

Short range interaction in $\pi J/\psi - D\bar{D}^*$ channel

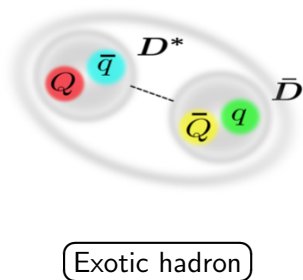
Yasuhiro Yamaguchi ¹

¹RIKEN, Japan

International workshop
“New aspects of the Hadron and Astro/Nuclear Physics”

National University of Uzbekistan, Tashkent 5-10 Nov. 2018

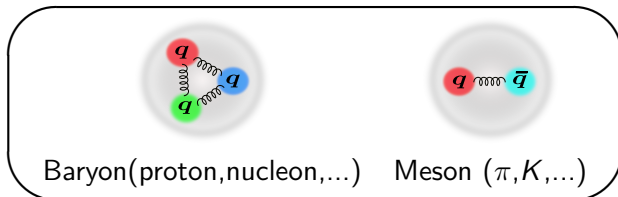
- Introduction
 - Exotic hadrons
 - $Z_c(3900)$
- Interaction model
 - Meson exchange model
 - Quark exchange model
- Summary



Description of Hadron structure

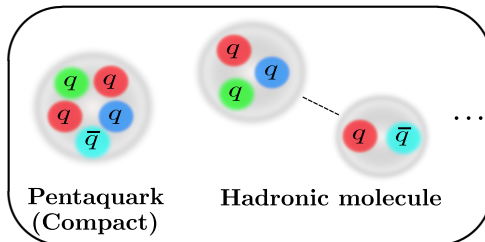
Introduction

- Ordinary Hadrons: Baryon (qqq) and Meson ($q\bar{q}$)



* q : "Constituent quark"

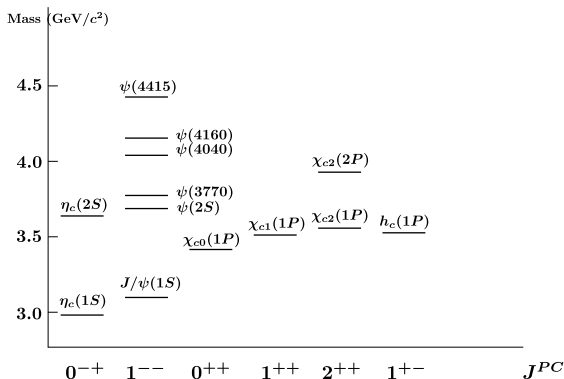
- Exotic Hadrons ($\neq qqq, q\bar{q}$): **Multiquark? Multihadron?**



Constituent quark picture and beyond

Introduction

▶ e.g. $c\bar{c}$ mesons (Charmonium)

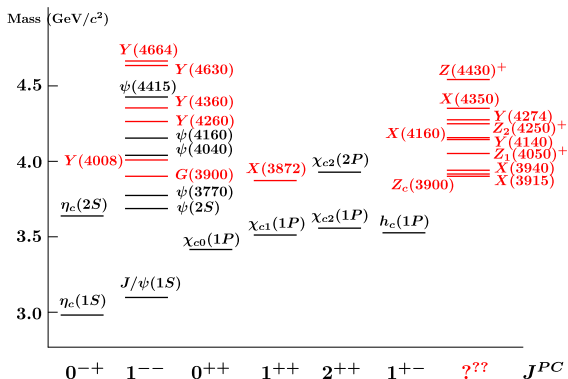


N. Brambilla, et al. Eur.Phys.J.C **71**(2011)1534, S. Godfrey and N. Isgur, PRD**32**(1985)189

Constituent quark picture and beyond

Introduction

- ▶ e.g. $c\bar{c}$ mesons (Charmonium) and **Unexpected X, Y, Z**



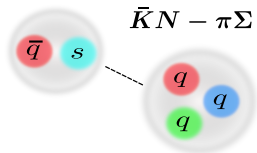
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- Exotics $\neq c\bar{c}$ have been observed in the Experiments (BaBar, Belle, BESIII, LHCb,...) \Rightarrow **Q. Structure? Physics?**

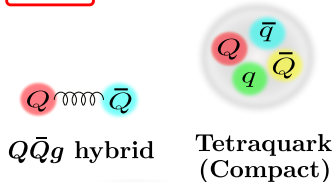
Many exotic candidate!! Many models!!

Introduction

$\Lambda(1405)$

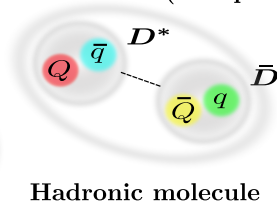
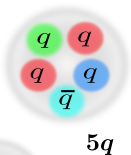
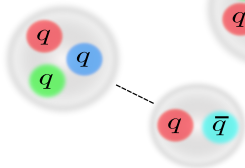


XYZ



Pentaquark P_c

$\bar{D}^{(*)}\Sigma_c^{(*)}$



BaBar, Belle, BESIII, LHCb, ...

