Hadronic Paschen-Back Effect in P-wave Charmonia under the strong magnetic field

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[1] “Hadronic Paschen-Back effect,” arXiv:1802.04971
[2] “Quarkonium radiative decays from the Hadronic Paschen-Back effect,” Phys. Rev. D**98**, 054017 (2018)

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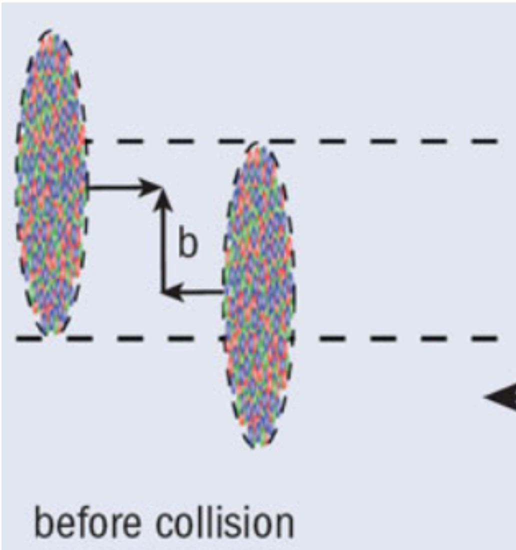
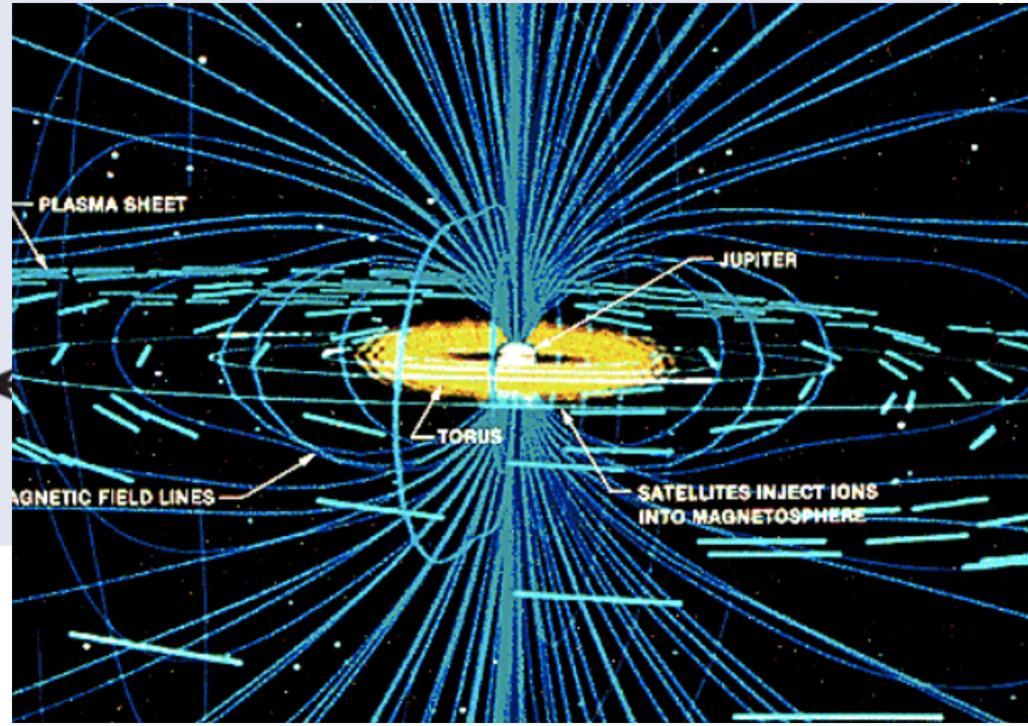
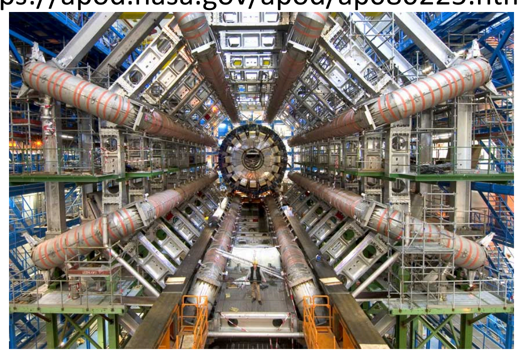
- Anisotropic decay
- Summary & Prospect

Introduction 1/2 – Relativistic Heavy Ion Collision and Magnetic Field –





https://apod.nasa.gov/apod/ap080225.html



↑ Large Hadron Collider (LHC)
and
Relativistic Heavy Ion Collider (RHIC)

1,000,000 times stronger

than MF on surface of neutron star[1,2]

[1] M. Obergaulinger, M. Aloy, arXiv:1711.09975
...oneev,
...925 (2009)

Next page :Effect about MF

Introduction 2/2 – Paschen-Back effect (PBE)–

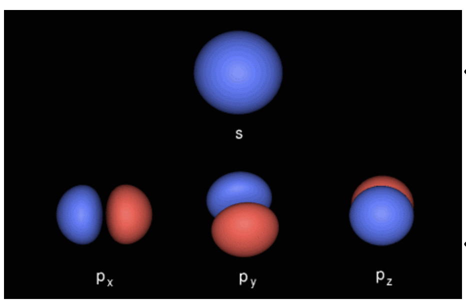
Zeeman effect { anomalous Zeeman eff. : split by S_z
normal Zeeman eff. : split by L_z

Paschen-Back effect : these in strong MF region↑

Eigenstates are separated by L_z and S_z in PB effect as MF gets stronger than the scale of *LS coupling*.

In a charmonium with $J_z = L_z + S_z = +1$,

↓WF's of an electron in hydrogen in vacuum



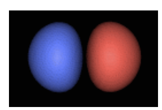
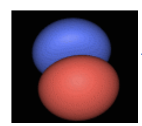
←S-wave (L=0)

← P-wave (L=1)

$L_z = +1$ 0 -1

http://electrons.wikidot.com/band-structure-theory-and-application-in-solids

(L_z, S_z): (1,0) (0,1)

 and  are mixed by 1:1 in vacuum

But in strong magnetic fields…

(1) they are separated

(2) eigenfunctions drastically deform

(3) Observables should change, too

Motivation

Extremely strong magnetic field is predicted in heavy ion collision (HIC) but its strength **has not been measured**

Charmonium ($\bar{c}c$) would be quickly produced in HIC
→ appropriate to measure quickly disappearing MF
P-wave system is expected to show PBE

Calculation

We calculate spectra, deformed wave functions (WF), and mixing ratios of **P-wave** charmonia in **strong** MF, neglecting thermal effects etc in HIC as *first step* for realistic calculation

Steps:

Constant magnetic field	+ time dependence	+ thermal effects	+ electrical field

→ Realistic calculation

Hamiltonian 1/3

Started from the form:

$$H = \sum_{i=1}^{2} \left[\frac{1}{2m_i} (\boldsymbol{p_i} - q_i \boldsymbol{A}(\boldsymbol{r}_i))^2 - \boldsymbol{\mu_i} \cdot \boldsymbol{B} + m_i \right] + V(r)$$

Chose symmetric gauge:

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{1}{2} \boldsymbol{B} \times \boldsymbol{r}$$

