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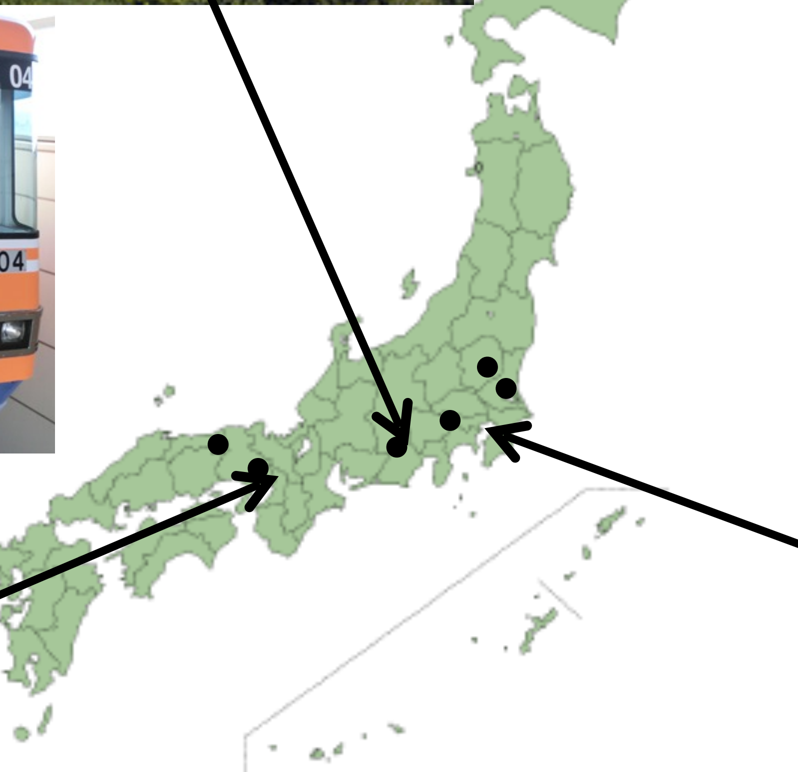
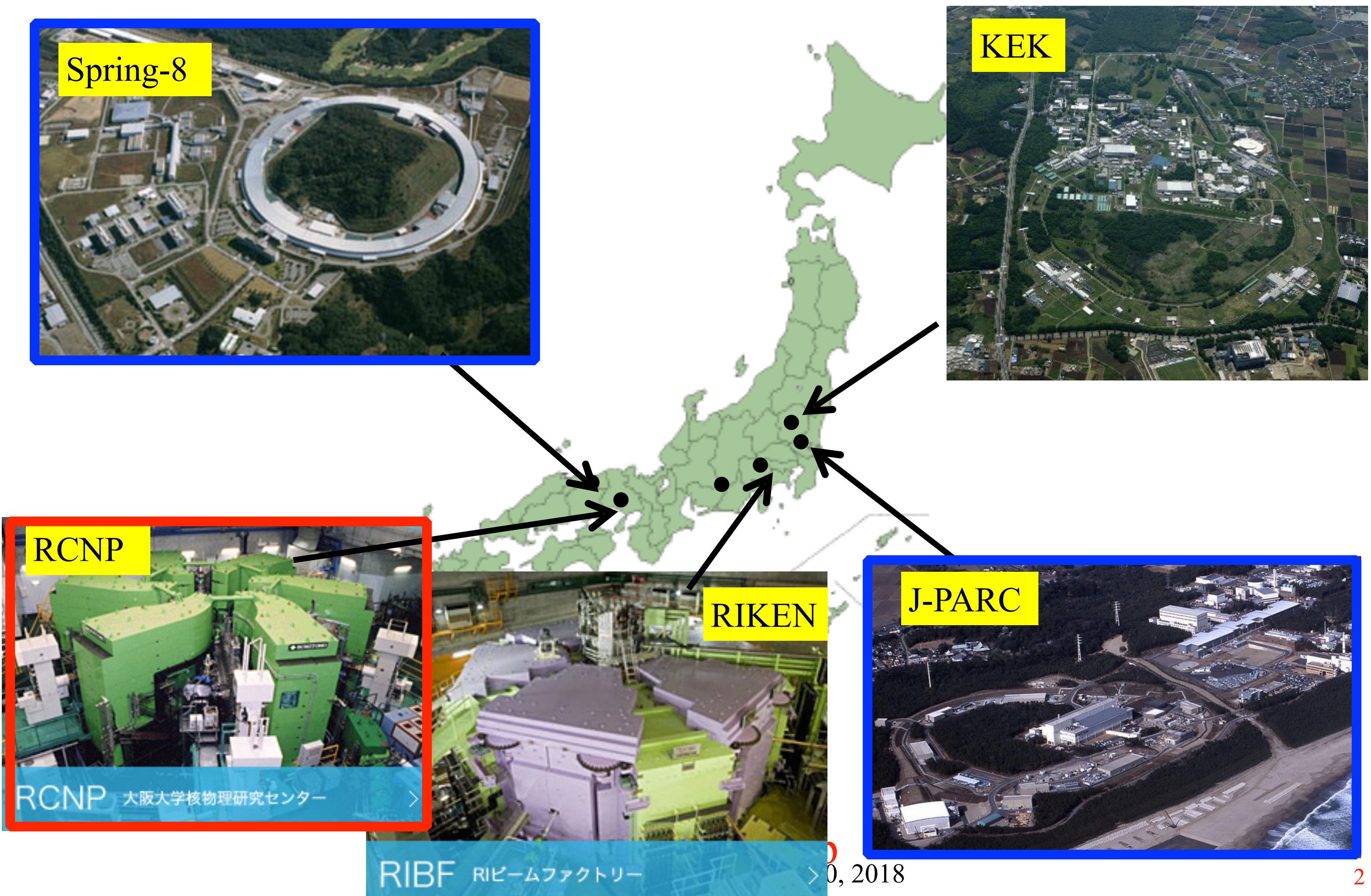