T. Hyodo, D. Jido, PPNP67(2012)55, N. Brambilla et al., Eur.Phys.J.C(2011)71,1534

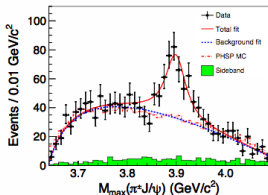
H.X.Chen, et al., Phys.Rept.639(2016)1,...

Charged Charmonium: $Z_c(3900)$

Introduction

- Charged Charmonium??
- $Y(4260) \rightarrow Z_c(3900)\pi \rightarrow J/\psi\pi\pi$

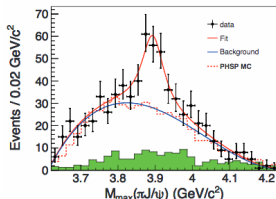
BESIII, PRL**110**(2013)252001



$$M = 3899.0 \pm 3.6_{sta} \pm 4.9_{sys} \text{ MeV}$$

$$\Gamma = 46 \pm 10_{sta} \pm 20_{sys} \text{ MeV}$$

Belle, PRL**110**(2013)252002



$$M = 3894.5 \pm 6.6_{sta} \pm 4.5_{sys} \text{ MeV}$$

$$\Gamma = 63 \pm 24_{sta} \pm 26_{sys} \text{ MeV}$$

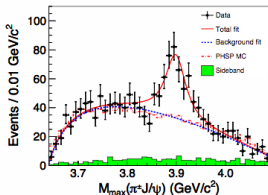
CLEO-c, PLB**727**(2013)366(2013), DØ, PRD**98**(2018)052010

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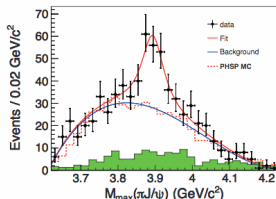
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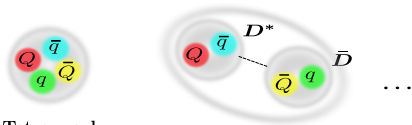
CLEO-c, PLB727(2013)366(2013), DØ, PRD98(2018)052010

▷ Ordinal Charmonium $c\bar{c}$: no electric charge.

⇒ $Z_c^+(3900)$: **Genuine Exotic State!?** $c\bar{c}u\bar{d}$

What is the structure of $Z_c(3900)$?

Introduction



Tetraquark
(Compact)

Hadronic molecule

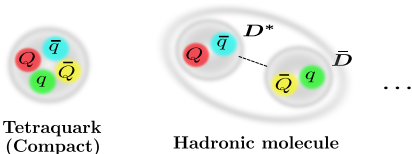
Multiquark states?

A. Hosaka *et al.* PTEP **2016** (2016) no.6, 062C01, D.-Y. Chen *et al.* PRD**88**(2013)036008,...

- Molecules? — $Z_c(3900)$ close to the $D\bar{D}^*$ threshold (~ 3875)

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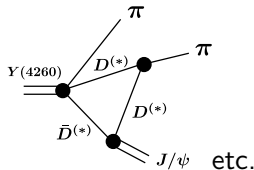
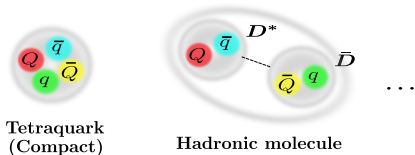
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 \Rightarrow Exotic state may be a **loosely bound state (resonance)** of the meson-meson.
 \rightarrow Analogous to atomic nuclei (Deuteron: $B \sim 2.2$ MeV)

What is the structure of $Z_c(3900)$?

Introduction



Multiquark states?

No bound state?

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- Molecules? — $Z_c(3900)$ close to the $D\bar{D}^*$ threshold (~ 3875)
 \Rightarrow Exotic state may be **a loosely bound state (resonance)** of the meson-meson.
 \rightarrow Analogous to atomic nuclei (Deuteron: $B \sim 2.2$ MeV)
- \Leftrightarrow Kinematical effect? **No bound state explanation**

D.-Y. Chen, X. Liu, T. Matsuki, PRD**88**(2013)036008, J. He, D.-Y. Chen, EPJC**78**(2018)94,...

$Z_c(3900)$: Lattice QCD (Numerical Experiments)

Introduction

- Lattice QCD simulation by HALQCD at $m_\pi = 410 - 700$ MeV
 \Rightarrow Coupled-channel $\pi J/\psi - \rho\eta_c - D\bar{D}^*$

PRL 117, 242001 (2016)

PHYSICAL REVIEW LETTERS

week ending
9 DECEMBER 2016

Fate of the Tetraquark Candidate $Z_c(3900)$ from Lattice QCD

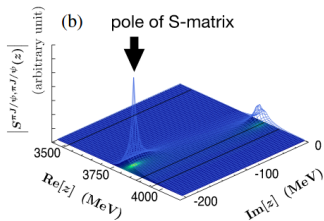
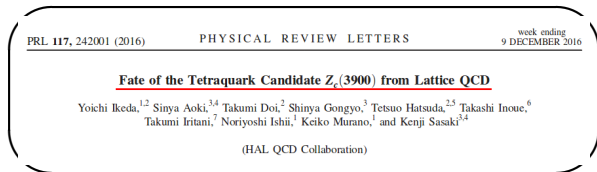
Yoichi Ikeda,^{1,2} Sinya Aoki,^{3,4} Takumi Doi,² Shinya Gongyo,³ Tetsuo Hatsuda,^{2,5} Takashi Inoue,⁶
Takumi Iritani,⁷ Noriyoshi Ishii,¹ Keiko Murano,¹ and Kenji Sasaki^{3,4}

