# How to restore Cosmic Censorship Conjecture: Can a test magnetic field serve as a cosmic censor?

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# Talk outline

1. Motivation

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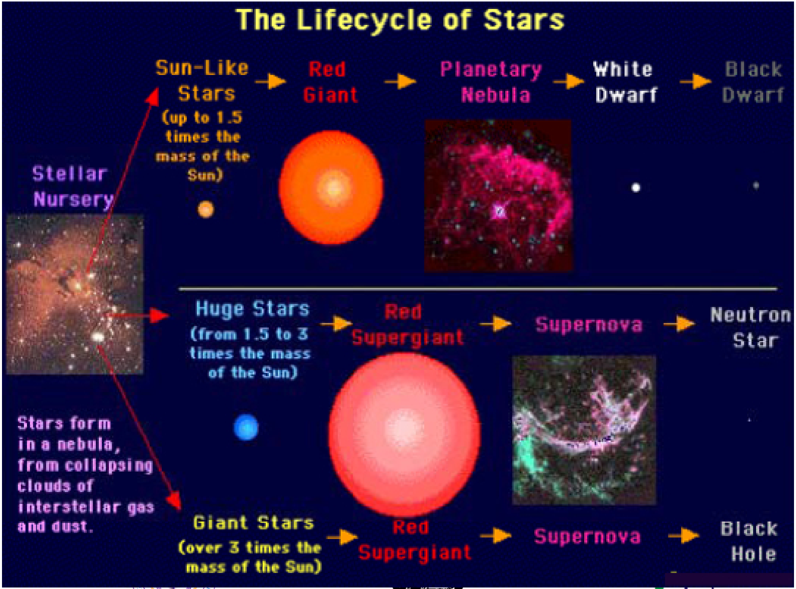
The Lifecycle of Stars
Sun-Like Stars
(up to 1.5 times the mass of the Sun)
Red Giant
Planetary Nebula
White Dwarf
Black Dwarf
Stellar Nursery
Huge Stars
(from 1.5 to 3 times the mass of the Sun)
Red Supergiant
Supernova
Neutron Star
Stars form in a nebula, from collapsing clouds of interstellar gas and dust.
Giant Stars
(over 3 times the mass of the Sun)
Red Supergiant
Supernova
Black Hole

# Cosmic Censorship Conjecture (CCC)

According to the Einstein gravity black holes is formed as a result of gravitational collapse. Later, Penrose proposed in 1969 that a singularity would always be hidden behind a horizon. This is called CCC. However, there exists no proof of CCC either way, true or false, and it remains as one of the most important open questions.

**Roger Penrose**

**Roger Penrose first formulated the cosmic censorship hypothesis in 1969.**

- R. Penrose, Riv. Nuovo Cim. **1**, 252 (1969);
- R. Penrose, Gen. Rel. Grav. **34**, 1141 (2002).

## Motivation

However, black holes have been still candidate BHs. The cosmic censership conjecture have remained unproven yet. If so, there appears new hypothesis, according to which the physical possibility of destroying the event horizon of BHs seems to be valid due to over-spinning or over-charging process.

**T. Jacobson & T.P. Sotiriou, Examined the physical possibility of over-spining a black hole with an infalling test particle, PRL, 2009**

## Approaching the issue from different prospectives

- R. Wald, Ann. Phys. **82**, 548 (1974) – The dynamics of overspinning process was examined by Wald for the first time.
- V. E. Hubeny, Phys. Rev. D **59**, 064013 (1999)– She investigated such process for near-extremal RN BH with a charged test particle.
- A. Saa and R. Santarelli, Phys. Rev. D 84, 027501 (2011).
- Z. Li and C. Bambi, Phys. Rev. D 87, 124022 (2013).

## The external magnetic field

Typically, in the astrophysical scenarios, black holes are surrounded with the magnetic field, which affects the motion of the surrounding electrically charged matter or particle. Then, the effect of an external magnetic field can be tested as an alternative tool to restore the cosmic censorship in the astrophysical context.