500 km


Tokyo
Sky Tree

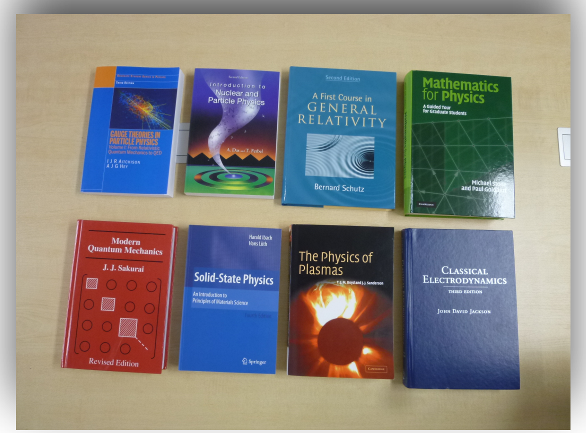
Facilities in Japan

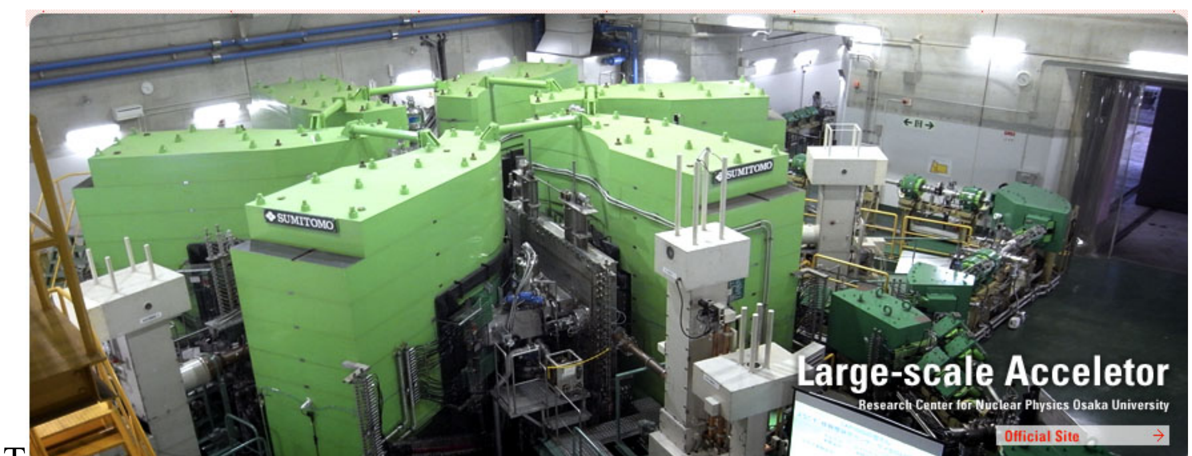


* RCNP Cyclotron Facility

Education
International Physics Course (IPC) for graduates

http://www.rcnp.osaka-u.ac.jp/~ipc/





Skyrmions and Kaon-Nucleon systems

Atsushi Hosaka
RCNP, Osaka University

Workshop on
New aspects of the Hadron and Astro/Nuclear Physics
National University of Uzbekistan, Tashkent, November 5-10, 2018

0. Hadrons beyond qq and qqq
1. Why Skyrmions
2. Kaons in the Skyrmion
3. Potentials, bound states, phaseshifts
4. Coupling to $\pi\Sigma$
5. Summary

0. Hadrons beyond $q\bar{q}$, qqq

Exotics

Conventional mesons, baryons

Gell-Mann (and Zweig's) quarks, PL8, 214, 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS

M. GELL-MANN
California Institute of Technology, Pasadena, California

anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest baryon configuration $(q\,q\,q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\,\bar{q})$ similarly gives just **1** and **8**.

$\Lambda(1405), J^P = 1/2^+$

• as the first candidate ***beyond the standard quark model***

baryon qqq ~ Molecule of $K(q\bar{q})N(qqq)$

Strong attraction as in Oka's talk today

Dalitz and Tuan in 1959

pointed out, the "repulsive" interactions, that is amplitudes of the type (a-) and (b-), predict the lower elastic scattering cross sections at very low energies, owing to their destructive interference with the Coulomb scattering, and are in accord with the trend found for the cross sections at the lowest energies in emulsion studies.[7] It will be pointed out here that this situation makes it quite probable that there should exist a resonant state for pion-hyperon scattering at an energy of about 20 Mev below the $K^- - p$ (c.m.) threshold energy. In the present discussion, charge-

• as a doorway states to dense hyperon (strange) matter ~ neutron stars

References

Dalitz and Tuan, Phys Rev Lett 2, 425 (1959)
Hyodo and Jido, Prog.Part.Nucl.Phys. 67 (2012) 55-98
…

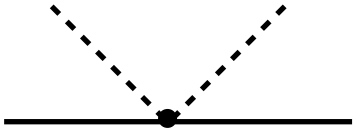
1. Why Skyrmions for $\Lambda(1405)$

Chiral interaction for MB

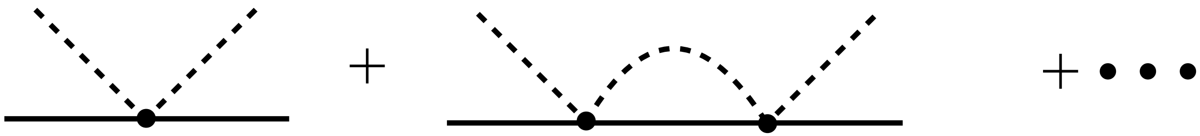
$$\mathcal{L}_1^B = \mathrm{Tr}\Big(\bar{B}(i\not{D} - M_0)B - D(\bar{B}\gamma^\mu\gamma_5\{A_\mu, B\}) - F(\bar{B}\gamma^\mu\gamma_5[A_\mu, B])\Big)$$

$$\mathcal{D}_\mu B = \partial_\mu B + i[V_\mu, B], \quad V_\mu = -\frac{i}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger)$$

$$\rightarrow \bar{B}\gamma^\mu B \times \phi\partial_\mu\phi$$



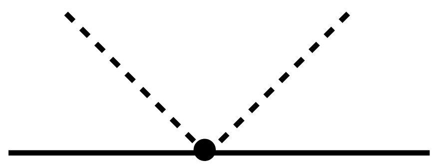
Point-like



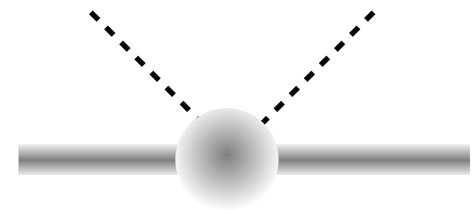
The loop diverges → Regularized by ***cutoff*** or ***subtraction constant***

