Magnetic fields of relativistic magnetized compact stars in the braneworld

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Outline

1. Introduction to compact stars

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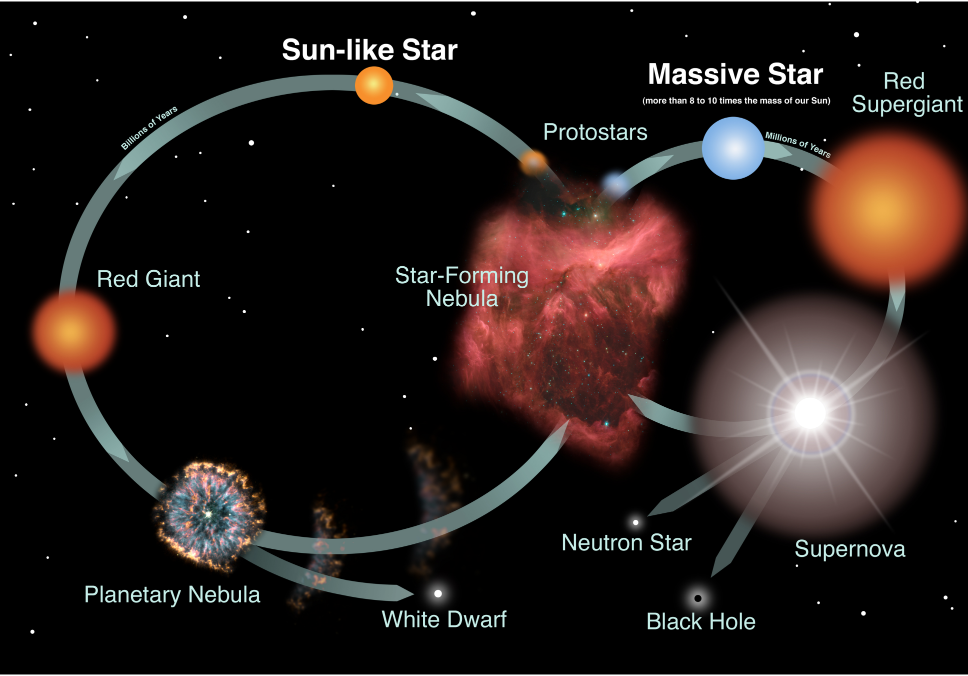
Sun-like Star
Massive Star
(more than 8 to 10 times the mass of our Sun)
Red Supergiant
Protostars
Billions of Years
Millions of Years
Red Giant
Star-Forming Nebula
Planetary Nebula
White Dwarf
Neutron Star
Supernova
Black Hole

Neutron Star

Mass: $M \simeq 1.1 - 3M_{\odot}$
Radius: $R \simeq 10\mathrm{km}$
Temperature: $T \simeq 10^9 - 10^{10}\mathrm{K}$
Magnetic field: $B \simeq 10^{10} - 10^{15}\mathrm{G}$
Due to conservation of angular momentum during collapse

$$\Omega R^2 = const \Rightarrow \Omega = \Omega_0 \left(R_0/R\right)^2 \propto 10^3 s^{-1}$$

Due to conservation of remnant mass during collapse

$$\rho R^3 = const \Rightarrow \rho = \rho_0 \left(R_0/R\right)^3 \propto 10^{15} g \cdot cm^{-3}$$

Compact gravitational object

$$R \approx 10km \ , \quad R_g = 2GM/c^2 \approx 5km \ , \left(1 - 2GM/c^2 r\right) \approx 0.5$$