## Five dimensional spacetime

Increasing the dimension order of spacetime can be also tested to restore the cosmic censorship conjecture.

S. Shaymatov, M. Patil, B. Ahmedov and P. S. Joshi, Destroying a near-extremal Kerr black hole with a charged particle: Can a test magnetic field serve as a cosmic censor?, **Phys. Rev. D**, 2015, V.91, 064025.
S. Shaymatov, B. Ahmedov and N. Dadhich, Cosmic censorship conjecture in five dimensional rotating black hole in Einstein gravity via test particle absorption, **arXiv:1809.10457**, 2018.

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**The metric of the Kerr geometry in the Boyer-Lindquist coordinates is given by**

$$ds^2 = -\left(\frac{\Delta - a^2\sin^2\theta}{\Sigma}\right)dt^2 - \frac{2a\sin^2\theta(r^2+a^2-\Delta)}{\Sigma}dtd\phi + \frac{(r^2+a^2)^2 - \Delta a^2\sin^2\theta}{\Sigma}\sin^2\theta d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2, \quad (1)$$

where $\Sigma = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 + a^2 - 2Mr$ with parameters $M$ and $a$.

**Black hole horizon**

$$r_{\mathrm{h}} = M + \sqrt{M^2 - a^2}, \quad (2)$$

exists only $M \geq a$ and black hole for which $a = M$ is referred to an extremal Kerr black hole.

To overspin/destroy black hole, the following condition must be satisfied for the Kerr BH

$$M^2 < a^2 \quad (3)$$

## Dynamics of overspinning process

There are two constants of motion associated with the particle $\delta E$ and $\delta J$, which are $\delta E \ll M$, $\delta J \ll J$ for test particle approximation. After falling particle into the BH, the final parameters are given by $M + \delta E$ and $J + \delta J$, respectively. One must start out with the near-extremal black hole $J/M^2 = a/M = 1 - 2\epsilon^2$, with $\epsilon \ll 1$ being a small dimensionless parameter.

$$(M + \delta E)^2 < \left(\frac{J + \delta J}{M + \delta E}\right)^2 . \qquad (4)$$

This yields the lower bound on the angular momentum $\delta J_{min}$

$$\delta J_{min} = 2M^2\epsilon^2 + 2M\delta E\epsilon + \delta E^2 , \qquad (5)$$

For particle to reach horizon, we have $\delta E \geq \Omega_+ \delta J$. This defines maximum threshold as

$$\delta J_{max} = (2 + 4\epsilon)M \ \delta E . \qquad (6)$$

Parameter space for for overspinning

$$\Delta J = \delta J_{max} - \delta J_{min} > 0 . \qquad (7)$$

**T. Jacobson & T.P. Sotiriou, PRL, 2009**

## Particle motion around near-extremal Kerr black hole

We analyze the behaviour of the effective potential to understand whether or not particle falls into the black hole,

$$\frac{\dot{r}^2}{2} + V_{\rm eff}(r, \tilde{\delta E}, \tilde{\delta J}) = 0, \tag{8}$$

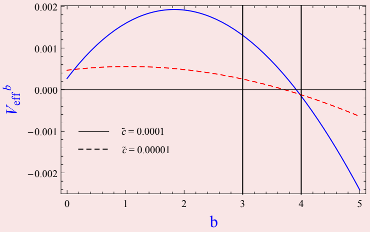
where $\tilde{\delta E} = \delta E/m$ and $\tilde{\delta J} = \delta J/m$, and $m$ is the rest mass of the particle.

