How to restore Cosmic Censorship Conjecture: Can a test magnetic field serve as a cosmic censor?

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Motivation
Talk outline

1. Motivation

2. Violation of cosmic censorship conjecture
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3. The effect of a high dimension and a test magnetic field on the CCC
1 Motivation

2 Violation of cosmic censorship conjecture

3 The effect of a high dimension and a test magnetic field on the CCC

4 Conclusion
Talk outline

1 Motivation

2 Violation of cosmic censorship conjecture

3 The effect of a high dimension and a test magnetic field on the CCC

4 Conclusion
The Lifecycle of Stars

Sun-Like Stars (up to 1.5 times the mass of the Sun) → Red Giant → Planetary Nebula → White Dwarf → Black Dwarf

Stellar Nursery

Stars form in a nebula, from collapsing clouds of interstellar gas and dust.

Huge Stars (from 1.5 to 3 times the mass of the Sun) → Red Supergiant → Supernova → Neutron Star

Giant Stars (over 3 times the mass of the Sun) → Red Supergiant → Supernova → Black Hole
According to the Einstein gravity black holes is formed as a result of gravitational collapse. Later, Penrose proposed in 1969 that a singularity would always be hidden behind a horizon. This is called CCC. However, there exists no proof of CCC either way, true or false, and it remains as one of the most important open questions.
Motivation

However, black holes have been still candidate BHs. The cosmic censorship conjecture have remained unproven yet. If so, there appears new hypothesis, according to which the physical possibility of destroying the event horizon of BHs seems to be valid due to over-spinning or over-charging process.

T. Jacobson & T.P. Sotiriou, Examined the physical possibility of over-spinning a black hole with an infalling test particle, PRL, 2009

Approaching the issue from different prospectives

- R. Wald, Ann. Phys. 82, 548 (1974) – The dynamics of overspinning process was examined by Wald for the first time.
The external magnetic field

Typically, in the astrophysical scenarios, black holes are surrounded with the magnetic field, which affects the motion of the surrounding electrically charged matter or particle. Then, the effect of an external magnetic field can be tested as an alternative tool to restore the cosmic censorship in the astrophysical context.

Five dimensional spacetime

Increasing the dimension order of spacetime can be also tested to restore the cosmic censorship conjecture.

Motivation

2. Violation of cosmic censorship conjecture

3. The effect of a high dimension and a test magnetic field on the CCC

4. Conclusion
The metric of the Kerr geometry in the Boyer-Lindquist coordinates is given by

\[ ds^2 = - \left( \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} \right) dt^2 - \frac{2a \sin^2 \theta (r^2 + a^2 - \Delta)}{\Sigma} dt d\phi \\
+ \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \sin^2 \theta d\phi^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2, \tag{1} \]

where \( \Sigma = r^2 + a^2 \cos^2 \theta \) and \( \Delta = r^2 + a^2 - 2Mr \) with parameters \( M \) and \( a \). 

Black hole horizon

\[ r_h = M + \sqrt{M^2 - a^2}, \tag{2} \]

exists only \( M \geq a \) and black hole for which \( a = M \) is referred to an extremal Kerr black hole.

To overspin/destroy black hole, the following condition must be satisfied for the Kerr BH

\[ M^2 < a^2 \tag{3} \]
Dynamics of overspinning process

There are two constants of motion associated with the particle $\delta E$ and $\delta J$, which are $\delta E \ll M$, $\delta J \ll J$ for test particle approximation. After falling particle into the BH, the final parameters are given by $M + \delta E$ and $J + \delta J$, respectively. One must start out with the near-extremal black hole $J/M^2 = a/M = 1 - 2\epsilon^2$, with $\epsilon \ll 1$ being a small dimensionless parameter.

\[
(M + \delta E)^2 < \left( \frac{J + \delta J}{M + \delta E} \right)^2 .
\]  

(4)

This yields the lower bound on the angular momentum $\delta J_{\text{min}}$

\[
\delta J_{\text{min}} = 2M^2\epsilon^2 + 2M\delta E\epsilon + \delta E^2 ,
\]  

(5)

For particle to reach horizon, we have $\delta E \geq \Omega_+\delta J$. This defines maximum threshold as

\[
\delta J_{\text{max}} = (2 + 4\epsilon)M \delta E.
\]  

(6)

Parameter space for for overspinning

\[
\Delta J = \delta J_{\text{max}} - \delta J_{\text{min}} > 0 .
\]  

(7)
Violation of cosmic censorship conjecture

Particle motion around near-extremal Kerr black hole

We analyze the behaviour of the effective potential to understand whether or not particle falls into the black hole,

\[ \frac{\dot{r}^2}{2} + V_{\text{eff}}(r, \tilde{\delta}E, \tilde{\delta}J) = 0, \quad (8) \]

where \( \tilde{\delta}E = \delta E/m \) and \( \tilde{\delta}J = \delta J/m \), and \( m \) is the rest mass of the particle.

