A unifying pion mean-field approach

:Electromagnetic properties of light and singly-heavy baryons

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ollaboration with Ghil-Seok Yang, Hyun-Chul Kim, M. Oka

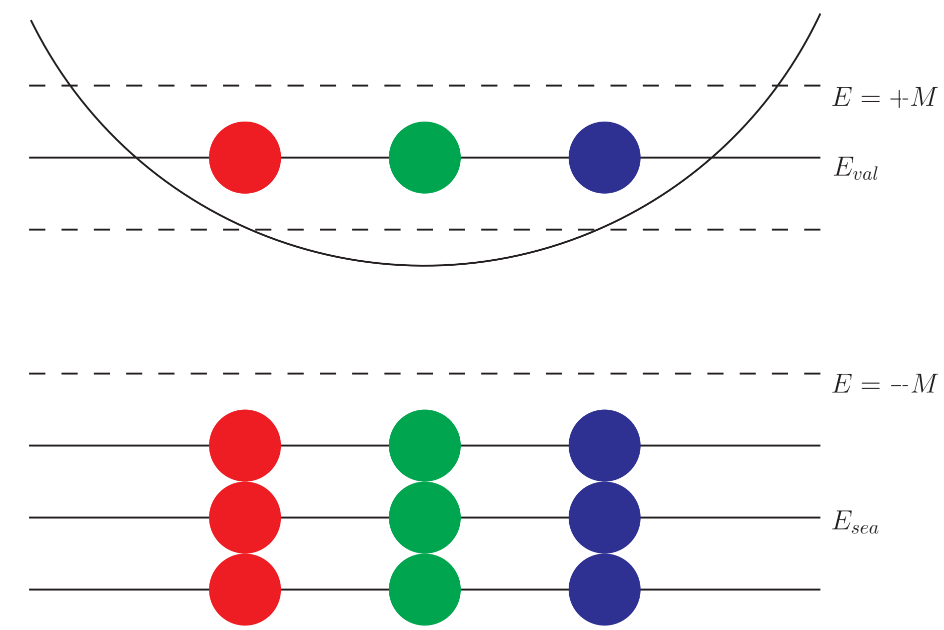
- ✓ A baryon can be viewed as a Nc valence quarks bound by mesonic mean field in the limit of large Nc - E.Witten
- ✓ The quantum fluctuation is suppressed by 1/Nc in that limit, which means that we ignore the interaction between mesons **(Mean-Field Approach)**
- ✓ It was realized by the Chiral Quark-Soliton Model.
- ✓ The chiral quark soliton model has successfully explained mass splittings and form factors of both light and singly-heavy baryons.

Light baryons

- Effective chiral action

$$S_{\text{eff}} = -N_c \mathrm{TrLn}(i\gamma_\mu \partial^\mu + i\hat{m} + iMU^{\gamma_5})$$

  - **Chiral symmetry without the mass term**
  - **Spontaneous chiral symmetry breaking**
  - **Explicit flavor SU(3) symmetry breaking**

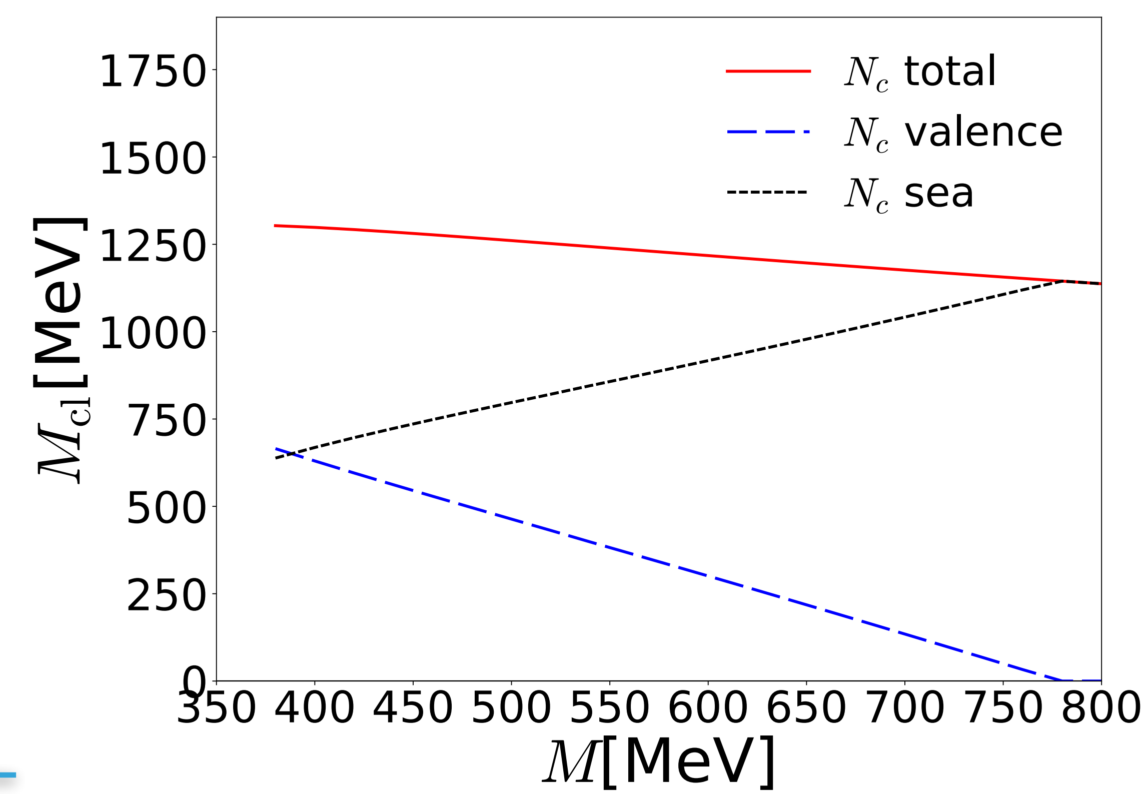


Nc Mean field

- Self-Consistent soliton solution

  - **Solving the saddle point equation (minimizing the effective chiral action).**

$$\frac{\delta S_{\text{eff}}(U)}{\delta U}\Big|_{U=U_c} = 0 \qquad M_{\text{cl}} \approx 1300 \text{ MeV}$$



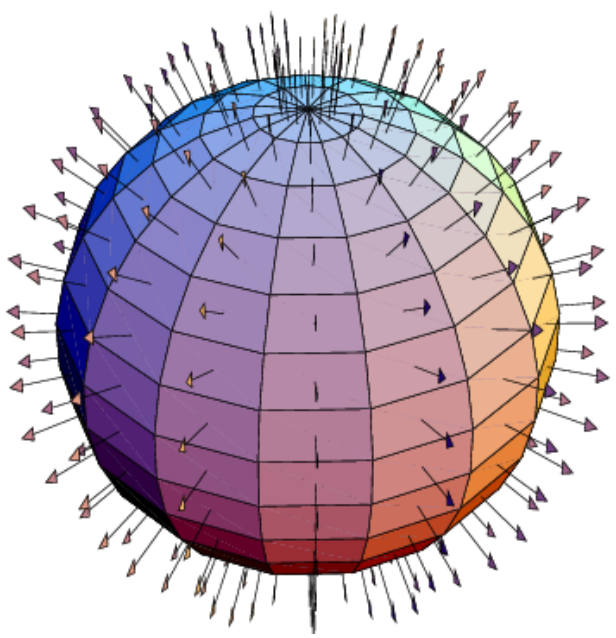
- Hedgehog Ansatz

$$U_c = e^{i\gamma_5 n^a \tau^a \pi(r)} \qquad U(\boldsymbol{r}) = \begin{pmatrix} U_c(\boldsymbol{r}) & 0 \\ 0 & 1 \end{pmatrix}.$$