~ δ-function attractive potential in 3-dim is unstable

Alternatives with hadron structure?



with
extended structure



The Skyrme model is a unified model for mesons and baryons based on

- Meson theory with baryons as solitons
- Expected in the large Nc limit
- Chiral symmetry incorporated

2. Kaons in the Skyrme model

A UNIFIED FIELD THEORY OF MESONS AND BARYONS

T. H. R. SKYRME †
A.E.R.E., Harwell, England

Nucl. Phys. 31, 556 (1962)

Received 29 September 1961

fields interacting through various types of coupling. The objective is the construction of a theory of self-interacting (boson) meson fields, which will admit states that have the phenomenological properties of (fermion) particles, interacting with mesons.

This programme is the obverse of the more fashionable endeavour to reduce the truly elementary particles to a set of spinor fields, out of which everything can be built by simple conjunction. It is a priori much less reasonable because, in particular, it is more difficult to construct half-integral representations of rotation groups out of integral than conversely; indeed it is patently impossible to do this within the limitations of a polynomial expansion. The hope that remains is that the particle-like states will be of a kind that cannot be reached by perturbation theory, and which cannot necessarily be discounted by general arguments. In the type of theory we are using the particle-like solutions have this character, arising from the fact that periodic, transcendental, functions

References

1) T. H. R. Skyrme, Nuclear Physics **31** (1962) 550
2) T. H. R. Skyrme, Proc. Roy. Soc. **262** (1961) 237
3) T. H. R. Skyrme, Proc. Roy. Soc. **260** (1961) 127

2. Kaons in the Skyrme model

Lagrangian with the WZW term

$$\mathscr{L} = \frac{f_\pi^2}{4}\mathrm{tr}\ \partial_\mu U \partial^\mu U^\dagger + \frac{1}{32e^2}\mathrm{tr}\ [\partial_\mu U U^\dagger,\ \partial_\nu U U^\dagger]^2 + L_{SB} + L_{WZW}$$

$$U(x) = \exp\left(i\lambda_a \phi_a(x)/f_\pi\right) \in SU(3)$$

- Effective theory of QCD in the large *Nc* with chiral symmetry
- Baryons emerge as ***topological solitons***
- Strong spin-isospin correlation forms the ***hedgehog***
- ***Nucleons as rotating hedgehogs***

Physical Review

VOL. 62, NOS. 3 AND 4 AUGUST 1 AND 15, 1942 SECOND SERIES

The Pseudoscalar Meson Field with Strong Coupling

W. PAULI, *Institute for Advanced Study, Princeton, New Jersey*

AND

S. M. DANCOFF, *Institute for Advanced Study, Princeton, New Jersey and University of Illinois, Urbana, Illinois*

(Received June 23, 1942)

The present paper treats the symmetrical and charged pseudoscalar theories of the meson field, using the strong coupling approximation; it restricts itself to the case of a single source. The energy levels of the excited states of the heavy particle and the scattering cross section for free mesons are computed by wave mechanical methods. An expression is also obtained for the magnetic moment of the proton or neutron. While the scattering cross section can, with reasonable assumptions, be brought into agreement with experimental values, the results for the magnetic moment are qualitatively at variance with the known values in that equal and opposite moments are predicted for proton and neutron.

Meson and baryon systems

$1/N_c$ expansion

1 N_c^{+1}: ***Baryons as pionic solitons*** $\sim$ 1 GeV

Strong spin (J) and isospin (I) coupling,

$\boldsymbol{K = J + I}$ is conserved $\rightarrow$ ***Hedgehog***

2 N_c^{0}: ***Meson fluctuations*** $\sim$ hundred MeV

3 N_c^{-1}: ***Rotating hedgehog for nucleons*** $\lesssim$ hundred MeV

$$E_{\rm rot} = \frac{J(J+1)}{2\mathcal{I}} \rightarrow \begin{cases} M_N - M_H \sim 75 \text{ MeV} \\ M_\Delta - M_N \sim 300 \text{ MeV} \end{cases}$$

Nuclear Physics B262 (1985) 365–382

BOUND-STATE APPROACH TO STRANGENESS IN THE SKYRME MODEL*

C.G. CALLAN and I. KLEBANOV

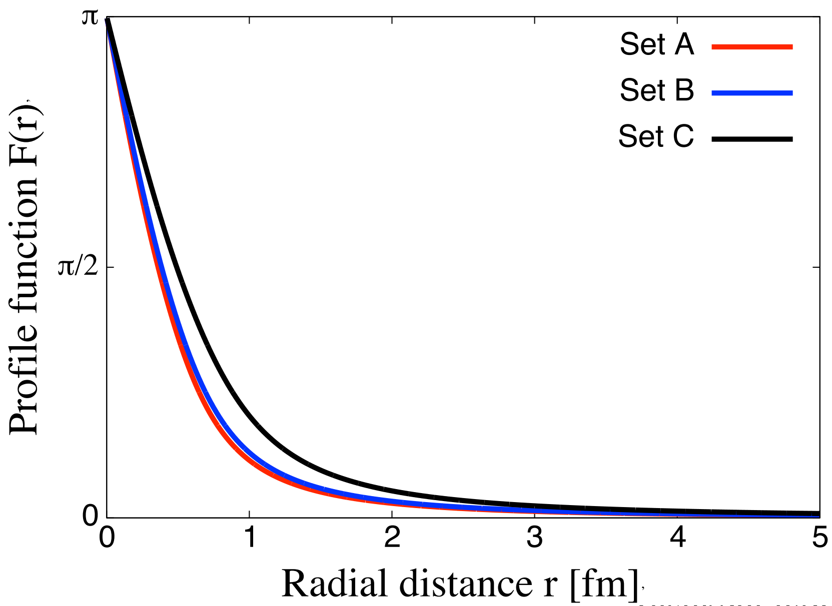
D's and F's. The basic baryon is a topological soliton mainly built out of the Goldstone pions. The simplest way for it to carry strangeness, etc., is for the baryon soliton to bind a meson carrying the appropriate quantum number. Whether or not,

Upon collective coordinate quantization, the kaon bound state carries spin $\frac{1}{2}$ and no isospin. These are precisely the quantum numbers of the strange quark. Thus, in the quark model language, we have added a heavy quark to $(N_c - 1)$ light quarks described by the soliton. The bound-state mode tells us the distribution of strangeness. It is the analog of the heavy-quark wave function in the non-relativistic quark model.

