HEAVY QUARKONIUM POTENTIAL AT NONZERO TEMPERATURE IN INSTATON LIQUID MODEL

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Introduction

- Direct contribution of instantons to the static singlet $Q\bar{Q}$ potential
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Direct contribution of instantons to the static $Q\bar{Q}$ potential at $T \neq 0$

Periodical solution of filed equations at nonzero temperature

- Harrington-Shepard(HS) caloron - the time periodic instanton solution

$$A_\mu = \Pi \frac{\bar{\eta}^a_{\mu\nu}\tau_a}{2i}\partial_\nu \Pi^{-1}$$

$$\Pi(r,t) = 1 + \frac{\pi\rho^2}{\beta r}\sinh\frac{2\pi r}{\beta}/(\cosh\frac{2\pi r}{\beta} - \cos\frac{2\pi t}{\beta})$$

- In small distances $r << \beta$ the finite-temperature instanton is identical to a zero-temperature instanton with a renormalized size

$$A^a_\mu \simeq \frac{2\rho'^2}{x^2}\frac{\bar{\eta}^a_{\mu\nu}x_\nu}{(x^2+\rho'^2)}, \qquad \rho'^2 = \rho^2/(1+\frac{\lambda^2}{3}), \qquad \lambda = \pi\rho T$$

Accuracy of the estimation

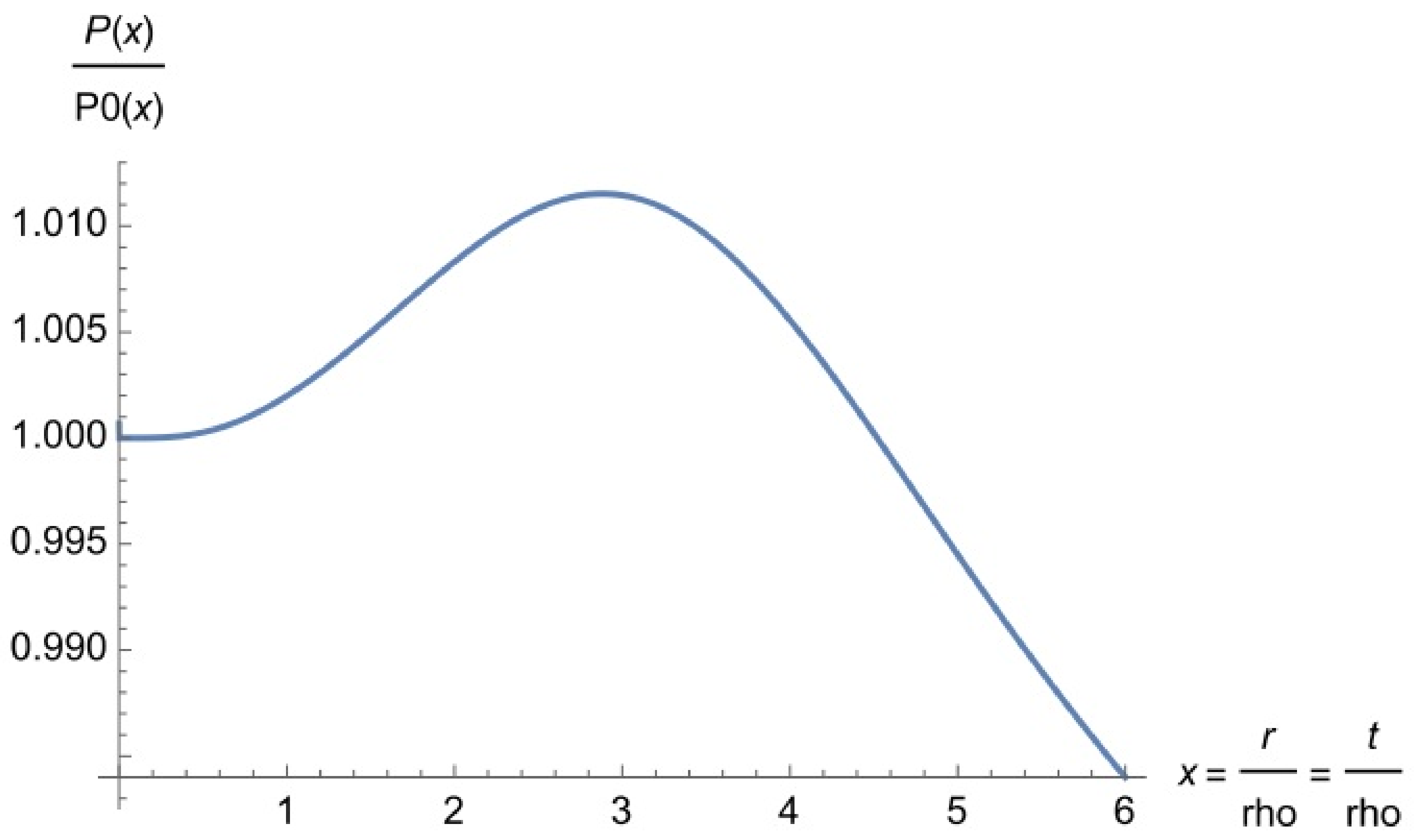


Figure: Profile function of HS-caloron. Here $\rho = 0.33\,fm$, $\beta = 6\rho$.

