Short range interaction in $\pi J/\psi - D\bar{D}^*$ channel

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“New aspects of the Hadron and Astro/Nuclear Physics”

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Outline

- Introduction
  - Exotic hadrons
  - $Z_c(3900)$
- Interaction model
  - Meson exchange model
  - Quark exchange model
- Summary
Description of Hadron structure

Introduction

- Ordinary Hadrons: Baryon \((qqq)\) and Meson \((q\bar{q})\)
- Baryon (proton, nucleon, ...) \(\rightarrow\) Meson \((\pi, K, ...)\)
  
  \* \(q\): “ Constituent quark”

- Exotic Hadrons \((\neq qqq, q\bar{q})\): Multiquark? Multihadron?

  - Pentaquark (Compact)
  - Hadronic molecule
Constituent quark picture and beyond

Introduction

▷ e.g. $c\bar{c}$ mesons (Charmonium)

Constituent quark picture and beyond

Introduction

▷ e.g. $c\bar{c}$ mesons (Charmonium) and Unexpected $X$, $Y$, $Z$

Exotics $\neq c\bar{c}$ have been observed in the Experiments (BaBar, Belle, BESIII, LHCb,...) ⇒ Q. Structure? Physics?
Many exotic candidates!! Many models!!

Introduction

H. X. Chen, et al., Phys. Rept. 639 (2016) 1, ...

6 Nov. 2018 Yasuhiro Yamaguchi (RIKEN)

New aspects of the Hadron and Astro/Nuclear Physics
Charged Charmonium: $Z_c(3900)$

**Introduction**

- Charged Charmonium??
- $Y(4260) \to Z_c(3900)\pi \to J/\psi\pi\pi$

BESIII, *PRL* **110**(2013)252001

Belle, *PRL* **110**(2013)252002

$M = 3899.0 \pm 3.6_{sta} \pm 4.9_{sys}$ MeV

$\Gamma = 46 \pm 10_{sta} \pm 20_{sys}$ MeV

$M = 3894.5 \pm 6.6_{sta} \pm 4.5_{sys}$ MeV

$\Gamma = 63 \pm 24_{sta} \pm 26_{sys}$ MeV

Charged Charmonium: \( Z_c(3900) \)

**Introduction**

- Charged Charmonium??
- \( Y(4260) \rightarrow Z_c(3900)\pi \rightarrow J/\psi \pi \pi \)

BESIII, PRL **110** (2013) 252001

Belle, PRL **110** (2013) 252002

\[
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\]

CLEO-c, PLB **727** (2013) 366(2013), \( \mathcal{D}_0 \), PRD **98** (2018) 052010

- Ordinal Charmonium \( c\bar{c} \): no electric charge.
- \(\Rightarrow Z_c^+(3900): \text{Genuine Exotic State}!? \ c\bar{c}ud\bar{d}\)
What is the structure of $Z_c(3900)$?

**Introduction**

**Multiquark states?**


- Molecules? — $Z_c(3900)$ close to the $D\bar{D}^*$ threshold ($\sim 3875$)
What is the structure of $Z_c(3900)$?

Introduction

Multiquark states?

- Molecules? — $Z_c(3900)$ close to the $D\bar{D}^*$ threshold ($\sim 3875$)
  
  $\Rightarrow$ Exotic state may be a loosely bound state (resonance) of the meson-meson.
  
  $\rightarrow$ Analogous to atomic nuclei (Deuteron: $B \sim 2.2$ MeV)

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Introduction

Molecules? — $Z_c(3900)$ close to the $D\bar{D}^*$ threshold ($\sim 3875$)

$\Rightarrow$ Exotic state may be a loosely bound state (resonance) of the meson-meson.

$\rightarrow$ Analogous to atomic nuclei (Deuteron: $B \sim 2.2$ MeV)

$\Leftrightarrow$ Kinematical effect? **No bound state explanation**

D.-Y. Chen, X. Liu, T. Matsuki, PRD**88**(2013)036008, J. He, D.-Y. Chen, EPJC**78**(2018)94, ...
Lattice QCD simulation by HALQCD at $m_\pi = 410 - 700$ MeV

Coupled-channel $\pi J/\psi - \rho \eta_c - D\bar{D}^*$
\[ Z_c(3900): \text{Lattice QCD (Numerical Experiments)} \]