The effective potential of radial motion of the particle

\[
V_{\text{eff}} = -\frac{\tilde{\delta}E^2}{2} \left[ 1 - \frac{3 + b\tilde{c}(b - 4) + 4b\epsilon + (4 + b^2)\epsilon^2}{r^2} \right.
+ \frac{2 - 2b\tilde{c}(b - 4) + 4b\epsilon - 4\tilde{c}(4 - b)\epsilon + 2(4 + b^2)\epsilon^2}{r^3} \left. \right], \quad (9)
\]

Assuming that \( \tilde{\delta}E \gg 1 \). One can easily write the Eq. (14) as

\[
V_{\text{eff}} = -\tilde{\delta}E^2 V_{\text{eff}}^b / 2, \quad \text{where } b \in [3, 4] \text{ is parametrization parameter.} \]
The dependence of value of the effective potential at given maximum radius $r_{max}$ on the parameter $b$ is plotted here.

Radial dependence of the effective potential on the radial motion of the charged particle moving around the nearextremal rotating black.
Talk outline

1. Motivation
2. Violation of cosmic censorship conjecture
3. The effect of a high dimension and a test magnetic field on the CCC
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The effect of a high dimension and a test magnetic field on the CCC

Five dimensional rotating black hole metric is given by

\[ ds^2 = -\frac{\Delta}{\Sigma} \left( dt - a \sin^2 \theta d\phi - b \cos^2 \theta d\psi \right)^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 \]

\[ + \frac{\sin^2 \theta}{\Sigma} \left[ (r^2 + a^2) d\phi - adt \right]^2 + \frac{\cos^2 \theta}{\Sigma} \left[ (r^2 + b^2) d\psi - bdt \right]^2 \]

\[ + r^2 \left( \cos^2 \theta + \sin^2 \phi \right) d\psi^2 , \tag{10} \]

where \( \Delta = \frac{(r^2 + a^2)(r^2 + b^2)}{r^2} - 2M \) and \( \Sigma = r^2 + a^2 \cos^2 \theta + b^2 \sin^2 \theta \). Here \( a \) and \( b \) are rotation parameters about the two rotation axes.

Parameter space available at all for overspinning

\[ \delta J_{\text{max}} = \left[ 1 + \sqrt{2} \left( \frac{1}{\beta} - 1 \right) \epsilon + \left( \frac{1}{\beta} - 1 \right) \epsilon^2 \right] \sqrt{2} M \delta E , \tag{11} \]

\[ \delta J_{\text{min}} = \sqrt{2} M^2 \epsilon^2 + 2\sqrt{2} M \delta E + \sqrt{2} \delta E^2 , \tag{12} \]

This clearly shows \( \Delta J < 0 \) indicating absence of parameter space for overspinning and so horizon cannot be destroyed. Thus CCC holds good for a five dimensional rotating black hole for linear order accretion process.
The effect of a high dimension and a test magnetic field on the CCC

Particle motion around near-extremal black holes

\[ \frac{\dot{r}^2}{2} + V_{\text{eff}}(r, \tilde{\delta}E, \tilde{\delta}J) = 0, \text{ where } \tilde{\delta}E = \frac{\delta E}{m} \text{ and } \tilde{\delta}J = \frac{\delta J}{m}. \]

The effective potential for the Kerr black hole

\[
V_{\text{eff}} = -\frac{1}{r} + \frac{\tilde{\delta}J^2}{2r^2} - \frac{(\tilde{\delta}J - a\tilde{\delta}E)^2}{r^3} + \frac{1}{2} \left( 1 - \tilde{\delta}E^2 \right) \left( 1 + \frac{a^2}{r^2} \right). \quad (13)
\]

The effective potential for 5-dimensional Kerr black hole

\[
V_{\text{eff}} = -\frac{1}{r^2} + \frac{\tilde{\delta}J^2}{2r^2} - \frac{(\tilde{\delta}J - a\tilde{\delta}E)^2}{r^4} + \frac{1}{2} \left( 1 - \tilde{\delta}E^2 \right) \left( 1 + \frac{a^2}{r^2} \right). \quad (14)
\]
The effect of a high dimension and a test magnetic field on the CCC