Used Cornell potential with SS, LS, and tensor coupling:

$$\begin{aligned} V(r) &= \sigma r - \frac{4}{3} \frac{\alpha_s}{r} + V_{\mathrm{SS}}(r) + V_{\mathrm{LS}} + V_{\mathrm{T}}, \\ V_{\mathrm{SS}}(r) &= \frac{32\pi\alpha_s}{9m_c^2} \delta(r) \boldsymbol{S}_1 \cdot \boldsymbol{S}_2, \\ V_{\mathrm{LS}} &= \frac{1}{m_c^2} \left(\frac{2\alpha_s}{r^3} - \frac{\sigma}{2r} \right) \boldsymbol{L} \cdot \boldsymbol{S}, \\ V_{\mathrm{T}} &= \frac{1}{m_c^2} \frac{4\alpha_s}{3r^3} \left[3(\boldsymbol{S}_1 \cdot \hat{\boldsymbol{r}})(\boldsymbol{S}_2 \cdot \hat{\boldsymbol{r}}) - \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 \right], \end{aligned}$$

} from relativistic correction

Reducing total Hamiltonian into relative one … (next page)

Hamiltonian 2/3

$$
\begin{aligned}
H_{\mathrm{rel}} &= H_{\mathrm{diag}} + H_{\mathrm{m.m.}} + V_{\mathrm{LS}} + V_{\mathrm{T}} + 2m_c, \\
H_{\mathrm{diag}} &= \left[-\frac{1}{2\mu}\nabla^2 + \frac{q^2 B^2}{8\mu}\rho^2\right] + \sigma r - \frac{4}{3}\frac{\alpha_s}{r} + \frac{32\pi\alpha_s}{9m_c^2}\delta(r)\left(\boldsymbol{S}_1\cdot\boldsymbol{S}_2\right), \\
H_{\mathrm{m.m.}} &= -\sum_{i=1}^{2}\left(\boldsymbol{\mu}_i\cdot\boldsymbol{B}\right), \\
V_{\mathrm{LS}} &= \frac{1}{m_c^2}\left(\frac{2\alpha_s}{r^3} - \frac{\sigma}{2r}\right)\boldsymbol{L}\cdot\boldsymbol{S}, \\
V_{\mathrm{T}} &= \frac{1}{m_c^2}\frac{4\alpha_s}{3r^3}\left[3(\boldsymbol{S}_1\cdot\hat{\boldsymbol{r}})(\boldsymbol{S}_2\cdot\hat{\boldsymbol{r}}) - \boldsymbol{S}_1\cdot\boldsymbol{S}_2\right],
\end{aligned}
$$

where we assumed $\boldsymbol{B} = (0,0,B)$,
and use cylindrical coordinate (ρ, z, ϕ)

But we can't start calculation!!

$\frac{1}{r^3}$ and $\delta(r)$ terms overcome ∇^2 ➡ "collapse" solutions appear

We have to smear potentials

$$
\begin{aligned}
\delta(r) &\rightarrow \left(\frac{\Lambda}{\sqrt{\pi}}\right)^3 e^{-\Lambda^2 r^2}, \\
\frac{1}{r^3} &\rightarrow A\frac{1-e^{-\Lambda^2 r^2}}{r^3},
\end{aligned}
$$

Hamiltonian 3/3

In summary, the relative Hamiltonian we solve in this study is as follows:

$$\begin{aligned}
H_{\rm rel} &= H_{\rm diag} + H_{\rm m.m.} + V_{\rm LS} + V_{\rm T} + 2m_c, \\
H_{\rm diag} &= \left[-\frac{1}{2\mu}\nabla^2 + \frac{q^2 B^2}{8\mu}\rho^2\right] + \sigma r - \frac{4}{3}\frac{\alpha_s}{r} + \frac{32\pi\alpha_s}{9m_c^2}\left(\frac{\Lambda}{\sqrt{\pi}}\right)^3 (\boldsymbol{S}_1\cdot\boldsymbol{S}_2)\, e^{-\Lambda^2 r^2}, \\
H_{\rm m.m.} &= -\sum_{i=1}^{2}(\boldsymbol{\mu}_i\cdot\boldsymbol{B}), \\
V_{\rm LS} &= \frac{1}{m_c^2}\left(2\alpha_s A_{\rm LS}\frac{1-e^{-\Lambda_{\rm LS}^2 r^2}}{r^3} - \frac{\sigma}{2r}\right)\boldsymbol{L}\cdot\boldsymbol{S}, \\
V_{\rm T} &= \frac{1}{m_c^2}4\alpha_s A_{\rm T}\frac{1-e^{-\Lambda_{\rm T}^2 r^2}}{3r^3}\left[3(\boldsymbol{S}_1\cdot\hat{\boldsymbol{r}})(\boldsymbol{S}_2\cdot\hat{\boldsymbol{r}}) - \boldsymbol{S}_1\cdot\boldsymbol{S}_2\right],
\end{aligned}$$

$\mu = m_c/2$:reduced mass, $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$

$(\sigma, \alpha_s, \Lambda, m_c)$=(0.1425 GeV^2, 0.5461, 1.0946 GeV, 1.4794 GeV)

$(\Lambda_{LS}, A_{LS}, \Lambda_T, A_T) = (0.2\mathrm{GeV}, 7.3, 1.2\mathrm{GeV}, 1.2)$

q : electrical charge of charm quarks Phys. Rev. D **72**, 054026(2005) Barnes, Godfrey, Swanson

Next page: How to solve the Hamiltonian

Calculation method :
Cylindrical Gaussian Expansion Method (CGEM)

Conventional GEM uses spherical Gaussian basis:

$$\Psi_{\text{spherical}}(r) = Ne^{-\alpha r^2}$$

cf.) E. Hiyama, Y.Kino and M. Kamimura, Prog.Part.Nucl.Phys.51 223 (2003).

Therefore … But now spherical symmetry violates by MF

Previous studies uses cylindrically symmetric one for S-wave:

$$\Psi^{S}_{\text{cylindrical}}(\rho, z, \phi) = Ne^{-\beta\rho^2-\gamma z^2}$$

K. Suzuki and T.Yoshida, Phys.Rev.D93, 051502 (2016).

※ β, γ : range parameters, spin functions omitted, N: normalization factor,

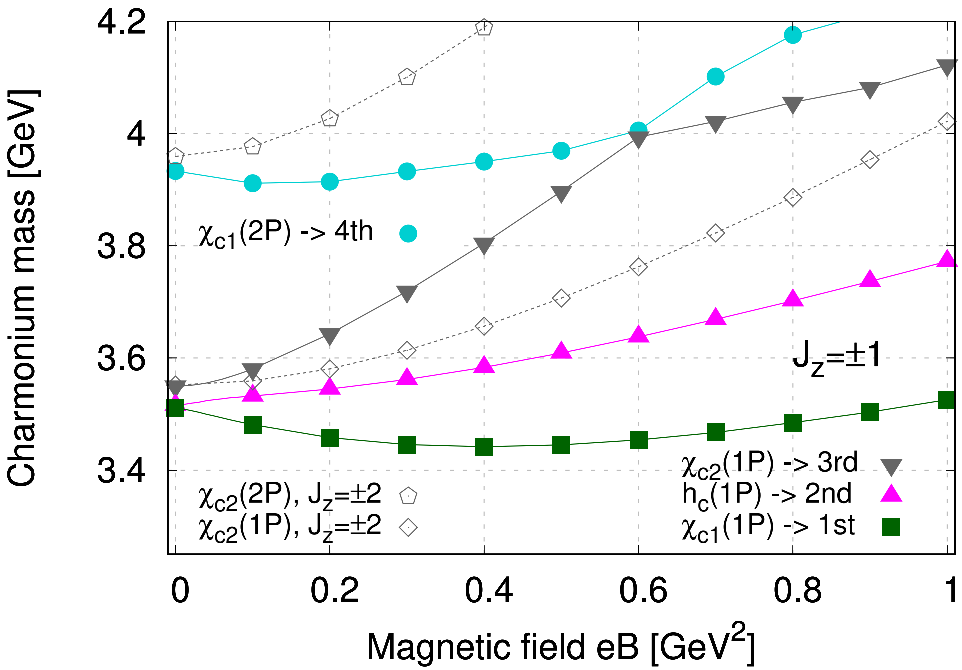
Furthermore …

This study: cylindrically symmetric one for P-wave:

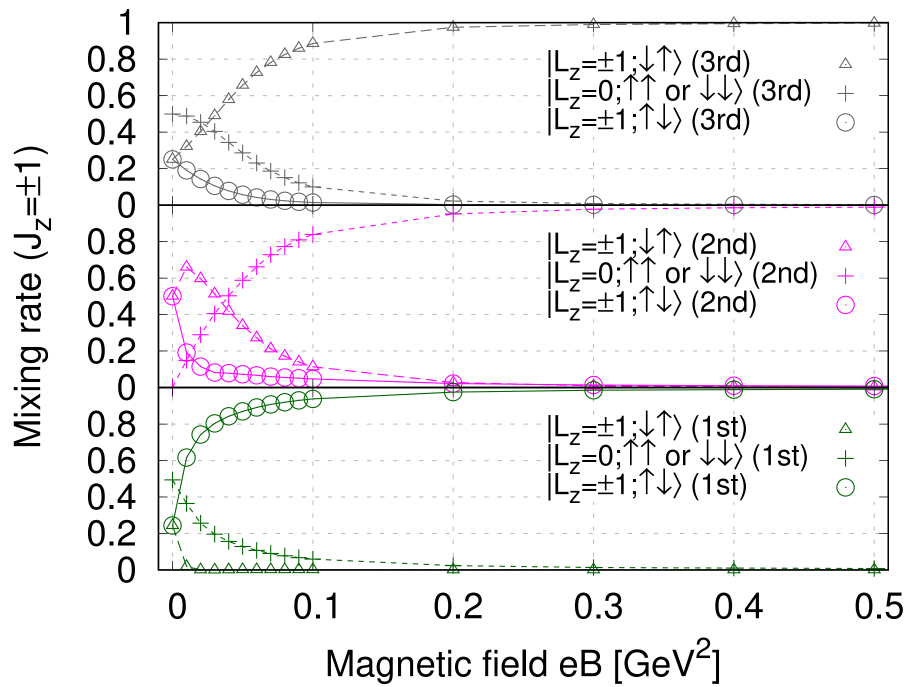
$$\Psi^{\mathrm{P}}_{\mathrm{cyl}}(\rho, z, \phi; L_z) = \begin{cases} N\ \ ze^{-\beta\rho^2-\gamma z^2} & \text{for } L_z = 0 \\ N(\mp\rho)e^{\pm i\phi}e^{-\beta\rho^2-\gamma z^2} & \text{for } \mathrm{L_z} = \pm 1 \end{cases}$$

Solving the Hamiltonian with CGEM, we see …

Results – mass for $J_z = \pm 1$



In vacuum



PB region

Particle: $|\mathrm{J;\,LS}\rangle$

χ_{c2}: $|2;11\rangle$

h_c: $|1;10\rangle$

χ_{c1}: $|1;11\rangle$

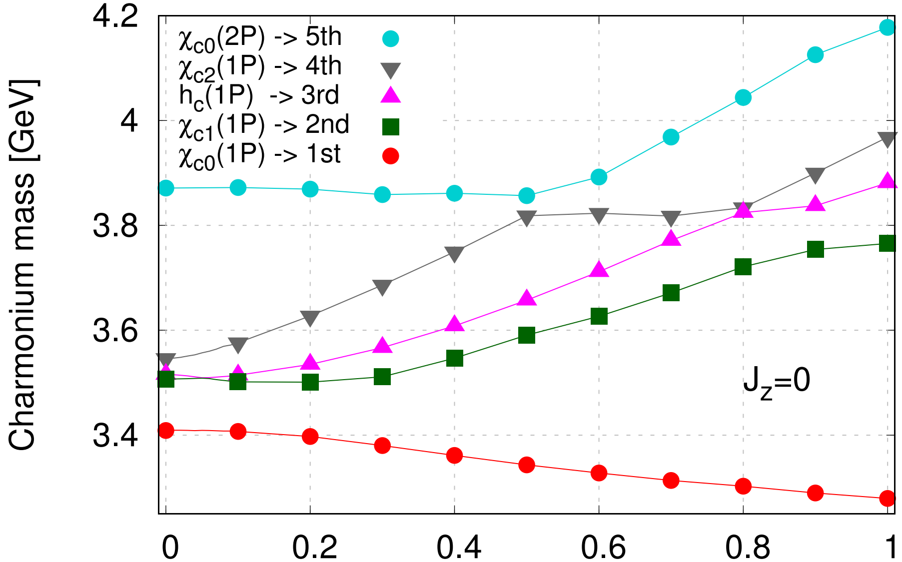
State: $|\mathrm{L_z;\,S_{1z}S_{2z}}\rangle$

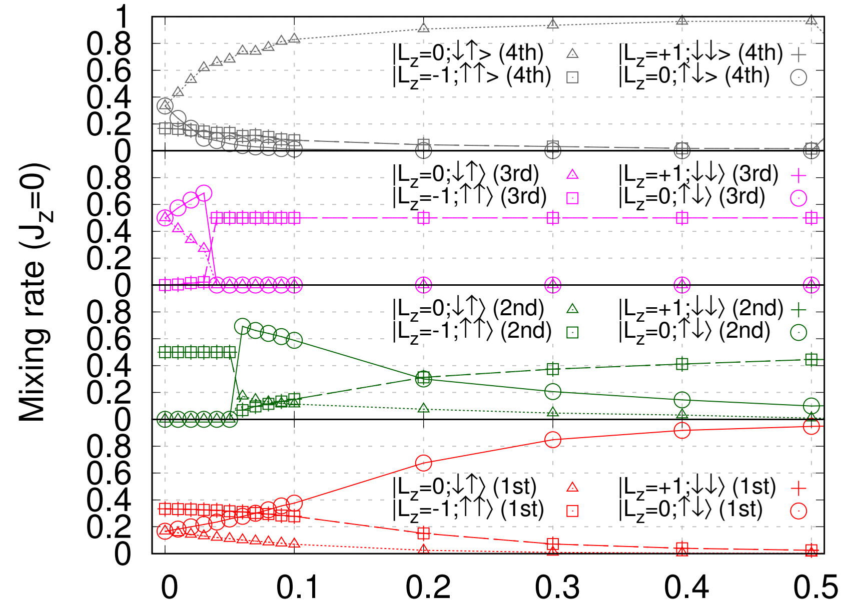
"3rd": $|\pm 1;\downarrow\uparrow\rangle$

"2nd": $|0;\uparrow\uparrow \text{ or } \downarrow\downarrow\rangle$

"1st": $|\pm 1;\uparrow\downarrow\rangle$

Results – mass for $J_z = 0$





OK, it's Hadronic Paschen-Back effect?

