MONOPOLES AND CONFINEMENT: NEW RESULTS AND STATE OF THE ART.

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PLAN OF THE TALK

- CONFINEMENT OF COLOR AND MONOPOLES
- MONOPOLES ON LATTICE
- GAUGE INVARIANCE AND MONOPOLES
- ORDER PARAMETER FOR CONFINEMENT
- DISCUSSION
QUARKS STRICTLY CONFINED IN NATURE

\[ \frac{n_q}{n_p} \leq 10^{-27} \quad \text{EXPECT } \approx 10^{-12} \]

\[ \frac{\sigma_q}{\sigma_{TOT}} \leq 10^{-15} \quad \text{EXPECT } O(1) \]

NATURAL EXPLANATION:
\[ n_q = 0 , \sigma_q = 0 \] PROTECTED BY SOME SYMMETRY.

DECONFINING TRANSITION A CHANGE OF SYMMETRY.

ALL THIS SHOULD BE BUILT-IN IN QCD,
GAUGE INVARIANCE CAN NOT BE BROKEN

\[ U_G(\vec{x}) = U(\vec{x})U_B(\vec{x}) \]

\[ U(\vec{x}) = 1 \text{ on } S_2 \quad U_B(\vec{x}) = 1 \text{ except on } S_2. \]

\( S_2 \) THE 2-DIMENSIONAL SURFACE AT SPATIAL INFINITY.

D.O.F. INVOLVED LIVE ON THE SURFACE AT SPATIAL INFINITY (TOPOLOGY).

TOPOLOGICAL EXCITATIONS IN (3+1)d MONOPOLES \([S_2 \to SU(2)]\), IN (2+1)d VORTICES \([S_1 \to U(1)]\)

CONFINEMENT BY DUAL SUPERCONDUCTIVITY OF THE VACUUM \(['t\text{Hooft , Mandelstam 1975 }\].

MONOPOLES DO EXIST IN HIGGS GAUGE SYSTEMS \(['t\text{Hooft , Polyakov ' 74 }\].
LATTICE A TOOL TO EXPLORE LARGE DISTANCES IN QCD.

THREE MAIN APPROACHES TO MONOPOLES

1) DETECT MONOPOLES IN LATTICE CONFIGURATIONS
   AND LOOK FOR MONOPOLE DOMINANCE [KANAZAWA ’90,
   BERLIN, ITEP ]

2) DEFINE A MONOPOLE CREATION OPERATOR, \( \mu \).
   \[ \langle \mu \rangle \equiv \langle 0 | \mu | 0 \rangle \]
   ORDER PARAMETER FOR DUAL SUPERCONDUCTIVITY, TO BE MEASURED ON LATTICE.
   \[ \langle \mu \rangle \neq 0 \ T \leq T_c, \langle \mu \rangle = 0 \ T > T_c \ [ \text{PISA, '94} ] \]

3) DETERMINE THE CONSTRAINT POTENTIAL OF THE MONOPOLE FIELD FROM THE MONOPOLES DETECTED IN
   LATTICE CONFIGURATIONS, AND LOOK FOR A MINIMUM AT NON-ZERO FIELD [ ITEP ].
2) AND 3) EQUIVALENT IN PRINCIPLE, BASED ON SYMMETRY. DIFFERENT TECHNIQUES.

1) INCONCLUSIVE FOR CONFINEMENT.

1), 2), 3) NEED TO IDENTIFY THE $U(1)$ SUBGROUP IN WHICH MONOPOLES LIVE (ABELIAN PROJECTION). THE ORIGINAL SYSTEM IN WHICH MONOPOLES WERE FOUND IS A HIGGS MODEL ['tHOOFT, POLYAKOV '74]: MAGNETIC CHARGE = WINDING NUMBER OF THE MAPPING $S_2 \rightarrow \vec{\Phi}$. ABELIAN PROJECTION $\equiv U(1)$ SUBGROUP WHICH LEAVES $\vec{\Phi}$ INVARIANT.

IN QCD NO HIGGS FIELD. LOOK FOR EFFECTIVE HIGGS. ANY FIELD IN THE ADJOINT REPRESENTATION CAN ACT AS EFFECTIVE HIGGS $\vec{\Phi}_{\text{eff}}$ ['tHOOFT '81] DIAGONALIZE DIFFERENT EFFECTIVE HIGGSES AND CREATE OR DETECT A $U(1)$ MONOPOLE.