(HAL QCD Collaboration)

$Z_c(3900)$: Lattice QCD (Numerical Experiments)

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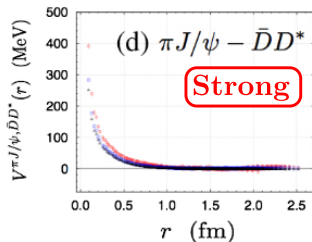
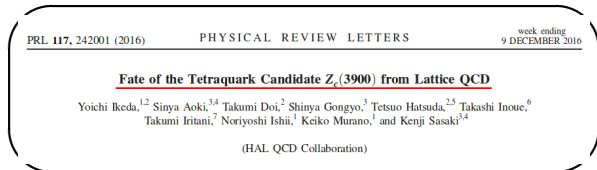


- ⇒ **Virtual state** is obtained.
- $Z_c(3900)$ is Threshold cusp

$Z_c(3900)$: Lattice QCD (Numerical Experiments)

Introduction

- Lattice QCD simulation by HALQCD at $m_\pi = 410 - 700$ MeV
- ⇒ Coupled-channel $\pi J/\psi - \rho\eta_c - D\bar{D}^*$



⇒ **Virtual state** is obtained.

- $Z_c(3900)$ is Threshold cusp induced by $\pi J/\psi - \bar{D}D^*$ **potential**

Charm flavor exchange?

Bound state? Threshold cusp? \rightarrow Hadron int.

Introduction

**Exotic structure:
Bound state? Cusp?**

Model of Hadron-hadron interaction

Introduction

- **Long-range force:** one π exchange potential (OPEP)
Lightest meson π , Importance in the nuclear force,
Heavy Quark Spin Symmetry ($0^- - 1^-$ mixing)

Model of Hadron-hadron interaction

Introduction

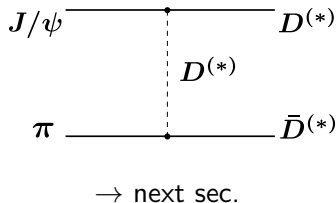
- **Long-range force:** one π exchange potential (OPEP)
Lightest meson π , Importance in the nuclear force,
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- **Short-range force:** Charm (c) exchange
- ▶ How can we understand **strong $\pi J/\psi - D\bar{D}^*$ potential?**

Model of Hadron-hadron interaction

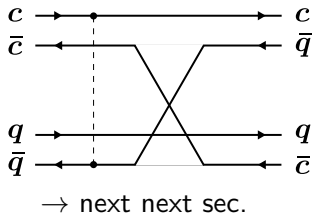
Introduction

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(a) $D^{(*)}$ meson exchange?

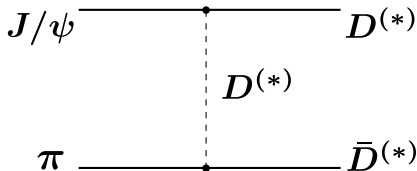


(b) Quark exchange?



Comparison between D exchange and Quark exchange

Meson exchange model



- Coupled channel: $\pi J/\psi - D\bar{D}^* - D^*\bar{D}^*$
- ▶ $D^{(*)}\bar{D}^{(*)} - D^{(*)}\bar{D}^{(*)}$: π exchange
- ▶ $\pi J/\psi - D^{(*)}\bar{D}^{(*)}$: $D^{(*)}$ exchange

Yasuhiro Yamaguchi (RIKEN), Yukihiro Abe (RCNP, Osaka Univ.),
Kenji Fukukawa (Suma Gakuen), Atsushi Hosaka (RCNP, Osaka Univ.),
in preparation

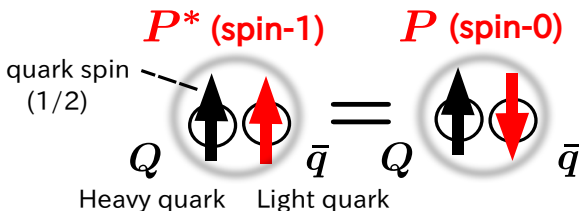
Heavy Quark Spin Symmetry and Mass degeneracy

Meson exchange model

Heavy Quark Spin Symmetry (HQS)

N.Isgur, M.B.Wise, PLB232(1989)113

- **Suppression of Spin-spin force** in $m_Q \rightarrow \infty$.
⇒ **Mass degeneracy** of hadrons with the different J
- e.g. $Q\bar{q}$ meson

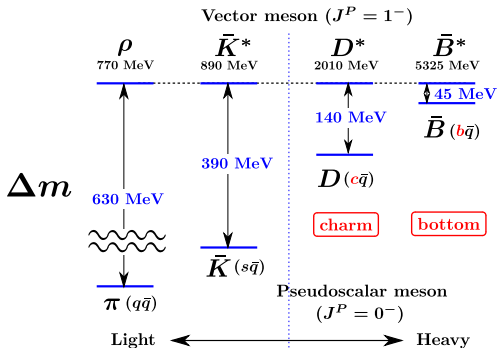


⇒ Mass degeneracy of spin-0 and spin-1 states!