Witten's embedding

√ **Trivial embedding**

√ **Hedgehog symmetry**



- Zero-mode quantization

$$\langle J_N(\boldsymbol{x}, T/2) J_N^\dagger(\boldsymbol{y}, -T/2)\rangle_0 \sim \Pi_N(T) \sim e^{-M_N T}$$

√ **Nucleon correlation function**

$$\rightarrow \int \mathcal{D}U[\cdots] \rightarrow \int \mathcal{D}A \int \mathcal{D}\vec{z}[\cdots]$$

√ **Saddle point approximation** and **Zero-mode quantization**

$$U(\vec{x}, t) = A(t) U_c(\vec{x} - \vec{z}(t)) A^\dagger(t)$$

√ **Nc valence quarks gives baryon number**.

$$\vec{J} + \vec{T} = 0 \qquad Y' = \frac{N_c}{3}$$

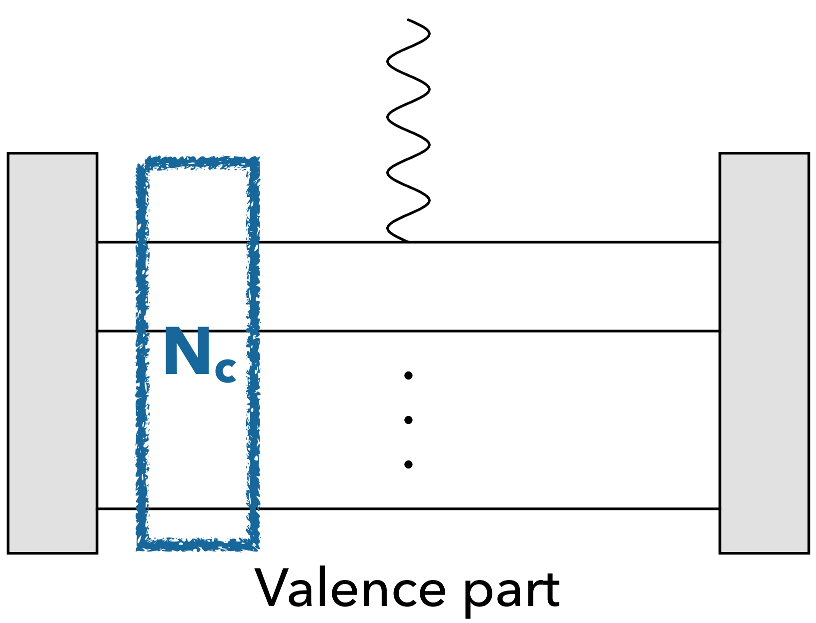
Quantization rule

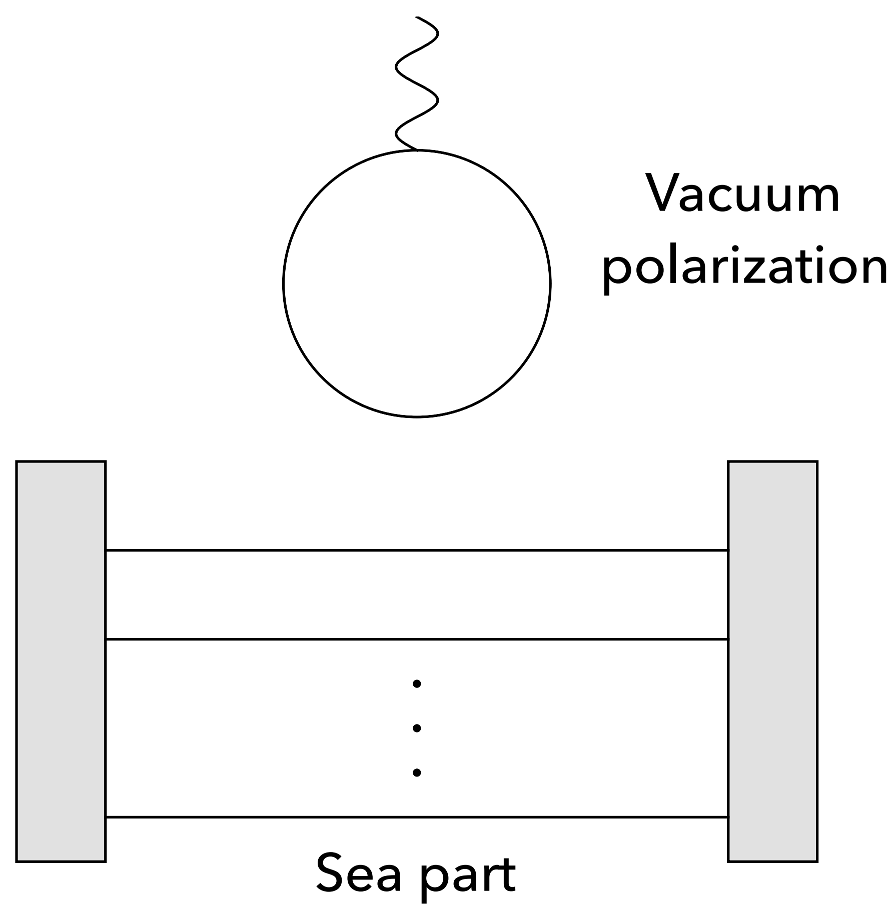
- Electromagnetic current

$$J_\mu(x) = \overline{\psi}(x)\gamma_\mu \hat{\mathcal{Q}}\psi(x) \qquad \text{where} \qquad \hat{\mathcal{Q}} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} = \frac{1}{2}\left(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right)$$

- The matrix elements of the electromagnetic form factors are found by using the **functional integral**.