Variational estimations in ILM at $T \neq 0$

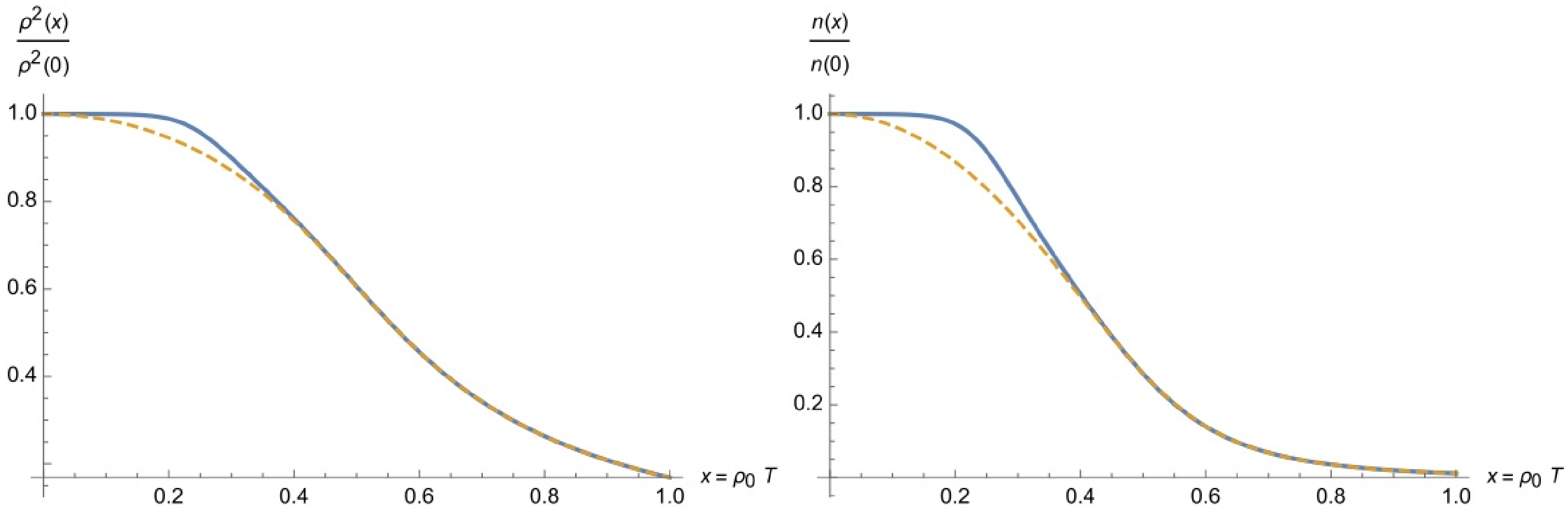


Figure: Temperature dependence of average size(left), and density of instantons(right) at $\bar{\rho}(0) = 0.33\ fm$, $n(0) = 1\ fm^{-4}$. Full line corresponds interpolation between no suppression below T_C and full suppression above $T_C = 150\ MeV$, with a width $T = 0.3T_C$. Dashed line correspond to the full suppression at the whole region of T.

A way of calculation of the direct contribution of instantons to the heavy-quarks potential

The static heavy-quarks potential is defined as the expectation value of the Wilson loop [D. Gross et al1981]

$$V(r) = -\lim_{T\to\infty} \frac{1}{T} \ln\langle 0|\mathrm{Tr}(W_C[A])|0\rangle$$

$$W_C[A] = P\exp\left(i\oint_C dz_\mu A_\mu(z)\right)$$

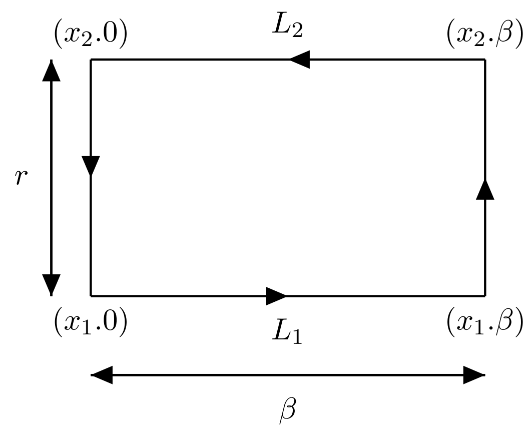


Figure: The rectangular Wilson loop with long sides $L_1 = (0, \beta), L_2 = (\beta, 0)$ and short sides $(x_1, x_2), (x_2, x_1)$.

