ASYMPTOTIC THEORY OF CHARGED PARTICLE TRANSFER REACTIONS AT SUB- AND ABOVE-BARRIER ENERGIES AND NUCLEAR ASTROPHYSICS

R. Yarmukhamedov[1] *, K. I. Tursunmakhatov[2] and N. Burtebaev[3]
[1] *Institute of Nuclear Physics, 100214 Tashkent, Uzbekistan*
[2] *Gulistan State University, Gulistan, Uzbekistan*
[3] *INP and IETPh, Almaty, Kazakhstan*
*email: rakhim@inp.uz;rakhim1943@mail.ru

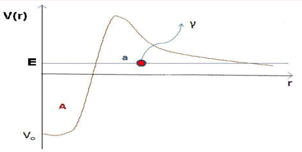
International Workshop " New aspects of the Hadron and Asto/Nuclear Physics",
November 5 –10, 2018, Tashkent, Uzbekstan

The discovery of the tunneling effect (G.Gamov, 1928) led astrophysicists to the following suspicion:

- nuclear reactions are the sources of the energy in stars and the Sun (A.S. Eddington, 1920; Atkinson and Houtermans, 1929)
- at relevant stellar temperatures only hydrogen could penetrate the Coulomb barrier with sufficient ease to include fusion

  $$A(p,\gamma)B \;\to\; 4p \to \alpha$$

  (Hydrogen burning $\to$ the pp-chain and the CNO-cycle)



The solar *pp*-chain:

$p + p \to d + e^+ + \nu_e$
$d + p \to {}^3\mathrm{He} + \gamma$
${}^3\mathrm{He} + {}^3\mathrm{He} \to \alpha + p + p$
${}^3\mathrm{He} + \alpha \to {}^7\mathrm{Be} + \gamma$
$({}^7\mathrm{Be} \to {}^7\mathrm{Li} + e^+ + \nu_e,\ {}^7\mathrm{Li} + p \to 2\alpha)$
${}^7\mathrm{Be} + p \to {}^8\mathrm{B} + \gamma$
$({}^8\mathrm{B} \to {}^8\mathrm{Be} + e^+ + \nu_e,\ {}^8\mathrm{Be} \to 2\alpha)$
$4p \to \alpha$, Q=26.7 MeV (E_{ν_e}=0.53–5.07 MeV).

$$\phi_\nu \sim \tilde{S}_{11}^{-2.5}\tilde{S_{33}}^{-0.3}\tilde{S_{34}}^{0.8}[1 + 3.47\tilde{S_{17}}\tilde{\tau}_{e7}^{-1}].$$

$\tilde{S_{ij}} = S_{ij}(E)/S_{ij}(0)$, $S_{ij} \longrightarrow i + j \to X + Y$
$\tau_{e7}(E) \longrightarrow {}^7\mathrm{Be} + e^- \to {}^7\mathrm{Li} + \nu_e$ at $0 \leq E \lesssim 25$ keV.

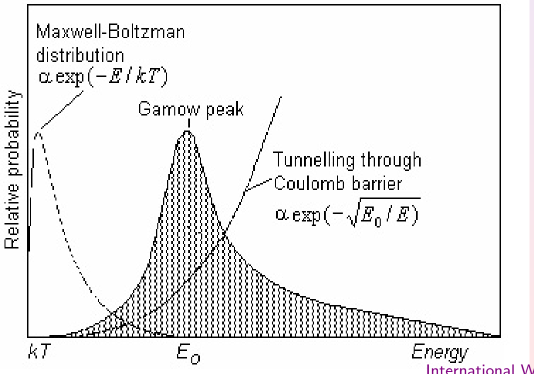
The "hot" pp-chain (CNO - cycle)(H.A.Bethe, 1938)

$^{12}C + p \to ^{13}N + \gamma$ ($^{13}N \to ^{13}C + e^{+} + \nu_e$)
$^{13}C + p \to ^{14}N + \gamma$
$^{14}N + p \to ^{15}O + \gamma$ ($^{15}O \to ^{15}N + e^{+} + \nu_e$)
$^{15}N + p \to ^{12}C + 4He$ ($4p \to ^{4}He$)
or
$^{15}N + p \to ^{16}O + \gamma$
$^{16}O + p \to ^{17}F + \gamma$($^{17}F \to ^{17}O + e^{+} + \nu_e$)
$^{17}O + p \to ^{14}N + ^{4}He$
(Q=26.8 MeV, T=10^8 K, E_ν=1.7 MeV)

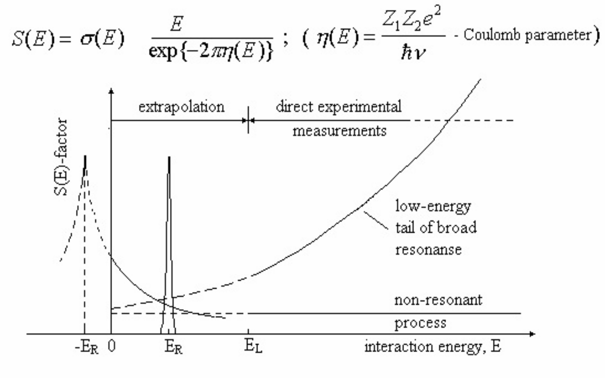
The most actual problems of the modern nuclear astrophysics:

- a reliable estimation of rates $N_A(\sigma v)$ different nuclear astrophysical processes

$$N_A(\sigma v) = N_A \left(\frac{8}{\pi \mu_{Ap}}\right)^2 (k_B T)^{-3/2} \int_0^{\infty} \sigma(E) exp[-E/k_B T] EdE \qquad (1)$$



- obtaining rather low energy cross sections $\sigma(E)$ (or equivalently its astrophysical S-factors $S(E)$ [$S(E) = Ee^{2\pi\eta}\sigma(E)$]) for these reactions



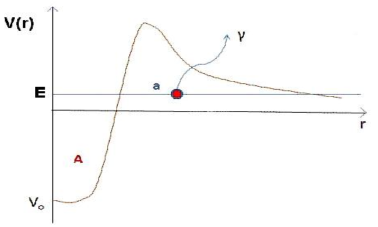
A critical analysis of low-energy astrophysical S-factors data made in the review works of
E.G. Adelberger et al. [Rev.Mod.Phys. 70(1998)1265;83(2011)195]
found out the following facts:

- The extrapolation of measured astrophysical S-factors to $E \lesssim 25$ keV is noticeable ambiguous
- The theoretical predictions for $S(E)$ at $E \lesssim 25$ keV depend noticeably on input parameters (for example on a form of the nuclear potential)
- the accuracy about 2-3% for $S(E)$ at stellar energies $E \lesssim 25$ keV is required for the Cosmological application [L. T. Baby, et al, Phys.Rev.Lett. 90(2003)022501]

Since 1995 considerable experimental and theoretical works have been performed to correctly determine rates of different astrophysical reactions rates of the pp-chain and the CNO-cycle $\Longrightarrow$ to remove these ambiguities.