Uh-huh, so ... 🤔

So what? 😕

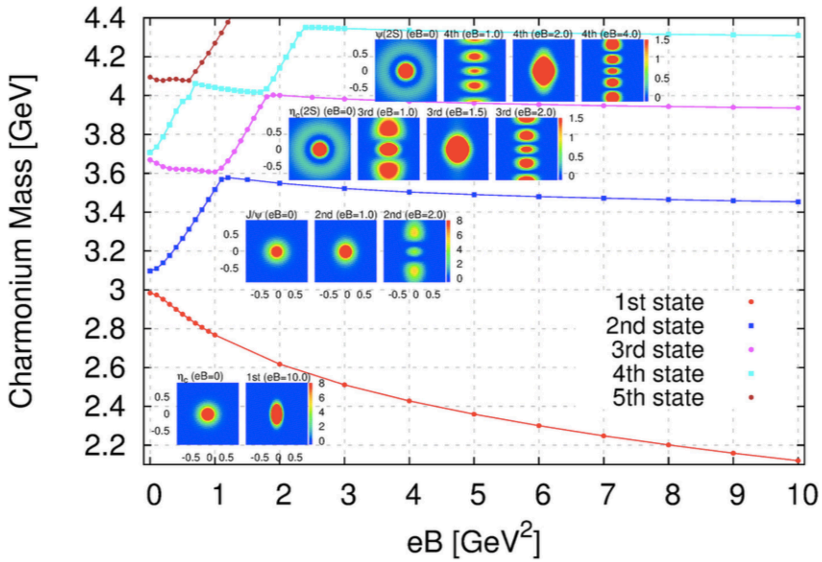
Comparing with another research …

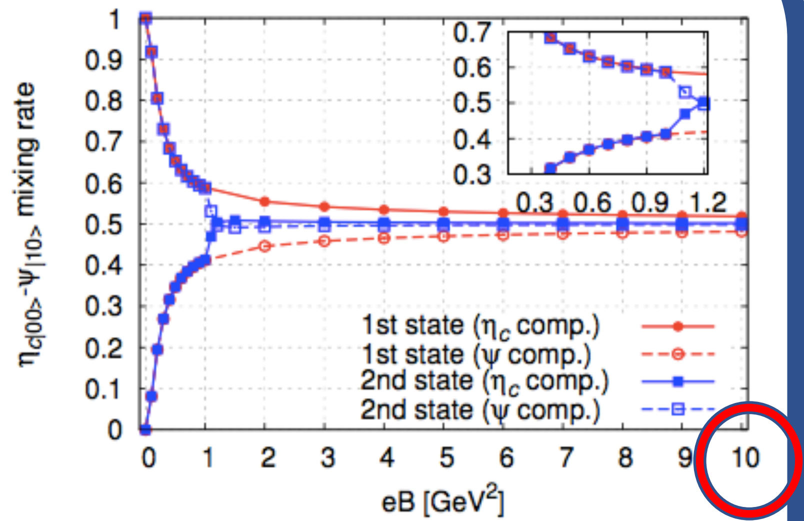
T. Yoshida and K. Suzuki, Phys. Rev. D94, 074043 (2016)