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DIFFICULT TO DETECT A CHARGE IN A SUPERCONDUCTOR BECAUSE OF SHIELDING.

MONOPOLES DETECTED ON LATTICE BY EXCESS OF FLUX (DIRAC STRING)[DEGRAND, TOUSSAINT ’81]. NUMBER AND LOCATION STRONGLY DEPEND ON THE CHOICE OF THE GAUGE (ABELIAN PROJECTION). MAGNETIC CHARGE A TOPOLOGICAL PROPERTY: SHOULD BE GAUGE INVARIANT.
MONOPOLE IS A CLASSICAL TOPOLOGICAL CONFIGURATION: WE SHALL PROVE THAT ITS EXISTENCE AND CREATION ARE ABELIAN PROJECTION INVARIANT CONCEPTS.

WHATEVER PROJECTION WE USE TO DEFINE $\mu$, IT CREATES A MONOPOLE IN ALL PROJECTIONS: $\langle \mu \rangle \neq 0$ MEANS DUAL SUPERCONDUCTIVITY IN ALL OF THEM.

LATTICE DETECTION HAS PROBLEMS, . COMPUTATION OF THE EFFECTIVE POTENTIAL PLAGUED BY LATTICE ARTEFACTS.
MONOPOLES AND GAUGE INVARIANCE-1

► MONOPOLES: SOLITONS OF THE SU(2) HIGGS MODEL WITH Φ IN THE ADJOINT REPRESENTATION.
[’t HOOFT 74, POLYAKOV 74]

\[
L = -\frac{1}{4} \tilde{G}_{\mu\nu} \tilde{G}^{\mu\nu} + \frac{1}{2} D_\mu \tilde{\Phi} D^\mu \Phi - \frac{m^2}{2} \tilde{\Phi} \Phi - \frac{\lambda}{4} (\tilde{\Phi} \Phi)^2
\]

\[
\Phi^a = \rho \frac{x^a}{|x|} H(\xi), \quad H(\xi)_{\xi \to \infty} \to 1, \quad H(0) = 0
\]

\[
A_0^a = 0, \quad A_i^a = -\frac{1}{g x^2} \epsilon_{iab} x_b [1 - K(\xi)], \quad K(\infty) = 0, \quad K(0) = 1
\]

\[
\xi = g \rho x, \quad \rho \equiv \sqrt{\left( -\frac{m^2}{\lambda} \right)}.
\]

► \(\Phi^a \big|_{x \to 0} \approx \text{constant} \cdot x^a\)

\[
\Phi^a_{x \to \infty} = \rho \frac{x^a}{x}, \quad Q^{\text{mag}} = \frac{1}{g} H(\infty)
\]

A MAPPING \(S_2 \to O(3)/U(1)\).
MAGNETIC CHARGE \(\equiv\) WINDING NUMBER.
MONOPOLES AND GAUGE INVARIANCE-2

- **UNITARY GAUGE:** \( \Phi \equiv \Phi^a \sigma_a \) DIAGONAL \( \rightarrow \Phi = |\Phi| \sigma_3 \).
  
  ON \( S_2 \) \( \Phi = \rho \sigma_3 \). (ABELIAN PROJECTION)

- **ABELIAN MAGNETIC FIELD IN UNITARY GAUGE**
  
  \[
  B_3^k \equiv \frac{1}{2} \epsilon_{kij} F_{ij}^3 \quad F_{ij}^3 = \partial_i A_j^3 - \partial_j A_i^3 = F_{ij}
  \]

  \[
  F_{ij} \equiv \hat{\Phi}[\hat{G}_{ij} - \frac{1}{g} D_i \hat{\Phi} \wedge D_j \hat{\Phi}] \quad [\hat{\Phi} \equiv \frac{\Phi}{|\Phi|}] \quad (t'\text{HOOFT TENSOR})
  \]

  \[
  \vec{B}^3 \approx_{r \to \infty} \frac{1}{g} \left[ \frac{r}{r^3} + \text{Dirac-string} \right] \quad (\text{MONOPOLE})
  \]

- **DIRAC STRING CAN BE PUT ALONG +z-AXIS.**
CONSIDER THE QUANTITY

\[ M = \oint_{c} \hat{\Phi} \vec{A} \, dx \]

c \quad A CLOSED PATH ON \( S_2 \) ENCIRCLING + z axis.

1) \( M = \) TOTAL MAGNETIC CHARGE

2) \( M \) IS GAUGE INVARIANT.