The main idea of these works[R.F.Christy,I.Duck, Nucl.Phys. 24(1961)89]

- the direct radiative capture $\mathrm{A}(a,\gamma)\mathrm{B}$ reaction proceeds mainly at $r \gtrsim r_N$

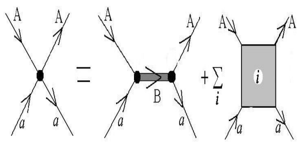


- The overall normalization of the $S_{dir}(E)$ can be correctly determined through the ANC (C_{Aa}) for $A + a \to B$:

The ANC for $A{+}a \to B$ determines the probability of the configuration $(A{+}a)$ in nucleus B at $r > r_N$.

$$I(\mathbf{r}) = <\Psi_B|\Psi_A\Psi_a> \sim C_{Aa} W_{-\eta_B;\, l_B+1/2}(2\kappa_{Aa} r) Y_{l_B \nu_B}(\hat{\mathbf{r}}) \text{ at } r > r_N$$

Nuclear vertex constants (NVC) $G_{Aa;\, l_B s_B}$ determine the virtual processes of the B$\leftrightarrow$ A+a type in the pole diagram



$$M^{l}_{l_{Aa} s_{Aa}}(E) = \frac{G^2_{Aa;\, l_{Aa} s_{Aa}}}{E + \varepsilon_{Ax}} + g^{l}_{l_{Aa} s_{Aa}}(E).$$

Here, $G^2_{Aa;\, l_{Aa} s_{Aa}}$ and $g^{l}_{l_{Aa} s_{Aa}}(E)$ are the residue of the partial amplitude at $E = -\varepsilon_{Aa}$ and a regular function at the point $E = -\varepsilon_{Aa}$, respectively.

$$\mathrm{res}\{M^{l}_{l_{Aa} s_{Aa}}(E)\} = \lim_{E \to -\varepsilon_{Aa}} (E + \varepsilon_{Aa}) M^{l}_{l_{Aa} s_{Aa}}(E) = G^2_{Aa;\, l_B s_B},$$

$$G_{Aa} = -i^{l_B + \eta_B} \frac{\pi^{1/2}}{\mu_{Aa}} C_{Aa}$$

The main methods used for determination of the ANC for $A + a \to B$ from the experimental data analysis

- The modified two-body (A and a) potential method (MTBPM) for the direct radiative capture $\mathrm{A}(a,\gamma)\mathrm{B}$ reactions proposed by S.B. Igamov and R. Yarmukhamedov [Nucl.Phys.A **781** (2007)247]
- The modified DWBA proposed by A.M. Mukhamedzhanov et al.[Phys.Rev.C 56(1997)1302] and S.V. Artemov, at al.[PAN 59(1996)428] $\Longrightarrow$ the zero-and first others of the perturbation theory over the parameter $V^{C}_{i,f}$ in the transition operator
- The asymptotic theories for peripheral transfer reactions based on combination of the dispersion theory and DWBA proposed by Sh.S. Kajumov, A.M. Mukhamedzhanov, R. Yarmukhamedov for neutron transfer [Z.Phys. A 331(1988)315] and authors of the present report for charged particle transfer
- the phase shift analysis based on combination of the dispersion theory and the effective-range expansion proposed by R. Yarmukhamedov, D. Baye, Phys.Rev.C 84(2011)024603

1. The modified two-body (A and a) potential method (MTBPM) for the direct radiative capture $\mathrm{A}(a,\gamma)\mathrm{B}$ reactions

$$M =< I(\mathbf{r})|O^{\mathcal{E}\,\mathcal{M}}(\mathbf{r})|\Psi_{Aa;\,\mathbf{k}}(\mathbf{r}) > \;\Longleftarrow\; Z_{Aa}^{1/2}\varphi_B(\mathbf{r}) \Longleftarrow I(\mathbf{r})$$

$$S_{dir}^{exp}(E) = C_{Aa}^2\mathcal{R}(E;\{b_{Aa}\}),\; \mathcal{R}(E;\{b_{d\alpha}\}) = \frac{\tilde{S}(E;\{b_{d\alpha}\})}{b_{d\alpha}^2},$$

$\varphi_B(\mathbf{r}) \rightarrow\; b_{Aa}W_{-\eta_B;\,l_B+1/2}(2\kappa_{Aa}r)Y_{l_B\nu_B}(\hat{\mathbf{r}})$ at $r > r_N$; $C_{Aa} = Z_{Aa}^{1/2}b_{Aa}$

S.B. Igamov and R. Yarmukhamedov, Nucl.Phys.A **781** (2007)247

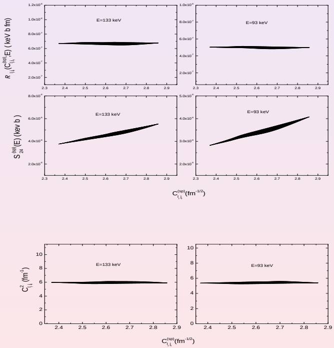
$$(C_{Aa}^{exp})^2 = S_{dir}^{exp}(E)/\mathcal{R}(E;\{b_{Aa}\})$$

if

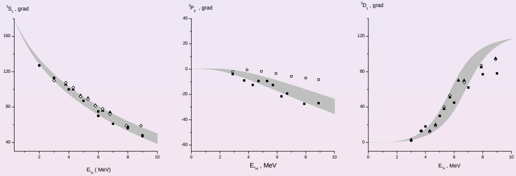
$$\mathcal{R}(E;\{b_{Aa}\}) = f(E) \text{ for } \{b_{Aa}\}_{min} \leq \{b_{Aa}\} \leq \{b_{Aa}\}_{max}.$$

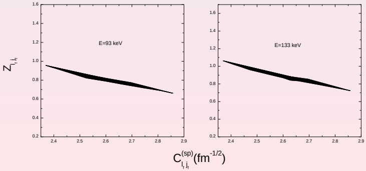
$S_{dir}(E) = Z_{Aa}\tilde{S}_{dir}(E;\{b_{Aa}\})$, at $Z_{Aa} = 1 \Longrightarrow S_{dir}(E) \Longrightarrow$ strong model dependent on $\{b_{Aa}\}$ for the phase equivalent of potentials.

- C. Angulo,et al. Nucl.Phys. A657(1999)[NACRE]; F. Hummache, at al. Phys.Rev.C 82(2010) 065803;S.B. Dubovichenko Phys. Atom. Nucl. 73 (2010)1526 and E.M. Tursunov et al. Phys. Atom. Nucl. 78 (2015)193 $\Longrightarrow d(\alpha,\gamma)^6\mathrm{Li}$ assuming $Z_\alpha = 1$(???).