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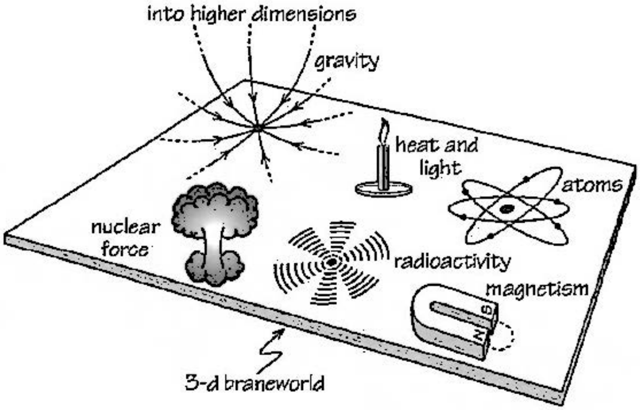
Braneworld


into higher dimensions
gravity
heat and
light
atoms
nuclear
force
radioactivity
magnetism
3-d braneworld

Spacetime of the star in the braneworld

- Gravitational field equations

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \kappa T_{\alpha\beta} ,$$

$$T_{\alpha\beta} = T_{(mat)\alpha\beta} + T_{(*)\alpha\beta} + T_{(em)\alpha\beta} ,$$

where $T_{(G)\alpha\beta}$ is a perfect fluid energy-momentum tensor

$$T_{(mat)\alpha\beta} = \rho u_\alpha u_\beta + p(g_{\alpha\beta} + u_\alpha u_\beta) ,$$

- The space–time metric for a rotating relativistic star is

$$ds^2 = -N^2 dt^2 + N^{-2} dr^2 - 2\omega(r) r^2 \sin^2\theta dt d\phi + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \ ,$$

The external vacuum solution of the field equations for outside the star (where $\rho = p = 0$) is given by [1]

$$N^2 = 1 - \frac{2M}{r} + \frac{Q^*}{r^2}, \ r \leq R \ , \tag{1}$$

where $\omega(r)$ is the Lense-Thirring angular velocity and outside the star is given by

$$\omega(r) \equiv \frac{d\phi}{dt} = -\frac{g_{0\phi}}{g_{\phi\phi}} = \frac{2J}{r^3}\left(1 - \frac{Q^*}{2Mr}\right) \ ,$$

$J = I(M,R)\Omega$ is the total angular momentum of metric source as measured from infinity and $I(M,R)$ its momentum of inertia.

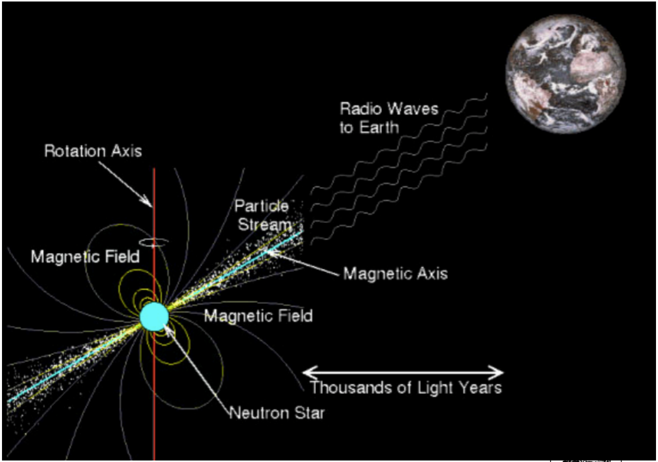
[1] N.Dadhich, et.al. *Phys. Lett. B*, **487**, 1 (2000)

Outline

Our approach

Star formation:

- Slowly rotation limit or linear approximation: $\mathcal{O}(\omega^2)$ and $\mathcal{O}(\Omega^2)$
- Stationary field: $\partial_t(\vec{E}, \vec{B}) = 0$
- Outside of NS is just vacuum
- Axial symmetric relativistic star



- General relativistic Maxwell equations

$$F_{[\alpha\beta,\gamma]} = 0\,, \qquad F^{\alpha\beta}{}_{;\beta} = 4\pi J^{\alpha}\ .$$

where

$$F_{\alpha\beta} \equiv 2u_{[\alpha}E_{\beta]} + \eta_{\alpha\beta\gamma\delta}u^{\gamma}B^{\delta}\ .$$

$F_{\alpha\beta}$ is the electromagnetic field tensor, J^{α} is the four–current, u^{α} is four–velocity of an observer and $\eta_{\alpha\beta\gamma\delta} = \sqrt{-g}\epsilon_{\alpha\beta\gamma\delta}$ is the pseudo-tensorial expression for the Levi-Civita symbol.

