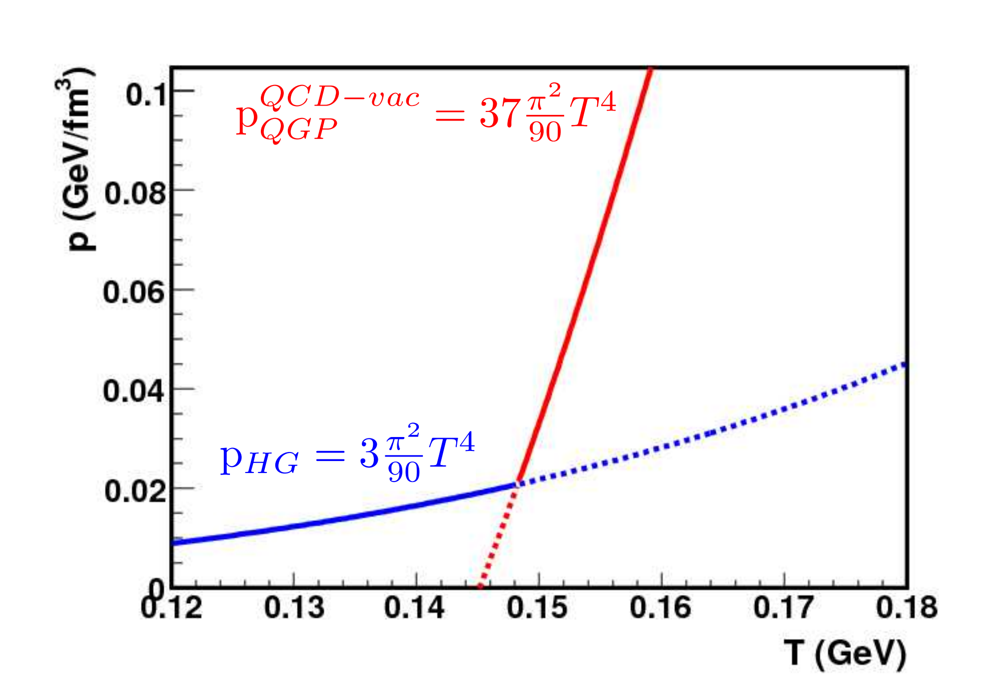
- **Thermodynamics of quarks and gluons**
- **The deconfinement transition in the (MIT) Bag Model**
- **The phase diagram of QCD**
- **Lattice QCD Global event properties**

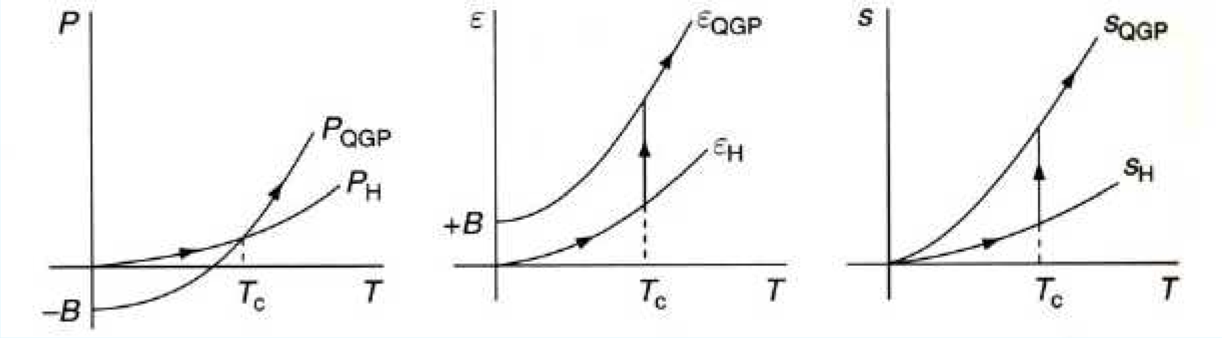
MIT Bag Model

- **Calculations on black board**
  - Introduction to Bag Model
  - Fermi-Dirac and Bose-Einstein Distribution
  - Degeneracy
  - Quark/anti-quark density
  - Quark-Gluon Plasma at $\mu = 0$
    - Energy density, pressure, particle density, entropy
    - Critical temperature