Callan-Klebanov for KN bound states

1	N_c^{+1}:	***Baryons as hedgehog solitons***	$E_{\rm sol} \sim 1$ GeV
2	N_c^{0}:	***"Kaon" bound*** to the ***Hedgehog*** ~ *KH*	
3	N_c^{-1}:	**Rotating *KH*** for ***hyperons***	

$$U_\pi = \begin{pmatrix} \exp[i\vec{\tau}\cdot\hat{r}\,F(r)] & 0 \\ 0 & 1 \end{pmatrix}$$



	F_π [MeV]	e	eF_π [MeV]
Set A	205	4.67	957
Set B	186	4.82	897
Set C	129	5.45	703

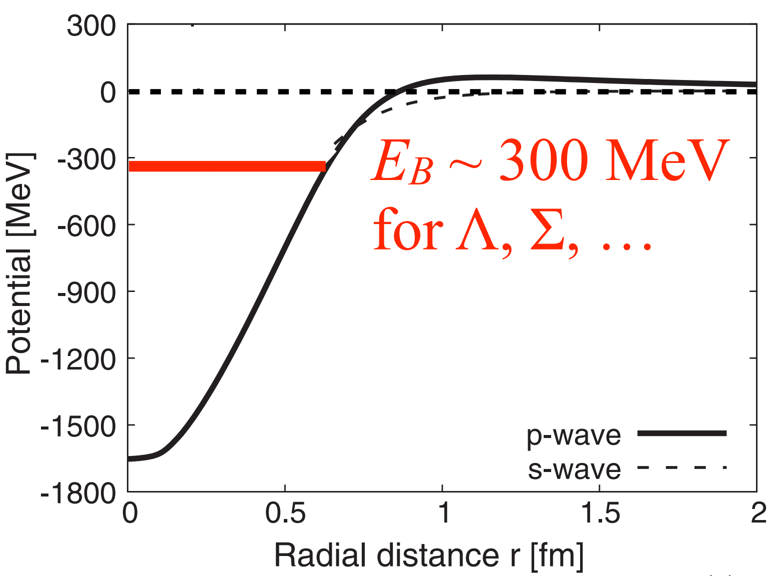
Set C: G. S. Adkins *et al.*, Nucl. Phys. B **228**, 552 (1983)

Soliton energy ~ 1 GeV
Baryon number list. ~ 0.5 fm

Callan-Klebanov for KN bound states

1	N_c^{+1}:	***Baryons as hedgehog solitons***	$E_{sol} \sim 1$ GeV
2	N_c^{0}:	***"Kaon" bound*** to the ***Hedgehog ~ KH***	$E_B \sim 300$ MeV
3	N_c^{-1}:	**Rotating *KH*** for ***hyperons***	$E_{rot} \sim 50$ MeV

$$U_{CK} = A(t)\sqrt{U_\pi}U_K\sqrt{U_\pi}A(t)^\dagger \qquad U_\pi = \begin{pmatrix} \exp[i\vec{\tau}\cdot\hat{r}\, F(r)] & 0 \\ 0 & 1 \end{pmatrix}$$



$$U_K = \exp\left(\frac{i}{f_\pi}\begin{pmatrix} 0 & \sqrt{2}K \\ \sqrt{2}K^\dagger & 0 \end{pmatrix}\right), \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$$

Kaon bound state

Distribution ~ 0.5 fm; Tightly bound

Rotation provides hyperon masses

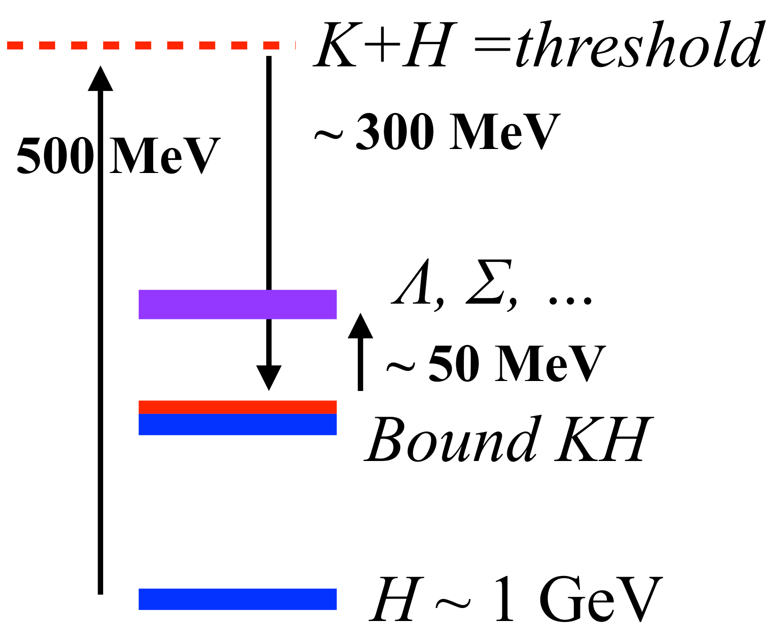
Callan-Klebanov for KN bound states

N_c^{+1}: ***Baryons as hedgehog solitons*** $E_{\rm sol} \sim 1$ GeV

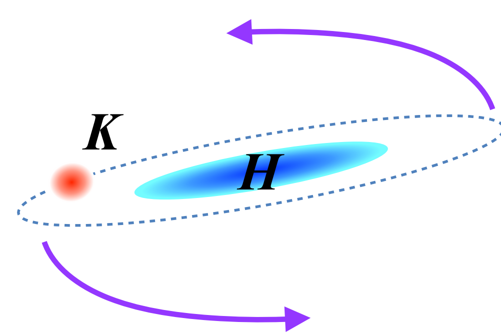
N_c^{0}: ***"Kaon" bound*** to the ***Hedgehog ~ KH*** $E_B \sim 300$ MeV