Mass degeneracy of heavy hadrons

Meson exchange model

- Mass difference between vector and pseudoscalar mesons.
($Q\bar{q}$, $q = u, d$)

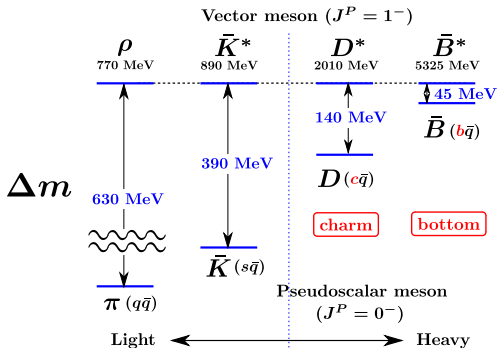


▷ Δm decreases when the quark mass increases.

Mass degeneracy of heavy hadrons

Meson exchange model

- Mass difference between vector and pseudoscalar mesons.
($Q\bar{q}$, $q = u, d$)



- Δm decreases when the quark mass increases.
 \Rightarrow **Degeneracy of Heavy hadrons!**

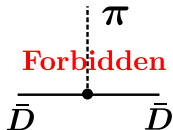
For $Z_c(3900)$, $D - D^*$ mixing $\Rightarrow D\bar{D}^* - D^*\bar{D}$ coupled-channel

Heavy hadron- π coupling

Meson exchange model

- Effective Lagrangians: Heavy hadron and π

R. Casalbuoni *et al.*, Phys.Rept.**281** (1997)145, T. M. Yan, *et al.*, PRD**46**(1992)1148



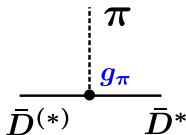
- ▶ Heavy meson: **($DD\pi$: Parity violation)**

Heavy hadron- π coupling

Meson exchange model

- Effective Lagrangians: Heavy hadron and π

R. Casalbuoni *et al.*, Phys.Rept.**281** (1997)145, T. M. Yan, *et al.*, PRD**46**(1992)1148



- Heavy meson: $\bar{D}^{(*)}\bar{D}^{(*)}\pi$ ($DD\pi$: Parity violation)

$$\mathcal{L}_{\pi HH} = -\frac{g_\pi}{2f_\pi} \text{Tr} [H\gamma_\mu\gamma_5\partial^\mu\hat{\pi}\bar{H}], \quad H = \frac{1+\not{v}}{2} [D_\mu^*\gamma^\mu - D\gamma_5]$$

- One coupling const. $g_\pi = 0.59$ (from $D^* \rightarrow D\pi$ decay)

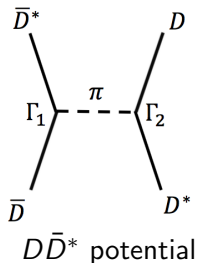
- Form factor (Hadron has **finite size**)

$$F(q^2) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - q^2}, \quad \Lambda_{\bar{D}} \sim 1130 \text{ MeV (by Quark model)}$$

One pion exchange potential in $D^{(*)}\bar{D}^{(*)}$

Meson exchange model

- One boson exchange potential (OBEP)



$DD^*\pi$ vertex induces OPEP
($DD\pi$ vertex violates the parity conservation)

OPEP

$$V^\pi = -\frac{1}{2} \left(\frac{g_\pi}{f_\pi} \right)^2 \left[\vec{S}_1 \cdot \vec{S}_2 C(r) + S_{12}(\hat{r}) T(r) \right] \vec{\tau}_1 \cdot \vec{\tau}_2$$

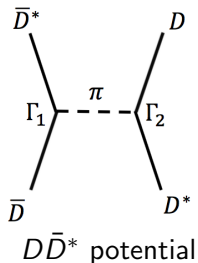
Comments

- HQS induces $D(0^-) - D^*(1^-)$ coupling \rightarrow OPEP works!

One pion exchange potential in $D^{(*)}\bar{D}^{(*)}$

Meson exchange model

- One boson exchange potential (OBEP) **with Tensor force!**



$DD^*\pi$ vertex induces OPEP
($DD\pi$ vertex violates the parity conservation)

OPEP

$$V^\pi = -\frac{1}{2} \left(\frac{g_\pi}{f_\pi} \right)^2 \left[\vec{S}_1 \cdot \vec{S}_2 C(r) + \mathbf{S}_{12}(\hat{r}) \mathbf{T}(r) \right] \vec{\tau}_1 \cdot \vec{\tau}_2$$