↓ mass spectra

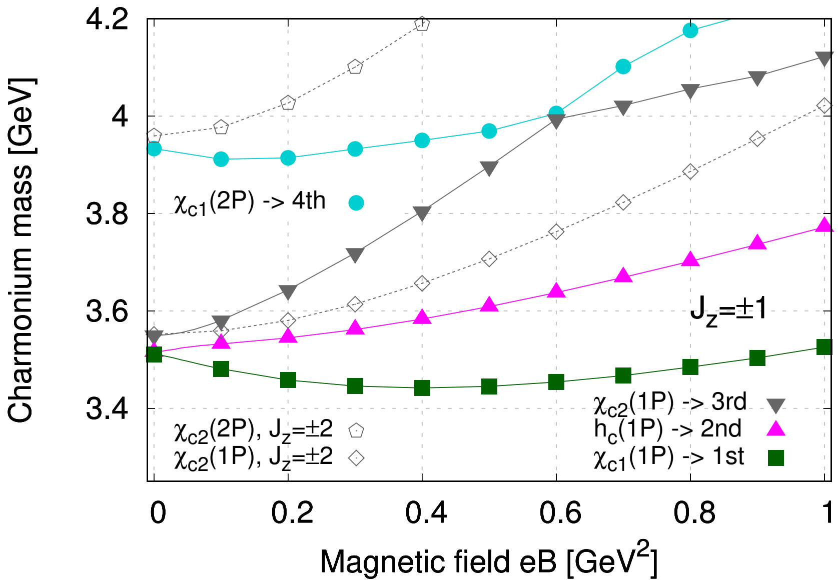
↓mixing ratios

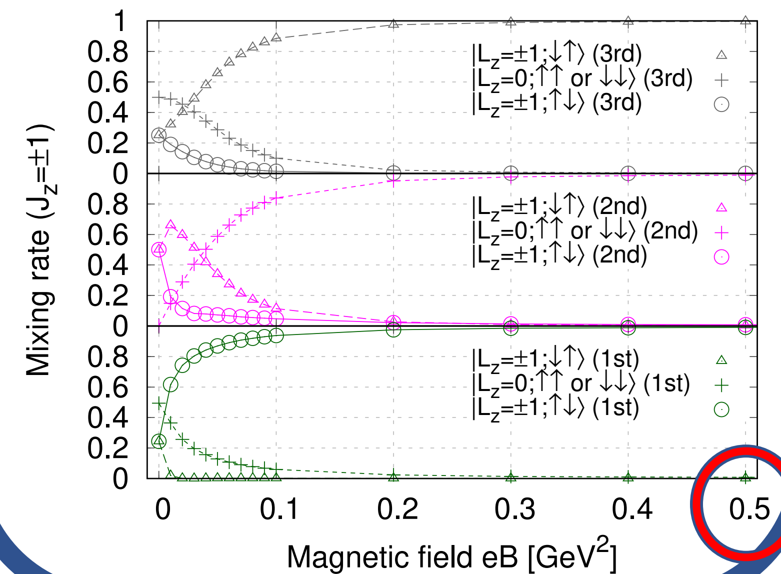
→ S-wave





→ P-wave





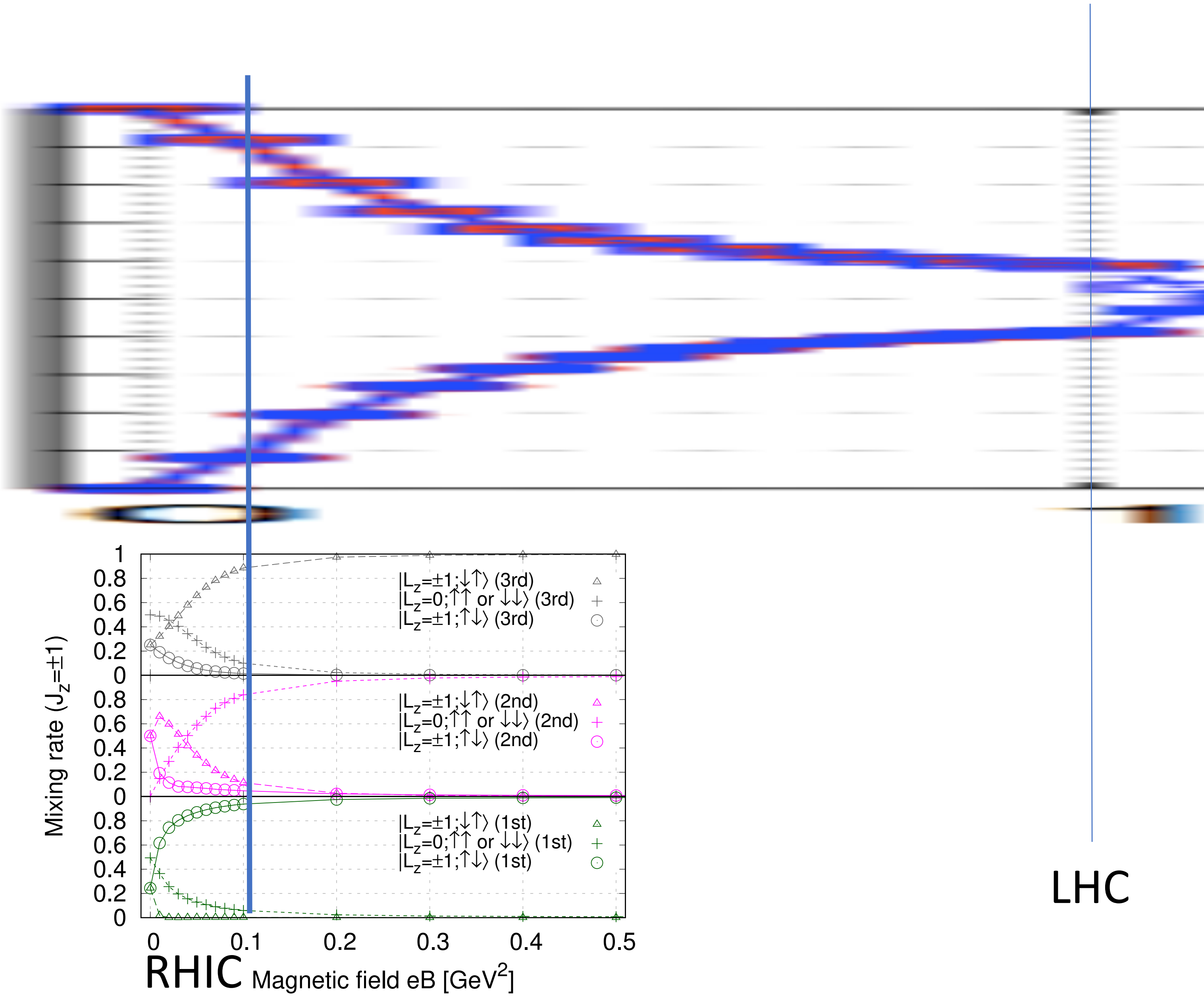
Comparing with another research …

→

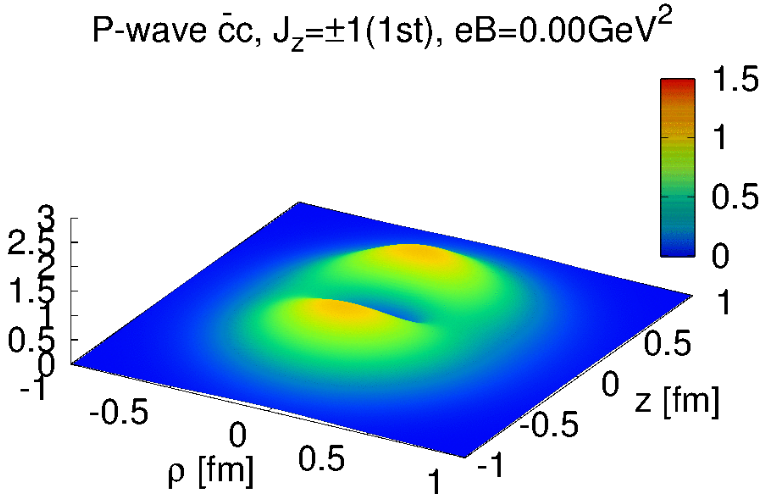
S-wave

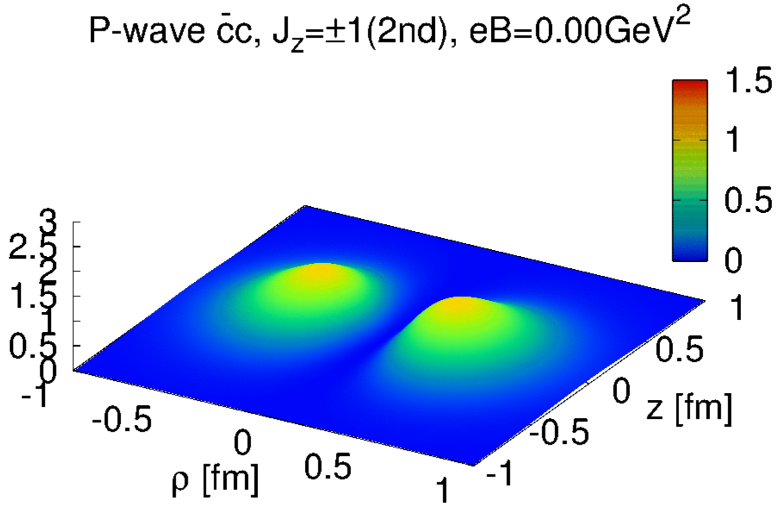
→

P-wave



Results ($J_z = \pm 1$, PB effect)





- WF's start to deform from $eB \sim 0.01\mathrm{GeV}^2$
- Max strength of MF in LHC: $|eB| \sim 1.0\mathrm{GeV}^2$
  RHIC: $|eB| \sim 0.1\mathrm{GeV}^2$
  SPS: $|eB| \sim 0.01\mathrm{GeV}^2$

⇒ Deformations of wave functions can be detectable

anisotropic decay – to observe PB effect –

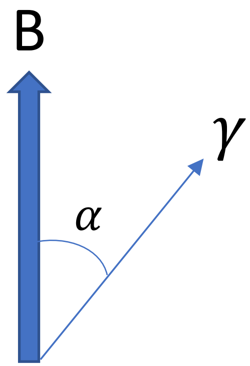
Photon radiation

E1 decay operator : $r \cdot \epsilon^{\pm}$

$\epsilon^{\pm} = \pm \frac{1}{\sqrt{2}}(1, \pm i, 0)$: polarization vector

with z-axis along photon momentum

(Take z-axis parallel to the magnetic field)