- Assuming the magnetic field to be dipolar we look for separable solutions of Maxwell equations in the form

$$
\begin{aligned}
B^{\hat{r}}(r,\theta,\phi,t) &= F(r)\left[\cos\chi\cos\theta+\sin\chi\sin\theta\cos\lambda\right],\\
B^{\hat{\theta}}(r,\theta,\phi,t) &= G(r)\left[\cos\chi\sin\theta-\sin\chi\cos\theta\cos\lambda\right],\\
B^{\hat{\phi}}(r,\theta,\phi,t) &= H(r)\sin\chi\sin\lambda, \qquad \lambda=\phi-\Omega t,
\end{aligned}
$$

- Maxwell equations for the radial part of the magnetic field as [2]

$$
(r^2F)_{,r}+2N^{-1}rG=0\,, \quad (NrG)_{,r}+F=0\ , \qquad H=G\ .
$$

[2] L.Rezzolla, B. Ahmedov, J. Miller, *Mon. Not. R. Astron. Soc.*, **322**, 723–740 (2001)

- Above equations can be expressed as a single, second order differential equation for the unknown function $F(r)$

$$\frac{d}{dr}\left[\left(1-\frac{2M}{r}+\frac{Q^*}{r^2}\right)\frac{d}{dr}\left(r^2F\right)\right]-2F=0\ ,$$

- The exact analytical form of the profile functions $F(r)$, $G(r)$ and $H(r)$ are given by following form

$$F(r)=-\frac{3\mu}{4\varrho^3M^3}\left[\frac{2\varrho M}{r}\left(1+\frac{M}{r}\right)+\left(1-\frac{Q^*}{r^2}\right)\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right]\ ,$$

$$G(r)=H(r)=\frac{3\mu}{4\varrho^3M^3}\left[\frac{2\varrho M}{rN}\left(1-\frac{M}{r}\right)+N\ln\left(\frac{r-M(1+\varrho)}{r-M(1-\varrho)}\right)\right]\ ,$$

where

$$\varrho=\left(1-\frac{Q^*}{M^2}\right)^{1/2}\ .$$

- In GR when $Q^* \to 0$ and $N^2 = 1 - 2M/r$ [3]

$$B^{\hat{r}} = -\frac{3}{4M^3}\left[\ln N^2 + \frac{2M}{r}\left(1 + \frac{M}{r}\right)\right](\cos\chi\cos\theta + \sin\chi\sin\theta\cos\lambda)\mu \ ,$$

$$B^{\hat{\theta}} = \frac{3N}{4M^2 r}\left[\frac{r}{M}\ln N^2 + \frac{1}{N^2} + 1\right](\cos\chi\sin\theta - \sin\chi\cos\theta\cos\lambda)\mu \ ,$$

$$B^{\hat{\phi}} = \frac{3N}{4M^2 r}\left[\frac{r}{M}\ln N^2 + \frac{1}{N^2} + 1\right](\sin\chi\sin\lambda)\mu \ .$$

- In the Newtonian approximation [4]

$$\lim_{M\to 0} F(r) = \frac{2\mu}{r^3} \ , \qquad \lim_{M\to 0} G(r) = \frac{\mu}{r^3} \ .$$

[3] L.Rezzolla, B. Ahmedov, J. Miller, *Mon. Not. R. Astron. Soc.*, **322**, 723–740 (2001)
[4] A. J. Deutsch, *Ann. Astrophys.*, **1**, 1 (1955)