**Introduction**

- Lattice QCD simulation by HALQCD at \( m_\pi = 410 - 700 \) MeV
  - Coupled-channel \( \pi J/\psi - \rho \eta_c - D\bar{D}^* \)

\[ \text{Fate of the Tetraquark Candidate } Z_c(3900) \text{ from Lattice QCD} \]

- Virtual state is obtained.
  - \( Z_c(3900) \) is Threshold cusp

Ikeda, *et al.*, PRL 117(2016)242001
$Z_c(3900)$: Lattice QCD (Numerical Experiments)

Introduction

- Lattice QCD simulation by HALQCD at $m_\pi = 410 - 700$ MeV
  - Coupled-channel $\pi J/\psi - \rho \eta_c - D\bar{D}^*$

- Virtual state is obtained.
- $Z_c(3900)$ is Threshold cusp induced by $\pi J/\psi - \bar{D}D^*$ potential

Charm flavor exchange?

Ikeda, et al., PRL 117 (2016) 242001
Exotic structure:
Bound state? Cusp?
Exotic structure: Bound state? Cusp?

Hadron-hadron interaction is important to understand the nature of exotic states! not only $Z_c$ but also others.
Model of Hadron-hadron interaction

Introduction

- **Long-range force:** one $\pi$ exchange potential (OPEP)
  - Lightest meson $\pi$, Importance in the nuclear force,
  - Heavy Quark Spin Symmetry ($0^- - 1^-$ mixing)
Model of Hadron-hadron interaction

Introduction

- **Long-range force**: one $\pi$ exchange potential (OPEP)
  
  Lightest meson $\pi$, Importance in the nuclear force,
  Heavy Quark Spin Symmetry ($0^- - 1^-$ mixing)

- **Short-range force**: Charm ($c$) exchange

  ▶ How can we understand strong $\pi J/\psi - D\bar{D}^*$ potential?
Model of Hadron-hadron interaction

**Introduction**

- **Long-range force:** one \( \pi \) exchange potential (OPEP)
  
  Lightest meson \( \pi \), Importance in the nuclear force,
  
  **Heavy Quark Spin Symmetry** \( (0^- - 1^- \text{ mixing}) \)

- **Short-range force:** Charm \( (c) \) exchange

  ▶ How can we understand strong \( \pi J/\psi - D\bar{D}^* \) potential?

  (a) \( D^{(*)} \) meson exchange?

  \[
  \begin{array}{ccc}
  J/\psi & \rightarrow & D^{(*)} \\
  \downarrow \pi & \text{D(*)} & \downarrow \bar{D}^{(*)}
  \end{array}
  \]

  \( \rightarrow \) next sec.

(b) Quark exchange?

\[
\begin{array}{ccc}
  c & \rightarrow & c \\
  \downarrow \bar{c} & \text{Quark exchange} & \downarrow \bar{q}
  \end{array}
\]

\[
\begin{array}{ccc}
  q & \rightarrow & q \\
  \downarrow \bar{q} & \rightarrow \text{next next sec.} & \downarrow \bar{c}
  \end{array}
\]

**Comparison between** \( D \) exchange and \( \text{Quark exchange} \)
Meson exchange model

\[ J/\psi \rightarrow D(\ast) \]
\[ \pi \rightarrow D(\ast) \]

- Coupled channel: \( \pi J/\psi - D\bar{D}\ast - D\ast\bar{D}\ast \)
- \( D(\ast)\bar{D}(\ast) - D(\ast)\bar{D}(\ast) \): \( \pi \) exchange
- \( \pi J/\psi - D(\ast)\bar{D}(\ast) \): \( D(\ast) \) exchange

Yasuhiro Yamaguchi (RIKEN), Yukihiro Abe (RCNP, Osaka Univ.), Kenji Fukukawa (Suma Gakuen), Atsushi Hosaka (RCNP, Osaka Univ.), in preparation
Heavy Quark Spin Symmetry (HQS)  

- **Suppression of Spin-spin force** in $m_Q \to \infty$.