For $L_z = 0$ state, we have

$$\langle S | r \cdot \epsilon^{\pm} | P; L_z = 0 \rangle \propto \sin\alpha$$

For $L_z = \pm 1$ state, we have

$$\langle S | r \cdot \epsilon^{\pm} | P; L_z = +1 \rangle \propto \cos\alpha \pm 1$$

$$\langle S | r \cdot \epsilon^{\pm} | P; L_z = -1 \rangle \propto \cos\alpha \mp 1$$

As $|eB|$ gets large, states are purified into $|\mathrm{L_z}; \mathrm{S_{1z}} \mathrm{S_{2z}}\rangle$ ones

There's possibility to see such anisotropic decay from each state

Summary

- Motivation: to measure MF in HIC
- Calculation: P-wave charmonia in strong MF

$$\hat{H}_{\rm rel} = \hat{H}_{\rm diag} + \hat{H}_{\rm m.m.} + \hat{V}_{\rm LS} + \hat{V}_{\rm T},$$

- We prepared basis for P-wave:

$$\Psi^{P}_{\rm cyl}(\rho, z, \phi) = N r Y^{L=1}_{L_z}(\theta, \phi) e^{-\beta\rho^2 - \gamma z^2}$$

- We confirmed PBE occurs also in hadronic system
- HPBE leads to anisotropic decay

Prospect

- Go toward the realistic calculation
  - To consider time-dependence, electrical field, thermal effects, …

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Calculation method 4/5

Generalized eigenvalue problem

Usual eigenvalue problem： $H\boldsymbol{c} = E\boldsymbol{c}$

When we use orthogonal basis functions $\Psi_i (i = 1, \cdots, n)$,

$$\begin{pmatrix} \langle \Psi_1 | \mathcal{H} | \Psi_1 \rangle & \cdots & \langle \Psi_1 | \mathcal{H} | \Psi_n \rangle \\ \vdots & \ddots & \vdots \\ \langle \Psi_\mathrm{n} | \mathcal{H} | \Psi_1 \rangle & \cdots & \langle \Psi_\mathrm{n} | \mathcal{H} | \Psi_n \rangle \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} = E \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix}$$

But now we use non-orthogonal basis: $\langle \Psi_\mathrm{i} | \Psi_\mathrm{j} \rangle \neq \delta_{ij}$,

$$\begin{pmatrix} \langle \Psi_1 | \mathcal{H} | \Psi_1 \rangle & \cdots & \langle \Psi_1 | \mathcal{H} | \Psi_n \rangle \\ \vdots & \ddots & \vdots \\ \langle \Psi_\mathrm{n} | \mathcal{H} | \Psi_1 \rangle & \cdots & \langle \Psi_\mathrm{n} | \mathcal{H} | \Psi_n \rangle \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} = E \begin{pmatrix} \langle \Psi_1 | \Psi_1 \rangle & \cdots & \langle \Psi_1 | \Psi_\mathrm{n} \rangle \\ \vdots & \ddots & \vdots \\ \langle \Psi_\mathrm{n} | \Psi_1 \rangle & \cdots & \langle \Psi_\mathrm{n} | \Psi_\mathrm{n} \rangle \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix}$$

The form of $H\boldsymbol{c} = EN\boldsymbol{c}$: generalized eigenvalue problem

➡ We use Cholesky decomposition

Calculation method 5/5
Cholesky decomposition

- If we have $N^{-1}H|\Psi\rangle = E|\Psi\rangle$,
  then $N^{-1}H$ is not symmetric matrix
  → technically difficult to diagonalize

Norm matrix N:real, symmetric, and positive
so that we can decompose N as

$$N = U^T U$$

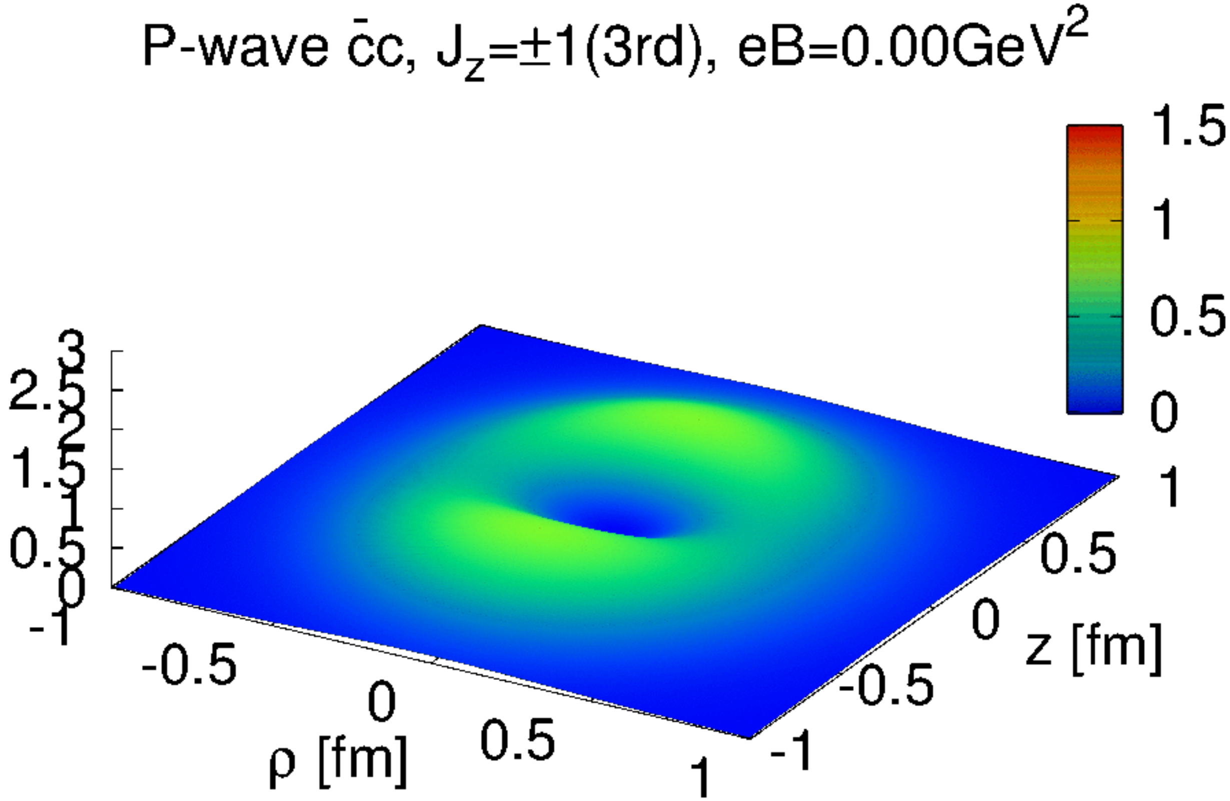
U:upper triangular matrix only w/ positive diagonal components

↓ Here,

- Eigenvalues by $H' := (U^T)^{-1} H U^{-1}$ is same as
  those by $H|\Psi\rangle = EN|\Psi\rangle$
- H : symmetric → H' : symmetric