N_c^{-1}: **Rotating *KH*** for ***hyperons*** $E_{\rm rot} \sim 50$ MeV

Variation ***before*** projection $E_{\rm sol} > E_B > E_{\rm rot}$



Hyperons as rotating [hedgehog and "kaon"]



Interpretation after projection

Hyperons as
rotating [hedgehog and "kaon"]

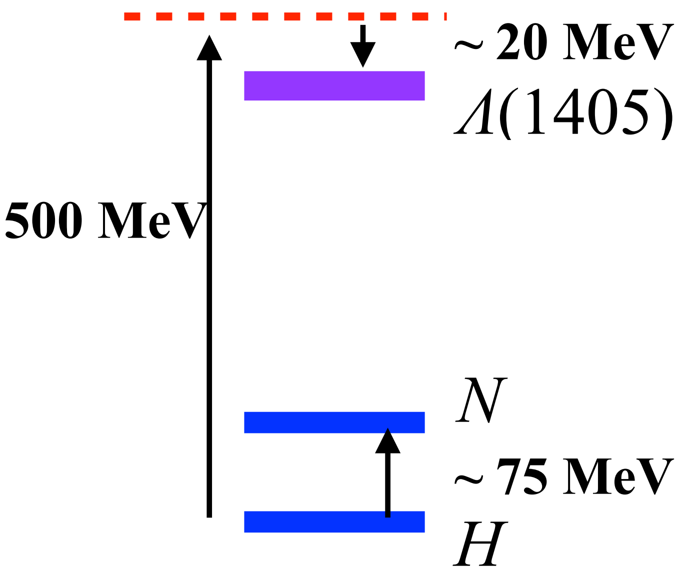
The kaon around the hedgehog has good $K = J + I$
After rotation,

Kaon acquires spin and behaves as a ***strange quark***
Rotating hedgehog behaves as a ***diquark***

We would like to have a method to look at the physical kaon and nucleon

$\Lambda(1405)$ as [rotating hedgehog] and kaon

T. Ezoe and A. Hosaka, Phys. Rev. D 94, no. 3, 034022 (2016).
T. Ezoe and A. Hosaka, Phys. Rev. D 96, no. 5, 054002 (2017).

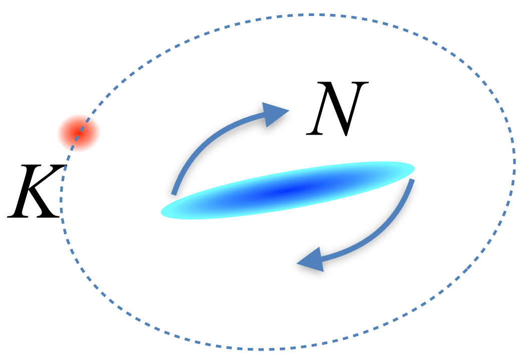


$$E_{\mathrm{sol}} > E_B > E_{\mathrm{rot}}$$

⇩

$$E_{\mathrm{sol}} > E_{\mathrm{rot}} > E_B$$

Variation ***after*** projection



Rotating hedgehog behaves as ***nucleon***
Kaon behaves as physical ***kaon***

Rotating hedgehog and Kaon

$$U \to A\sqrt{U_\pi}A^\dagger U_K A\sqrt{U_\pi}A^\dagger$$

cf: $U_{CK} = A(t)\sqrt{U_\pi}K_K\sqrt{U_\pi}A(t)^\dagger$

$$U_\pi = \begin{pmatrix} \exp[i\vec{\tau}\cdot\hat{r}\ F(r)] & 0 \\ 0 & 1 \end{pmatrix} \qquad U_K = \exp\left(\frac{i}{f_\pi}\begin{pmatrix} 0 & \sqrt{2}K \\ \sqrt{2}K^\dagger & 0 \end{pmatrix}\right), \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$$

Rotating hedgehog and Kaon

$$U \rightarrow A\sqrt{U_\pi}A^\dagger U_K A\sqrt{U_\pi}A^\dagger$$

cf: $U_{CK} = A(t)\sqrt{U_\pi}K_K\sqrt{U_\pi}A(t)^\dagger$

$$U_\pi = \begin{pmatrix} \exp[i\vec{\tau}\cdot\hat{r}\,F(r)] & 0 \\ 0 & 1 \end{pmatrix} \qquad U_K = \exp\left(\frac{i}{f_\pi}\begin{pmatrix} 0 & \sqrt{2}K \\ \sqrt{2}K^\dagger & 0 \end{pmatrix}\right), \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$$

$$\begin{aligned} L_{KN} \quad = \quad & (D_\mu K)^\dagger D^\mu K - K^\dagger a_\mu^\dagger a^\mu K - m_K^2 K^\dagger K \\ & + \frac{1}{(eF_\pi)^2}\left\{ -K^\dagger K \mathrm{tr}\left[\partial_\mu \tilde{U}\tilde{U}^\dagger, \partial_\nu \tilde{U}\tilde{U}^\dagger\right]^2 - 2\left(D_\mu K\right)^\dagger D_\nu K \mathrm{tr}\left(a^\mu a^\nu\right) \right. \\ & \left. \qquad - \frac{1}{2}\left(D_\mu K\right)^\dagger D^\mu K \mathrm{tr}\left(\partial_\nu \tilde{U}^\dagger \partial^\nu \tilde{U}\right) + 6\left(D_\nu K\right)^\dagger \left[a^\nu, a^\mu\right] D_\mu K \right\} \\ & + \frac{3i}{F_\pi^2} B^\mu \left[\left(D_\mu K\right)^\dagger K - K^\dagger \left(D_\mu K\right)\right] \end{aligned}$$

$$\mathscr{L}_{KN} = \partial_\mu K^\dagger \partial^\mu K - m_K^2 K^\dagger K - 2m_K K^\dagger V(\vec{x})K$$