Calculation of Wilson Loop

$$\langle W_c[A]\rangle = \langle\langle T|\bar{S}|0\rangle\rangle$$

[D. Diakonov, U. Petrov Nucl. Phys. B 245,1984]

- The quark propagator $\bar{S} = \left\langle (i\hat{p} + im + \sum_I \hat{A}_I)^{-1} \right\rangle$
- Free propagator $S_0 = -(i\hat{\partial} + im)^{-1}$
- After averaging over position, orientation and size we get Pobylitsa equation for quark propagator

$$\bar{S}^{-1} - S_0^{-1} = \frac{N}{2VN_c} \cdot Tr_c \left(\int d^4 z_I (\bar{S} - \hat{A}_I^{-1})^{-1} + \int d^4 z_{\bar{I}} (\bar{S} - \hat{A}_{\bar{I}}^{-1})^{-1} \right)$$

- Potential from Wilson loop

$$V(r,T) = \frac{n(T)}{2N_c} \int d^3 z_I \mathrm{tr}_c \Big[1 - P\exp\left(i\int_{-\infty}^{\infty} A_{I4}(\vec{r}_1, x_4) dx_4 \right) \times P\exp\left(-i\int_{-\infty}^{\infty} A_{I4}(\vec{r}_2, x_4') dx_4' \right) \Big]\Big|_{z_{I,4}=0} + (I \to \bar{I}), \quad \vec{r}_{1,2} = \vec{x}_{1,2} - \vec{z}_I.$$

Expression for static $Q\bar{Q}$ potential

- Static $Q\bar{Q}$ potential

$$V_C(r,T) = \frac{4\pi\rho(T)^3 n(T)}{N_C} I\left(\frac{r}{\rho(T)}\right)$$

where $I(x)$- dimensionless integral which can be calculated numerically

- At small distances

$$V_C(r,T) \simeq \frac{4\pi\bar{\rho}(T)^3 n(T)}{N_c}\left(1.345\frac{r^2}{\bar{\rho}(T)^2} - 0.501\frac{r^4}{\bar{\rho}(T)^4}\right)$$

- At large distances

$$V_C(r,T) \simeq 2\Delta M_Q - \frac{2\pi^3\bar{\rho}(T)^4 n(T)}{N_c}\frac{1}{r}$$