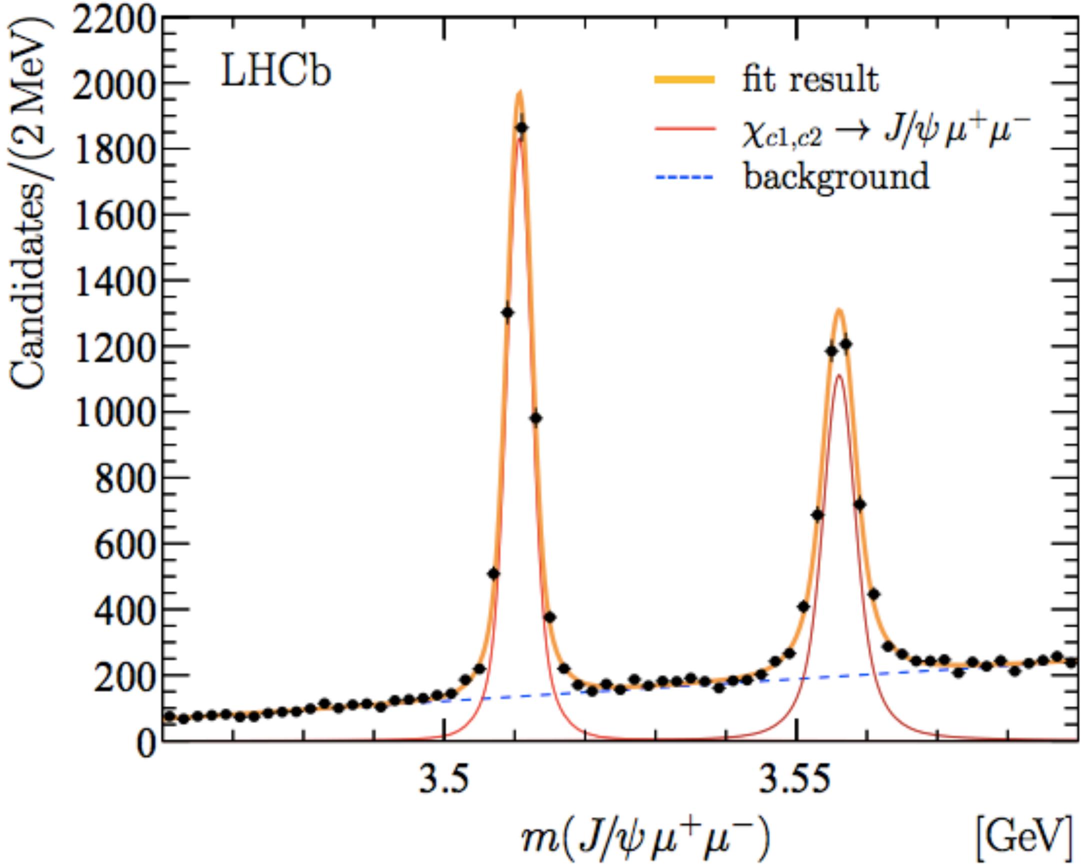
We can separate the calculation

Slide in case : PB effect on $J_z = \pm 1$, 3rd



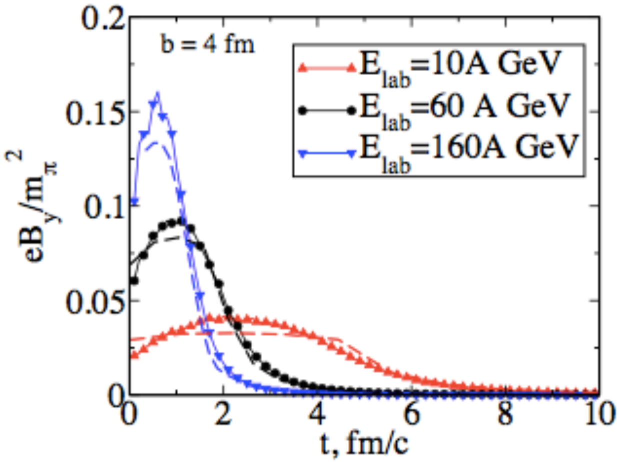
P波チャーモニウムの観測

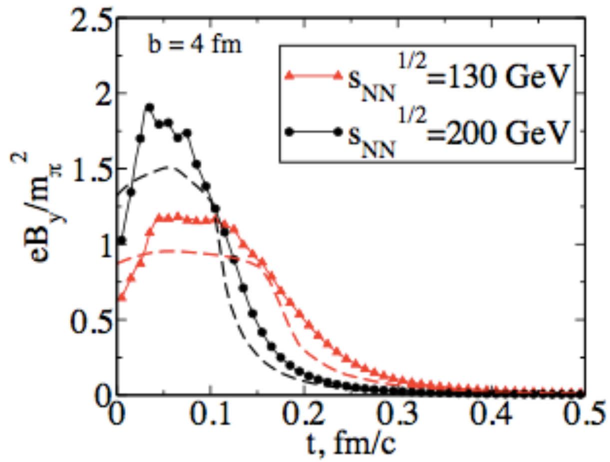
$\chi_{c1,c2} \to J/\Psi\mu^+\mu^-$ の崩壊ははっきり観測できる



1709.04247

LHC, RHIC, SPS での磁場の詳細



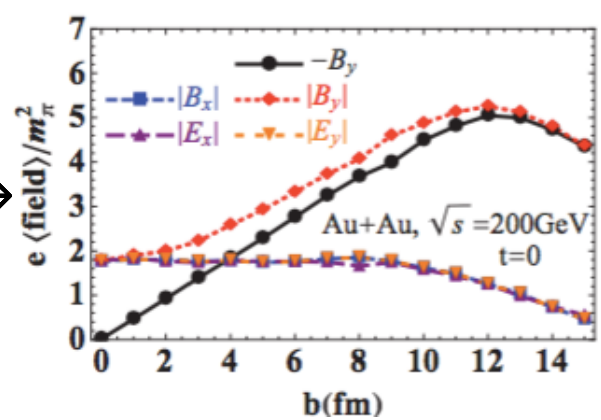


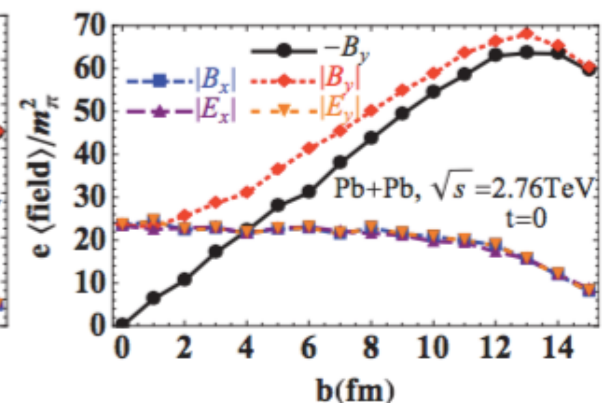
←磁場の時間発展
(衝突パラメータ固定)

