MONOPOLES AND CONFINEMENT: NEW RESULTS AND STATE OF THE ART.

Adriano Di Giacomo

Pisa University and INFN, Italy.

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PLAN OF THE TALK

- CONFINEMENT OF COLOR AND MONOPOLES
- MONOPOLES ON LATTICE
- GAUGE INVARIANCE AND MONOPOLES .
- ORDER PARAMETER FOR CONFINEMENT
- DISCUSSION

CONFINEMENT OF COLOUR-1

QUARKS STRICTLY CONFINED IN NATURE

- $\frac{n_q}{n_p} \leq 10^{-27}$ EXPECT $\approx 10^{-12}$
- $\frac{\sigma_q}{\sigma_{TOT}} \leq 10^{-15}$ EXPECT $O(1)$

NATURAL EXPLANATION :
$n_q = 0$, $\sigma_q = 0$ PROTECTED BY SOME SYMMETRY.

DECONFINING TRANSITION A CHANGE OF SYMMETRY.

ALL THIS SHOULD BE BUILT-IN IN QCD,

CONFINEMENT OF COLOUR -2

- GAUGE INVARIANCE CAN NOT BE BROKEN

$$U_G(\vec{x}) = U(\vec{x}) U_B(\vec{x})$$

$$U(\vec{x}) = 1 \text{ on } S_2 \qquad U_B(\vec{x}) = 1 \text{ except on } S_2.$$

  S_2 THE 2-DIMENSIONAL SURFACE AT SPATIAL INFINITY.

- D.O.F. INVOLVED LIVE ON THE SURFACE AT SPATIAL INFINITY (TOPOLOGY).
- TOPOLOGICAL EXCITATIONS IN (3+1)d MONOPOLES $[S_2 \to SU(2)]$, IN (2+1)d VORTICES $[S_1 \to U(1)]$
- CONFINEMENT BY DUAL SUPERCONDUCTIVITY OF THE VACUUM ['tHooft , Mandelstam 1975].
- MONOPOLES DO EXIST IN HIGGS GAUGE SYSTEMS ['tHooft , Polyakov ' 74].

MONOPOLES ON LATTICE-1

LATTICE A TOOL TO EXPLORE LARGE DISTANCES IN QCD.
THREE MAIN APPROACHES TO MONOPOLES

1) DETECT MONOPOLES IN LATTICE CONFIGURATIONS AND LOOK FOR MONOPOLE DOMINANCE [KANAZAWA '90, BERLIN, ITEP]

2) DEFINE A MONOPOLE CREATION OPERATOR, μ .
$\langle\mu\rangle \equiv \langle 0|\mu|0\rangle$ ORDER PARAMETER FOR DUAL SUPERCONDUCTIVITY, TO BE MEASURED ON LATTICE.
$\langle\mu\rangle \neq 0 \;\; T \leq T_c$, $\langle\mu\rangle = 0 \;\; T > T_c$ [PISA, '94]

3) DETERMINE THE CONSTRAINT POTENTIAL OF THE MONOPOLE FIELD FROM THE MONOPOLES DETECTED IN LATTICE CONFIGURATIONS, AND LOOK FOR A MINIMUM AT NON-ZERO FIELD [ITEP].

MONOPOLES ON LATTICE-2

- 2) AND 3) EQUIVALENT IN PRINCIPLE, BASED ON SYMMETRY. DIFFERENT TECHNIQUES.
- 1) INCONCLUSIVE FOR CONFINEMENT.
- 1), 2), 3) NEED TO IDENTIFY THE $U(1)$ SUBGROUP IN WHICH MONOPOLES LIVE (ABELIAN PROJECTION). THE ORIGINAL SYSTEM IN WHICH MONOPOLES WERE FOUND IS A HIGGS MODEL ['tHOOFT, POLYAKOV '74] : MAGNETIC CHARGE $=$ WINDING NUMBER OF THE MAPPING $S_2 \to \vec{\Phi}$. ABELIAN PROJECTION $\equiv U(1)$ SUBGROUP WHICH LEAVES $\vec{\Phi}$ INVARIANT.
  IN *QCD* NO HIGGS FIELD. LOOK FOR EFFECTIVE HIGGS. ANY FIELD IN THE ADJOINT REPRESENTATION CAN ACT AS EFFECTIVE HIGGS $\vec{\Phi}_{eff}$['tHOOFT '81]
  DIAGONALIZE DIFFERENT EFFECTIVE HIGGSES AND CREATE OR DETECT A $U(1)$ MONOPOLE.