NA49

Quark-Gluon Plasma with μ = 0: Critical Temperature





Quark-Gluon Plasma
Yagi et al.
Cambridge University Press
2005

Quark-Gluon Plasma with μ ≠ 0: Energy and Particle Number Density of the Quarks

For $\mu_q \neq 0$ a solution in closed form can be found for $\varepsilon_q + \varepsilon_{\bar{q}}$ but not for ε_q and $\varepsilon_{\bar{q}}$ separately: Chin, PL 78B (1978) 552

$$\varepsilon_q + \varepsilon_{\bar{q}} = g_q \times \left(\frac{7\pi^2}{120} T^4 + \frac{\mu_q^2}{4} T^2 + \frac{\mu_q^4}{8\pi^2} \right)$$

Accordingly one finds for the quark density

$$n_q - n_{\bar{q}} = g_q \times \left(\frac{\mu_q}{6} T^2 + \frac{\mu_q^3}{6\pi^2} \right)$$

From this the net baryon density can be determined as (for $g_q = 12$):

$$n_B = \frac{n_q - n_{\bar{q}}}{3} = \frac{2\mu_q}{3} T^2 + \frac{2\mu_q^3}{3\pi^2} = \frac{2\mu_B}{9} T^2 + \frac{2\mu_B^3}{81\pi^2} \qquad (\mu_B = 3\mu_q)$$

Quark-Gluon Plasma with μ ≠ 0: Critical Temperature

Energy density in a QGP with μ ≠ 0 (without particle interactions):

$$\varepsilon_{\mathrm{QGP}} = \varepsilon_q + \varepsilon_{\bar{q}} + \varepsilon_g = \frac{37\pi^2}{30}T^4 + 3\mu_q^2 T^2 + \frac{3\mu_q^4}{2\pi^2}$$

Condition for QGP stability:

$$p_{\mathrm{QGP}} = \frac{1}{3}\varepsilon_{\mathrm{QGP}} \stackrel{!}{=} B \quad \Rightarrow T_c(\mu_q)$$

Condition for QGP:
QGP-pressure ≥ pressure of the QCD-vacuum
(similar, but not identical, to the previous condition $p_{\mathrm{HG}} = p_{\mathrm{QGP}}$)

Critical temperature / quark potential:

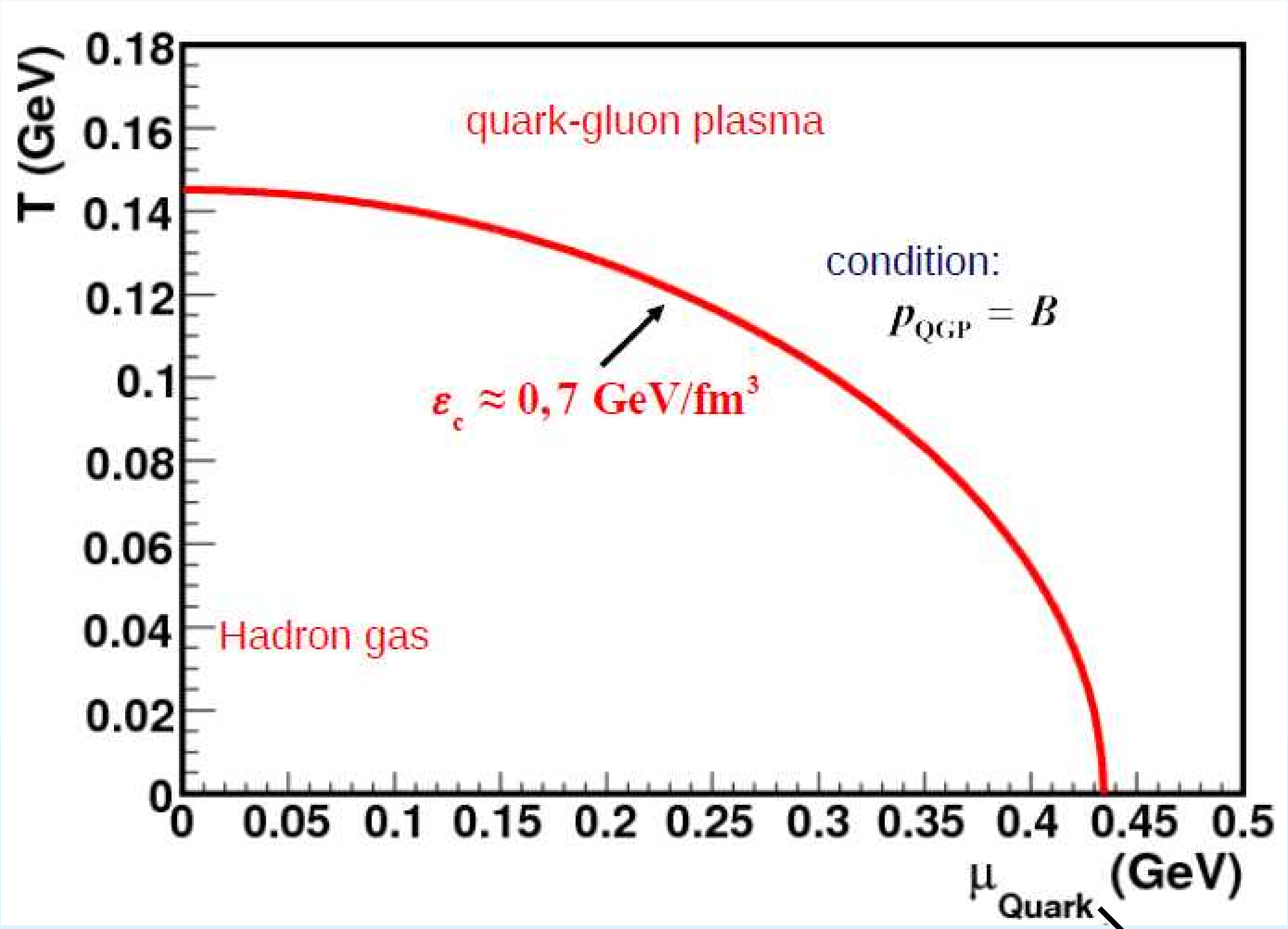
$$T_c(\mu_q = 0) = \left(\frac{90B}{37\pi^2}\right)^{1/4}$$