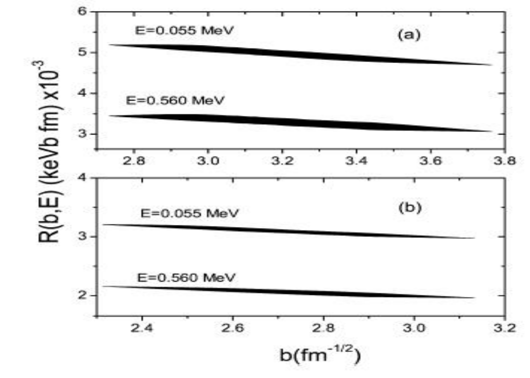


The *s*, *p* and *d* phase shifts of the$d\alpha$ scattering (upper) and the spectroscopic factors ($Z_{d\alpha}$) *for the phase shift equivalent potentials* as a function of the different potential

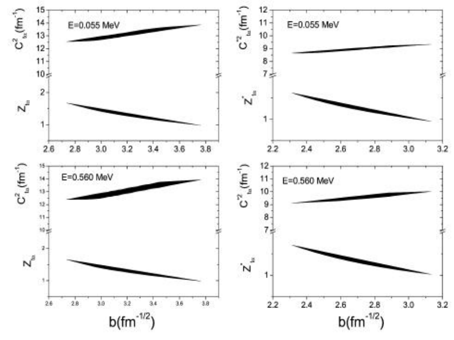


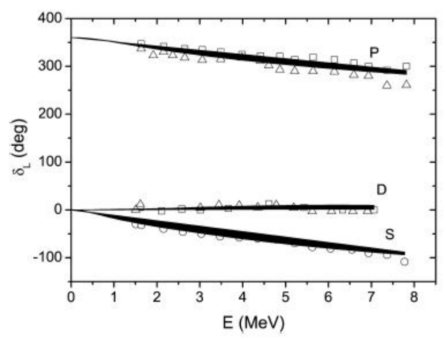


- C. Angulo, et al. Nucl.Phys. A657(1999)[NACRE];S.B. Dubovichenko Phys. Atom. Nucl. 73 (2010)1526 and E.M. Tursuntov et al. Phys.Rev. C 97 (2018) 035802 $\Longrightarrow t(\alpha,\gamma)^7\mathrm{Li}$ assuming $Z_\alpha = 1.$(???)



C. Angulo, et al. Nucl.Phys. A657(1999)[NACRE];S.B. Dubovichenko PAN.73(2010)1526 and E.M. Tursunov et al.Phys.Rev.C97(2018)035802 $\Longrightarrow t(\alpha,\gamma)^7\mathrm{Li} \Longrightarrow$ assuming $Z_{t\alpha} = 1$ (???).

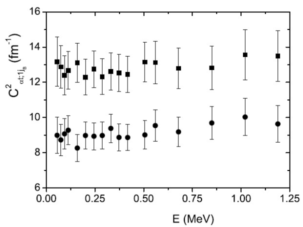


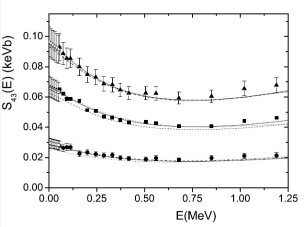


S.B. Igamov and R. Yarmukhamedov, Nucl.Phys.A **781** (2007)247 [1]

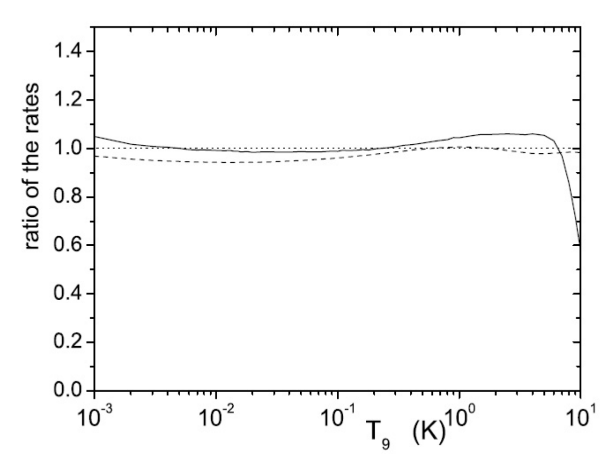
The astrophysical S factors for $t(\alpha,\gamma)^7\mathrm{Li}$;
Exp. data:C.R. Brune et al.
PhR.C50(1994)2205:

The ANCs for $\alpha + t \to {}^7\mathrm{Li}$:



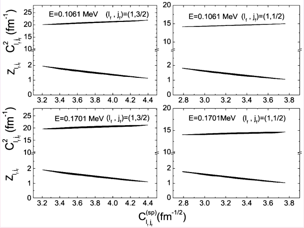


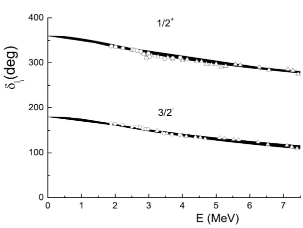
$C^2_{t\alpha;\,g.s.}$=12.74±1.10 and 12.74[1] fm^{-1}
$C^2_{t\alpha;\,exc.}$= 9.00±0.09 and 9.00 [2] fm^{-1}; $S_{tot}(0)$=0.097±0.010 keV·b
[2]. R. Yarmukhamedov, D. Baye, Phys.Rev.C 84(2011)024603



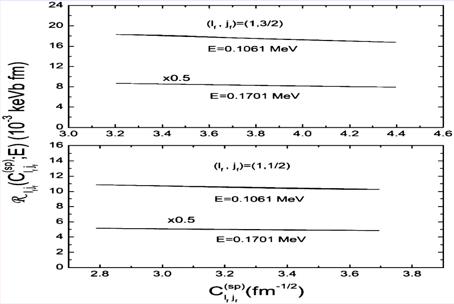
The ratio of the $t(\alpha,\gamma)^7\mathrm{Li}$ reaction rate $N_A(\sigma\vartheta)$ of Igam-Yarmukh[Nucl. Phys.(2007)] to that taken from NACRE [Nucl.Phys.(1999)] (solid line) and to that taken from C.R.Brune et al.[PhR. C50(1994)] (dashed line).

C. Angulo, et al. Nucl.Phys. A657(1999)[NACRE];S.B. Dubovichenko Phys. Atom. Nucl. 73 (2010)1526 and E.M. Tursunov et al. Phys.Rev. C 97 (2018) 035802 $\Longrightarrow$ $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ putting $Z_{Aa} = 1$(???). [Q.I. Tursunmahatov, R. Yarmukhamedov, Phys.Rev.C 85(2012)045807]



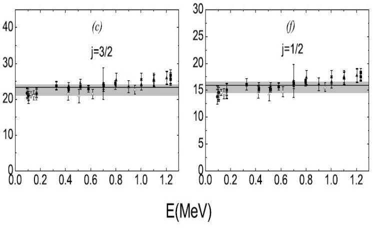


[Q.I. Tursunmahatov, R. Yarmukhamedov, Phys.Rev.C 85(2012)045807



Q. I. Tursunmahatov and R. Yarmukhamedov, Phys.Rev. C 85(2012)045807

The ANCs for $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}$ from the ${}^3\mathrm{He}(\alpha,\gamma){}^7\mathrm{Be}$ analysis



$C^2_{{}^3\mathrm{He}\,\alpha;g.s.}$=21.3±0.3 and 24.1±0.2 fm^{-1} for Sets I and II, respectively.