  $\Rightarrow$ **Mass degeneracy** of hadrons with the different $J$

- e.g. $Q\bar{q}$ meson

\[ P^* \text{ (spin-1)} \quad P \text{ (spin-0)} \]

\[ Q \bar{q} \quad \Rightarrow \text{Mass degeneracy of spin-0 and spin-1 states!} \]
Mass degeneracy of heavy hadrons
Meson exchange model

- Mass difference between vector and pseudoscalar mesons. 
  \((Q\bar{q}, q = u, d)\)

\[\Delta m\]

\(\rho\)  
770 MeV

\(\vec{K}^*\)  
890 MeV

\(D^*\)  
2010 MeV

\(\vec{B}^*\)  
5325 MeV

For \(Z_c(3900)\), \(D_D\) mixing
\(D_D\) coupled-channel

\(\Delta m\) decreases when the quark mass increases.
Mass degeneracy of heavy hadrons
Meson exchange model

- Mass difference between vector and pseudoscalar mesons.
  \((Q\bar{q}, q = u, d)\)

\[\Delta m\]

\(\rho\) 770 MeV
\(\bar{K}^*\) 890 MeV
\(D^*\) 2010 MeV
\(\bar{B}^*\) 5325 MeV

\(\Delta m\) decreases when the quark mass increases.

\(\Rightarrow\) Degeneracy of Heavy hadrons!

For \(Z_c(3900)\), \(D - D^*\) mixing \(\Rightarrow\) \(D\bar{D}^* - D^*\bar{D}^*\) coupled-channel
Effective Lagrangians: Heavy hadron and $\pi$


$\pi$ forbidden

Heavy meson: ($DD\pi$: Parity violation)
Heavy hadron-π coupling
Meson exchange model

- **Effective Lagrangians**: Heavy hadron and π
  

\[ \pi \]

\[ g_\pi \]

\[ \bar{D}^{(*)} \quad \bar{D}^* \]

- Heavy meson: \( \bar{D}^{(*)} \bar{D}^{(*)}\pi \) (\(DD\pi\): Parity violation)

\[ \mathcal{L}_{\pi HH} = -\frac{g_\pi}{2f_\pi} \text{Tr} \left[ H \gamma_\mu \gamma_5 \partial^\mu \pi \bar{H} \right], \quad H = \frac{1+\gamma^5}{2} \left[ \bar{D}^*_\mu \gamma^\mu - D \gamma_5 \right] \]

- One coupling const. \( g_\pi = 0.59 \) (from \( D^* \rightarrow D\pi \) decay)

- Form factor (Hadron has **finite size**)

\[ F(q^2) = \frac{\Lambda^2 - m^2_\pi}{\Lambda^2 - q^2}, \quad \Lambda_{\bar{D}} \sim 1130 \text{ MeV} \text{ (by Quark model)} \]
One pion exchange potential in $D^{(*)} \bar{D}^{(*)}$

Meson exchange model

- One boson exchange potential (OBEP)

$\bar{D}^* \pi$ vertex induces OPEP
($DD\pi$ vertex violates the parity conservation)

\[
V = -\frac{1}{2} \left( g_{\pi} \right)^2 \left\{ \vec{S}_1 \cdot \vec{S}_2 C(r) + S_{12}(r) T(r) \right\} \vec{\tau}_1 \cdot \vec{\tau}_2
\]

DD* potential

Comments

- HQS induces $D(0^-) - D^*(1^-)$ coupling $\rightarrow$ OPEP works!
One pion exchange potential in $D^{(*)} \bar{D}^{(*)}$

Meson exchange model

- One boson exchange potential (OBEP) with Tensor force!

\[ \bar{D}^{*} \pi \] vertex induces OPEP
($DD_{\pi}$ vertex violates the parity conservation)

\[
V^\pi = -\frac{1}{2} \left( \frac{g_\pi}{f_\pi} \right)^2 \left[ \vec{S}_1 \cdot \vec{S}_2 C(r) + S_{12}(\hat{r}) T(r) \right] \vec{\tau}_1 \cdot \vec{\tau}_2
\]

**Comments**

- HQS induces $D(0^-) - D^*(1^-)$ coupling $\rightarrow$ OPEP works!
- Tensor force $T(r) \Rightarrow$ the driving force in atomic nuclei

\[ S_{12}(\hat{r}) = 3(\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r}) - \vec{S}_1 \cdot \vec{S}_2 \rightarrow S-D$ mixing
**Heavy meson exchange potential**