$$\mu_q^c(T = 0) = (2\pi^2 B)^{1/4} = 0.43\,\mathrm{GeV}$$

$$n_B^c(T = 0) = \frac{2}{3\pi^2}(2\pi^2 B)^{3/4} = 0.72\,\mathrm{fm}^{-3} \approx 5 \times n_{\mathrm{nucleus}}$$

Possibly reached in neutron stars

Quark-Gluon Plasma with μ ≠ 0: Phase Diagram of the Non-Interacting QGP



$\mu_B = 3 \cdot \mu_{\text{Quark}}$

Drawbacks of the Bag Model

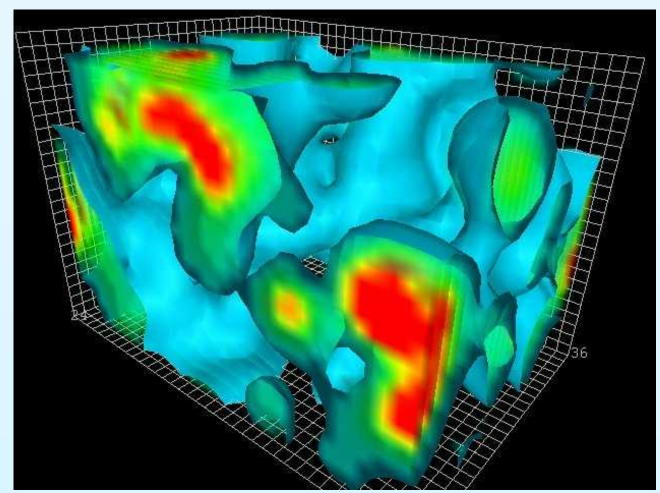
- Neglecting interactions
- Only u and d quarks
- 1st order phase transition at high T

Lattice QCD (I)

- QCD asymptotically free at extremely large T and/or small distances
- Cannot use perturbation theory to calculate, e.g., properties of hadrons
- Instead solve QCD numerically at zero and finite temperature by putting gauge fields on a space-time lattice ↔ “lattice QCD”
- First-principle non-perturbative calculation
- Lattice needs to be big, e.g. 163 x 32

Example of a machine for lattice QCD:
JUGENE in Jülich (294,912 processor cores, ~ 1 PetaFLOPS)

Snapshot of fluctuating quark and gluon fields on a discrete space-time lattice:

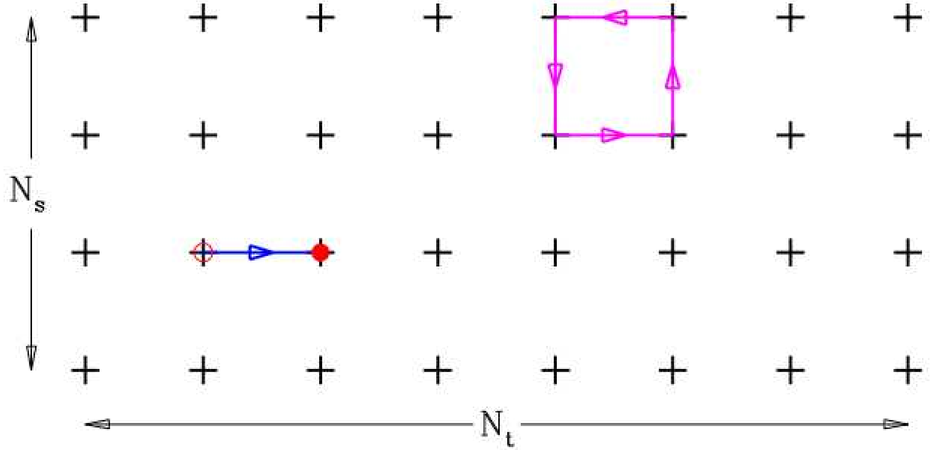


http://www.physics.adelaide.edu.au/theory/staff/leinweber/index.html

Lattice QCD (II)

Lattice spacing $a, \quad a^{-1} \sim \Lambda_{\mathrm{UV}}, \quad x_\mu = n_\mu a$

Finite volume $L^3 \cdot T, \quad N_s = L/a, \quad N_t = T/a$



(anti)quarks: $\psi(x),\ \overline{\psi}(x)$ lattice sites

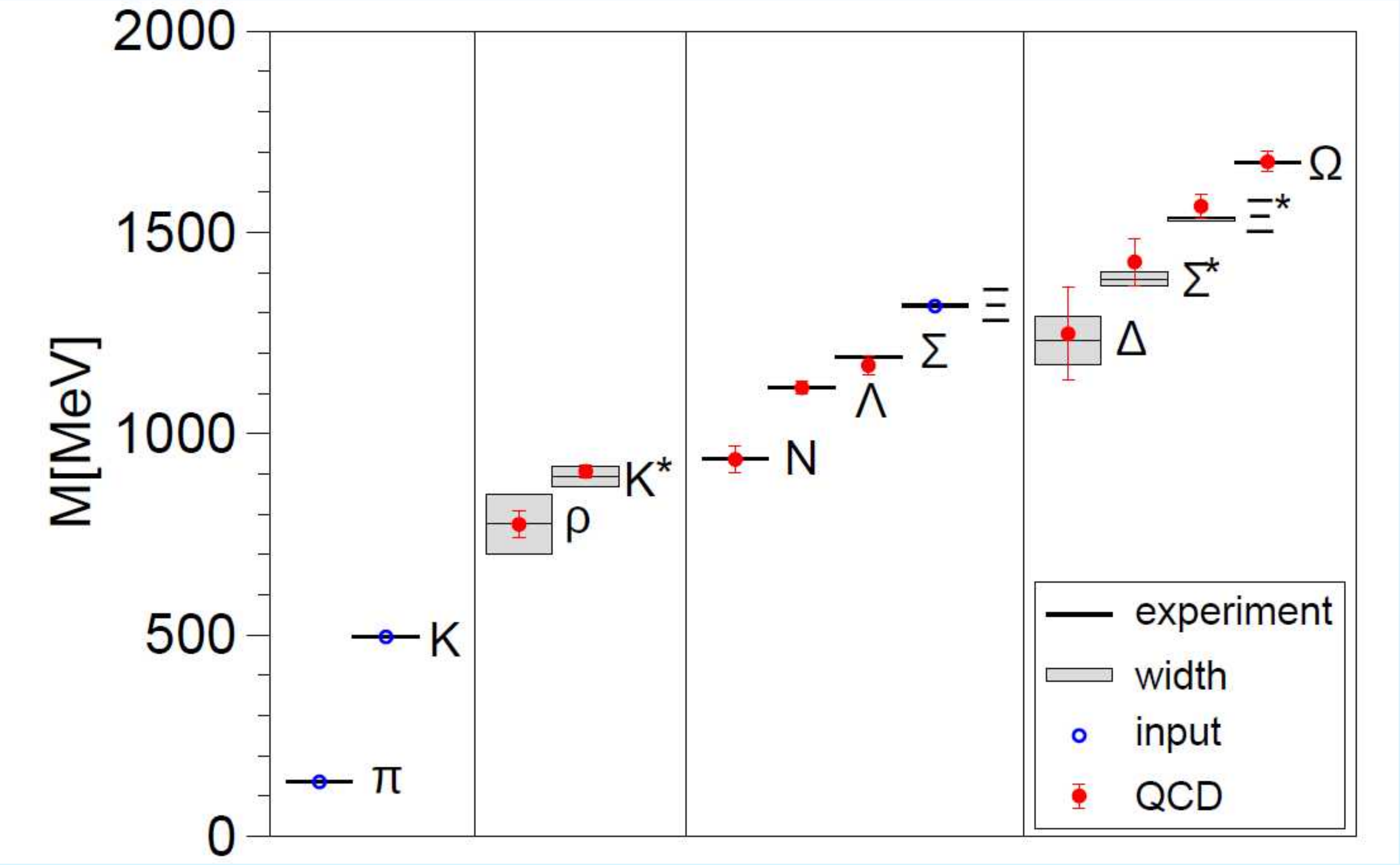
gluons: $U_\mu(x) = \mathrm{e}^{aA_\mu(x)} \in \mathrm{SU}(3)$ links