$C^2_{{}^3\mathrm{He}\,\alpha;exc}$=14.6±0.2 and 16.3±0.2 fm^{-1} for Sets I and II, respectively.

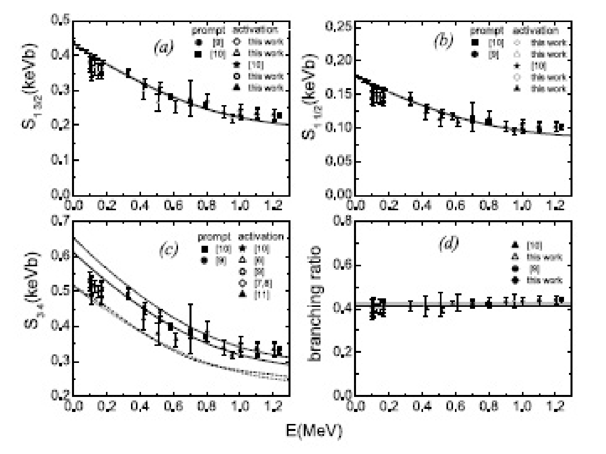
$C^2_{{}^3\mathrm{He}\,\alpha;g.s.}=23.3^{+1.0}_{-2.3}$ fm^{-1} and $C^2_{{}^3\mathrm{He}\,\alpha;exc.} = 15.9^{+0.6}_{-1.5}$ for Set I + Set II.

$C^2_{{}^3\mathrm{He}\,\alpha;g.s.}$=21.3±0.3 fm^{-1} [1]; $C^2_{{}^3\mathrm{He}\,\alpha;g.s.}$=21.3±0.3 fm^{-1} [1]

$Z_{{}^3\mathrm{He}\alpha;\,g.s.}/Z_{t\alpha;\,g.s.}=1.34^{+0.13}_{-0.18}$ and $Z_{{}^3\mathrm{He}\alpha;\,g.s.}/Z_{t\alpha;exc.}=1.26^{+0.16}_{-0.19}$

1. R. Yarmukhamedov and D. Baye, Phys.Rev.C 84(2011)024603

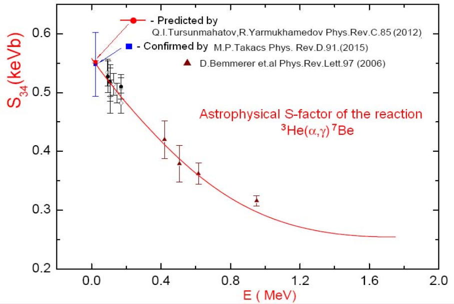
The astrophysical S factors for $^{3}\mathrm{He}(\alpha,\gamma)^{7}\mathrm{Be}$. In (c):Set I and Set II.



$S(0)$=0.562±0.008 keV·b and S(23 keV)=0.552±0.008 keV·b for Set I
$S(0)$=0.628±0.006 keV·b and S(23 keV)=0.619±0.008 keV·b for Set II
$S(0)=0.613^{+0.026}_{-0.063}$ keV·b and $S(23\ \mathrm{keV})=0.601^{+0.030}_{-0.072}$ keV·b for Sets I+II

The astrophysical S factors for $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ for Set I

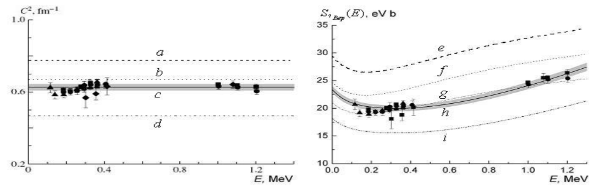
S(23 keV)=0.552±0.008 keV·b for Set I for Set I predicted by Q. I. Tursunmahatov and R. Yarmukhamedov[Phys.Rev. C 85(2012)045807]



$S(23^{+6}_{-5}$ keV)=0.548±0.054 keV·b confirmed by Marcell P. Taräcs et al.[Phys.Rev. D 91(2015)123526]

S.B. Igamov and R. Yarmukhamedov, Phys.Atom.Nucl.71(2008)1740[1]

$^7\mathrm{Be} + p \to {}^8\mathrm{B}$; $S(E)$ for $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$;



The ANC: C^2=0.626±0.017(c) [1] and 0.613±0.060 [2] fm^{-1};

(e) and (f): S(0)=23.4(0.6)(h) [1];22.8(2.2)[2];29.45(e) and 24.65 (f) [3]; 20.8(1.6)(g)[4] and 18.2(1.8)(i) [5] eV·b

Exp. data: LUNA COLLABORATION (2003); PhRev. C67 (2003)

[2]. O. Tojiboev et al. Phys.Rev.C 94(2016)054616

[3]. P.Descouvemont,PR(2004)

[4]. E.G. Adelberger et al., Rev .Mod.Ph. (2011)

[5]. G. Tabacaru et al.Ph .Rev. C73(2006).

The FIRST CONCLUSION

If a radiative capture $A(a, \gamma)B$ is peripheral and the astrophysical S factor ($S^{\rm exp}(E)$) is accurately measured, then the MTBPM makes it possible

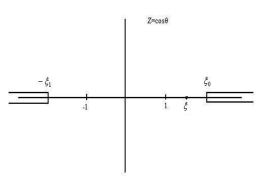
- to determine the ANC for $A + a \rightarrow B$ with a high precision
- combination of the dispersion theory with the effective-range expansion it makes it possible to determine the ANC for $A + a \rightarrow B$ corresponding to near subthreshold states of the B nucleus from the phase shift analysis for Aa scattering at low energies
- the ANC value obtained for $A + a \rightarrow B$ can be used for extrapolating $S^{\rm exp}(E)$ with a high precision at solar energy region ($0 \leq E \lesssim 25$ keV)

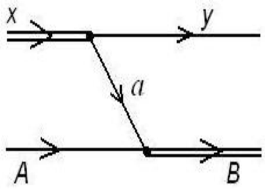
2. Asymptotic theory for the peripheral sub- and above-barrier charged particle transfer reaction

$A(x, y)B[x = (y + a), \ \ B = (A + a)]$

The main idea:

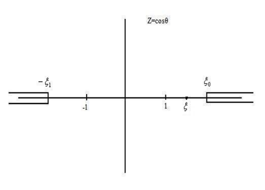
- the peripheral $A(x, y)B$ reaction is governed by the nearest singularity (ξ) of the reaction amplitude ($M(E_i, \cos\theta)$) at $\cos\theta = \xi > 1$
- the dominant pole played by this nearest singularity is the result of the peripheral nature of the considered reaction at least in the main peak of the angular distribution $Z \sim 1$.