where ΔM_Q - heavy-quark mass correction

Results of numerical calculatios

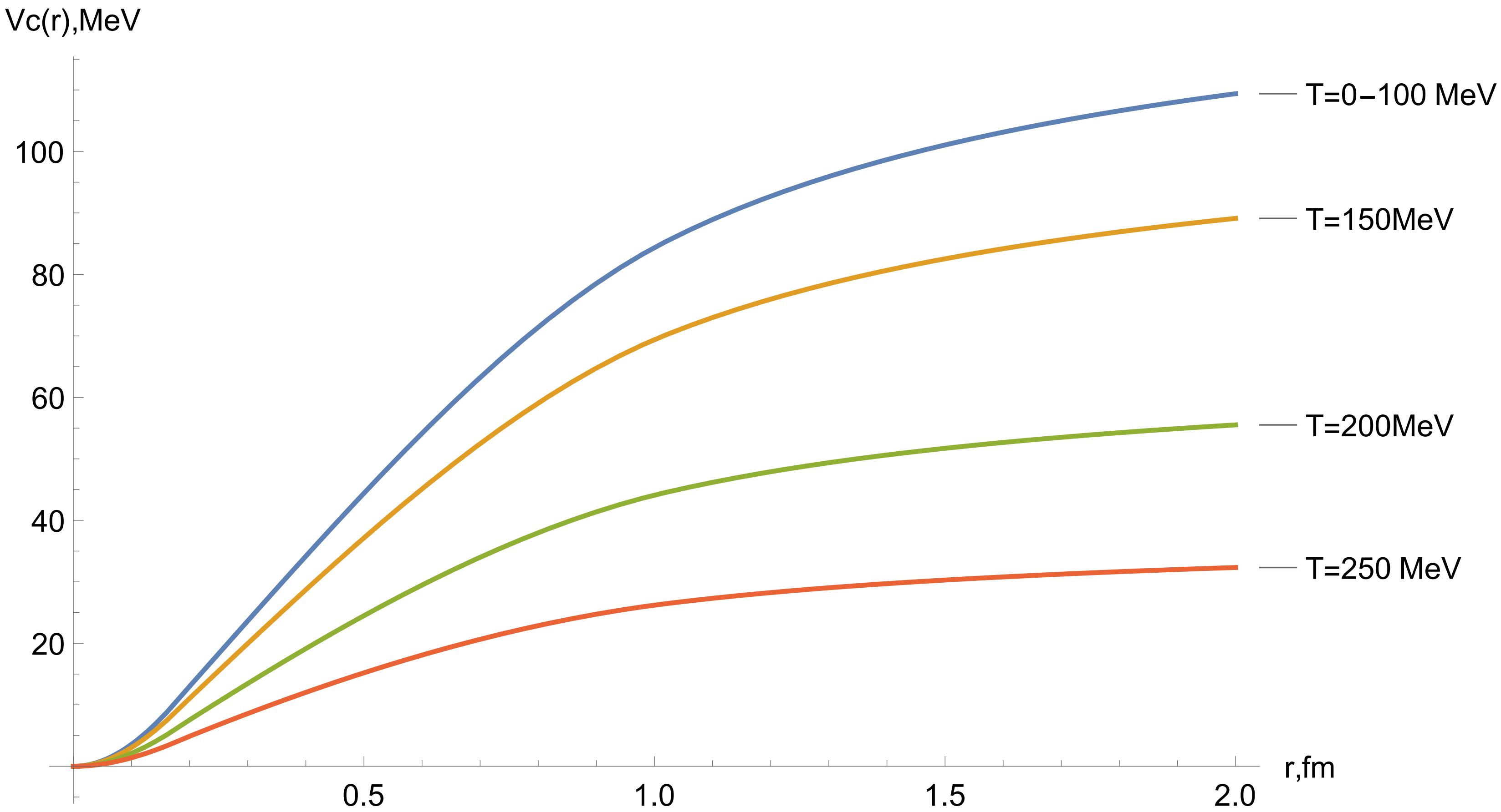


Figure: Direct contribution of instantons to the static $Q\bar{Q}$ potential

One gluon exchange contribution to the potential at $T \neq 0$

Idea

- Perturbative potential
  $V_{one\ gl.}(r,T) = \lambda \cdot \bar{\lambda} g^2 \int \frac{d^3k}{(2\pi)^3} \exp(i\vec{k}\vec{r}) D_{44}(k)$

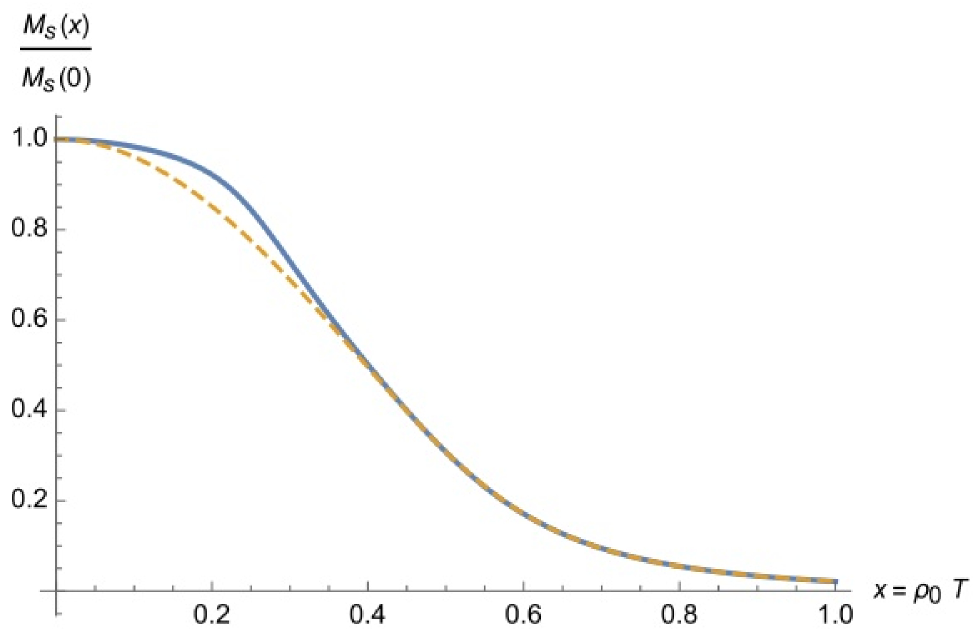
$$D_{44}(k) = (\vec{k}^2 + M_{el}(\vec{k},T)^2)^{-1}$$