field tensor: $P_{\mu\nu}(x) = U_\mu(x)U_\nu(x + a\hat{\mu})U_\mu^\dagger(x + a\hat{\nu})U_\nu^\dagger(x)$ "plaquettes"

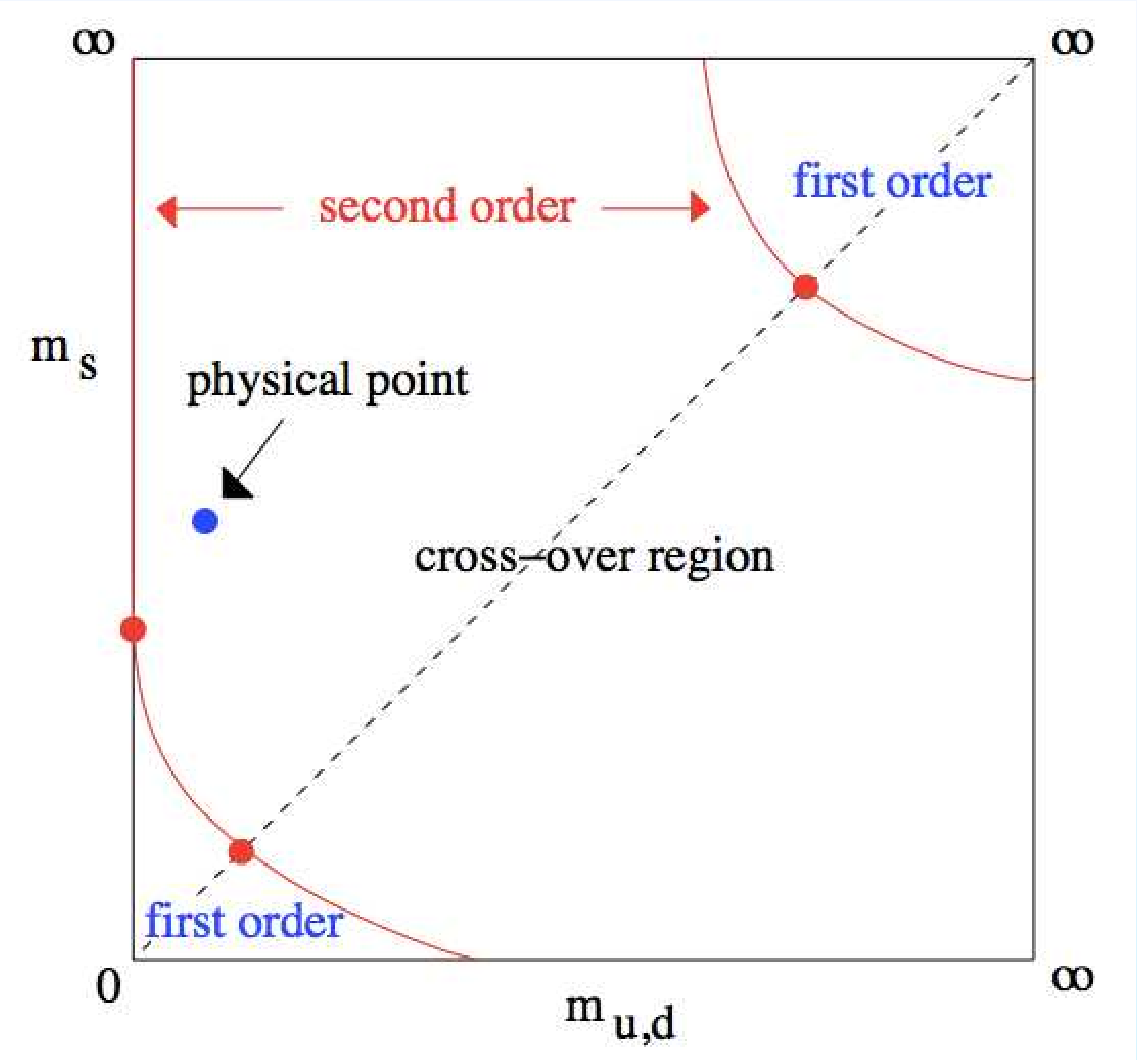
from Hartmut Wittig: Lattice QCD - Introduction and Results