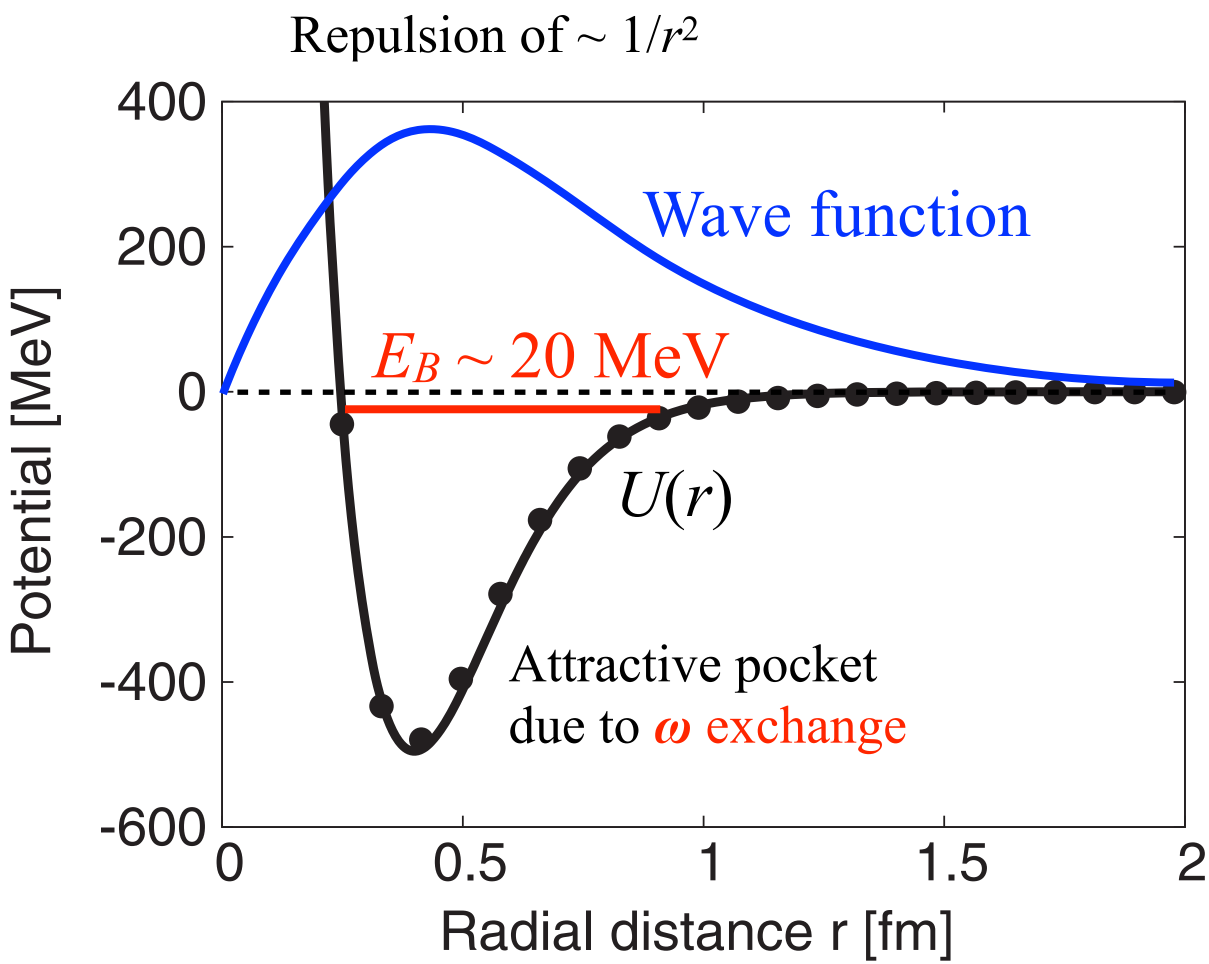
3. Potentials, bound states, phaseshifts

3. Potentials, bound states, phaseshifts

$$U(\overrightarrow{x}) = U_0^c(r) + U_0^\tau \vec{\tau}_K \cdot \vec{\tau}_N + \left(U_0^{LS}(r) + U_\tau^{LS}(r)\vec{\tau}_K \cdot \vec{\tau}_N\right)$$

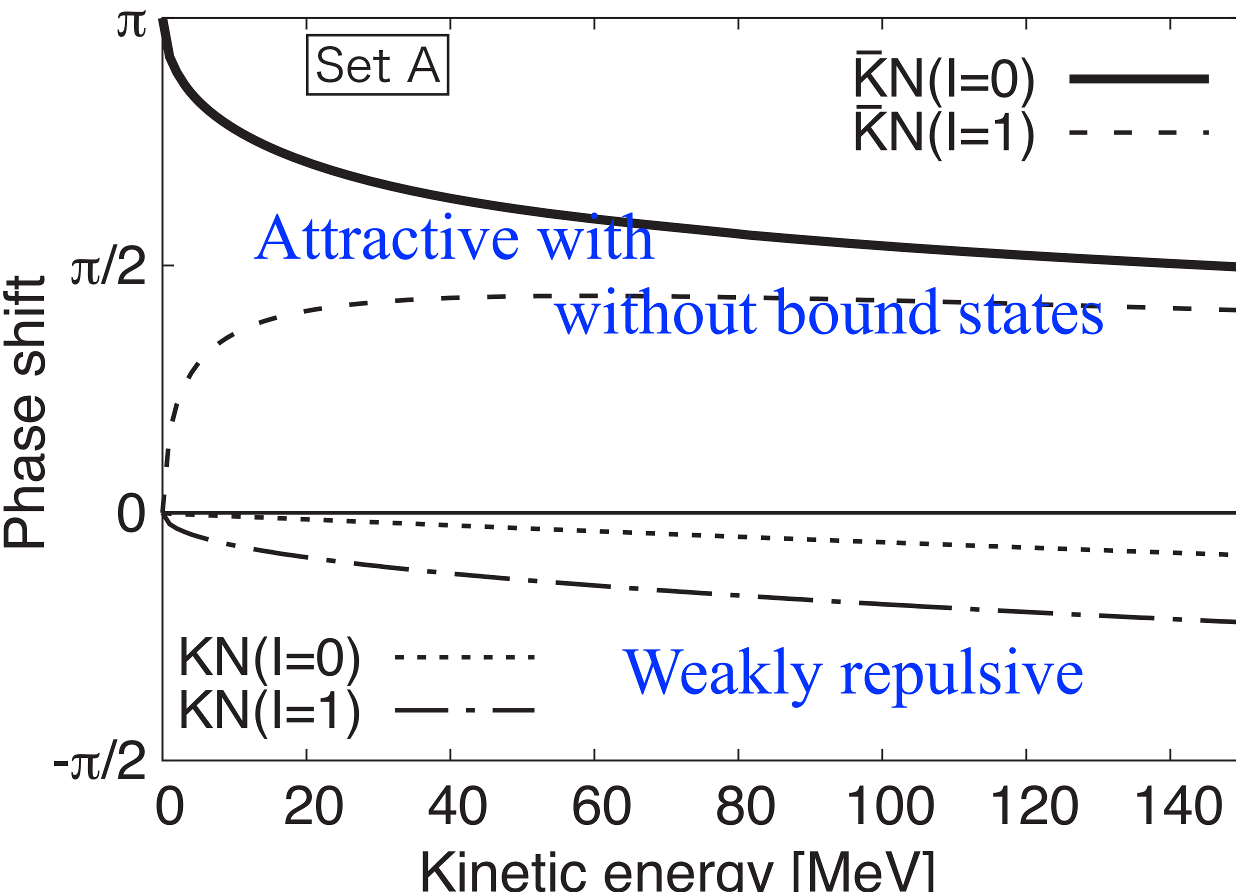
- Contains central and LS, isospin dep. and indep.
  $\rightarrow$ State-dependent
- Non-local, energy dependent
- For S-wave K^{bar}-N:
  Middle range ***attraction*** and short range ***repulsion***
  Attraction from ω, Repulsion ~ p-wave like hedgehog
  Attraction: $(I = 0) > (I = 1)$ $\overrightarrow{\pi}(\overrightarrow{x}) = \hat{r}F(r)$
  Accommodates a weakly bound Kaon in $I = 0$
- For S-wave K-N:
  Weakly repulsive foer $I = 0, 1$