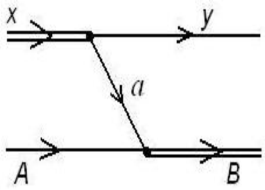




The main idea:

- the peripheral $A(x,y)B$ reaction is governed by the nearest singularity (ξ) of the reaction amplitude ($M(E_i, \cos\theta)$) at $\cos\theta = \xi > 1$
- the dominant pole played by this nearest singularity is the result of the peripheral nature of the considered reaction at least in the main peak of the angular distribution

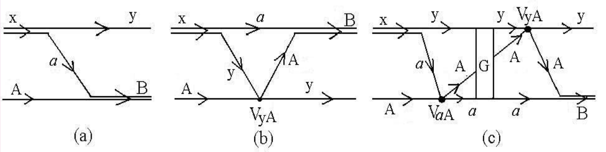




Within the strict three-body model the amplitude has the form as

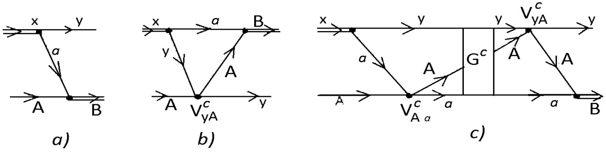
$$M^{TB} = <\chi_{k_f}^{(-)}(\mathbf{r}_f) I_{Aa}(\mathbf{r}_{Aa}) \mid V^{TB} \mid I_{ay}(\mathbf{r}_{ay}) \chi_{k_i}^{(+)}(\mathbf{r}_i)>,$$

where $V^{TB} = \Delta V_f + \Delta V_i G \Delta V_f = \Delta V_i + \Delta V_f G \Delta V_i$ is the three-body (A, a and y) transition operator; G is the operator of the three-body (A, a and y) Green function and $\Delta V_f = V_{ay} + V_{Ay} - V_f$, $\Delta V_i = V_{Aa} + V_{Ay} - V_i$



$$V^{TB} \Rightarrow V_{\rm post}^{TBAT} = V_{ay}^{N} + \Delta V_f^C + \Delta V_i^C G_C \Delta V_f^C, \;\; \Delta V_f^C = V_{ay}^C + V_{yA}^C - V_f^C,$$

$$\Delta V_f \approx V_{ay}^N + \Delta V_f^C + \Delta V_f^C G^C \Delta V_i^C;\ \Delta V_f^C = V_{ay}^C + V_{Ay}^C - V_f^C;$$



$$M_{l;\,pole}^{\rm DWBA}(E_i) \simeq \pi^{1/2}\frac{m_a}{k_i k_f}\frac{G_{Aa}G_{ay}}{(\tau^2-1)^{1/2}}\tilde N_{pole}^{\rm DWBA}\frac{e^{-l\ln\tau}}{l^{1/2+\eta_\alpha+\eta_\beta-i(\eta_i+\eta_f)}}, \qquad l \gg 1,$$

$$M_{l;\,post}^{\rm DWBA}(E_i) \simeq \pi^{1/2}\frac{m_f}{k_i k_f}\frac{G_{Aa}G_{ay}}{(\tau^2-1)^{1/2}}\tilde N_{post}^{\rm DWBA}\frac{e^{-l\ln\tau}}{l^{1/2+\eta_\alpha+\eta_\beta-i(\eta_i+\eta_f)}}, \qquad l \gg 1,$$

$$M_{l}^{\rm TB}(E_i) \simeq \pi^{1/2}\frac{m_f}{k_i k_f}\frac{G_{Aa}G_{ay}}{(\tau^2-1)^{1/2}}\tilde N^{\rm TB}\frac{e^{-l\ln\tau}}{l^{1/2+\eta_\alpha+\eta_\beta-i(\eta_i+\eta_f)}}, \qquad l \gg 1,$$

$\tau = \xi + (\xi^2-1)^{1/2} > 1.$ $M_l^{\rm TB}(E_i) = \tilde{\mathcal{R}}^{\rm TB} M_{l;\,pole}^{\rm DWBA}(E_i)$ at $l \gg 1$.

$$\tilde{\mathcal{R}}^{\rm TB} = \tilde N^{\rm TB}/\tilde N_{pole}^{\rm DWBA}.$$

$$M^{(s)\,\mathrm{TB}}(E_i,\,\cos\theta) = \mathcal{R}^{\mathrm{TB}} M^{(s)\,\mathrm{DWBA}}_{\mathrm{pole}}(E_i,\,\cos\theta).$$

$$M^{\mathrm{DWBA}}_{\mathrm{pole}}(E_i,\,cos\theta) = \int d\mathbf{r}_i d\mathbf{r}_f \chi^{(-)^*}_{\mathbf{k}_f}(\mathbf{r}_f) I^*_{Aa}(\mathbf{r}_{Aa}) V_{ay}(\mathbf{r}_{ay}) I_{ay}(\mathbf{r}_{ay}) \chi^{(+)}_{\mathbf{k}_i}(\mathbf{r}_i)$$

$\mathbf{r}_i \equiv \mathbf{r}_{xA}$, $\mathbf{r}_f \equiv \mathbf{r}_{yB}$, $\mathbf{r}_{ay} = \bar{a}\mathbf{r}_i - \bar{b}\mathbf{r}_f$, $\mathbf{r}_{Aa} = -\bar{c}\mathbf{r}_i + \bar{d}\mathbf{r}_f$,
$\bar{a} = \mu_{Ax}/m_a$, $\bar{b} = \mu_{Ax}/\mu_{Aa}$, $\bar{c} = \mu_{By}/\mu_{ay}$ and $\bar{d} = \mu_{By}/m_a$.

$$M^{\mathrm{DWBA}}_{\mathrm{pole}}(E_i,\,cos\theta) \Longrightarrow M^{(s)\,\mathrm{DWBA}}_{\mathrm{pole}}(E_i,\,\cos\theta)$$

$$M_{\rm pole}^{\rm DWBA}(E_i,\,cos\theta)\,=\,\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{d\mathbf{k}'}{(2\pi)^3}\chi_{\mathbf{k}_f}^{(+)}(\mathbf{k}')\mathcal{M}_{\rm pole}^{\rm DWBA}(\mathbf{k}',\,\mathbf{k})\chi_{\mathbf{k}_i}^{(+)}(\mathbf{k}),$$

$$\mathcal{M}_{\rm pole}^{\rm DWBA}(\mathbf{k}',\,\mathbf{k})\,=\,\sum_{M_a}\langle\mathbf{k}',\,I_{Aa}(\mathbf{q}_{Aa})|V_{ay}(\mathbf{q}_{ay})|I_{ay}(\mathbf{q}_{ay}),\,\mathbf{k}\rangle$$

$$=\,-\sum_{M_a}\frac{M_{ay}(\mathbf{q}_{ay})M_{Aa}^{*}(\mathbf{q}_{Aa})}{\frac{q_{Aa}^2}{2\mu_{Aa}}\,+\,\varepsilon_{Aa}}$$

$M_{ay}(\mathbf{q}_{ay})$ and $M_{Aa}(\mathbf{q}_{Aa})$ are the matrix elements (the amplitudes) for the virtual decays for $x\,\rightarrow\,y\,+\,a$ and for $B\,\rightarrow\,A\,+\,a$, respectively.