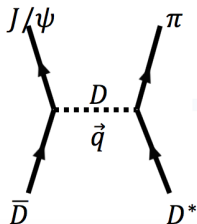
Comments

- HQS induces **$D(0^-) - D^*(1^-)$ coupling** \rightarrow OPEP works!
- Tensor force $T(r) \Rightarrow$ **the driving force** in atomic nuclei
 $S_{12}(\hat{r}) = 3(\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r}) - \vec{S}_1 \cdot \vec{S}_2 \rightarrow S$ - D mixing

Heavy meson exchange potential

Meson exchange model

- $D^{(*)}$ meson exchange potential in $\pi J/\psi - D^{(*)} \bar{D}^{(*)}$



$\pi J/\psi - D\bar{D}^*$ potential

D exchange

$$V^D = \frac{2}{3} \frac{g_\psi g_\pi}{f_\pi \sqrt{E_\pi}} \left[\vec{S}_1 \cdot \vec{S}_2 C(r) + S_{12}(\hat{r}) T(r) \right]$$

D^* exchange

$$V^{D^*} = \frac{2}{3} \frac{g_\psi g_\pi}{f_\pi \sqrt{E_\pi}} \left[2\vec{S}_1 \cdot \vec{S}_2 C(r) - S_{12}(\hat{r}) T(r) \right]$$

$g_\psi = 8.0$ (Assuming VMD), $\Lambda_\psi = 2.2$ GeV

A. Deandrea, G. Nardulli and A. D. Polosa, PRD**68**(2003)034002

Comments

- Spin-spin ($\vec{S}_1 \cdot \vec{S}_2$) and Tensor (S_{12}) terms
- Energy-dependence ($1/\sqrt{E_\pi}$)

Numerical results: Phase shift

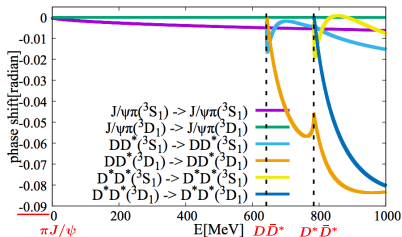
Meson exchange model

- We found...

Numerical results: Phase shift

Meson exchange model

- We found... **No Bound state, No Resonance**
Very Small phase shift $|\delta| < 0.09$ [rad]

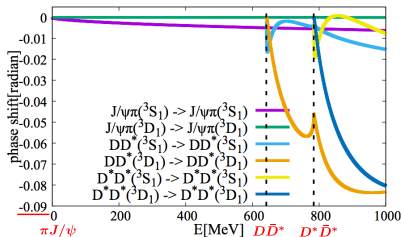


- $D^{(*)}\bar{D}^{(*)}$ channel: **Small** contribution from OPEP
- $\pi J/\psi$ channel: $D^{(*)}$ exchange is **Negligible**

Numerical results: Phase shift

Meson exchange model

- We found... **No Bound state, No Resonance**
Very Small phase shift $|\delta| < 0.09$ [rad]



- $D^{(*)}\bar{D}^{(*)}$ channel: **Small** contribution from OPEP
Why?: Isospin factor $\vec{\tau}_1 \cdot \vec{\tau}_2$, **-3** ($I = 0$), but $Z_c: +1$ ($I = 1$)
- $\pi J/\psi$ channel: $D^{(*)}$ exchange is **Negligible**
Why?: Volume Integral $V_C^D(\vec{q}^2 = 0) = 3.14 \text{ GeV}^{-2}$
 $\leftrightarrow V_{NN}^\sigma \sim \mathbf{3.00 \times 10^2 \text{ GeV}^{-2}}$

D meson exchange

Meson exchange model

- No resonance is found
- ↔ The strong $\pi J/\psi - D\bar{D}^*$ contribution is not explained.

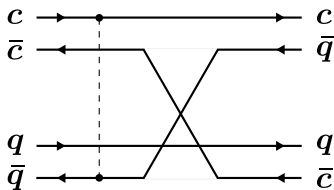
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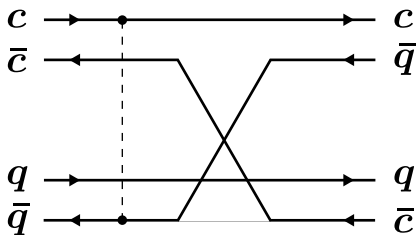
↓ Another Short range force

Quark exchange interaction!



→next section

Quark exchange model]



Meson-meson scattering by the quark exchange

- Only $\pi J/\psi - D\bar{D}^*$ channel

Yasuhiro Yamaguchi (RIKEN), Yukihiro Abe (RCNP, Osaka Univ.),
Kenji Fukukawa (Suma Gakuen), Atsushi Hosaka (RCNP, Osaka Univ.),
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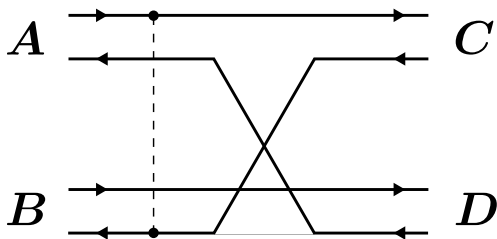
Quark exchange model