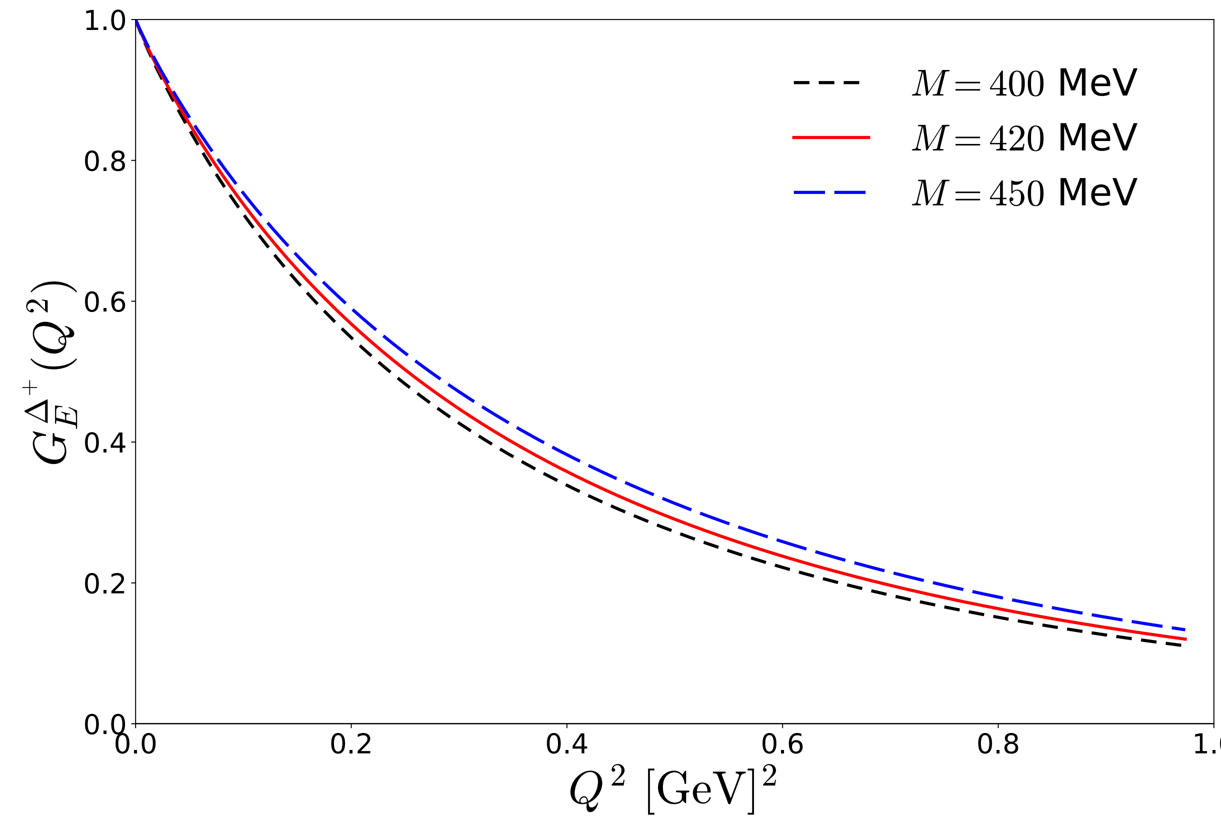
Three-point correlation function for a baryon form factor





ELECTROMAGNETIC PROPERTIES OF BARYON DECUPLET

- ✓ All form factors are insensitive to the value of constituent quark mass.
- ✓ The strange current quark mass was fixed by reproducing mass splittings of the lowest-lying light baryons.



$$G_{E0}(Q^2) = \int d^3z\, j_0(kr)\mathcal{G}_{E0}(\boldsymbol{z})$$

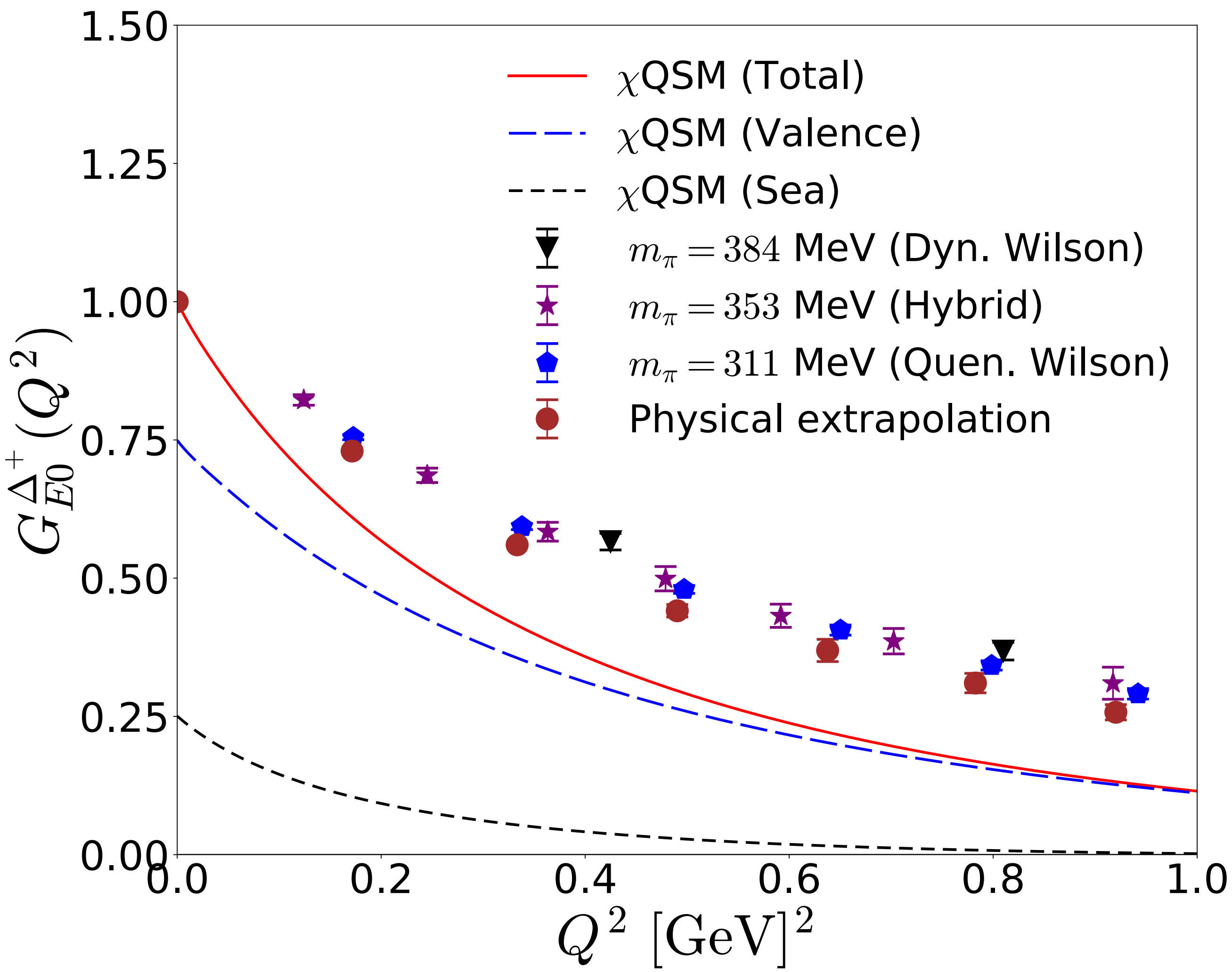
$$G_{E2}(Q^2) = 6\sqrt{5}\frac{M_B^2}{|\boldsymbol{q}|^2}\int d^3z\, j_2(|\boldsymbol{q}||\boldsymbol{z}|)\mathcal{G}_{E2}(\boldsymbol{z})$$