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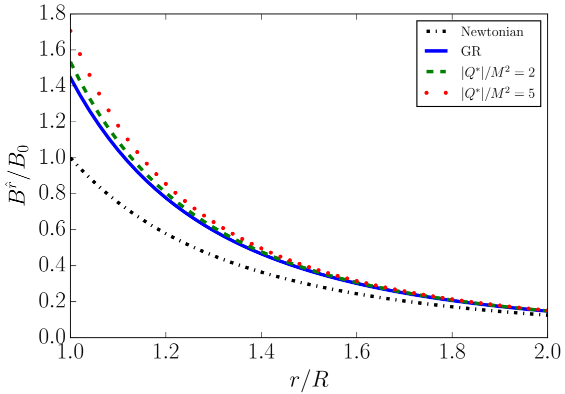
- The radial $\eta = r/R$ dependence of the normalized radial component of the magnetic field for several values of the brane parameter Q^*/M^2 when $\epsilon = 0.4$, $\chi = 0$ and $\theta = 0$, where $B_0 = 2\mu/R^3$.



- Assume that the oblique rotating magnetized star is observed as radio pulsar through magnetic dipole radiation. Then the luminosity of the relativistic star in the case of a purely dipolar radiation, and the power radiated in the form of dipolar electromagnetic radiation

$$L^*_{em} \equiv \int_{\partial\Sigma} P^{\hat{r}} dS = \frac{1}{4\pi} \int_{\partial\Sigma} \left(\vec{E} \times \vec{B}\right)^{\hat{r}} dS = \frac{\Omega^4_{_R} R^6 B^{*2}_R}{6c^3} \sin^2\chi \ ,$$

where the magnetic field amplification $B^*_R = F_{_R} B_0$ at the stellar surface and partly to the increase in the effective rotational angular velocity produced by the gravitational redshift as $\Omega = N_R \Omega_R$.

- The magnetic field strength at the surface of the star:

$$B_{\rm R}^* \quad = \quad f(\epsilon, Q^*) B_0 \ ,$$

with

$$f(\epsilon, Q^*) \quad = -\frac{3}{\varrho^3 \epsilon^3}\left[\varrho\epsilon\left(1+\frac{\epsilon}{2}\right)+\left(1-\frac{\epsilon^2(1-\varrho^2)}{4}\right)\ln\left(\frac{2-\epsilon(1+\varrho)}{2-\epsilon(1-\varrho)}\right)\right] \ ,$$

where $r = R, \quad B_0 = 2\mu/R^3, \quad \epsilon = 2M/R, \quad \chi = \theta = 0$

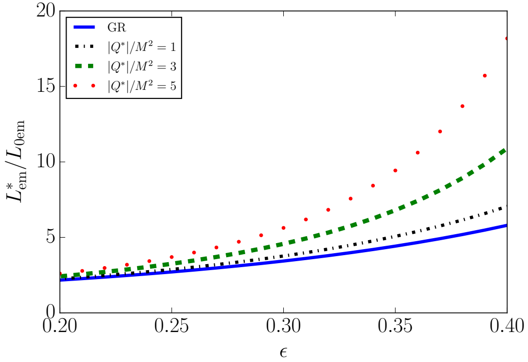
- When compared with the equivalent Newtonian expression for the rate of electromagnetic energy loss through dipolar radiation

$$(L_{em})_{\rm Newt} = \frac{\Omega^4 R^6 B_0^2}{6c^3} \sin^2\chi \ .$$

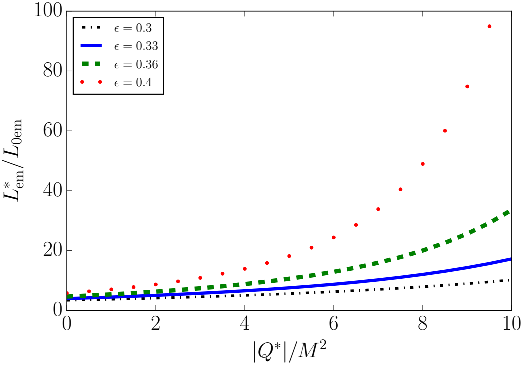
- In the presence of brane parameter the rate of energy loss through dipolar electromagnetic radiation is

$$\frac{L_{em}}{(L_{em})_{\rm Newt}} = \left(\frac{\Omega_R}{\Omega}\right)^4 \left(\frac{B_R^*}{B_0}\right)^2 = \left(\frac{f}{N_R^{*2}}\right)^2 .$$