**Meson exchange model**

- $D(\ast)$ meson exchange potential in $\pi J/\psi - D(\ast)\bar{D}(\ast)$

\[ V^D = \frac{2}{3} g_\psi g_\pi \left[ \vec{S}_1 \cdot \vec{S}_2 C(r) + S_{12}(\vec{r}) T(r) \right] \]

\[ V^{D\ast} = \frac{2}{3} g_\psi g_\pi \left[ 2\vec{S}_1 \cdot \vec{S}_2 C(r) - S_{12}(\vec{r}) T(r) \right] \]

$g_\psi = 8.0$ (Assuming VMD), $\Lambda_\psi = 2.2$ GeV


**Comments**

- Spin-spin ($\vec{S}_1 \cdot \vec{S}_2$) and Tensor ($S_{12}$) terms
- Energy-dependence ($1/\sqrt{E_\pi}$)
Numerical results: Phase shift
Meson exchange model

- We found...

Why?

- Isospin factor $\vec{1}_1 \vec{1}_2$, but $Z_c: +1$ ($I = 1$)

- Why?
  - Volume Integral $V_D C (\vec{q}^2 = 0) = 3.14 \text{ GeV}^2$
  - $V_{NN} = 3.00 \times 10^2 \text{ GeV}^2$

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Numerical results: Phase shift
Meson exchange model

We found... **No Bound state, No Resonance**
Very Small phase shift $|\delta| < 0.09$ [rad]

$D(*)\bar{D}(*)$ channel: **Small** contribution from OPEP

$\pi J/\psi$ channel: $D(*)$ exchange is **Negligible**
Numerical results: Phase shift
Meson exchange model

- We found... No Bound state, No Resonance
  Very Small phase shift $|\delta| < 0.09$ [rad]

- $D^{(*)} \bar{D}^{(*)}$ channel: Small contribution from OPEP
  Why?: Isospin factor $\vec{r}_1 \cdot \vec{r}_2, -3$ ($I = 0$), but $Z_c: +1$ ($I = 1$)

- $\pi J/\psi$ channel: $D^{(*)}$ exchange is Negligible
  Why?: Volume Integral $V_C^D(q^2 = 0) = 3.14$ GeV$^{-2}$
  \[ \leftrightarrow V_{NN}^{\sigma} \sim 3.00 \times 10^2 \text{ GeV}^{-2} \]
No resonance is found

\[ \pi J/\psi - D\bar{D}^* \] contribution is not explained.
No resonance is found

\[ \pi J/\psi - D\bar{D}^* \] contribution is not explained.

Another Short range force

Quark exchange interaction!

\[ c \rightarrow \bar{c} \rightarrow q \rightarrow \bar{q} \rightarrow c \]

→next section
Quark exchange model

Meson-meson scattering by the quark exchange

- Only $\pi J/\psi - D\bar{D}^*$ channel

Yasuhiro Yamaguchi (RIKEN), Yukihiro Abe (RCNP, Osaka Univ.), Kenji Fukukawa (Suma Gakuen), Atsushi Hosaka (RCNP, Osaka Univ.), in preparation
Quark exchange model
Quark exchange interaction

- **Born-order quark-exchange diagram**


  \[ AB \rightarrow CD \text{ scattering} \quad \mathcal{M}_{fi} \propto \langle C, D | H_1 | A, B \rangle \]

- Ingredients: Meson Wavefunctions \((A, B, C, D)\)
  Quark interaction (Quark Model)

- Born amplitude \(\Rightarrow\) Meson-meson Potential can be obtained
Quark Hamiltonian (One gluon exchange + Linear potentials)


\[ H_{ij}^q = K_q + \left( -\frac{3}{4} br + \frac{\alpha_s}{r} - C \right) \vec{F}_i \cdot \vec{F}_j \]

\[ - \frac{8\pi\alpha_h}{3m_i m_j} \left( \frac{\sigma^3}{\pi^{3/2}} e^{-\sigma^2 r_{ij}^2} \right) \vec{S}_i \cdot \vec{S}_j \vec{F}_i \cdot \vec{F}_j \]

Parameters are fixed to reproduce the mass of mesons

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_q )</td>
<td>0.375 GeV</td>
</tr>
<tr>
<td>( \alpha_s )</td>
<td>0.857</td>
</tr>
<tr>
<td>( b )</td>
<td>0.154 GeV(^{-2} )</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.70 GeV</td>
</tr>
<tr>
<td>( m_c )</td>
<td>1.9 GeV</td>
</tr>
<tr>
<td>( \alpha_h )</td>
<td>0.840</td>
</tr>
<tr>
<td>( C )</td>
<td>-0.4358 GeV</td>
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</table>
Meson Wavefunction
Quark exchange interaction