3) \( M \) IS ABELIAN PROJECTION INDEPENDENT.

PROOF of 1) BY STOKES THEOREM

\[ M = \int_{S} d\vec{\sigma} \cdot \vec{\nabla} \wedge (\hat{\Phi}^a \vec{A}^a) = \int_{S} d\vec{\sigma} \vec{B} \]

SINCE \( \vec{\nabla} \hat{\Phi}^a = 0 \) IN THE UNITARY GAUGE.
PROOF OF 2)

\[
M' \equiv \oint_c \hat{\Phi}' \hat{A}'_i dx_i = \oint_c (\hat{\Phi} \hat{A}_i + \frac{i}{g} \hat{\Phi} \partial_i R^\dagger) dx_i = M \quad [ \hat{\Phi}' = R \hat{\Phi} ]
\]

\[
\oint_c \hat{\Phi} \partial_i R^\dagger = 0 \quad \text{SINCE } \vec{\nabla} \hat{\Phi} = 0 \quad \text{AND } R \text{ SINGLE VALUED, AND IN ANY CASE IF THE TRANSFORMATION IS GLOBAL.}
\]

PROOF OF 3)

SUPPOSE THERE ARE TWO (OR MORE) HIGGS FIELDS \( \hat{\Phi} \) AND \( \hat{\Phi}' \) OF COLOR ORIENTATIONS \( \hat{\Phi} \) AND \( \hat{\Phi}' \) EACH WITH ITS MONOPOLE SOLUTIONS. ON THE SURFACE \( S_2 \) AT SPACIAL INFINITY AN \( \tilde{R}(\vec{n}) \) EXISTS SUCH THAT \( \hat{\Phi}'(\vec{n}) = \tilde{R}(\vec{n}) \hat{\Phi}(\vec{n}) \), AND WITH IT A GLOBAL TRANSFORMATION FROM THE UNITARY GAUGE OF \( \hat{\Phi} \) TO THAT OF \( \hat{\Phi}' \). FROM 2) \( M = M' \) THE MAGNETIC CHARGE IS ABELIAN PROJECTION INDEPENDENT.

CREATING A MONOPOLE IS AN ABELIAN PROJECTION INDEPENDENT OPERATION.
IN QCD NO FUNDAMENTAL HIGGS FIELD. EFFECTIVE HIGGS FIELD NEEDED TO DEFINE MONOPOLES [E.G. $A_4 = iA_0$ IN THE GAUGE $\partial_0 A_0 = 0$].

PROOF OF PROJECTION INDEPENDENCE STAYS UNCHANGED BEING BASED ON SYMMETRY.

MAGNETIC CHARGE PROJECTION INVARIANT

$\mu$ creates a monopole in all abelian projections. If $\langle \mu \rangle \neq 0$ dual superconductivity in all of them.

NUMBER OF MONOPOLES PROJECTION INVARIANT (PROBLEM WITH LATTICE DETECTION)

GENERALIZED TO HIGHER GAUGE GROUPS LIKE $SU(N)$ [A.D.G. ’16].
ABOUT THE EFFECTIVE HIGGS FIELD IN QCD -1

- LOOK FOR AN EFFECTIVE HIGGS FIELD IN QCD, \( \Phi_{\text{eff}}(\vec{A}_\mu) \), IN THE PHASE IN WHICH MONOPOLES CAN EXIST AS SOLITONS (DECONFINED). RECALL

\[
\Phi^a_{\text{eff}}(\vec{A}_\mu(\vec{x})) \xrightarrow{\vec{x} \to 0} \text{constant}\cdot x^a \quad \Phi^a_{\text{eff}}(\vec{A}_\mu(\vec{x})) \xrightarrow{\vec{x} \to \infty} = \rho \hat{x}^a
\]

\( A_0^a = 0, \quad A_i^a = -\frac{1}{gx^2} \varepsilon_{iab} x_b [1 - K(\xi)], \quad K(\infty) = 0, \quad K(0) = 1 \)

NO WAY TO HAVE CONSISTENCY AT \( x \to \infty \) IF \( \vec{A}_0 \equiv 0 \).