Hadron Spectrum from Lattice QCD

S. Dürr, Z.Fodor et al., Budapest-Marseille–Wuppertal Coll.,Science 322 (2008) 1225



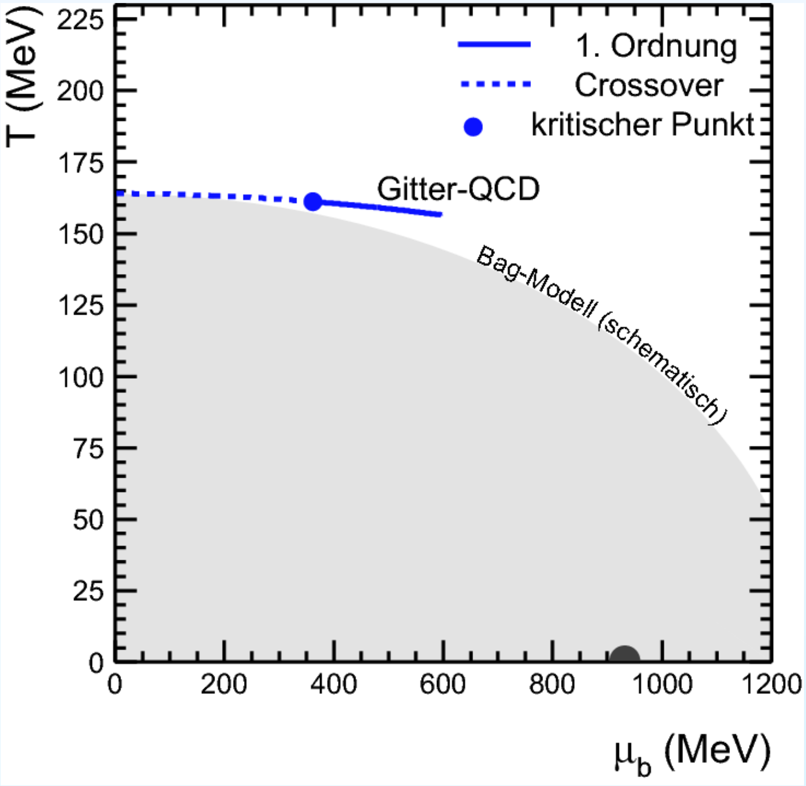
Nature of the Critical Behavior in QCD



The nature of the transition depends sensitively on the quark masses.