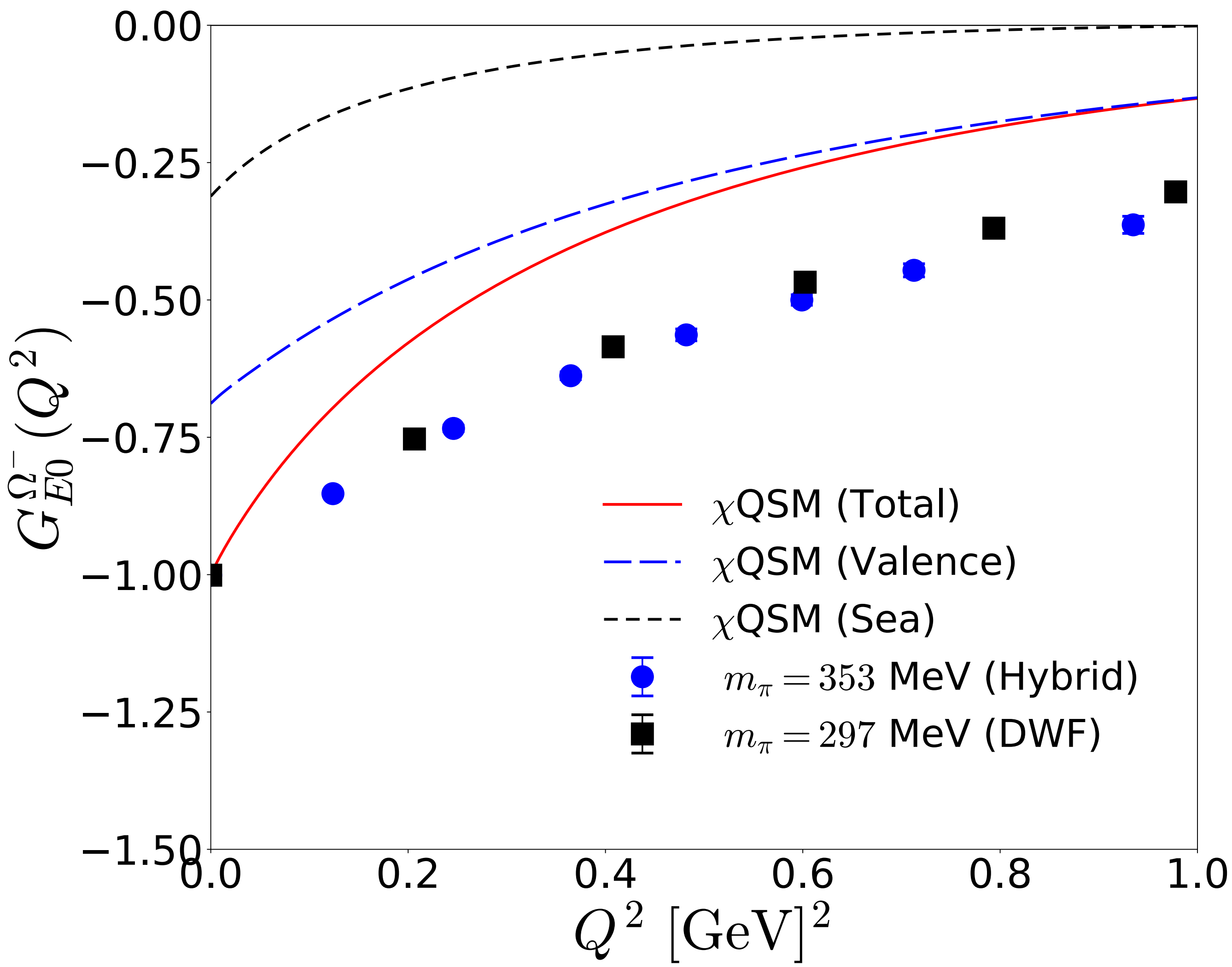
$$G_{M1}(Q^2) = -i2\sqrt{6\pi}\frac{M_B}{|\boldsymbol{q}|}\int d^3z\, j_1(|\boldsymbol{q}||\boldsymbol{z}|)\mathcal{G}_{M1}(\boldsymbol{z})$$

$$G_{M3}(Q^2) = -i20\sqrt{21\pi}\frac{M_B^3}{|\boldsymbol{q}|^3}\int d^3z\, j_3(|\boldsymbol{q}||\boldsymbol{z}|)\mathcal{G}_{M3}(\boldsymbol{z})$$

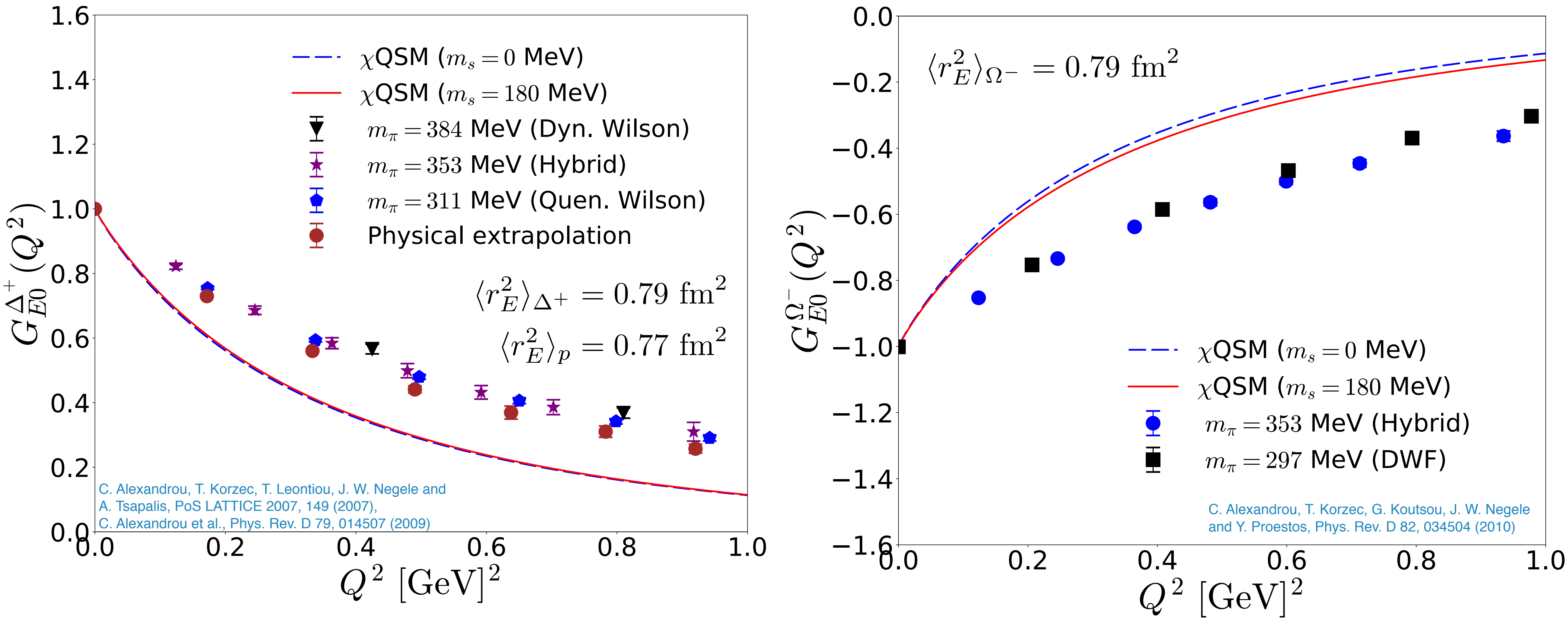
- ✓ Due to the hedgehog structure, the **magnetic octupole form factor is found to be zero.**

ELECTROMAGNETIC PROPERTIES OF BARYON DECUPLET





- ✓ The electric form factors of the baryon decuplet satisfy the **SU(3) Gell-Mann-Nishijima formula**. (symmetry-conserving quantization)
- ✓ The results from lattice QCD are known to fall off more slowly.
- ✓ The effects of the explicit SU(3) symmetry breaking are marginal.
- ✓ The **charge radii** of the baryon decuplet were also computed.