Potential and bound state for Λ(1405)



F_π [MeV]	e	B.E. [MeV]	$\sqrt{\langle r_N^2 \rangle}$ [fm]	$\sqrt{\langle r_K^2 \rangle}$ [fm]
205	4.67	19.9	0.43	1.30

Phase shifts, $J^P = 1/2^-$



cf) K̄N channel

a_0 = -1.81 + i 0.92 fm

a_1 = 0.48 + i 0.87 fm

(M. Mai and U. G. Meißner, [arXiv:1202.2030 [nucl-th]].)

$$a_0 = -1.56 \text{ fm}$$
$$a_1 = 3.38 \text{ fm}$$
$$a_{\bar{K}N} = \frac{1}{2}(a_0 + a_1) = 0.91 \text{ fm}$$

cf) KN channel

a_0 = 0.00 fm

a_1 = -0.33 fm

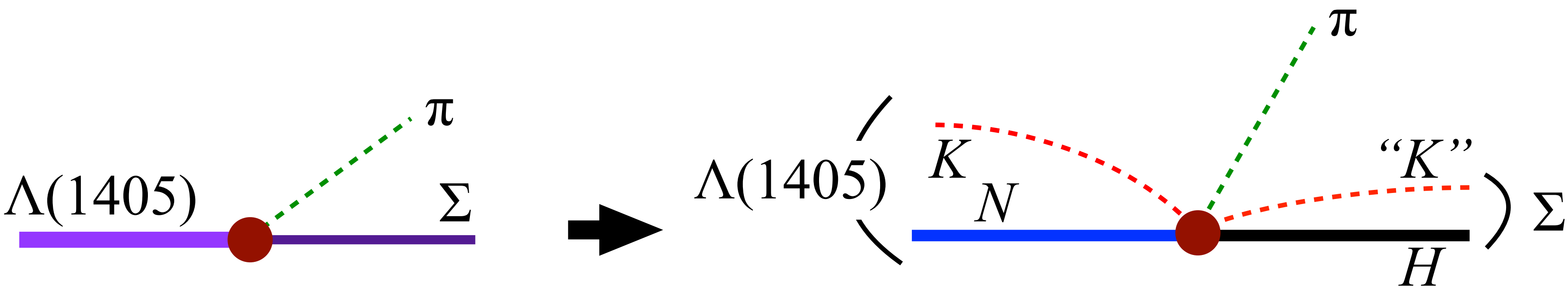
(J. S. Hyslop *et al.*, Phys. Rev. D, **46**, 961 (1992).)

$$a_0 = -0.03 \text{ fm}$$
$$a_1 = -0.46 \text{ fm}$$

4. Coupling $\Lambda(1405) \to \pi\Sigma$

4. Coupling $\Lambda(1405) \to \pi\Sigma$

as a ***Feshbach resonance*** of $K^{\mathrm{bar}}N \to \pi\Sigma$



$$\mathscr{L} = \partial_\mu \pi \; J_5^\mu$$

$$\langle \Sigma \,|\, J_5^\mu \,|\, \Lambda(1405) \rangle \sim \langle \{\bar{K}\} H \,|\, J_5^\mu \,|\, \bar{K} N \rangle$$

$$\langle \Sigma \,|\, J_5^\mu \,|\, \Lambda(1405) \rangle \sim \bar{\psi}_\Sigma \, \gamma^\mu \, \psi_{\Lambda(1405)}$$

In the non-relativistic limit,
the leading contribution is

$$\to \bar{\psi}_\Sigma \, \gamma^0 \, \psi_{\Lambda(1405)}$$

Axial current

$$J^{(2),5,a=3}_{\mu=0} = i\frac{F_\pi^2}{16}\left(\frac{2\sqrt{2}}{F_K}\right)^2 \mathrm{tr}\left[\tau^3 X_0^{(kin)}\right] - \frac{i}{16e^2}\left(\frac{2\sqrt{2}}{F_K}\right)^2 \mathrm{tr}\left[\tau^3 X_0^{(Skyrme)}\right]$$
$$+\frac{N_c}{96\pi^2}\left(\frac{2\sqrt{2}}{F_K}\right)^2 \epsilon^{ijk}\mathrm{tr}\left[\tau^3 X_{0ijk}^{(WZ)}\right]$$