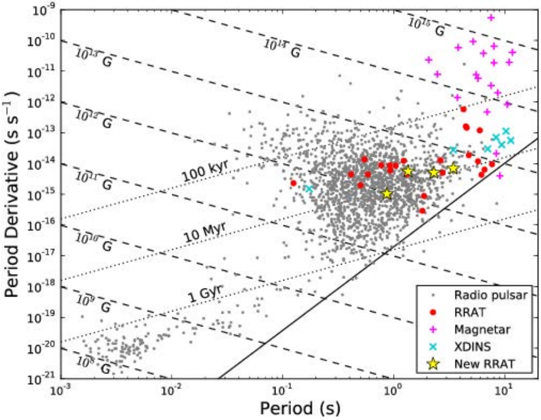
- The dependence of the energy losses $L^*_{\rm em}/L_{\rm 0em}$ from the compactness ϵ of the star for several value of the brane parameter Q^*/M^2.



- The dependence of the energy losses $L^*_{\rm em}/L_{\rm 0em}$ from the brane parameter Q^*/M^2 for several value of the compactness ϵ of the star.



- $P - \dot{P}$ diagram for the observable pulsars and magnetars from the paper [5].



For typical radio pulsar $B \simeq 10^{12}\mathrm{G}$, and $\dot{P} \simeq 10^{-15}\mathrm{ss}^{-1}$.

$$|Q^*| \lesssim 3 \times 10^{11}\mathrm{cm}^2 \ (|Q^*|/M^2 \simeq 8)$$

[5] C.Karako–Argaman, et-al, *Astrophys. J.*, **809**, 67 (2015)

Thank You

$T_{(*)\alpha\beta}$ is the energy-momentum tensor arises from extra dimension (braneworld)

$$\begin{aligned} T_{(*)\alpha\beta} &= \frac{1}{2\lambda^*}\rho^2 u_\alpha u_\beta + \frac{1}{2\lambda^*}\rho\left(\rho+2p\right)\left(g_{\alpha\beta}+u_\alpha u_\beta\right) \\ &+ \frac{1}{2\lambda^* k^4}\left[\mathcal{U}u_\alpha u_\beta + \mathcal{P}r_\alpha r_\beta + \left(\mathcal{U}-\mathcal{P}\right)\left(g_{\alpha\beta}+\frac{1}{3}u_\alpha u_\beta\right)\right] \ , \end{aligned}$$

where u^μ is the four-velocity of the medium, r^μ is a unit radial vector, $\rho(r)$ and $p(r)$ are matter energy density and pressure, respectively. The nonlocal bulk effects are carried by the nonlocal energy density $\mathcal{U}(r)$ and nonlocal pressure $\mathcal{P}(r)$. The quantity λ^* is the brane tension parameter and the standard general relativity is recovered in the case of when $\lambda^* \to \infty$.

$$T_{(em)\alpha\beta} = \frac{1}{4\pi}\left(F_{\alpha\mu}F^\mu_\beta - \frac{1}{4}g_{\alpha\beta}F_{\mu\nu}F^{\mu\nu}\right) \ ,$$

The energy-momentum tensor for the electromagnetic field which is very small ($T_{(em)\alpha\beta} \ll T_{(G)\alpha\beta} + T_{(*)\alpha\beta}$) with compare to other two terms of the total energy-momentum tensor.