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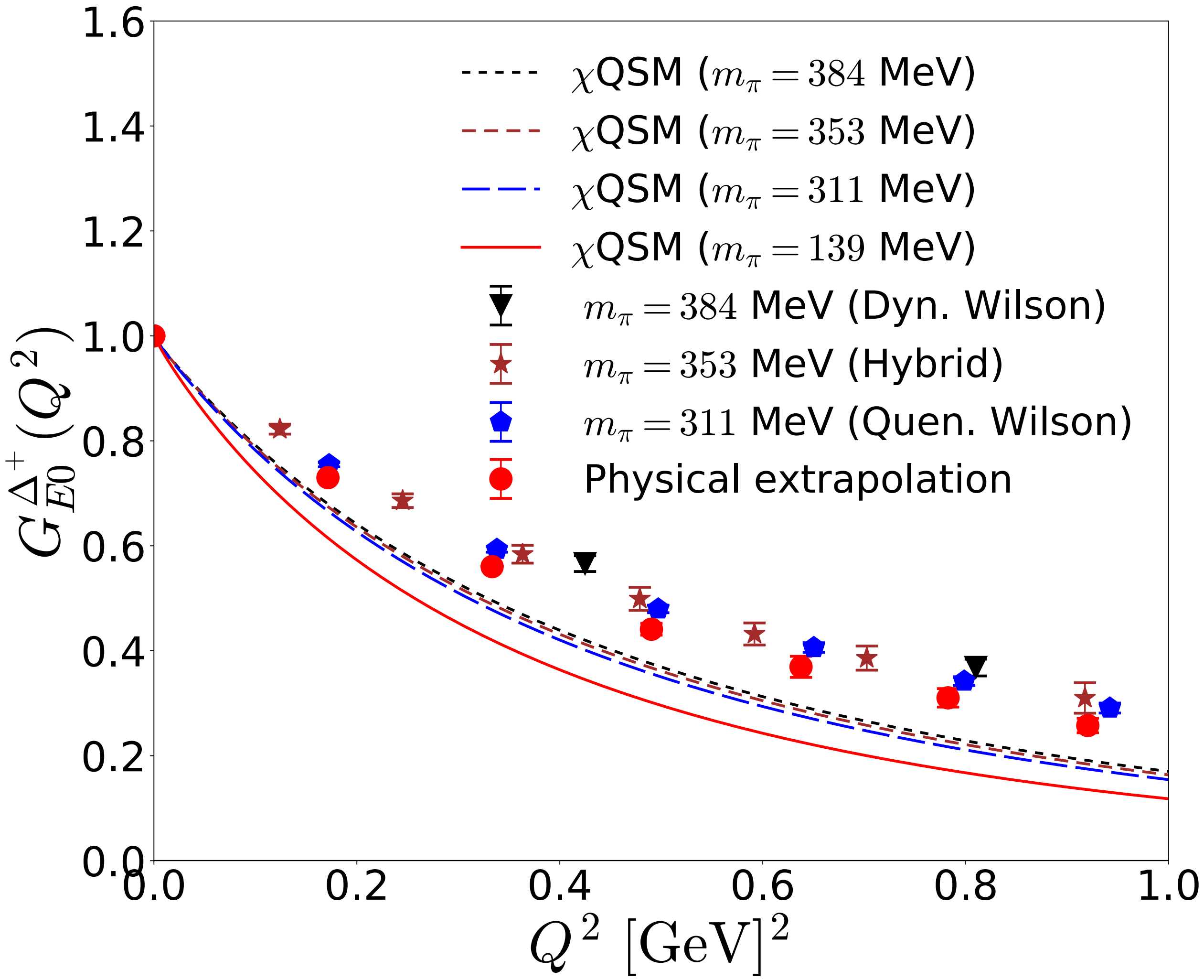