- Temperature dependent gluon mass

$$M_g(\vec{k},T) = \left[\frac{24\pi^2 \bar{\rho}'^2(T) n(T)}{N_c^2 - 1}\right] F(k,T),$$

$$F(0,0) = 1, \quad F(k,T) \leq F(k,0) = k\bar{\rho} K_1(k\bar{\rho})$$

Dynamical Gluon Mass at nonzero temperature



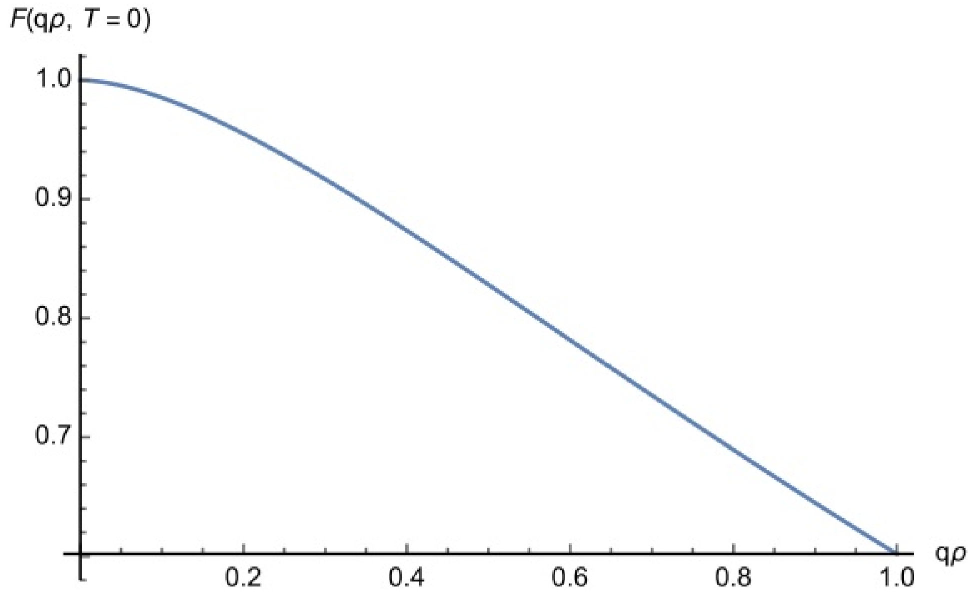


Figure: Left side – T-dependence of "electric" gluon dynamical mass $M_{el}(0, T)/M(0,0)$. Solid line – modified $A_{N_c} \to A_{N_c}\Theta_{\Delta x}(x - x_c)$. At small $T \leq T_c$ full line correspond to the $M_{el}(0, T)/M_{el}(0,0) = (1 - 1/6\,\pi^2\bar{\rho}_0^2 T^2)$. Dashed line here correspond to the full suppression at the whole region of T (A_{N_c} is not modified). Here $M_{el}(0,0) = 362\,MeV$ at the phenomenological values of $\bar{\rho}(0) = 1/3\,fm$ and $n(0) = 1\,fm^{-4}$. Right side – form-factor of dynamical mass $F(q,0)$.