Quark exchange interaction

- **Born-order quark-exchange diagram**

T. Barnes and E. S. Swanson, PRD46(1992)131. Swanson, Ann. Phys. 220(1992)73.

- $AB \rightarrow CD$ scattering $\mathcal{M}_{fi} \propto \langle C, D | H_I | A, B \rangle$



- Ingredients: Meson Wavefunctions(A, B, C, D)
Quark interaction (Quark Model)
- Born amplitude \Rightarrow Meson-meson Potential can be obtained

Quark Model

Quark exchange interaction

- Quark Hamiltonian (One gluon exchange + Linear potentials)

Barnes and Swanson, PRD**46**(1992)131.; Swanson, Ann. Phys. **220**(1992)73.

$$H_{ij}^q = K_q + \left(-\frac{3}{4}br + \frac{\alpha_s}{r} - C \right) \vec{F}_i \cdot \vec{F}_j - \frac{8\pi\alpha_h}{3m_i m_j} \left(\frac{\sigma^3}{\pi^{3/2}} e^{-\sigma^2 r_{ij}^2} \right) \vec{S}_i \cdot \vec{S}_j \vec{F}_i \cdot \vec{F}_j$$

- Parameters are fixed to reproduce the mass of mesons

Table: Quark Model Parameters from Ann.Phys.**220**(1992)73.

$m_q = 0.375 \text{ GeV}$	$m_c = 1.9 \text{ GeV}$
$\alpha_s = 0.857$	$\alpha_h = 0.840$
$b = 0.154 \text{ GeV}^{-2}$	$C = -0.4358 \text{ GeV}$
$\sigma = 0.70 \text{ GeV}$	

Meson Wavefunction

Quark exchange interaction

- Single Gaussian Approximation (**Simple**)

$$\psi(r) = (4\pi\lambda)^{-3/4} \exp\left(-\frac{r^2}{8\lambda}\right)$$

- λ is determined to minimize $E(\lambda) = \langle \psi | H^q | \psi \rangle$

	$(m [\text{GeV}], \lambda [\text{GeV}^{-2}])$		$(m [\text{GeV}], \lambda [\text{GeV}^{-2}])$		$(m [\text{GeV}], \lambda [\text{GeV}^{-2}])$
π	(0.258, 0.854)	D	(1.876, 0.965)	η_c	(2.826, 0.261)
ρ	(0.782, 2.549)	D^*	(2.016, 1.298)	J/ψ	(2.910, 0.290)

▷ π wavefunc. \Rightarrow Single Gaussian is not enough

Meson Wavefunction

Quark exchange interaction

- Single Gaussian Approximation (**Simple**)

$$\psi(r) = (4\pi\lambda)^{-3/4} \exp\left(-\frac{r^2}{8\lambda}\right)$$

- λ is determined to minimize $E(\lambda) = \langle \psi | H^q | \psi \rangle$

	$(m [\text{GeV}], \lambda [\text{GeV}^{-2}])$		$(m [\text{GeV}], \lambda [\text{GeV}^{-2}])$		$(m [\text{GeV}], \lambda [\text{GeV}^{-2}])$
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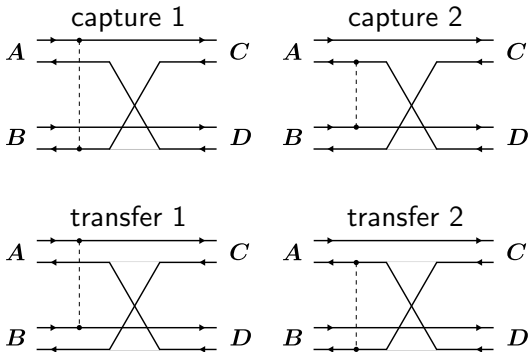
- ▷ π **wavefunc.** \Rightarrow Single Gaussian is not enough
- \rightarrow We use $\lambda = 2.20 \text{ GeV}^{-2}$ by T. Barnes and E. S. Swanson
($\pi\pi$ phase shift is reproduced)

Single Gaussian Wavefunc. is obtained \rightarrow Amplitude

Scattering Amplitude

Quark exchange interaction

- Born quark exchange diagrams T. Barnes and E. S. Swanson, PRD46, 131 (1992).
Quark interaction between Mesons \Rightarrow Four diagrams

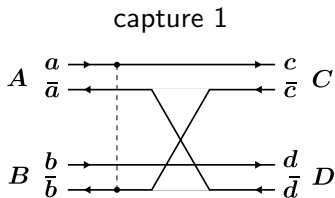


- Scattering Amplitude $\mathcal{M}_{fi} \propto \langle C, D | H^q | A, B \rangle$

$$\mathcal{M}_{fi}^{tot} = \mathcal{M}_{fi}^{capture1} + \mathcal{M}_{fi}^{capture2} + \mathcal{M}_{fi}^{transfer1} + \mathcal{M}_{fi}^{transfer2}$$

Scattering Amplitude

Quark exchange interaction



▶ Meson momenta: A, B, C, D

▶ Quark momenta:
 $a, \bar{a}, b, \bar{b}, c, \bar{c}, d, \bar{d}$

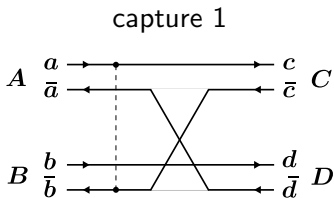
▶ Conservation:
 $A + B = C + D,$
 $\bar{a} = \bar{d}, b = d$