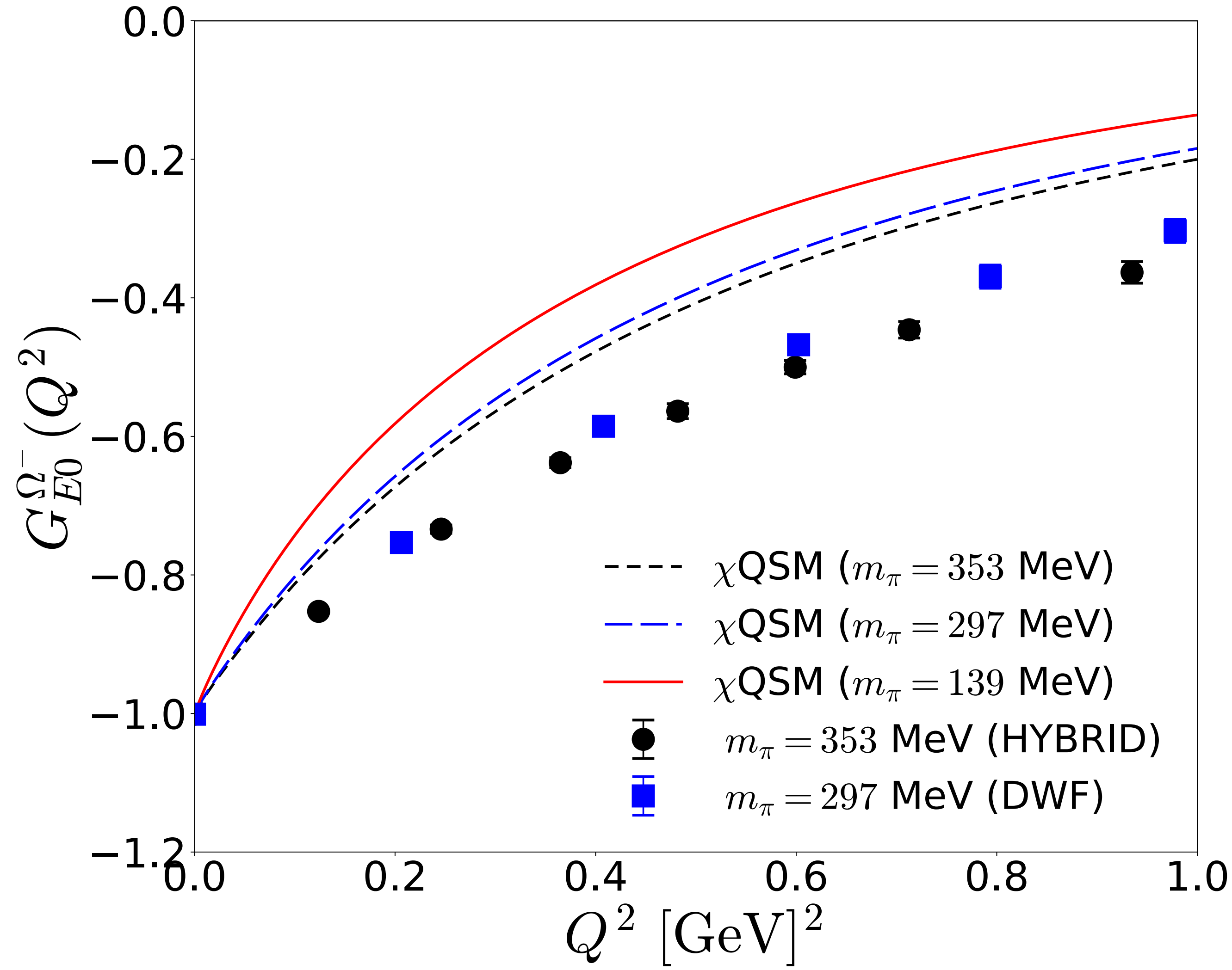

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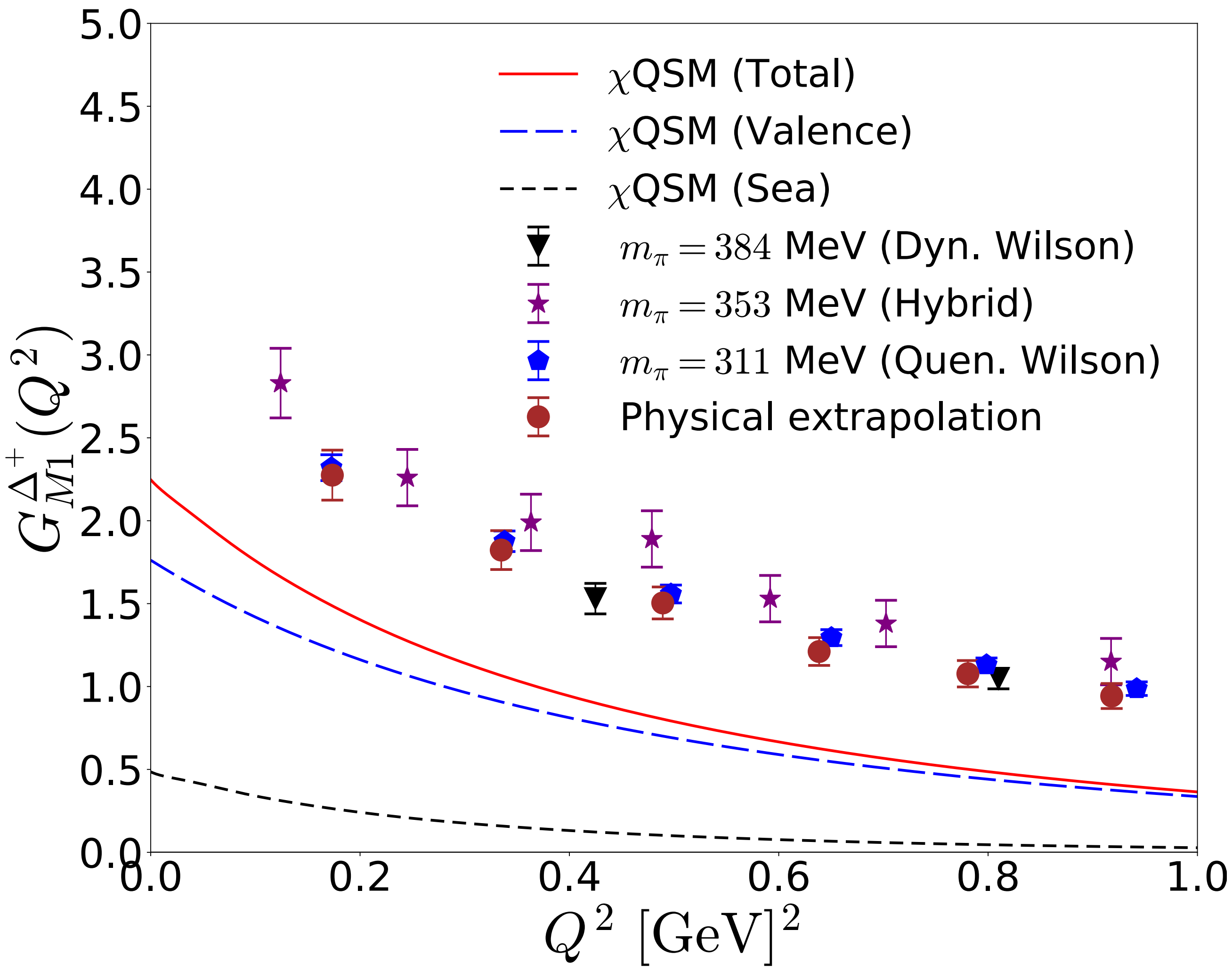

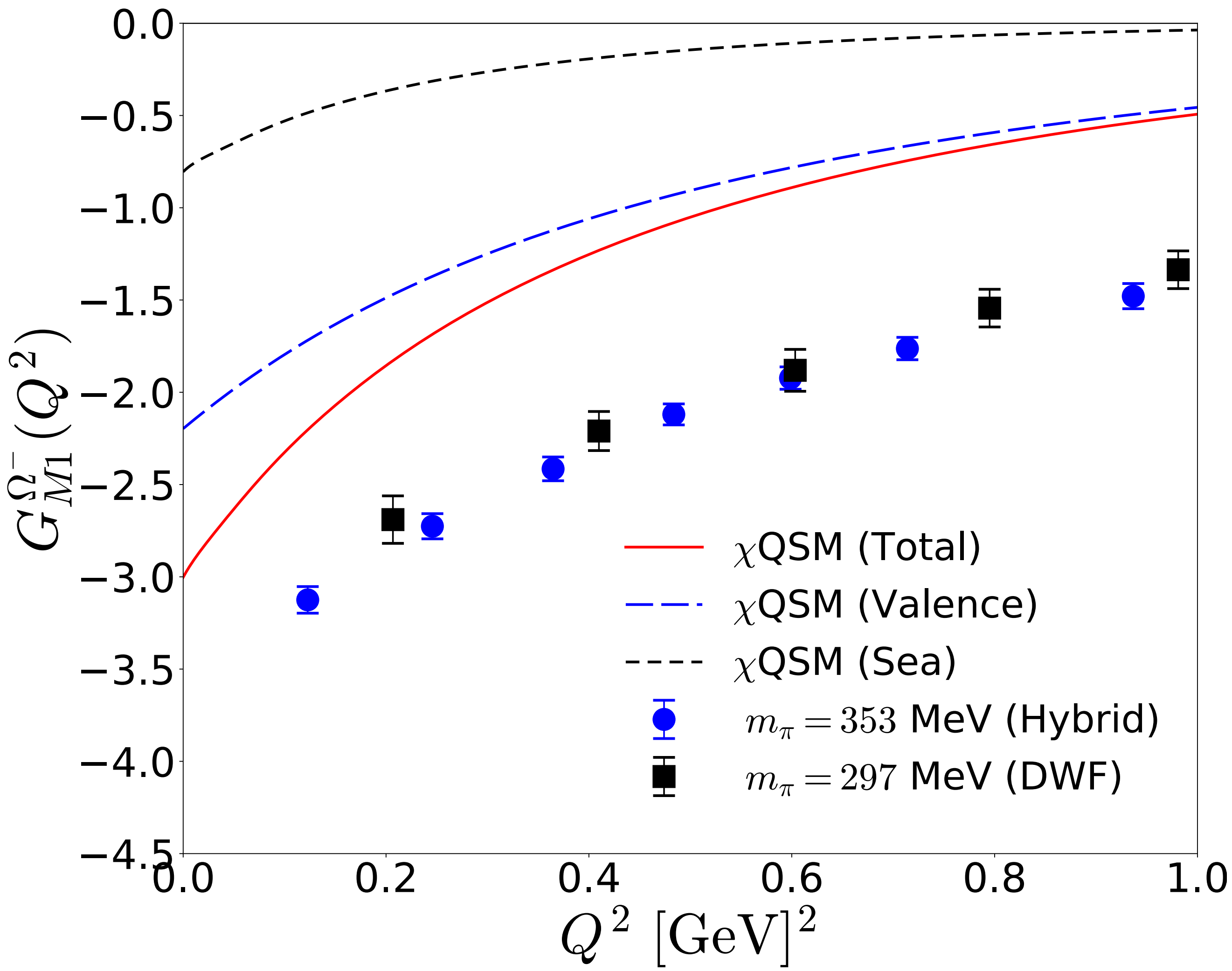

- ✓ The effects of the explicit SU(3) symmetry breaking are marginal.
- ✓ The **magnetic moments** of the baryon decuplet can be found from the magnetic form factors.
- ✓ The mass of the nuclear magneton could be replaced by that of the soliton to improve the results. However, we didn't do that.