the partial expansion

$$\mathcal{M}^{\rm DWBA}_{\rm pole}(\mathbf{k}',\,\mathbf{k}) \Rightarrow \Longrightarrow \tilde{\mathcal{M}}^{\rm DWBA}_{{\rm pole};\,\alpha_B\alpha_x}(\mathbf{k}',\,\mathbf{k}) = I^*_{Aa;\,\alpha_B}(\mathbf{q}_{Aa})W_{ay;\,\alpha_x}(\mathbf{q}_{ay})$$

$$I_{Aa;\,\alpha_B}(\mathbf{q}_{Aa}) = -2\mu_{Aa}\frac{W_{Aa;\,\alpha_B}(\mathbf{q}_{ya})}{q^2_{Aa}+\kappa^2_{Aa}},$$

and

$$W_{Aa;\,\alpha_B}(\mathbf{q}_{Aa}) = \sqrt{4\pi}G_{Aa;\,l_Bj_B}(q_{Aa})Y_{l_B\mu_B}(\hat{\mathbf{q}}_{Aa}),$$

$$W_{ay;\,\alpha_x}(\mathbf{q}_{ay}) = \sqrt{4\pi}G_{ay;\,l_xj_x}(q_{ay})Y_{l_x\mu_x}(\hat{\mathbf{q}}_{ay})$$

are the reduced vertex functions for the virtual decays $B \rightarrow A + a$ and $x \rightarrow y + a$, respectively.

$$W^{\rm (C)}_{\beta\gamma;\,\alpha_\alpha}(\mathbf{q}_{\beta\gamma}) \simeq W^{({\rm C};\,s)}_{\beta\gamma;\,\alpha_\alpha}(\mathbf{q}_{\beta\gamma}) \Rightarrow \left(\frac{\mathbf{q}^2_{\beta\gamma}+\kappa^2_{\beta\gamma}}{4i\kappa^2_{\beta\gamma}}\right)^{\eta_{\beta\gamma}} G_{\beta\gamma;\,l_\alpha j_\alpha}Y_{l_\alpha\nu_\alpha}(\hat{\mathbf{q}}_{\beta\gamma})$$

$$M_{\rm pole}^{\rm DWBA}(E_i,\, cos\theta) \simeq\; M_{\rm pole}^{(s)\,{\rm DWBA}}(E_i,\, cos\theta)$$

$$\Longrightarrow \int d\mathbf{r}_i d\mathbf{r}_f \Psi_{\mathbf{k}_f}^{*(-)}(\mathbf{r}_f) I_{Aa;\,\alpha}^{*(as)}(\mathbf{r}_{Aa}) W_{ay;\,\alpha_x}^{(as)}(\mathbf{r}_{ay}) \Psi_{\mathbf{k}_i}^{(+)}(\mathbf{r}_i).$$

$$W_{ay;\,\alpha_x}^{(as)}(\mathbf{r}_{ay}) = -\frac{\sqrt{2}\eta_{ay}}{\pi} G_{ay;\,l_x j_x} \left(\frac{\kappa_{ay}}{r_{ay}}\right)^{3/2} \frac{K_{l_x+3/2+\eta_{ay}}(\kappa_{ay} r_{ay})}{(2i\kappa_{ay} r_{ay})^{\eta_{ay}}} i^{-l_x} Y_{l_x \nu_x}(\hat{\mathbf{r}}_{ay})$$

$$\approx\; V_{ay}^{C}(r_{ay}) I_{ay;\,\alpha_x}^{(as)}(r_{ay}) Y_{l_x \nu_x}(\hat{\mathbf{r}}_{ay}),$$

for $r_{ya} \gtrsim R_x$ and

$$I_{Aa;\,\alpha_B}^{*(as)}(\mathbf{r}_{Aa}) \;=\; -\frac{\sqrt{2}}{\pi} G_{Aa;\,l_B j_B} \left(\frac{\mu_{Aa}^2 \kappa_{Aa}}{r_{Aa}}\right)^{1/2} \frac{K_{l_B+1/2+\eta_{Aa}}(\kappa_{Aa} r_{Aa})}{(2i\kappa_{Aa} r_{Aa})^{\eta_{Aa}}} i^{-l_B} Y_{l_B \nu_B}^{*}(\hat{\mathbf{r}}_{Aa}$$

$$\approx C_{Aa\,\alpha_B} \frac{\exp\{-\kappa_{Aa} r_{Aa} - \eta_{Aa} \ln{(2\kappa_{Aa} r_{Aa})}\}}{r_{Aa}} Y_{l_B \nu_B}^{*}(\hat{\mathbf{r}}_{Aa}),$$

for $r_{Aa} \gtrsim R_B$.

$$M_{\rm pole}^{\rm DWBA}(E_i,\, cos\theta) \Longrightarrow \sum_{J,\,M}\sum_{l_i l_f} ...A^{\rm pole}_{JMl_x l_B l_i l_f}\,Y_{l_f M}(\theta,0)$$

$$A^{\rm pole}_{JMl_x l_B l_i l_f} \Longrightarrow \mathcal{B}^{\rm pole}_{l_x l_B l_i l_f \lambda_1 \sigma_1}(k_i,\, k_f) \Longrightarrow$$

$$\int_{R_i^{\rm ch}}^{\infty} dr_i r_i^{\lambda_1+\sigma_1+1}\Psi_{l_i}({\rm r}_i;\, k_i)\int_{R_f^{\rm ch}}^{\infty} dr_f r_f^{\lambda_2+\sigma_2+1}\Psi_{l_f}({\rm r}_f;\, k_f)\tilde{\mathcal{A}}_{l_B l_x l}(r_i,\, r_f),$$

$$\mathcal{B}^{\rm pole}_{l_x l_B l_i l_f \lambda_1 \sigma_1}(k_i, k_f) \Longrightarrow \tilde{\mathcal{B}}^{\rm TB}_{l_x l_B l_i l_f \lambda_1 \sigma_1}(k_i, k_f) = \mathcal{N}^{\rm TB}_{l_i l_f}(E_i)\mathcal{B}^{\rm pole}_{l_x l_B l_i l_f \lambda_1 \sigma_1}(k_i, k_f),$$

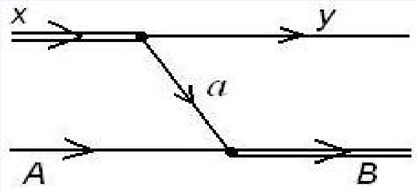
$$\mathcal{N}^{\rm TB}_{l_i l_f}(E_i) = \begin{cases} 1, & \text{for } l_i, < L_0 \text{ and } l_f < L_0; \\ \tilde{\mathcal{R}}^{\rm TB}(E_i), & \text{for } l_i \geq L_0,\ l_f \geq L_0, \end{cases}$$