H. Satz,
The Thermodynamics of
Quarks and Gluons, arXiv:0803.1611

Quark-Gluon Plasma with μ ≠ 0: Phase Diagram from Lattice-QCD



Z. Fodor, S.D. Katz,
JHEP 404 (2004) 50 [hep-lat/0402006].

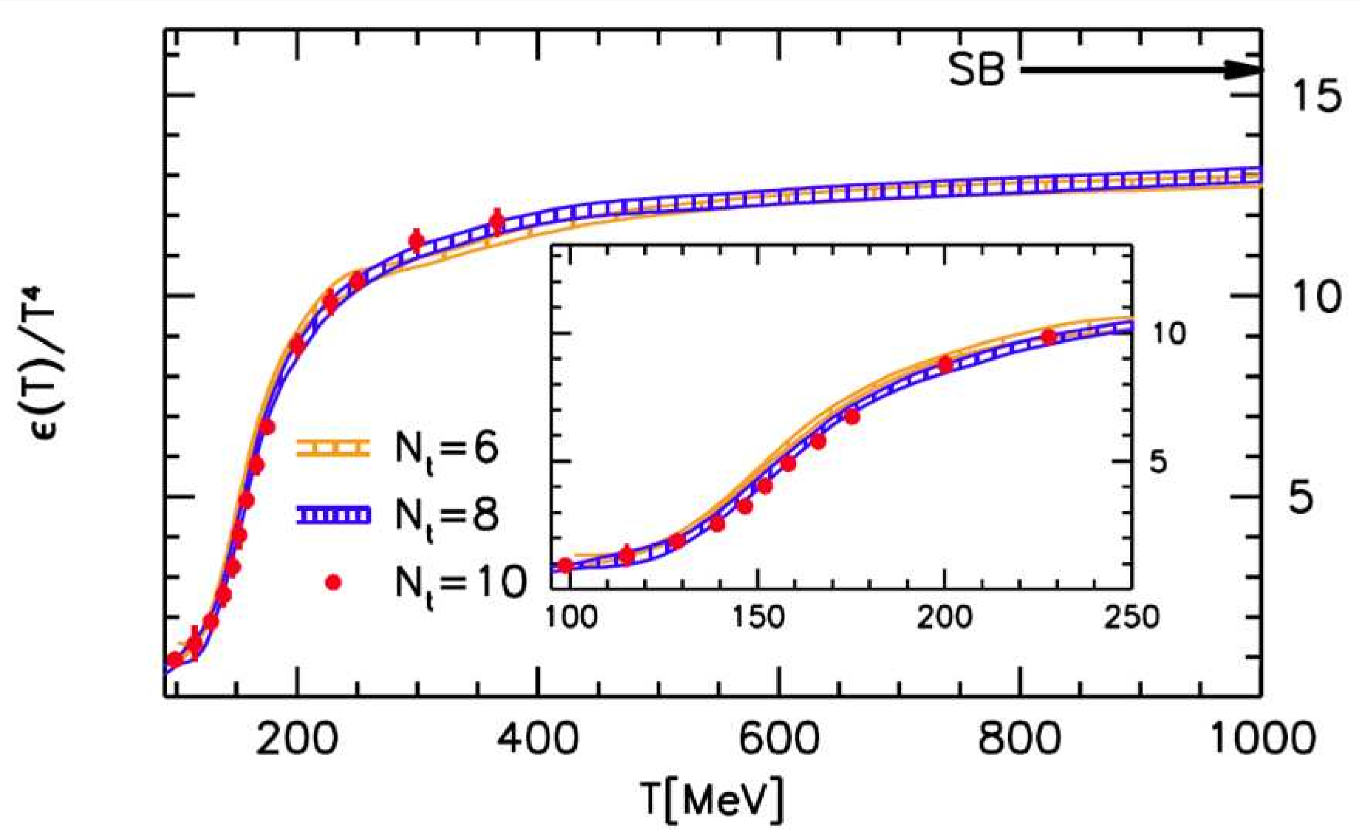
- **Lattice-calculations for $\mu_b \neq 0$**
  - numerically very expensive
- **Some calculations suggest a critical point (with large theoretical uncertainties):**
  - T = 162 MeV
  - μ_b = 340 MeV

The existence and exact position of the critical point remains an open question

Lattice Results

Wuppertal-Budapest Collaboration:
arXiv:1309.5258 [hep-lat]
HotQCD Collaboration:
arXiv:1407.6387 [hep-lat]

- **Latest lattice results for T_c (μ = 0):**
  - $T_c \approx$ 150 MeV
- **Example: ε/T^4 vs. T from Wuppertal-Budapest Collaboration:**



Take-Home Message

- **When treated as a ultra-relativistic ideal gas, parameters for the transition hadron Gas ↔ QGP are:**
  - T_c (μ_b=0) ≈ 150 MeV
  - $\mu_{b,c}$(T=0) = 3 $\mu_{Quark,c}$(T=0) ≈ 1.3 GeV
    (this is approximately five times the density of „normal“ nuclear matter)
- **Lattice QCD calculations show that for temperatures up to several times T_c the assumption of an ideal gas is a poor approximation**
- **Transition temperature from Lattice QCD (as of 2013/2014):**
  - T_c (μ_b=0) ≈ 150 MeV
  - At low μ_b a cross-over transition
  - More structure in the phase diagram, maybe a critical point