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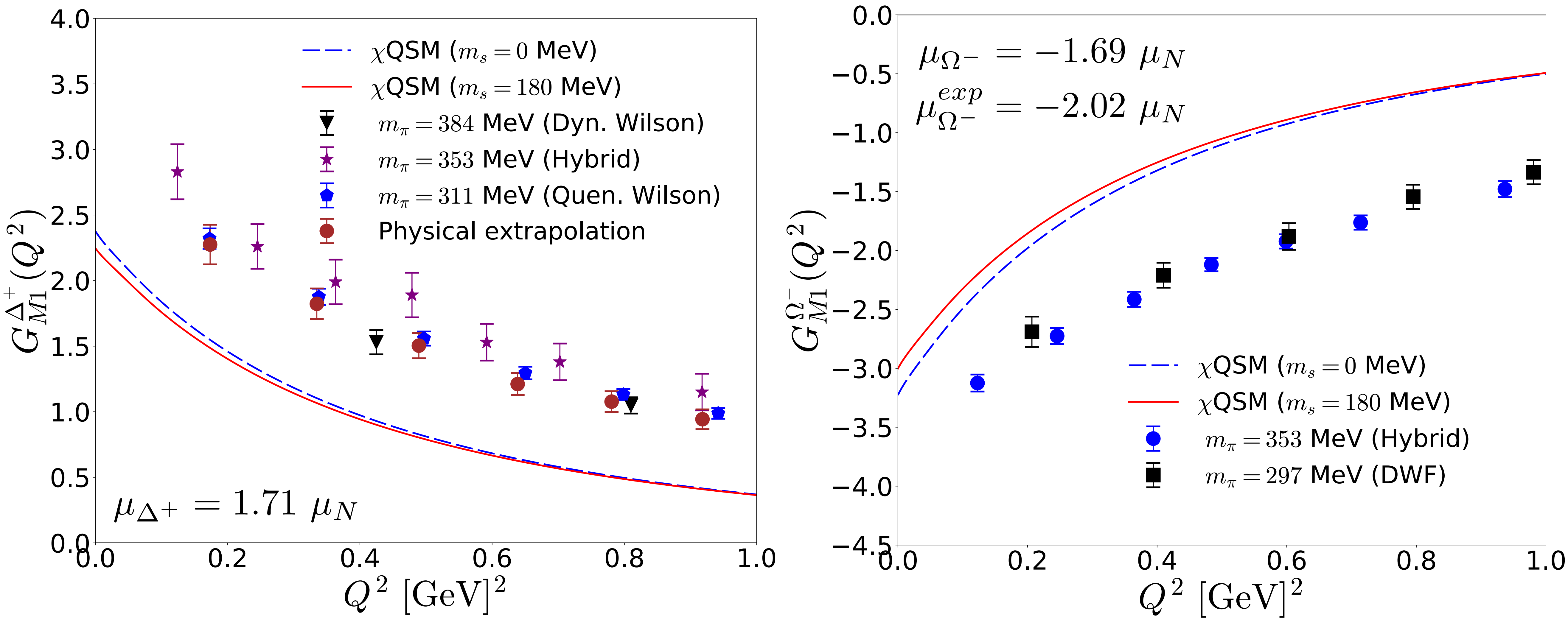

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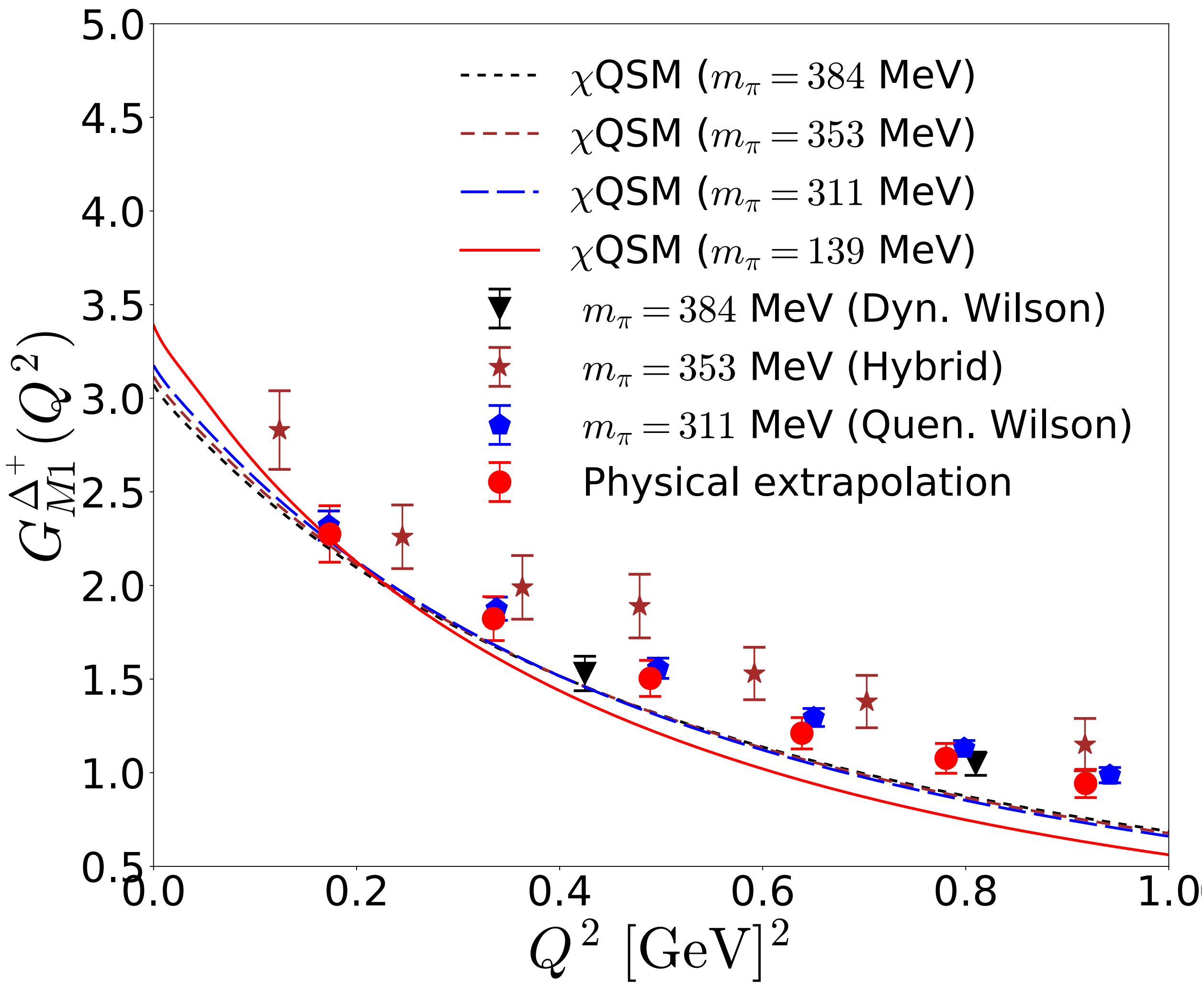