## The effective potential of radial motion of the particle

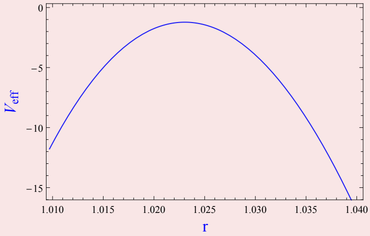
$$\begin{aligned} V_{eff} &= -\frac{\tilde{\delta E}^2}{2}\Bigg[1 - \frac{3 + b\tilde{c}(b-4) + 4b\epsilon + (4+b^2)\epsilon^2}{r^2} \\ &+ \frac{2 - 2b\tilde{c}(b-4) + 4b\epsilon - 4\tilde{c}(4-b)\epsilon + 2(4+b^2)\epsilon^2}{r^3}\Bigg], \end{aligned} \tag{9}$$

Assuming that $\tilde{\delta E} >> 1$. One can easily write the Eq. (14) as $V_{eff} = -\tilde{\delta E}^2 V^b_{eff}/2$, where $b \in [3, 4]$ is parametrization parameter.

The dependence of value of the effective potential at given maximum radius $r_{max}$ on the parameter $b$ is plotted here.




Radial dependence of the effective potential on the radial motion of the charged particle moving around the nearextremal rotating black.



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## Five dimensional rotating black hole metric is given by

$$
\begin{aligned}
ds^2 &= -\frac{\Delta}{\Sigma}\left(dt - a\sin^2\theta d\phi - b\cos^2\theta d\psi\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 \\
&+ \frac{\sin^2\theta}{\Sigma}\left[(r^2+a^2)d\phi - adt\right]^2 + \frac{\cos^2\theta}{\Sigma}\left[(r^2+b^2)d\psi - bdt\right]^2 \\
&+ r^2\left(\cos^2\theta + \sin^2\phi\right)d\psi^2 \ , \qquad (10)
\end{aligned}
$$

where $\Delta = \frac{(r^2+a^2)(r^2+b^2)}{r^2} - 2M$ and $\Sigma = r^2 + a^2\cos^2\theta + b^2\sin^2\theta$. Here $a$ and $b$ are rotation parameters about the two rotation axes.

## Parameter space available at all for overspinning

$$
\delta J_{max} = \left[1 + \sqrt{2\left(\frac{1}{\beta} - 1\right)}\,\epsilon + \left(\frac{1}{\beta} - 1\right)\epsilon^2\right]\sqrt{2}\,M\,\delta E\,, \qquad (11)
$$

$$
\delta J_{min} = \sqrt{2}\,M^2\,\epsilon^2 + 2\sqrt{2}\,M\,\delta E + \sqrt{2}\,\delta E^2\,, \qquad (12)
$$

This clearly shows $\Delta J < 0$ indicating absence of parameter space for overspinning and so horizon cannot be destroyed. Thus CCC holds good for a five dimensional rotating black hole for linear order accretion process.

## Particle motion around near-extremal black holes

$\frac{\dot{r}^2}{2} + V_{\rm eff}(r, \delta\tilde{E}, \delta\tilde{J}) = 0$, where $\delta\tilde{E} = \delta E/m$ and $\delta\tilde{J} = \delta J/m$.

## The effective potential for the Kerr black hole

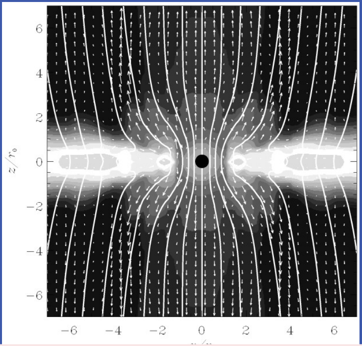
$$V_{eff} = -\frac{1}{r} + \frac{\delta\tilde{J}^2}{2r^2} - \frac{\left(\delta\tilde{J} - a\delta\tilde{E}\right)^2}{r^3} + \frac{1}{2}(1 - \delta\tilde{E}^2)\left(1 + \frac{a^2}{r^2}\right). \quad (13)$$

## The effective potential for 5-dimensional Kerr black hole

$$V_{eff} = -\frac{1}{r^2} + \frac{\delta\tilde{J}^2}{2r^2} - \frac{\left(\delta\tilde{J} - a\delta\tilde{E}\right)^2}{r^4} + \frac{1}{2}(1 - \delta\tilde{E}^2)\left(1 + \frac{a^2}{r^2}\right). \quad (14)$$

# Magnetic Field of BH

## BH in external magnetic field




Typically, in the astrophysical context, black holes are surrounded with a magnetic field that would exert a Lorentz force on the charged particle affecting its motion (Wald, 1974).