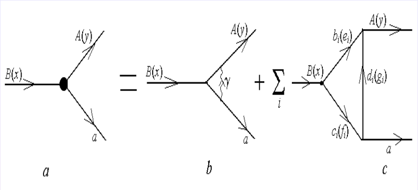
where $L_0 \sim k_i R_i^{\rm ch}$ (or $\sim k_f R_f^{\rm ch}$), $R_i^{\rm ch} = R_A + R_x$ and $R_f^{\rm ch} = R_B + R_y$.

$$\frac{d\sigma}{d\Omega} = C^2_{Aa}C^2_{ay}\tilde{\sigma}^{\rm TB}(E_i,\, \cos\theta;\, R_i^{\rm ch},\, R_f^{\rm ch}) \Longrightarrow d\sigma^{\rm exp}/d\Omega, \ \text{at } \theta \sim \theta_{\rm peak}.$$

RESULTS OF APPLICATIONS TO THE SPECIFIC SUB- AND ABOVE-BARRIER REACTIONS

- $^{9}\mathrm{Be}(^{10}\mathrm{B},^{9}\mathrm{Be})^{10}\mathrm{B}$ at $E_{^{10}\mathrm{B}}$= 100 MeV [A. M. Mukhamedzhanov et al. Phys. Rev.C 56(1997)1302] $\Longrightarrow$ ANC for $^{9}\mathrm{Be}+p\rightarrow\ ^{10}\mathrm{B}$ $\Longrightarrow {}^{9}\mathrm{Be}(p,\gamma)^{10}\mathrm{B}$
- $^{11}\mathrm{B}^{12}\mathrm{C},^{11}\mathrm{B})^{12}\mathrm{C}$ at $E_{^{12}\mathrm{C}}$= 87 MeV [R. M. DaVries. Phys. Rev.C 8(1973)951] $\Longrightarrow$ ANC for $^{10}\mathrm{B}+p\rightarrow\ ^{12}\mathrm{C}\Longrightarrow\ ^{11}\mathrm{B}(p,\gamma)^{12}\mathrm{C}$
- $^{16}\mathrm{O}(^{3}\mathrm{He},d)^{17}\mathrm{F}$ at $E_{^{3}\mathrm{He}}$= 29.75 MeV; [C. A. Gagliardi at al. Phys. Rev.C 59(1999)1149]$\Longrightarrow$ ANC for $^{16}\mathrm{O}+p\rightarrow\ ^{17}\mathrm{F}\Longrightarrow\ ^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$
- $^{19}\mathrm{F}(p,\ \alpha)^{16}\mathrm{O}$ at sub-barrier energies E_p= 250; 350 and 450 keV; [H. Herndl. Phys. Rev.C 44(1991)R952]$\Longrightarrow$ ANC for $^{16}\mathrm{O}+t\rightarrow\ ^{19}\mathrm{F}$ $\Longrightarrow$ the total cross sections

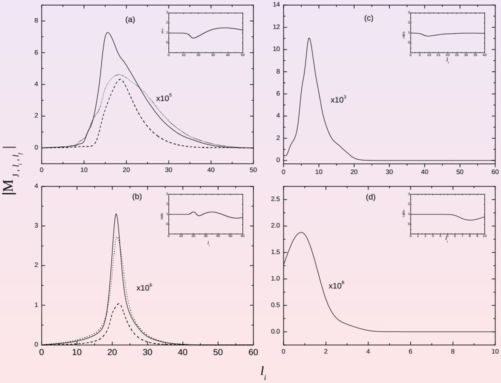




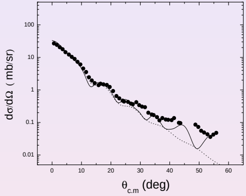
Reaction $A(x,y)B$	$E_x^{\rm lab}$ MeV	The vertex $B \to A + a$ $(x \to y + a)$	ξ $(\kappa,\ {\rm fm}^{-1})$	b_i (e_i)	c_i (f_i)	d_i (g_i)	$\kappa_i(\bar\kappa_i)$, ${\rm fm}^{-1}$	ξ_i $(\bar\xi_i)$
$^9{\rm Be}(^{10}{\rm B},{}^9{\rm Be})^{10}{\rm B}$	100	$^{10}{\rm B} \to {}^9{\rm Be} + p$	1.020(0.534)	$^8{\rm Be}$	d	n	0.940	1.064
				$^6{\rm Li}$	$^4{\rm He}$	t	2.024	1.479
				n	$^9{\rm Be}$	$^8{\rm Be}$	0.802	4.169
$^{11}{\rm B}(^{12}{\rm C},{}^{11}{\rm B})^{12}{\rm C}$	87	$^{12}{\rm C} \to {}^{11}{\rm B} + p$	1.037(0.840)	$^{10}{\rm B}$	d	n	2.131	1.264
				$^8{\rm Be}$	$^4{\rm He}$	t	2.059	1.384
				n	$^{11}{\rm C}$	$^{10}{\rm B}$	1.618	16.020
$^{16}{\rm O}(^3{\rm He},d)^{17}{\rm F}({\rm g.s.})$	29.7	$^{17}{\rm F} \to {}^{16}{\rm O} + p$	1.065(0.165)	$^{14}{\rm N}$	$^3{\rm He}$	d	2.696	3.253
				$^{13}{\rm N}$	$^4{\rm He}$	t	2.645	3.508
				p	$^{16}{\rm O}$	$^{15}{\rm N}$	0.905	49.551
		$(^3{\rm He} \to d + p)$	1.065(0.420)	(p)	(d)	(n)	(0.652)	(1.562)
$^{19}{\rm F}(p,\alpha)^{16}{\rm O}$	0.250	$^{19}{\rm F} \to {}^{16}{\rm O} + t$	13.648(1.194)	$^{15}{\rm N}$	α	d	1.522	19.720
	0.350		11.544(1.194)				1.522	16.647
	0.450		10.190(1.194)				1.522	14.665