• Amplitude

$$\rightarrow \int \int d^3 a d^3 c \phi_C^*(2\vec{c} - \vec{C}) \phi_D^*(2\vec{a} - 2\vec{A} - \vec{C}) V(\vec{a} - \vec{c}) \phi_A(2\vec{a} - \vec{A}) \phi_B(2\vec{a} - \vec{A} - 2\vec{C})$$

Scattering Amplitude

Quark exchange interaction



▷ Meson momenta: A, B, C, D

▷ Quark momenta:
 $a, \bar{a}, b, \bar{b}, c, \bar{c}, d, \bar{d}$

▷ Conservation:
 $A + B = C + D,$
 $\bar{a} = \bar{d}, b = d$

• Amplitude

$$\rightarrow \int \int d^3a d^3c \phi_C^*(2\vec{c} - \vec{C}) \phi_D^*(2\vec{a} - 2\vec{A} - \vec{C}) V(\vec{a} - \vec{c}) \phi_A(2\vec{a} - \vec{A}) \phi_B(2\vec{a} - \vec{A} - 2\vec{C})$$

• Potentials (momentum space)

Coulomb: $V^{Coul}(q) = -\frac{\alpha_s}{2\pi^2} \frac{1}{\vec{q}^2}$, **Hyperfine:** $V^{Hyp}(q) = -\frac{8\pi\alpha_h}{3m_i m_j} e^{-\vec{q}^2/4\sigma^2}$

Linear (Regularized):

$$V^{Lin}(r) = br \times e^{-\epsilon r} \rightarrow V^{Lin}(q) = b \left[\frac{-8\pi}{(\vec{q}^2 + \epsilon^2)^2} + \frac{32\pi\epsilon^2}{(\vec{q}^2 + \epsilon^2)^3} \right]$$

Cross Section (Born term): $\pi J/\psi - D\bar{D}^*$

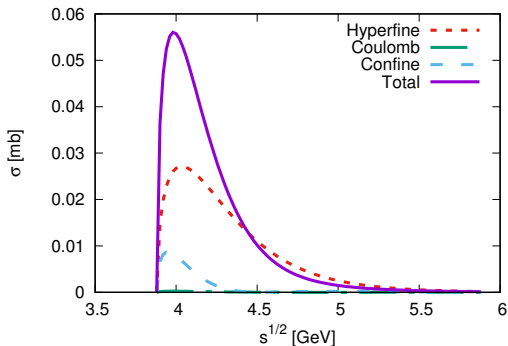
Numerical Result

- $\pi J/\psi - D\bar{D}^*$: Amplitude
- ⇒ Cross section $\propto |(\text{Coulomb}) + (\text{Confine}) + (\text{Hyperfine})|^2$

Cross Section (Born term): $\pi J/\psi - D\bar{D}^*$

Numerical Result

- $\pi J/\psi - D\bar{D}^*$: Amplitude
- ⇒ Cross section $\propto |(\text{Coulomb}) + (\text{Confine}) + (\text{Hyperfine})|^2$



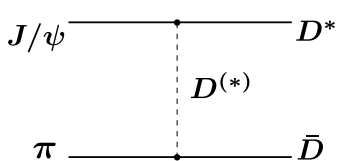
- Dominant role of the Hyperfine (Spin-spin) term
⇔ Minor role of the Coulomb term.

Cross Section: Quark exchange vs $D^{(*)}$ exchange

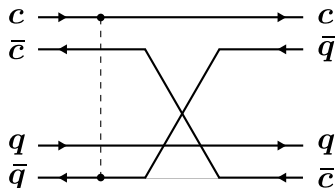
Quark exchange interaction

- Comparing results of Quark exchange and $D^{(*)}$ exchange

(a) $D^{(*)}$ meson exchange



(b) Quark exchange

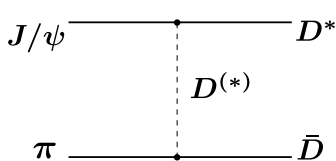


Cross Section: Quark exchange vs $D^{(*)}$ exchange

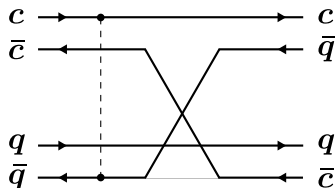
Quark exchange interaction

- Comparing results of Quark exchange and $D^{(*)}$ exchange

(a) $D^{(*)}$ meson exchange



(b) Quark exchange



$$\mathcal{L}_{\pi HH} = -\frac{g_\pi}{2f_\pi} \text{Tr} [H \gamma_\mu \gamma_5 \partial^\mu \hat{\pi} \bar{H}], \quad \mathcal{L}_\psi = g_\psi \text{Tr} \left[\mathcal{J} \bar{H}_2 \overset{\leftrightarrow}{\partial}_\mu \gamma^\mu \bar{H}_1 \right]$$