$B$ takes constant value at infinity and is aligned with axis of symmetry

$$\begin{aligned} A_t &= -\frac{1}{2\Sigma}\left\{aB\left[\Delta\left(1+cos^2\theta\right)+\left(r^2-a^2\right)\sin^2\theta\right]-2aB\left(\Sigma-2Mr\right)\right\}, \\ A_r &= A_\theta = 0\ , \\ A_\varphi &= \frac{1}{\Sigma}\left\{\frac{B}{2}\left[\Delta a^2\left(1+cos^2\theta\right)+r^4-a^4\right]-2QMBa^3\right\}\sin^2\theta\ . \end{aligned} \tag{15}$$

## The effective potential of radial motion of the particle in the presence of external magnetic field

We analyze the effective potential in order to understand the effect of magnetic field on the process of destroying a black hole. The effective potential can be written in the form

$$V_{eff} = -\frac{\delta\tilde{E}^2}{2}\left(V^b_{eff} + V^\beta_{eff}\right), \tag{16}$$

where $V^\beta_{eff}$ is defined as

$$\begin{aligned} V^\beta_{eff} &= \beta\Bigg[\frac{2+b\epsilon}{\delta\tilde{E}} - \frac{8\tilde{c}-2b\tilde{c}+\beta(1-4\epsilon^2)}{2\delta\tilde{E}^2} - \frac{(4+2b\epsilon)/\delta\tilde{E}-(8\tilde{c}-2b\tilde{c})/\delta\tilde{E}^2}{r} \\ &+ \frac{(16b\tilde{c}-64\tilde{c}+3\beta)/\delta\tilde{E}^2}{4r^2} - \frac{\beta(1-8\epsilon^2)/\delta\tilde{E}^2}{2r^3} \\ &+ \frac{(2+b\epsilon-8\epsilon^2)/\delta\tilde{E}-(4b\tilde{c}-16\tilde{c}+2\beta)(\epsilon/\delta\tilde{E})^2}{r^2}\Bigg], \end{aligned} \tag{17}$$

and $V^b_{eff}$ was given in the earlier. Where $\beta = \frac{qBMG}{mc^4}$ and $\tilde{c} = q^2/2m$ are magnetic and charge parameters.

## Magnetic and gravitational forces acting on a charged particle motion

In a flat space-time (in the absence of gravity) a particle with charge $q$ and the rest mass mass $m$ in the magnetic field $B$ has the characteristic cyclotron frequency

$$\Omega_c = \frac{qB}{mc}\,, \tag{18}$$

Let us write the Keplerian frequency of a particle orbiting a black hole of mass $M$,

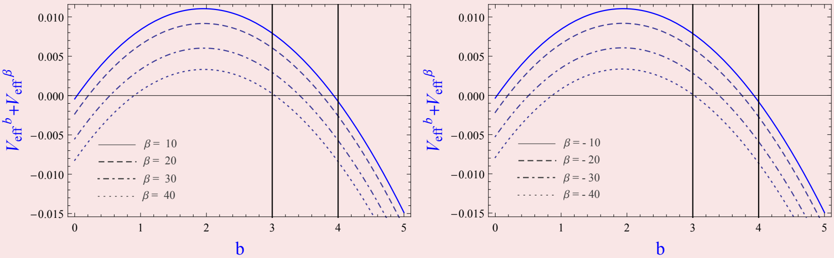
$$\Omega_K = \frac{\sqrt{GM}}{r^{3/2}}\,. \tag{19}$$