- IF \( \vec{A}_0 \neq 0 \) \( \partial_0 \vec{A}_0 = 0 \) AT LEAST ONE SOLUTION EXISTS, NAMELY \( \Phi_{\text{eff}} = \vec{A}_0 \) [JULIA ZEE ’75], ”BPS” SOLUTION. ABEL. PROJ. : POLYAKOV LINE \( P(\vec{x}) = \exp \left( \frac{i g}{\ell} \vec{A}_0(\vec{x}) \right) \)

- ANY OPERATOR IN THE ADJOINT REPRESENTATION BEHAVES AS A \( \Phi_{\text{eff}} \) NEAR A ZERO [’tHOOFT ’81] AND CAN ACT AS EFFECTIVE HIGGS OF A MONOPOLE. NOT CLEAR IN THE CONFINED PHASE.
 DEFINE THE ORDER PARAMETER -1

\[ \mu(\vec{x}, t) = \exp\left( i \int d^3y \vec{E}_\perp(\vec{y}, t) \cdot \vec{A}_\perp(\vec{y} - \vec{x}) \right) \]

\[ \vec{A}_\perp(\vec{y} - \vec{x}) : \text{THE FIELD IN } \vec{y} \text{ PRODUCED BY A MONOPOLE SITTING \text{ IN TRANSVERSE GAUGE}.} \]

\[ \exp(ipa)|x\rangle = |x+a\rangle \rightarrow \mu(\vec{x}, t)|\vec{A}_\perp^3(\vec{z}, t)\rangle = |\vec{A}_\perp^3(\vec{z}, t) + \vec{A}_\perp^0(\vec{z} - \vec{x})\rangle \]

\[ \langle \mu(\beta) \rangle = \langle 0|\mu 0 \rangle = \frac{Z(\beta(S+\Delta S))}{Z(\beta S)} \quad \beta \equiv \frac{2N}{g^2} \]

\[ \rho(\beta) \equiv \frac{\partial \ln(\langle \mu(\beta) \rangle)}{\partial \beta} = \langle S \rangle_S - \langle (S + \Delta S) \rangle_{S+\Delta S} \]

\[ \langle \mu(\beta) \rangle = \exp \left( \int_0^\beta \rho(\beta') d\beta' \right) \quad \mu(0) = 1 \]
CREATING A MONOPOLE IS ABELIAN PROJECTION INDEPENDENT OPERATION → CREATE IT ALONG FORMAL 3-AXIS IN COLOUR SPACE USED IN THE UPDATING PROCEDURE, AS DONE IN [ PISA, BARI ].

2) AN INFRARED PROBLEM FOUND IN THE ORIGINAL DEFINITION OF ORDER PARAMETER [ COSSU et al '07 ]. SOLVED IN [ BONATI et al '12 ] :

\[
\langle 0 | \mu_0 \rangle \rightarrow \mu' = \frac{\langle 0 | \mu_0 \rangle}{\sqrt{\langle 0 | 0 \rangle \langle \mu_0 | \mu_0 \rangle}}
\]

A RIGOROUS TEST OF DUAL SUPERCONDUCTIVITY.

SU(2) PURE GAUGE \( \mu' \neq 0 \) IN CONFINED \( \mu' = 0 \) IN DECONFINED. CORRECT SCALING AT THE TRANSITION AND CLUSTER PROPERTY. [ BONATI et al '12 ] HIGHER GROUPS AND QUARKS IN PROGRESS.
ORDER PARAMETER FOR CONFINEMENT IN QCD, $\langle \mu \rangle$ OF AN OPERATOR $\mu$ CREATING A MONOPOLE. $\langle \mu \rangle \neq 0$ IN THE CONFINED PHASE, $\langle \mu \rangle = 0$ IN THE DECONFINED PHASE, IN ALL ABELIAN PROJECTIONS. CAN USE THE SIMPLEST CHOICE TO DEFINE $\mu$.

NUMERICALLY TESTED FOR SU(2) PURE GAUGE. CORRECT CRITICAL INDEXES AT THE TRANSITION [BONATI et al ’12 ]. CHECK POSSIBLE BY USE OF AN INDEPENDENT ORDER PARAMETER, THE STRING TENSION IN THE HEAVY $\bar{q} - q$ SYSTEM.

DETECTION OF MONOPOLES ON LATTICE APPARENTLY INCONSISTENT WITH PROJECTION INDEPENDENCE OF THE EXISTENCE OF MONOPOLES: POSSIBLY PLAGUED BY LATTICE ARTEFACTS POLLUTING THE DETERMINATION OF THE CONSTRAINT POTENTIAL [POLIKARPOV ’97].
THANK YOU