Int. J. Mod. Phys. A24 (2009) 5925-5932

衝突時の磁場の
衝突パラメータ依存性→

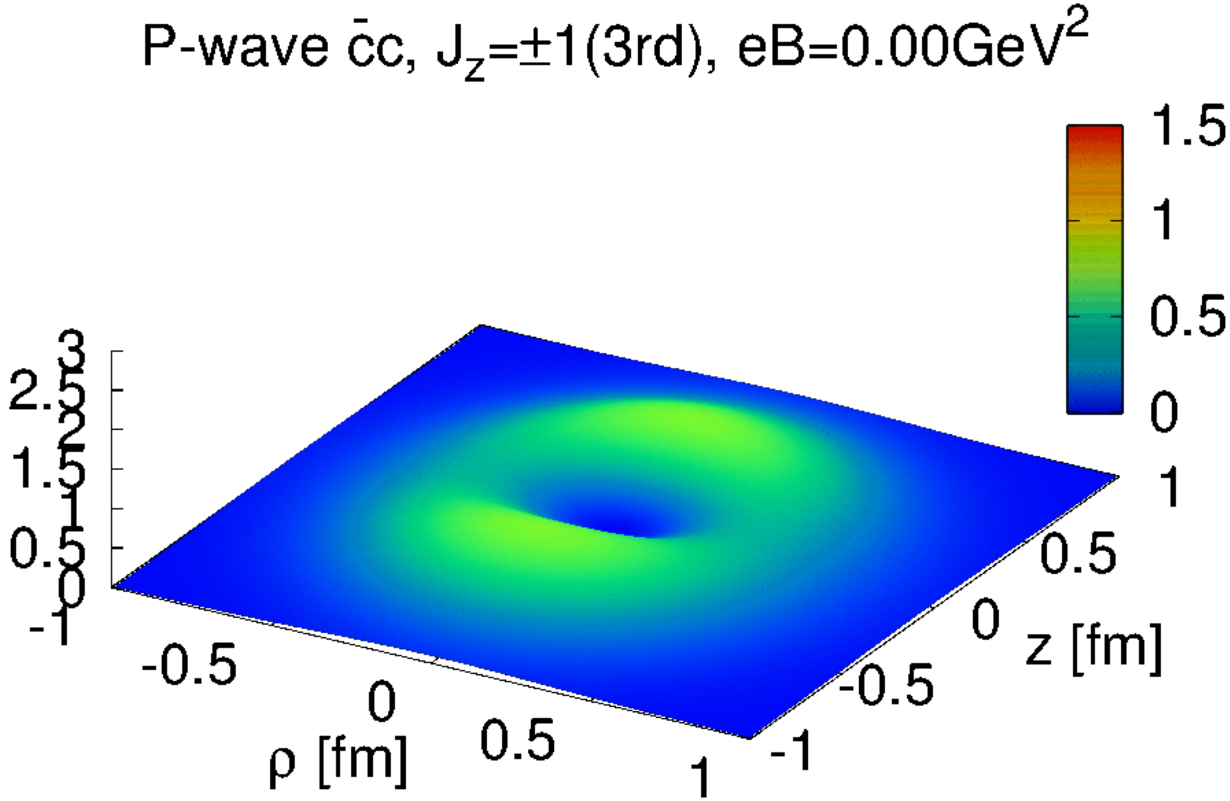
Phys. Rev. C **85**, 044907 (2012)



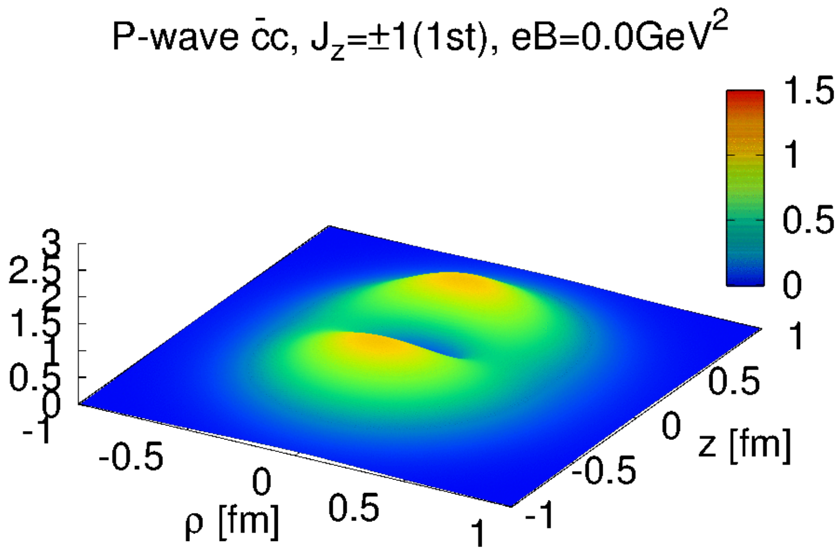


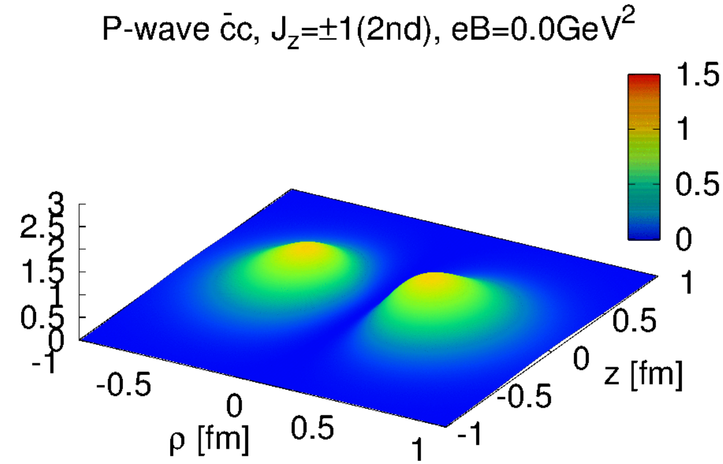
↑SPS の磁場　　↑RHIC の磁場　　↑LHCの磁場

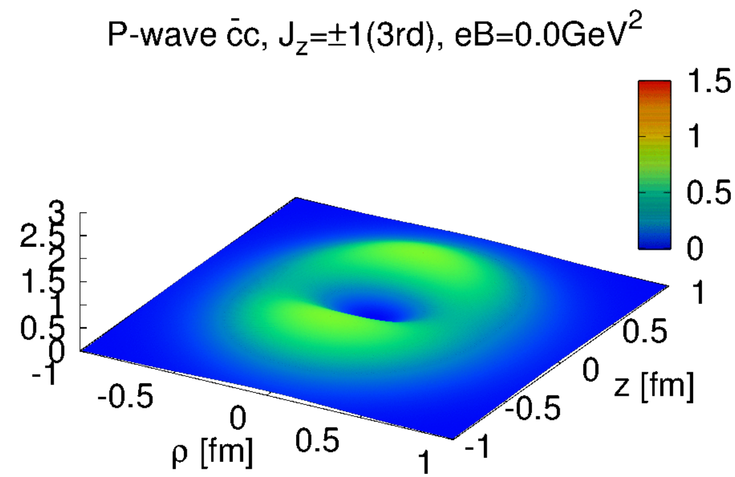
予備スライド：$J_z = \pm 1$, 3rdでのパッシェンバック効果



Slide in case : results on $J_z = \pm 1$
0.0, 0.1, …, 1.0 GeV のGIF(1^{st}~3^{rd})







Slide in case : results on $J_z = \pm 1$
GIF of 0.0, 0.1, …, 1.0 GeV (4th)