MONOPOLES ON LATTICE-3

- DIFFICULT TO DETECT A CHARGE IN A SUPERCONDUCTOR BECAUSE OF SHIELDING.
- MONOPOLES DETECTED ON LATTICE BY EXCESS OF FLUX (DIRAC STRING)[DEGRAND, TOUSSAINT '81]. NUMBER AND LOCATION STRONGLY DEPEND ON THE CHOICE OF THE GAUGE (ABELIAN PROJECTION). MAGNETIC CHARGE A TOPOLOGICAL PROPERTY : SHOULD BE GAUGE INVARIANT.

MONOPOLES ON LATTICE-4

- MONOPOLE IS A CLASSICAL TOPOLOGICAL CONFIGURATION : WE SHALL PROVE THAT ITS EXISTENCE AND CREATION ARE ABELIAN PROJECTION INVARIANT CONCEPTS.

  WHATEVER PROJECTION WE USE TO DEFINE μ, IT CREATES A MONOPOLE IN ALL PROJECTIONS: $\langle\mu\rangle \neq 0$ MEANS DUAL SUPERCONDUCTIVITY IN ALL OF THEM.

- LATTICE DETECTION HAS PROBLEMS, . COMPUTATION OF THE EFFECTIVE POTENTIAL PLAGUED BY LATTICE ARTEFACTS.

MONOPOLES AND GAUGE INVARIANCE-1

- MONOPOLES: SOLITONS OF THE $SU(2)$ HIGGS MODEL WITH Φ IN THE ADJOINT REPRESENTATION. ['t HOOFT 74, POLYAKOV 74]

$$L = -\frac{1}{4}\vec{G}_{\mu\nu}\vec{G}_{\mu\nu} + \frac{1}{2}D_\mu\vec{\Phi}D_\mu\vec{\Phi} - \frac{m^2}{2}\vec{\Phi}\vec{\Phi} - \frac{\lambda}{4}(\vec{\Phi}\vec{\Phi})^2$$

$$\Phi^a = \rho\frac{x^a}{|\vec{x}|}H(\xi), \quad H(\xi)_{\xi\to\infty} \to 1, \quad H(0) = 0$$

$$A_0^a = 0, \;\; A_i^a = -\frac{1}{gx^2}\epsilon_{iab}x_b[1 - K(\xi)], \quad K(\infty) = 0, \;\; K(0) = 1$$

$$\xi = g\rho x, \qquad \rho \equiv \sqrt{(-\tfrac{m^2}{\lambda})}\ .$$

- ${\Phi^a}_{\vec{x}\to 0} \approx constant.x^a \quad {\Phi^a}_{x\to\infty} = \rho\frac{x^a}{x} \quad Q^{mag} = \frac{1}{g}H(\infty)$

  A MAPPING $S_2 \to O(3)/U(1)$.
  MAGNETIC CHARGE $\equiv$ WINDING NUMBER.

MONOPOLES AND GAUGE INVARIANCE-2

- UNITARY GAUGE: $\Phi \equiv \Phi^a \sigma_a$ DIAGONAL $\rightarrow$ $\Phi = |\vec{\Phi}| \sigma_3$.
  ON S_2 $\Phi = \rho \sigma_3$. (ABELIAN PROJECTION)

- ABELIAN MAGNETIC FIELD IN UNITARY GAUGE

$$B^3_k \equiv \frac{1}{2} \epsilon_{kij} F^3_{ij} \qquad F^3_{ij} = \partial_i A^3_j - \partial_j A^3_i = F_{ij}$$

$$F_{ij} \equiv \hat{\Phi}[\vec{G}_{ij} - \frac{1}{g} D_i \hat{\Phi} \wedge D_j \hat{\Phi}] \quad [\hat{\Phi} \equiv \frac{\vec{\Phi}}{|\vec{\Phi}|}] \quad (t' HOOFT \ \ TENSOR)$$

$$\vec{B}^3 \approx_{r \rightarrow \infty} \frac{1}{g} [\frac{\vec{r}}{r^3} + Dirac - string] \qquad (\text{ MONOPOLE })$$

- DIRAC STRING CAN BE PUT ALONG +z-AXIS.

MONOPOLES AND GAUGE INVARIANCE-3

- CONSIDER THE QUANTITY

$$M = \oint_c \hat{\Phi} \vec{A}_i dx_i.$$

c A CLOSED PATH ON S_2 ENCIRCLING $+$ z axis.