Results of calculations

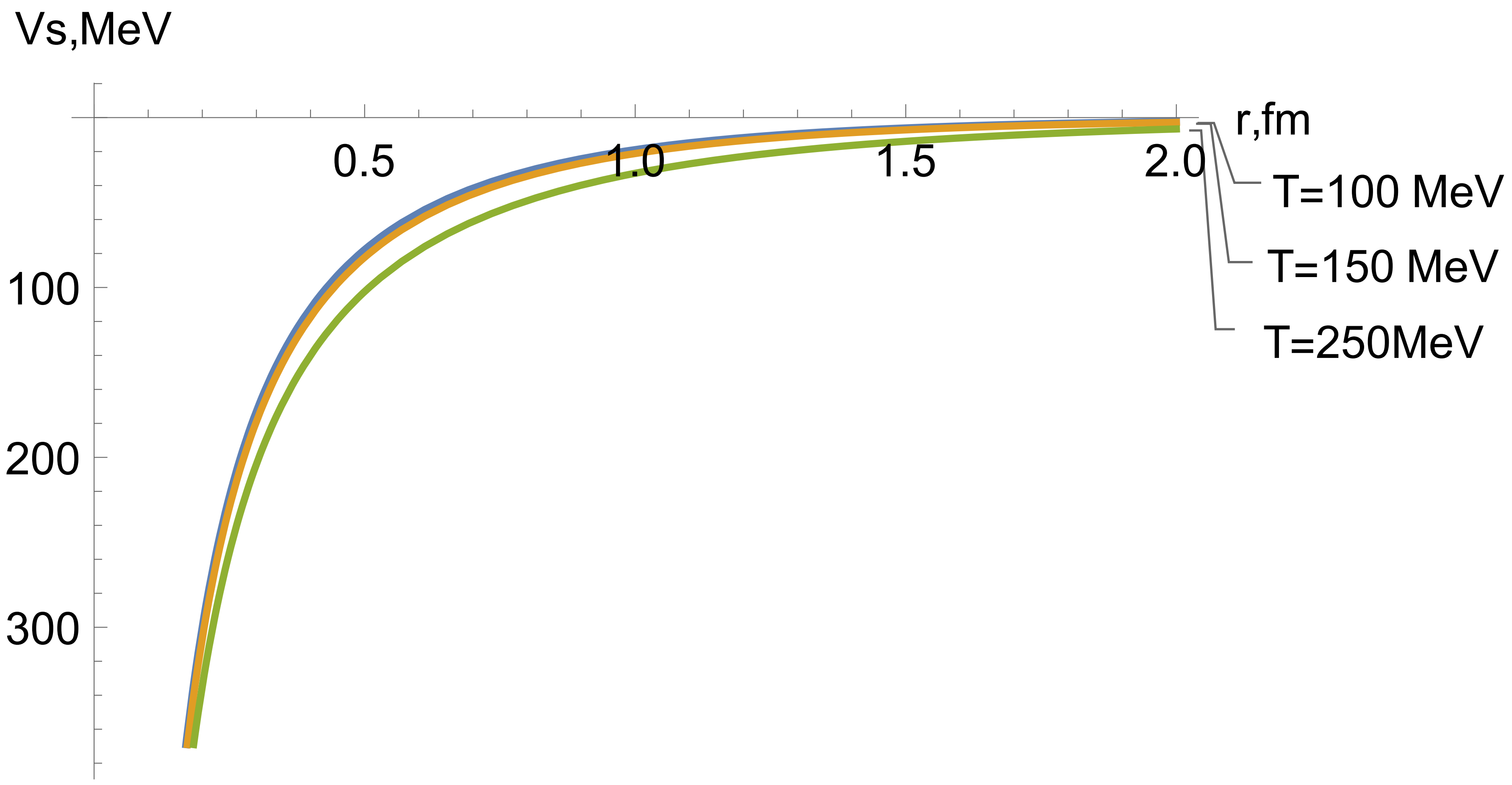


Figure: One gluon exchange contribution to $Q\bar{Q}$ potential in color singlet state. Constants was chosen as $g^2/4\pi = 0.3$.