Here, the ratio of these frequencies $\Omega_c/\Omega_K$ is of the order of

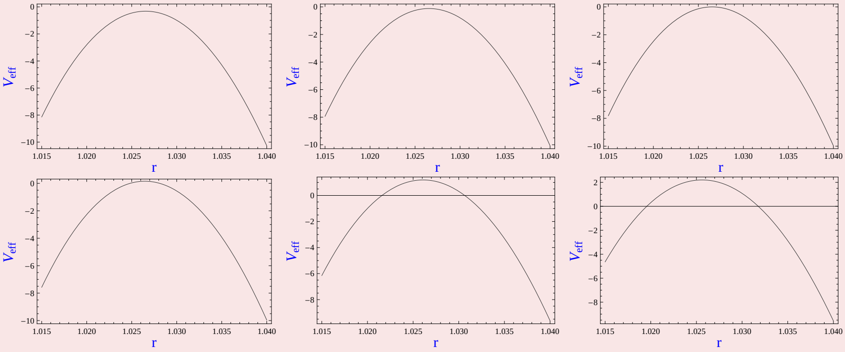
$$\beta \equiv \frac{qBMG}{mc^4}\,. \tag{20}$$

We deal with this parameter so as to understand the effect of magnetic and gravitational forces on the charged particle motion around the black hole.

The dependence of the effective potential at the maximum point $r_{max}$ near the extremal rotating black hole placed in a magnetic field of strength B on the parametrization parameter $b$ for both the negative $\beta < 0$ and positive $\beta > 0$ cases for the different values of magnetic parameter $\beta$ in the case in which the charge parameter $\tilde{c} = 10^{-3}$.



Radial dependence of the effective potential on the radial motion of the charged particle moving around the near extremal rotating black hole immersed in a magnetic field of strength $B$ for the different values of magnetic parameter $\beta$. For this figure, $\beta = 0$ (a), $\beta = 0.01$ (b), $\beta = 0.1$ (c), $\beta = 1$ (d), $\beta = 5$ (e), and $\beta = 10$ (f) in the case in which the charge parameter $\tilde{c} = 10^{-3}$ and $b = 3.9863$.



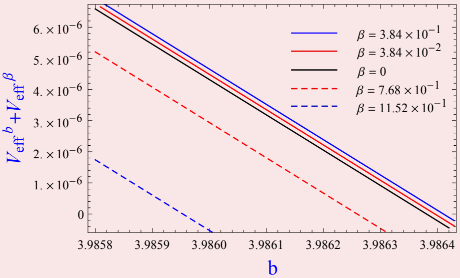
Analyzing the backreaction of the magnetic field on the background. Comparing the backreaction of the magnetic field to that of the test particle for the chosen value, $\epsilon = 0.01$, the critical value of the magnetic field is given by

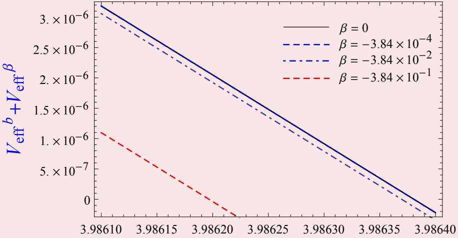
$$B_{cr} \sim 0.6872. \tag{21}$$

The value of parameter $\beta_{cr}$ at which we have $b_{cr} = 3$ and magnetic field starts acting as a cosmic censor is tabulated here. The result is similar for different nonextremal cases.

| | | | $b_{cr} = 3$ | | |
|---|---|---|---|---|---|
| $m$ | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| $\tilde{c}$ | $10^{-3}$ | $10^{-4}$ | $5 \times 10^{-5}$ | $10^{-5}$ | $5 \times 10^{-6}$ |
| | | | $\epsilon = 0.01$ | | |
| $\beta_{cr}$ | 37.0809 | 12.3265 | 9.1618 | 5.4369 | 4.7714 |
| | −36.8021 | −12.3011 | −9.1490 | −5.4342 | −4.7699 |
| $B/B_{cr}$ | 9.6514 | 10.1457 | 10.6644 | 14.1512 | 17.5631 |
| | 9.5788 | 10.1248 | 10.6495 | 14.1442 | 17.5576 |
| | | | $\epsilon = 0.0001$ | | |
| $\beta_{cr}$ | 37.8413 | 11.9930 | 8.4816 | 3.7937 | 2.6827 |
| | −37.8242 | −11.9937 | −8.4819 | −3.7938 | −2.6828 |
| $B/B_{cr}$ | 9.8493 | 9.8711 | 9.8727 | 9.8742 | 9.8748 |
| | 9.8448 | 9.8717 | 9.8730 | 9.8745 | 9.8752 |