- Single Gaussian Approximation (Simple)

\[ \psi(r) = (4\pi\lambda)^{-3/4} \exp \left( -\frac{r^2}{8\lambda} \right) \]

- \( \lambda \) is determined to minimize \( E(\lambda) = \langle \psi | H^q | \psi \rangle \)

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<th>( m ) [GeV], ( \lambda ) [GeV(^{-2})]</th>
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<td>( \pi ) (0.258, 0.854)</td>
<td>( D ) (1.876, 0.965)</td>
<td>( \eta_c ) (2.826, 0.261)</td>
</tr>
<tr>
<td>( \rho ) (0.782, 2.549)</td>
<td>( D^* ) (2.016, 1.298)</td>
<td>( J/\psi ) (2.910, 0.290)</td>
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▷ \( \pi \) wavefunc. ⇒ Single Gaussian is not enough
Meson Wavefunction
Quark exchange interaction

- Single Gaussian Approximation \textbf{(Simple)}

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<td></td>
<td>( J/\psi )</td>
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</tbody>
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\( \pi \) wavefunc. \( \Rightarrow \) Single Gaussian is not enough

\( \rightarrow \) We use \( \lambda = 2.20 \text{ GeV}^{-2} \) by T. Barnes and E. S. Swanson
(\( \pi \pi \) phase shift is reproduced)

\textbf{Single Gaussian Wavefunc. is obtained }\rightarrow\textbf{ Amplitude}
Born quark exchange diagrams T. Barnes and E. S. Swanson, PRD 46, 131 (1992).

Quark interaction between Mesons ⇒ Four diagrams

- Capture 1
  - $A$ to $C$ and $B$ to $D$
  - $C$ to $A$ and $D$ to $B$

- Capture 2
  - $A$ to $C$ and $B$ to $D$
  - $C$ to $A$ and $D$ to $B$

- Transfer 1
  - $A$ to $D$ and $B$ to $C$
  - $C$ to $B$ and $D$ to $A$

- Transfer 2
  - $A$ to $D$ and $B$ to $C$
  - $C$ to $B$ and $D$ to $A$

Scattering Amplitude $\mathcal{M}_{fi} \propto \langle C, D | H^q | A, B \rangle$

$$\mathcal{M}_{fi}^{tot} = \mathcal{M}_{fi}^{capture1} + \mathcal{M}_{fi}^{capture2} + \mathcal{M}_{fi}^{transfer1} + \mathcal{M}_{fi}^{transfer2}$$
Scattering Amplitude
Quark exchange interaction

Meson momenta: $A, B, C, D$
Quark momenta: $a, \bar{a}, b, \bar{b}, c, \bar{c}, d, \bar{d}$
Conservation:
$A + B = C + D$, $\bar{a} = \bar{d}$, $b = d$

Amplitude

$$\rightarrow \int \int d^3ad^3c \phi_C^*(2\vec{c} - \vec{C}) \phi_D^*(2\vec{a} - 2\vec{A} - \vec{C}) V(\vec{a} - \vec{c}) \phi_A(2\vec{a} - \vec{A}) \phi_B(2\vec{a} - \vec{A} - 2\vec{C})$$
Scattering Amplitude

Quark exchange interaction

capture 1

Meson momenta: A, B, C, D

Quark momenta: a, "a, b, "b, c, "c, d, "d

Conservation:
A + B = C + D,
"a = "d, b = d

Amplitude
\[ \rightarrow \int \int d^3a d^3c \phi^*_C(2\vec{c} - \vec{C})\phi^*_D(2\vec{a} - 2\vec{A} - \vec{C})V(\vec{a} - \vec{c})\phi_A(2\vec{a} - \vec{A})\phi_B(2\vec{a} - 2\vec{A} - 2\vec{C}) \]

Potentials (momentum space)

Coulomb: \( V^{\text{Coul}}(q) = -\frac{\alpha_s}{2\pi^2} \frac{1}{q^2} \), Hyperfine: \( V^{\text{Hyp}}(q) = -\frac{8\pi\alpha_h}{3m_i m_j} e^{-q^2/4\sigma^2} \)