- 1) $M =$ TOTAL MAGNETIC CHARGE
- 2) M IS GAUGE INVARIANT.
- 3) M IS ABELIAN PROJECTION INDEPENDENT.
- PROOF of 1) BY STOKES THEOREM

$$M = \int_S d\vec{\sigma}.\vec{\nabla} \wedge (\hat{\Phi}^a \vec{A}^a) = \int_S d\vec{\sigma} \vec{B}$$

SINCE $\vec{\nabla}\hat{\Phi}^a = 0$ IN THE UNITARY GAUGE.

PROOF OF 2)

$$M' \equiv \oint_c \hat{\Phi}' \vec{A}'_i dx_i = \oint_c (\hat{\Phi} \vec{A}_i + \frac{i}{g} \hat{\Phi} \partial_i R^\dagger) dx_i = M \quad [\ \vec{\Phi}' = R \vec{\Phi}]$$

$\oint_c \hat{\Phi} \partial_i R^\dagger = 0$ SINCE $\vec{\nabla} \hat{\Phi} = 0$ AND R SINGLE VALUED, AND IN ANY CASE IF THE TRANSFORMATION IS GLOBAL.

PROOF OF 3)

SUPPOSE THERE ARE TWO (OR MORE) HIGGS FIELDS $\vec{\Phi}$ AND $\vec{\Phi}'$ OF COLOR ORIENTATIONS $\hat{\Phi}$ AND $\hat{\Phi}'$ EACH WITH ITS MONOPOLE SOLUTIONS.

ON THE SURFACE S_2 AT SPACIAL INFINITY AN $\bar{R}(\vec{n})$ EXISTS SUCH THAT $\hat{\Phi}'(\vec{n}) = \bar{R}(\vec{n}) \hat{\Phi}(\vec{n})$, AND WITH IT A GLOBAL TRANSFORMATION FROM THE UNITARY GAUGE OF $\vec{\Phi}$ TO THAT OF $\vec{\Phi}'$. FROM 2) $M = M'$ THE MAGNETIC CHARGE IS ABELIAN PROJECTION INDEPENDENT.

CREATING A MONOPOLE IS AN ABELIAN PROJECTION INDEPENDENT OPERATION.

MONOPOLES AND GAUGE INVARIANCE-5

- IN *QCD* NO FUNDAMENTAL HIGGS FIELD. EFFECTIVE HIGGS FIELD NEEDED TO DEFINE MONOPOLES [E.G. $A_4 = iA_0$ IN THE GAUGE $\partial_0 A_0 = 0$].

  PROOF OF PROJECTION INDEPENDENCE STAYS UNCHANGED BEING BASED ON SYMMETRY.

- MAGNETIC CHARGE PROJECTION INVARIANT

  $\longrightarrow \quad \mu$ CREATES A MONOPOLE IN ALL ABELIAN PROJECTIONS. IF $\langle\mu\rangle \neq 0$ DUAL SUPERCONDUCTIVITY IN ALL OF THEM.

- NUMBER OF MONOPOLES PROJECTION INVARIANT (PROBLEM WITH LATTICE DETECTION)

- GENERALIZED TO HIGHER GAUGE GROUPS LIKE $SU(N)$ [A.D.G. '16].

- LOOK FOR AN EFFECTIVE HIGGS FIELD IN *QCD* , $\vec{\Phi}_{eff}(\vec{A}_\mu)$, IN THE PHASE IN WHICH MONOPOLES CAN EXIST AS SOLITONS (DECONFINED). RECALL

$$\Phi^a_{eff}(\vec{A}_\mu(\vec{x}))_{\vec{x}\to 0} \approx constant.x^a \qquad \Phi^a_{eff}(\vec{A}_\mu(\vec{x}))_{\vec{x}\to\infty} = \rho\hat{x}^a$$

$$A^a_0 = 0, \;\; A^a_i = -\frac{1}{gx^2}\epsilon_{iab}x_b[1 - K(\xi)], \quad K(\infty) = 0, \;\; K(0) = 1$$

NO WAY TO HAVE CONSISTENCY AT $x \to \infty$ IF $\vec{A}_0 = 0$.