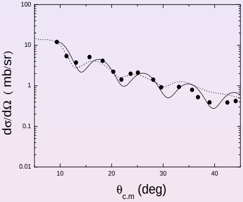
$A(x, y)B$	E_x, MeV	$\tilde{N}^{\rm DWBA}_{\rm pole}$ ($\tilde{N}^{\rm DWBA}_{\rm post}$)	$\tilde{N}^{\rm TB}$	$\tilde{\mathcal{R}}^{\rm TB}$ ($\tilde{\mathcal{R}}^{\rm TB}_{\rm post}$) [$\tilde{\mathcal{R}}^{\rm DWBA}_{\rm post}$]
^{9}Be(^{10}B,^{9} Be)^{10}B	100	0.339 - i·2.664 (0.5154 - i·4.0530)	-4.117	-0.193 - i·1.521 (-0.1270 - i·0.9995) [1.521 + i·1.300x10^{-15}]
^{11}B(^{12}C,^{11} B)^{12}C	87	-0.911 + i·0.835 (-1.260 + i·1.154)	-1.714	1.023 + i·0.937 (0.7399 + i·0.6777) [1.382 + i·5.200x10^{-16}]
^{16}O(^{3}He, d)^{17}F(g.s.)	29.75	261.48 + i·435.04 (279.68 + i·465.32)	-590.36	-0.599 + i·0.996 (-0560 + i·0.932) [1.069 - i·1.600x10^{-15}]
^{16}O(^{3}He, d)^{17}F(0.429 MeV)		(-2.96 - i·4.75)x10^{15} ((-0.725 - i·1.160)x10^{15})	-1.33x10^{9}	(1.26 - i·2.05)x10^{-7} ((5.14 - i·8.26)x10^{-7}) [0.245 - i·1.300x10^{-11}]
^{19}F(p, α)^{16}O	0.250	(-1.360 + i·0.453)x10^{-03} ((-0.148 + i·4.940)x10^{-4})	-1.68x10^{-3}	1.112 + i·0.370 (1.023 + i·0.340) [1.088 - i·7.150x10^{-18}]
	0.350	(-3.20 + i·1.14)x10^{-3} ((-3.480 - i·1.240)x10^{-3})	-3.98x10^{-3}	1.104+ i·0.394 (1.014 + i·0.361) [1.088 - i·6.005x10^{-18}]
	0.450	(-5.41 + i·2.04)x10^{-3} ((-5.893 - i·2.217)x10^{-3})	-6.78x10^{-3}	1.097+ i·0.412 (1.008 + i·0.379) [1.088]

$^{9}\mathrm{Be}(^{10}\mathrm{B},^{9}\mathrm{Be})^{10}\mathrm{B}$ at $E_{^{10}\mathrm{B}}$= 100 MeV (*a*)
$^{11}\mathrm{B}^{12}\mathrm{C},^{11}\mathrm{B})^{12}\mathrm{C}$ at $E_{^{12}\mathrm{C}}$= 87 MeV(*b*)
$^{16}\mathrm{O}(^{3}\mathrm{He},d)^{17}\mathrm{F(g.s.)}$ at $E_{^{3}\mathrm{He}}$= 29.75 MeV(*c*)
$^{19}\mathrm{F}(p,\alpha)^{16}\mathrm{O}$ at E_p= 250 keV (*d*)



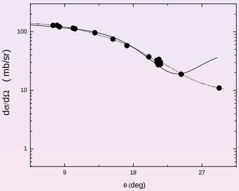
$^{9}\mathrm{Be}(^{10}\mathrm{B},^{9}\mathrm{Be})^{10}\mathrm{B}$ at $E_{^{10}\mathrm{B}}$= 100 MeV (left) and $^{11}\mathrm{B}^{12}\mathrm{C},^{11}\mathrm{B})^{12}\mathrm{C}$ at $E_{^{12}\mathrm{C}}$= 87 MeV(right)

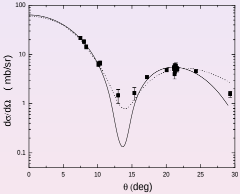




$C^2_{p^9Be}$=4.35±0.19 fm^{-1}[$S_{dir}(0)$=0.17±0.01 keV·b] (the present work); 5.06±0.46 fm^{-1} [$S_{dir}(0)$=0.21±0.02 keV·b; (A. M. Mukhamedzhanov, et al., Phys.Rev. C 56(1997)1302]. $C^2_{p^{11}B}$=311.6±13.3 fm^{-1} [$S_{dir}(0)$=0.19±0.01 keVb] (the present work).

$^{16}\mathrm{O}(^{3}\mathrm{He}, d)^{17}\mathrm{F}$ at $E_{^3\mathrm{He}}$= 29.75 MeV: $^{17}\mathrm{F}$(g.s.)(left) and $^{17}\mathrm{F}$(0.429 MeV)(right)

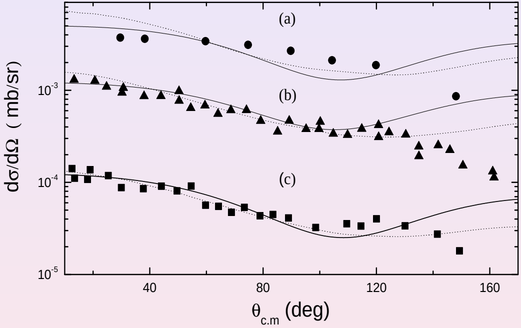




$C^2_{p^{16}O;g.s.}$=1.21±0.12(our work);1.08±0.10[1];1.09±0.11[2];1.09 fm^{-1} [3]
$C^2_{p^{16}O;exc}$=6216±632(our work);6490±680 [1];5700±225[2];5700 fm^{-1}[3]
$S_{dir;\,g.s.}(0)$=0.44±0.04(our work);0.40±0.04 [1] and 0.40±0.04 [2]
$S_{dir;\,exc}(0)$=9.89±1.01(our work);9.8±1.0 [1] and 9.45±1.0 [2]
$S_{tot}(0)$=10.34±1.06(our work);10.2±1.0 [1] and 9.85±1.00 [2]

[1]. C.A. Gagliardi, et al., Phys.Rev. C 59(1999)1149]
[2]. C.V. Artemov, et al., Bull.RAN 73(2009)165
the $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$ analysis
[3]. R. Yarmukhamedov, D. Baye, Phys.Rev. C 84 (2011)024603
the phase shift analysis for the $p^{16}\mathrm{O}$ scattering

$^{19}\mathrm{F}(p,\alpha)^{16}\mathrm{O}$ at sub-barrier energies E_p= 250 (c; 350 (b) and 450 (a) keV



$C^2_{t^{16}O}$=618.1±100.1 (c);605.0±70.3 (b) and 544.8±64.4 (a) fm^{-1} [583.5±46.2 fm^{-1} (w.m.)] $\Longrightarrow$ the total cross sections

The SECOND CONCLUSION

- Within the strong three-body (A, a and y) Schrödinger formalism combined with the dispersion method, a new asymptotic theory is proposed for the peripheral sub- and above-barrier charged-particle transfer $A(x, y)B$ reaction, where $x = y + a$, $B = A + a$ and a is the transferred particle.
- the DCS is directly parametrized in the terms of the product of the squared ANCs for $y + a \to x$ and $A + a \to B$ being adequate to physics of the surface reaction.
- The proposed asymptotic theory can be considered as a generalization of the "post"-approximation and/or the "post" form of the DWBA, where the contribution of the three-body (A, a and y) Coulomb dynamics of the transfer mechanism to the reaction amplitude is correctly taken into account in all others of the parameter of the perturbation theory over $V^C_{i,f}$

- the asymptotic theory has been applied to the analysis of the experimental differential cross sections of the specific above- and sub-barrier peripheral reactions corresponding to the proton and triton transfer mechanisms, respectively.
- it is demonstrated that it gives an adequate description of both angular distributions in the corresponding main peaks of the angular distributions and the absolute values of the specific ANCs (NVCs).
- the ANCs were also applied to calculations of the specific nuclear-astrophysical radiative proton capture reactions and the new values of the astrophysical S factors extrapolated at stellar energies were obtained.

THANKS FOR YOUR ATTENTION !!!