Linear (Regularized):
\[ V^{\text{Lin}}(r) = br \times e^{-\varepsilon r} \rightarrow V^{\text{Lin}}(q) = b \left[ \frac{-8\pi}{(\vec{q}^2 + \varepsilon^2)^2} + \frac{32\pi\varepsilon^2}{(\vec{q}^2 + \varepsilon^2)^3} \right] \]
Cross Section (Born term): $\pi J/\psi - D\bar{D}^*$

**Numerical Result**

- $\pi J/\psi - D\bar{D}^*$: Amplitude

  $\Rightarrow$ Cross section $\propto |(\text{Coulomb}) + (\text{Confine}) + (\text{Hyperfine})|^2$
Cross Section (Born term): $\pi J/\psi - D\bar{D}^*$

Numerical Result

- $\pi J/\psi - D\bar{D}^*$: Amplitude
  $\Rightarrow$ Cross section $\propto |(\text{Coulomb}) + (\text{Confinement}) + (\text{Hyperfine})|^2$

- Dominant role of the Hyperfine (Spin-spin) term
  $\iff$ Minor role of the Coulomb term.
Comparing results of Quark exchange and $D^{(*)}$ exchange

(a) $D^{(*)}$ meson exchange

(b) Quark exchange
Comparing results of Quark exchange and $D^{(*)}$ exchange

(a) $D^{(*)}$ meson exchange

(b) Quark exchange

\[ \mathcal{L}_{\piHH} = -\frac{g_\pi}{2f_\pi} \text{Tr} \left[ H \gamma_\mu \gamma_5 \partial^\mu \bar{H} \right], \quad \mathcal{L}_\psi = g_\psi \text{Tr} \left[ J \bar{H}_2 \partial_\mu \gamma^\mu \bar{H}_1 \right] \]

\[
\begin{array}{c}
D \text{ exchange} \\
V^D = \frac{2}{3} \frac{g_\psi g_\pi}{f_\pi \sqrt{E_\pi}} \left[ \vec{S}_1 \cdot \vec{S}_2 C(r) + S_{12}(\vec{r}) T(r) \right]
\end{array}
\]

\[
\begin{array}{c}
D^* \text{ exchange} \\
V^{D^*} = \frac{2}{3} \frac{g_\psi g_\pi}{f_\pi \sqrt{E_\pi}} \left[ 2\vec{S}_1 \cdot \vec{S}_2 C(r) - S_{12}(\vec{r}) T(r) \right]
\end{array}
\]

$g_\pi = 0.59 \ (D^* \to D\pi) \quad g_\psi = 8.0 \ (Assuming \ VMD), \quad \Lambda_D = 1.1 \text{ GeV}, \quad \Lambda_\psi = 2.2 \text{ GeV}
Comparing results of Quark exchange and $D^{(*)}$ exchange

(i) Quark ex vs $D^{(*)}$ ex

![Graph showing cross section comparison](image-url)
Comparing results of Quark exchange and $D(\ast)$ exchange

(i) Quark ex. vs $D(\ast)$ ex.

(ii) $D(\ast)$ ex. (Zoom)

$D(\ast)$ exchange:
Comparing results of Quark exchange and $D(\ast)$ exchange

(i) Quark ex. vs $D(\ast)$ ex.

(ii) $D(\ast)$ ex. (Zoom)

$D(\ast)$ exchange: $\sigma < 3.5 \times 10^{-8}$ mb

Large difference between Quark exchange and $D(\ast)$ exchange
Many exotic states near the threshold.

→ Understanding the hadron-hadron interaction is needed.

Charged charmonium $Z_c(3900)$ has been discussed as the Hadronic molecules or the threshold cusp.

OPEP contribution is not strong. $D^{(*)}$ meson exchange is negligible.

Quark exchange interaction is introduced as Short range $\pi J/\psi - D^{(*)}\bar{D}^{(*)}$ potential. We find Large difference between results from Quark exchange and $D^{(*)}$ meson exchange.
Future Work

\[ \pi J/\psi - D(\ast)\bar{D}(\ast) \] potential

- Single Gaussian → Multi-Gaussian (Especially \( \pi \))
- Beyond Born-order → \( T = V + VGT \)
  ⇒ To compare the Exp. and Lattice result
- Introducing \( \rho \eta_c, \psi' \pi, \ldots \)
- Bottom Sector: \( Z_b(10610) \) and \( Z_b(10650) \) ⇒ \( \pi \gamma - B\bar{B}^* \)

Thank you for your kind attention.