Magnetic Field of BH

Typically, in the astrophysical context, black holes are surrounded with a magnetic field that would exert a Lorentz force on the charged particle affecting its motion (Wald, 1974).

\[ B \text{ takes constant value at infinity and is aligned with axis of symmetry} \]

\begin{align*}
A_t &= -\frac{1}{2\Sigma} \left\{ aB \left[ \Delta (1 + \cos^2 \theta) + (r^2 - a^2) \sin^2 \theta \right] - 2aB (\Sigma - 2Mr) \right\}, \\
A_r &= A_\theta = 0, \\
A_\phi &= \frac{1}{\Sigma} \left\{ \frac{B}{2} \left[ \Delta a^2 (1 + \cos^2 \theta) + r^4 - a^4 \right] - 2QMBa^3 \right\} \sin^2 \theta . \quad (15)
\end{align*}
The effect of a high dimension and a test magnetic field on the CCC

The effective potential of radial motion of the particle in the presence of external magnetic field

We analyze the effective potential in order to understand the effect of magnetic field on the process of destroying a black hole. The effective potential can be written in the form

$$ V_{eff} = -\frac{\delta E^2}{2} \left( V_{eff}^b + V_{eff}^\beta \right), \quad (16) $$

where $V_{eff}^\beta$ is defined as

$$ V_{eff}^\beta = \beta \left[ \frac{2 + b\epsilon}{\delta E} - \frac{8\tilde{c} - 2b\tilde{c} + \beta(1 - 4\epsilon^2)}{2\delta E^2} - \frac{(4 + 2b\epsilon)/\delta E - (8\tilde{c} - 2b\tilde{c})/\delta E^2}{r} ight. $$

$$ + \left. \frac{(16b\tilde{c} - 64\tilde{c} + 3\beta)/\delta E^2}{4r^2} - \frac{\beta(1 - 8\epsilon^2)/\delta E^2}{2r^3} \right. $$

$$ + \left. \frac{(2 + b\epsilon - 8\epsilon^2)/\delta E - (4b\tilde{c} - 16\tilde{c} + 2\beta)(\epsilon/\delta E)^2}{r^2} \right], \quad (17) $$

and $V_{eff}^b$ was given in the earlier. Where $\beta = \frac{qBMG}{mc^4}$ and $\tilde{c} = \frac{q^2}{2m}$ are magnetic and charge parameters.
Magnetic and gravitational forces acting on a charged particle motion

In a flat space-time (in the absence of gravity) a particle with charge $q$ and the rest mass $m$ in the magnetic field $B$ has the characteristic cyclotron frequency

$$\Omega_c = \frac{qB}{mc},$$

(18)

Let us write the Keplerian frequency of a particle orbiting a black hole of mass $M$,

$$\Omega_K = \frac{\sqrt{GM}}{r^{3/2}}.$$

(19)

Here, the ratio of these frequencies $\Omega_c/\Omega_K$ is of the order of

$$\beta \equiv \frac{qBMG}{mc^4}.$$

(20)

We deal with this parameter so as to understand the effect of magnetic and gravitational forces on the charged particle motion around the black hole.
The dependence of the effective potential at the maximum point $r_{max}$ near the extremal rotating black hole placed in a magnetic field of strength $B$ on the parametrization parameter $b$ for both the negative $\beta < 0$ and positive $\beta > 0$ cases for the different values of magnetic parameter $\beta$ in the case in which the charge parameter $\tilde{c} = 10^{-3}$. 

![Graph showing the dependence of the effective potential on $b$ for different values of $\beta$.]
The effect of a high dimension and a test magnetic field on the CCC

Radial dependence of the effective potential on the radial motion of the charged particle moving around the near extremal rotating black hole immersed in a magnetic field of strength $B$ for the different values of magnetic parameter $\beta$. For this figure, $\beta = 0$ (a), $\beta = 0.01$ (b), $\beta = 0.1$ (c), $\beta = 1$ (d), $\beta = 5$ (e), and $\beta = 10$ (f) in the case in which the charge parameter $\tilde{c} = 10^{-3}$ and $b = 3.9863$. 