\Downarrow

$$\boxed{D \text{ exchange}} \quad v^D = \frac{2}{3} \frac{g_\psi g_\pi}{f_\pi \sqrt{E_\pi}} \left[\vec{S}_1 \cdot \vec{S}_2 C(r) + S_{12}(\hat{r}) T(r) \right]$$

$$\boxed{D^* \text{ exchange}} \quad v^{D^*} = \frac{2}{3} \frac{g_\psi g_\pi}{f_\pi \sqrt{E_\pi}} \left[2\vec{S}_1 \cdot \vec{S}_2 C(r) - S_{12}(\hat{r}) T(r) \right]$$

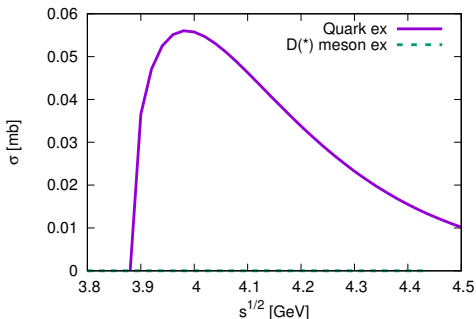
$$g_\pi = 0.59 \quad (D^* \rightarrow D\pi) \quad g_\psi = 8.0 \quad (\text{Assuming VMD}), \quad \Lambda_D = 1.1 \text{ GeV}, \quad \Lambda_{\psi^*} = 2.2 \text{ GeV}$$

Cross Section: Quark exchange vs $D^{(*)}$ exchange

Quark exchange interaction

- Comparing results of Quark exchange and $D^{(*)}$ exchange

(i) Quark ex vs $D^{(*)}$ ex

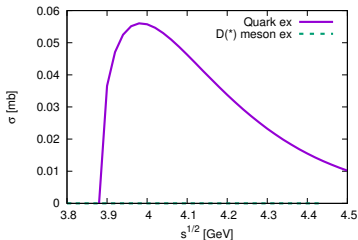


Cross Section: Quark exchange vs $D^{(*)}$ exchange

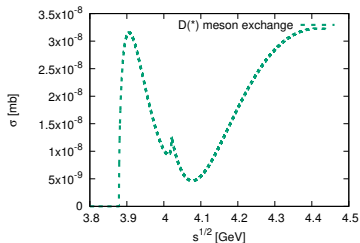
Quark exchange interaction

- Comparing results of Quark exchange and $D^{(*)}$ exchange

(i) Quark ex. vs $D^{(*)}$ ex.



(ii) $D^{(*)}$ ex. (Zoom)



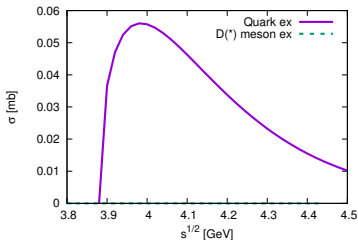
- $D^{(*)}$ exchange:

Cross Section: Quark exchange vs $D^{(*)}$ exchange

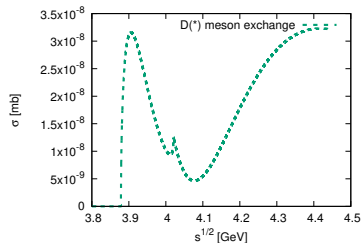
Quark exchange interaction

- Comparing results of Quark exchange and $D^{(*)}$ exchange

(i) Quark ex. vs $D^{(*)}$ ex.

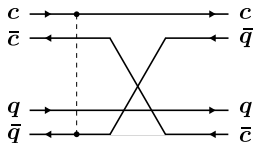
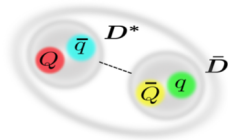


(ii) $D^{(*)}$ ex. (Zoom)



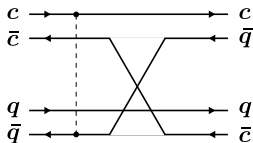
- $D^{(*)}$ exchange: $\sigma < 3.5 \times 10^{-8}$ mb
- Large difference between Quark exchange and $D^{(*)}$ exchange

Summary



- Many exotic states near the threshold.
→ Understanding **the hadron-hadron interaction** is needed.
- Charged charmonium $Z_c(3900)$ has been discussed as the Hadronic molecules or the threshold cusp.
- OPEP contribution is not strong. $D^{(*)}$ meson exchange is **negligible**.
- Quark exchange interaction is introduced as Short range $\pi J/\psi - D^{(*)}D^{(*)}$ potential.
We find **Large difference** between results from Quark exchange and $D^{(*)}$ meson exchange.

$\pi J/\psi - D^{(*)} \bar{D}^{(*)}$ potential



- Single Gaussian \rightarrow Multi-Gaussian (Especially π)
- Beyond Born-order $\rightarrow T = V + VGT$
 \Rightarrow To compare the Exp. and Lattice result
- Introducing $\rho\eta_c, \psi'\pi, \dots$
- Bottom Sector: $Z_b(10610)$ and $Z_b(10650) \Rightarrow \pi\Upsilon - B\bar{B}^*$

Thank you for your kind attention.