$$X_0^{(kin)} = \frac{1}{2}\left\{\tilde{\xi}K\dot{K}^\dagger\tilde{\xi}^\dagger - \tilde{\xi}\dot{K}K^\dagger\tilde{\xi}\tilde{\xi}^\dagger\right\} - \left\{\tilde{\xi}\leftrightarrow\tilde{\xi}^\dagger\right\}$$

$$\begin{aligned}X_0^{(Skyrme)} = & \left(\frac{1}{2}\left[\tilde{R}^i,\left[\tilde{R}_i,\tilde{\xi}\dot{K}K^\dagger\tilde{\xi}^\dagger - \tilde{\xi}K\dot{K}^\dagger\tilde{\xi}^\dagger\right]\right]\right.\\ & -\tilde{R}^i\tilde{U}\partial_i\left(\tilde{\xi}^\dagger K\right)\dot{K}^\dagger\tilde{\xi}^\dagger + 2\tilde{U}\partial^i\left(\tilde{\xi}^\dagger K\right)\dot{K}^\dagger\tilde{\xi}^\dagger\tilde{R}_i\\ & \left.+\tilde{\xi}\dot{K}\left\{K^\dagger\tilde{\xi}\partial^i\tilde{U} - \partial^i\left(K^\dagger\tilde{\xi}^\dagger\right)\right\}\tilde{R}_i - 2\tilde{R}^i\tilde{\xi}\dot{K}\left\{K^\dagger\tilde{\xi}\partial_i\tilde{U}^\dagger - \partial_i\left(K^\dagger\tilde{\xi}^\dagger\right)\right\}\right)\\ & -\left(\tilde{\xi}\leftrightarrow\tilde{\xi}^\dagger\right)\end{aligned}$$

$$\begin{aligned}X_{0ijk}^{(WZ)} = & \left(\frac{1}{2}\left[\tilde{R}_i\tilde{R}_j\left\{\tilde{U}\partial_k\left(\tilde{\xi}^\dagger K\right)K^\dagger\tilde{\xi}^\dagger - \tilde{\xi}K\partial_k\left(K^\dagger\tilde{\xi}\right)\tilde{U}^\dagger\right\}\right.\right.\\ & \left.+\tilde{R}_i\left\{\tilde{U}\partial_j\left(\tilde{\xi}^\dagger K\right)K^\dagger\tilde{\xi}^\dagger - \tilde{\xi}K\partial_j\left(K^\dagger\tilde{\xi}\right)\tilde{U}^\dagger\right\}\tilde{R}_k + \left\{\tilde{U}\partial_i\left(\tilde{\xi}^\dagger K\right)K^\dagger\tilde{\xi}^\dagger - \tilde{\xi}K\partial_i\left(K^\dagger\tilde{\xi}\right)\tilde{U}^\dagger\right\}\tilde{R}_j\tilde{R}_k\right]\\ & \left.-\left[\tilde{U}\partial_i\left(\tilde{\xi}^\dagger K\right)\left\{K^\dagger\tilde{\xi}\partial_j\tilde{U}^\dagger - \partial_j\left(K^\dagger\tilde{\xi}^\dagger\right)\right\}\tilde{R}_k + \tilde{R}_i\tilde{U}\partial_j\left(\tilde{\xi}^\dagger K\right)\left\{K^\dagger\tilde{\xi}\partial_k\tilde{U}^\dagger - \partial_k\left(K^\dagger\tilde{\xi}^\dagger\right)\right\}\right]\right)\\ & +\left(\xi\leftrightarrow\xi^\dagger\right)\end{aligned}$$

Wave functions

$$|\Lambda(1405)\rangle = |[\bar{K}_{I=1/2} N_{I=1/2}]^{I=0}\rangle = \sqrt{\frac{1}{2}}|p_\uparrow K^-\rangle + \sqrt{\frac{1}{2}}|n_\uparrow \bar{K}^0\rangle$$

$$|\Sigma\rangle = |[d_{J=1} s_{J=1/2}]^{J=1/2}\rangle = \sqrt{\frac{2}{3}}|d_{+1}\ s_\downarrow\rangle + \sqrt{\frac{1}{3}}|d_0\ s_\uparrow\rangle$$

$$\begin{aligned} A &= a_0 + i\tau_i a_i \\ &= i\pi \begin{pmatrix} -|n\uparrow\rangle & -|n\downarrow\rangle \\ |p\uparrow\rangle & |p\downarrow\rangle \end{pmatrix} \end{aligned} \qquad i = 1, 2, 3, \quad \sum_{\mu=0}^{3} {a_\mu}^2 = 1$$

・Nucleon ($I = J = 1/2$)[3]

$$\begin{aligned} |p\uparrow\rangle &= \frac{1}{\pi}(a_1 + ia_2) \\ |p\downarrow\rangle &= -\frac{i}{\pi}(a_0 - ia_3) \\ |n\uparrow\rangle &= \frac{i}{\pi}(a_0 + ia_3) \\ |n\downarrow\rangle &= -\frac{1}{\pi}(a_1 - ia_2) \end{aligned}$$

・Di-quark ($I = J = 1$)

$$\begin{aligned} |d_{I_3=0,J_3=0}\rangle &= \sqrt{\frac{3}{2}}\frac{i}{\pi}({a_0}^2 - {a_1}^2 - {a_2}^2 + {a_3}^2) \\ |d_{I_3=0,J_3=1}\rangle &= \frac{\sqrt{3}}{\pi}(a_1 + ia_2)(a_0 + ia_3) \end{aligned}$$

Decay of Λ(1405)

When writing

$$\mathscr{L}_{eff} = g_{\Lambda^*\Sigma\pi}\, \bar{\psi}_{\Sigma}\, \psi_{\Lambda(1405)}\, \pi$$

$$\Gamma_{\Lambda^*\to\pi\Sigma} = \frac{1}{2\pi}\frac{E_{\Sigma}+M_{\Sigma}}{2M_{\Lambda(1405)}}\,|g_{\Lambda^*\Sigma\pi}|^2$$

Numerical calculation is going ~ few MeV ~ Narrow

5. Summary

- Skyrme model can describe the basic features of $K^{(bar)}N$ systems
- S-wave K^{bar}-N potential has an attractive pocket and repulsion
- It allows a shallow bound state of K^{bar}-N
- It may provide $\Lambda(1405)$ as a Feshbach resonance decaying into $\pi\Sigma$