![Graphs showing the radial dependence of the effective potential for different values of magnetic parameter $\beta$.]
Analyzing the backreaction of the magnetic field on the background. Comparing the backreaction of the magnetic field to that of the test particle for the chosen value, $\epsilon = 0.01$, the critical value of the magnetic field is given by

$$B_{cr} \sim 0.6872.$$  \hspace{1cm} (21)

The value of parameter $\beta_{cr}$ at which we have $b_{cr} = 3$ and magnetic field starts acting as a cosmic censor is tabulated here. The result is similar for different nonextremal cases.

<table>
<thead>
<tr>
<th>$b_{cr} = 3$</th>
<th>$m$</th>
<th>$\tilde{c}$</th>
<th>$\epsilon = 0.01$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0001</td>
<td>$10^{-3}$</td>
<td>37.0809</td>
</tr>
<tr>
<td></td>
<td>0.0001</td>
<td>$10^{-4}$</td>
<td>$5 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>$\epsilon = 0.0001$</td>
<td>$\beta_{cr}$</td>
<td>$10.1457$</td>
</tr>
<tr>
<td></td>
<td>$B/B_{cr}$</td>
<td>$9.6514$</td>
<td>10.1248</td>
</tr>
</tbody>
</table>
The value of the effective potential at the maximum as a function of $b$ is plotted for positive as well as negative values of $\beta$ and for charge parameter $\tilde{c} = 10^{-3}$. The magnetic field is sufficiently smaller than the critical value. $b_c r$ initially increases, and then it decreases. We have $b_c r > 3$. Thus, when backreaction of the magnetic field can be ignored, it does not serve as a cosmic censor.
Talk outline

1. Motivation
2. Violation of cosmic censorship conjecture
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4. Conclusion
• Beyond the threshold value of the magnetic field it is not possible for a charged particle with the appropriate values of geodesic parameters to enter the black hole and turn it into the naked singularity. We find that when the magnetic field acts as a cosmic censor its backreaction is slightly larger than that of the test particle.

• The magnetic field plays a very crucial role in black holes vicinity not only in getting more information about black holes but also in preventing them from being destroyed by particle absorption.

• Five dimensional rotating BH cannot be overspun. This happens because the minimum threshold particle angular momentum required for overspinning turns out to be greater than the maximum threshold allowed for particle to reach black hole horizon. Thus there is no parameter space available at all for overspinning, and hence five dimensional rotating BH obeys CCC.
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Thank you for your kind attention
### Magnetic Fields around Black Holes


\[ B \approx 10^8 \text{Gauss, for } M \approx 10 M_\odot \]
\[ B \approx 10^4 \text{Gauss, for } M \approx 10^9 M_\odot \]

A.K. Baczko, R. Schulz, M. Kadler, E. Ros, et. al. A Highly magnetized twin-jet base pinpoints a supermassive black hole, AA, 593, A47 (2016) – The observation of the twin-jet system of NGC 1052 at 86 GHz with the Global mm-VLBI Array, deriving the magnetic field that would be between \( B \approx 200 \text{ Gauss} – 8.3 \times 10^4 \text{ Gauss} \)

- Wald (1971) – exact solution for BH immersed in MF.
- Blandford & Znajek (1977) – extraction of energy of Kerr BH immersed in MF.
- Morozova, Rezzolla & Ahmedov (2014). – a rotating black hole moving in an asymptotically uniform magnetic test field
Analyzing the backreaction of the magnetic field on the background

We calculate the square root of the difference of the Kretschmann scalar at the horizon for extremal and nearextremal geometries, subtract, and take a square root,

\[ K = (K_1 - K_2)^{1/2} = 6 \sqrt[4]{2} \left[ 2 \sqrt[2]{2} \left( \frac{1 - 7x^6 + 35x^4 - 21x^2}{(1 + x^2)^7} \right)^{1/2} (\epsilon)^{1/2} \right. 

\left. - \frac{7 - 2x^{10} + 47x^8 - 224x^6 + 434x^4 - 182x^2}{(1 + x^2)^8} \right] ^{1/2} (\epsilon)^{3/2} \right) , \quad (22) \]

and at the horizon the density of the energy momentum tensor of the magnetic field is given by

\[ T = \frac{B^2 \left( 25 + 77x^2 + x^4 - 99x^6 - 109x^8 + 11x^{10} + 63x^{12} + 27x^{14} + 4x^{16} \right)}{2 \left( 1 + x^2 \right)^8 \left( 3 + x^2 \right)} \] . \quad (23) \]

Comparing the backreaction of the magnetic field to that of the test particle for the chosen value, \( \epsilon = 0.01 \), the critical value of the magnetic field is given by

\[ B_{cr} \sim 0.6872. \quad (24) \]