- IF $\vec{A}_0 \neq 0$ $\partial_0 \vec{A}_0 = 0$ AT LEAST ONE SOLUTION EXISTS , NAMELY $\vec{\Phi}_{eff} = \vec{A}_0$ [JULIA ZEE '75] , "BPS" SOLUTION. ABEL. PROJ. : POLYAKOV LINE $P(\vec{x}) = \exp\left(\frac{ig}{T}\vec{A}_0(\vec{x})\right)$

- ANY OPERATOR IN THE ADJOINT REPRESENTATION BEHAVES AS A $\vec{\Phi}_{eff}$ NEAR A ZERO ['tHOOFT '81] AND CAN ACT AS EFFECTIVE HIGGS OF A MONOPOLE. NOT CLEAR IN THE CONFINED PHASE.

THE ORDER PARAMETER -1

- DEFINITION OF μ [A.D.G. et al '95 $\rightarrow$]

$$\mu(\vec{x},t) = \exp\left(i \int d^3y \vec{E}^3_\perp(\vec{y},t).\vec{A}^0_\perp(\vec{y}-\vec{x})\right)$$

$\vec{A}^0_\perp(\vec{y}-\vec{x})$: THE FIELD IN $\vec{y}$ PRODUCED BY A MONOPOLE SITTING in $\vec{x}$ (IN TRANSVERSE GAUGE).

$$\exp(ipa)|x\rangle = |x+a\rangle \;\rightarrow\; \mu(\vec{x},t)|\vec{A}^3_\perp(\vec{z},t)\rangle = |\vec{A}^3_\perp(\vec{z},t)+\vec{A}^0_\perp(\vec{z}-\vec{x})\rangle$$

- $$\langle\mu(\beta)\rangle = \langle 0|\mu 0\rangle = \frac{Z\big(\beta(S+\Delta S)\big)}{Z\big(\beta S\big)} \qquad \beta \equiv \frac{2N}{g^2}$$

$$\rho(\beta) \equiv \frac{\partial ln(\langle\mu(\beta)\rangle)}{\partial\beta} = \langle S\rangle_S - \langle(S+\Delta S)\rangle_{S+\Delta S}$$

$$\langle\mu(\beta)\rangle = \exp\left(\int_0^\beta \rho(\beta')d\beta'\right) \qquad \mu(0)=1$$

ORDER PARAMETER -2

- CREATING A MONOPOLE IS ABELIAN PROJECTION INDEPENDENT OPERATION $\rightarrow$ CREATE IT ALONG FORMAL 3-AXIS IN COLOUR SPACE USED IN THE UPDATING PROCEDURE, AS DONE IN [PISA , BARI].
- 2) AN INFRARED PROBLEM FOUND IN THE ORIGINAL DEFINITION OF ORDER PARAMETER [COSSU et al '07]. SOLVED IN [BONATI et al '12] :

$$\langle 0|\mu 0\rangle \rightarrow \mu' = \frac{\langle 0|\mu 0\rangle}{\sqrt{(\langle 0|0\rangle \langle \mu 0|\mu 0\rangle)}}$$

A RIGOROUS TEST OF DUAL SUPERCONDUCTIVITY.

- $SU(2)$ PURE GAUGE $\mu' \neq 0$ IN CONFINED $\mu' = 0$ IN DECONFINED. CORRECT SCALING AT THE TRANSITION AND CLUSTER PROPERTY. [BONATI et al '12] HIGHER GROUPS AND QUARKS IN PROGRESS.

DISCUSSION

- ORDER PARAMETER FOR CONFINEMENT IN *QCD*, *vev* $\langle\mu\rangle$ OF AN OPERATOR μ CREATING A MONOPOLE. $\langle\mu\rangle \neq 0$ IN THE CONFINED PHASE, $\langle\mu\rangle = 0$ IN THE DECONFINED PHASE, IN ALL ABELIAN PROJECTIONS. CAN USE THE SIMPLEST CHOICE TO DEFINE μ.
- NUMERICALLY TESTED FOR $SU(2)$ PURE GAUGE. CORRECT CRITICAL INDEXES AT THE TRANSITION [BONATI et al '12]. CHECK POSSIBLE BY USE OF AN INDEPENDENT ORDER PARAMETER, THE STRING TENSION IN THE HEAVY $\bar{q} - q$ SYSTEM.
- DETECTION OF MONOPOLES ON LATTICE APPARENTLY INCONSISTENT WITH PROJECTION INDEPENDENCE OF THE EXISTENCE OF MONOPOLES: POSSIBLY PLAGUED BY LATTICE ARTEFACTS POLLUTING THE DETERMINATION OF THE CONSTRAINT POTENTIAL [POLIKARPOV '97].

THANK YOU