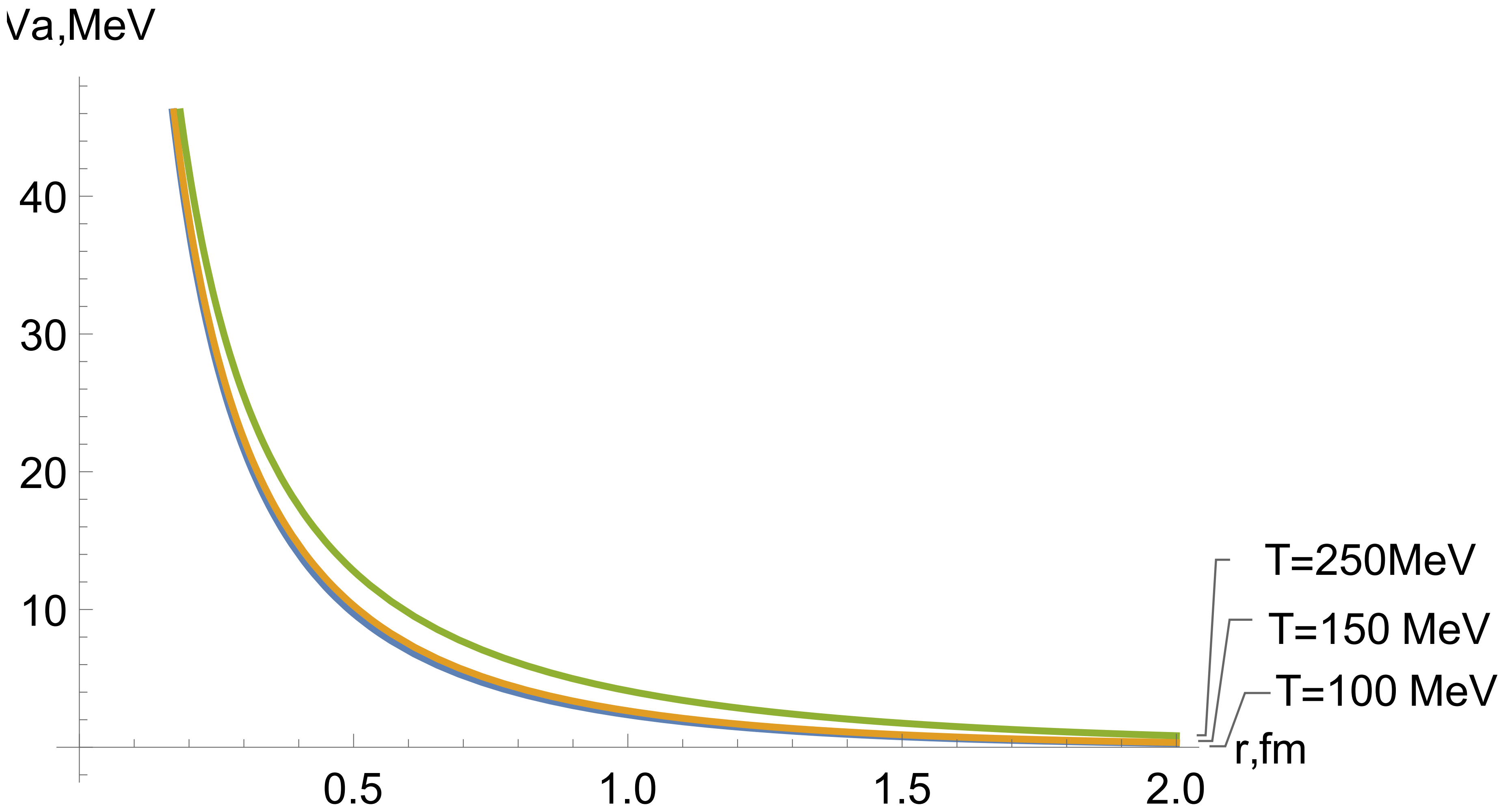


Figure: One gluon exchange contribution to $Q\bar{Q}$ potential in color octet state. Constants was chosen as $g^2/4\pi = 0.3$.

Heavy quarkonium potential as a sum of direct instanton and one gluon exchange contribution at $T \neq 0$