The value of the effective potential at the maximum as a function of $b$ is plotted for positive as well as negative values of $\beta$ and for charge parameter $\tilde{c} = 10^{-3}$. The magnetic field is sufficiently smaller than the critical value. $b_c r$ initially increases, and then it decreases. We have $b_c r > 3$. Thus, when backreaction of the magnetic field can be ignored, it does not serve as a cosmic censor.



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- Beyond the threshold value of the magnetic field it is not possible for a charged particle with the appropriate values of geodesic parameters to enter the black hole and turn it into the naked singularity. We find that when the magnetic field acts as a cosmic censor its backreaction is slightly larger than that of the test particle.
- The magnetic field plays a very crucial role in black holes vicinity not only in getting more information about black holes but also in preventing them from being destroyed by particle absorbtion.
- Five dimensional rotating BH cannot be overspun. This happens because the minimum threshold particle angular momentum required for overspinning turns out to be greater than the maximum threshold allowed for particle to reach black hole horizon. Thus there is no parameter space available at all for overspinning, and hence five dimensional rotating BH obeys CCC.

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# Thank you for your kind attention

## Magnetic Fields around Black Holes

**M.Yu. Piotrovich, N.A. Silant'ev, Yu.N. Gnedin, and T.M. Natsvlishvili (2010) – The strength of magnetic field for supermassive and stellar black holes**

$$B \approx 10^8 \text{Gauss}, \quad \text{for} \quad M \approx 10 M_\odot$$
$$B \approx 10^4 \text{Gauss}, \quad \text{for} \quad M \approx 10^9 M_\odot$$

A.K. Baczko, R. Schulz, M. Kadler, E. Ros, et. al. A Highly magnetized twin-jet base pinpoints a supermassive black hole, **AA**, 593, A47 (2016) – The observation of the twin-jet system of NGC $1052$ at $86\ GHz$ with the Global mm-VLBI Array, deriving the magnetic field that would be between $B \approx 200\ \text{Gauss} - 8.3 \times 10^4\ \text{Gauss}$

- Wald (1971) – exact solution for BH immersed in MF.
- Blandford & Znajek (1977) – extraction of energy of Kerr BH immersed in MF.
- V. P. Frolov, (2012). – extraction of energy of Weakly magnetized black holes.
- Morozova, Rezzolla & Ahmedov (2014). – a rotating black hole moving in an asymptotically uniform magnetic test field

## Analyzing the backreaction of the magnetic field on the background

We calculate the square root of the difference of the Kretschmann scalar at the horizon for extremal and nearextremal geometries, subtract, and take a square root,

$$\begin{aligned} K &= (K_1 - K_2)^{1/2} = 6\sqrt[4]{2}\Bigg[2\sqrt{2}\left(\frac{1-7x^6+35x^4-21x^2}{(1+x^2)^7}\right)^{1/2}(\epsilon)^{1/2} \\ &- \frac{7-2x^{10}+47x^8-224x^6+434x^4-182x^2}{(1+x^2)^8\left(\frac{1-7x^6+35x^4-21x^2}{(1+x^2)^7}\right)^{1/2}}(\epsilon)^{3/2}\Bigg], \end{aligned} \tag{22}$$

and at the horizon the density of the energy momentum tensor of the magnetic field is given by

$$T = \frac{B^2\left(25+77x^2+x^4-99x^6-109x^8+11x^{10}+63x^{12}+27x^{14}+4x^{16}\right)}{2\left(1+x^2\right)^8\left(3+x^2\right)}. \tag{23}$$

Comparing the backreaction of the magnetic field to that of the test particle for the chosen value, $\epsilon = 0.01$, the critical value of the magnetic field is given by

$$B_{cr} \sim 0.6872. \tag{24}$$