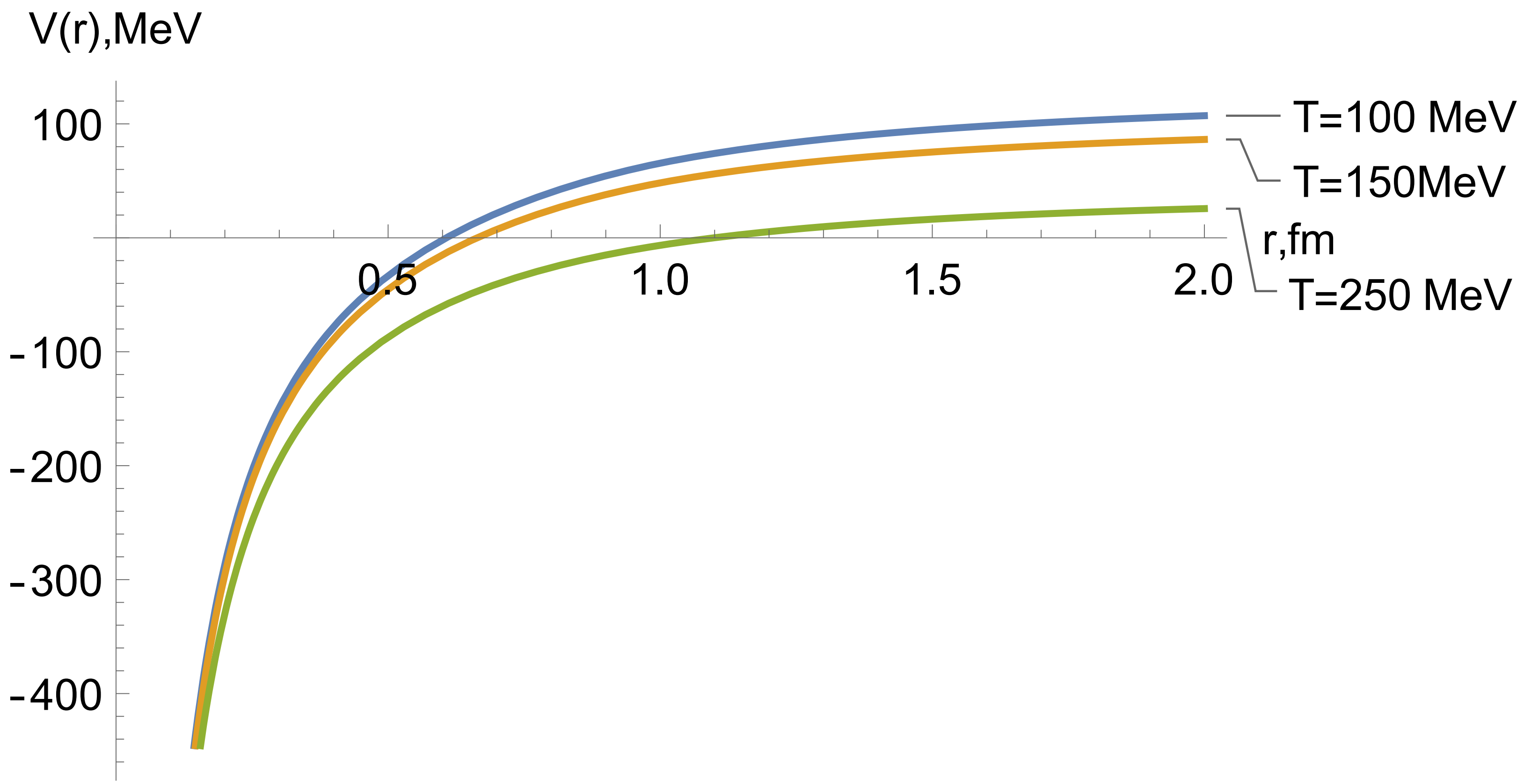


Figure: For color singlet $Q\bar{Q}$ potential

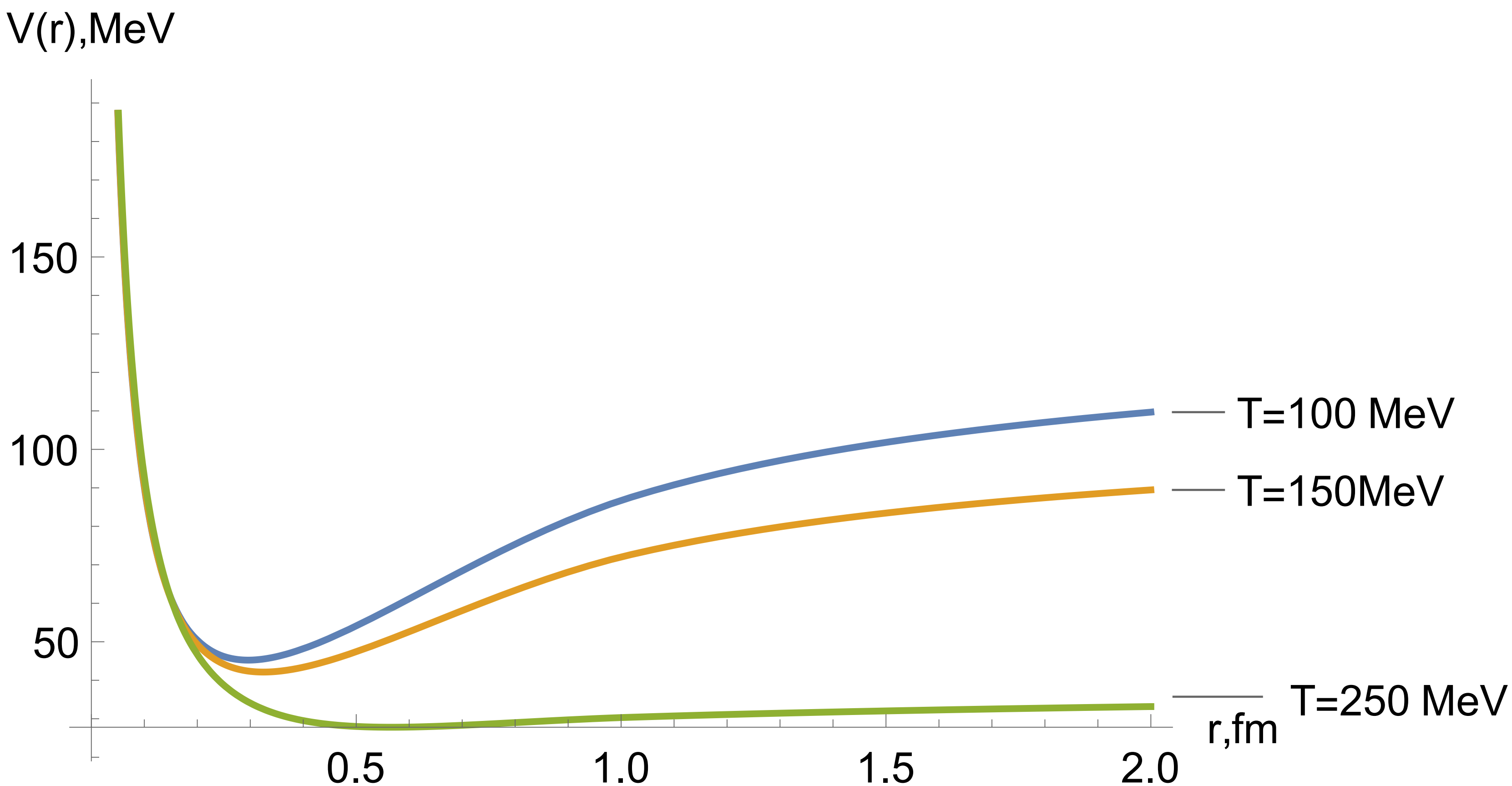


Figure: For color octet $Q\bar{Q}$ potential

Conclusion

- Hot QCD can be considered as $T = 0$ QCD with modified parameters
- Direct contribution of potential rises with the increase of distance while in large distances levels off: $V_c(r)_{r\to\infty} \to 2\Delta M_Q$
- It falls down when temperature becomes higher
- One gluon exchange contribution is Yukawa-like potential
- Because of weak temperature dependence of gluon mass there was observed one gluon exchange contribution to the $Q\bar{Q}$ potential with small differences in several temperatures
- Full HQP in color singlet state behaves as attractive potential in small r while it is repulsive in large r
- We can observe potential well in region $r \sim \bar{\rho}$ in color octet state of HQP

Thanks for attention!!!