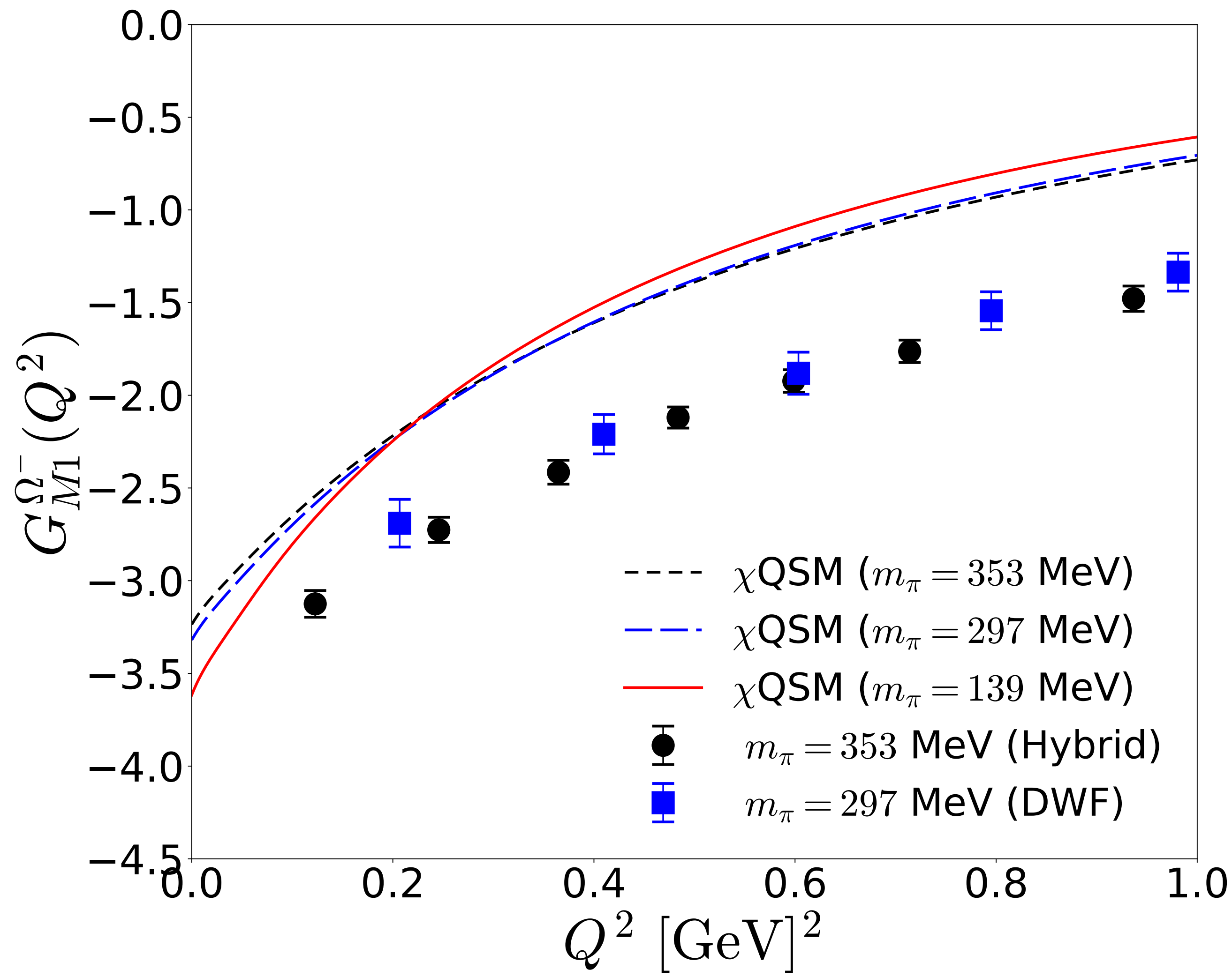

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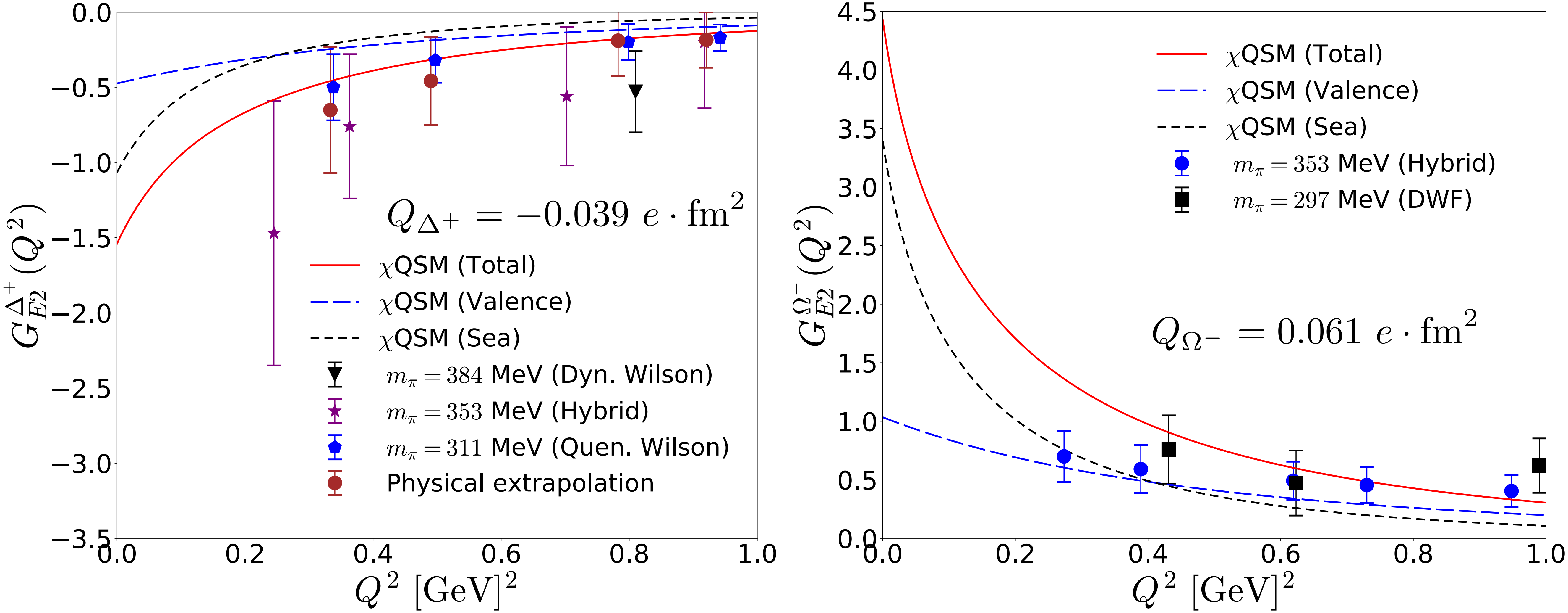

- In the case of the electric quadrupole form factor, the vacuum polarization contribution is dominant. It can be interpreted as follows: The members of the baryon decuplet may be deformed by the pion mean fields while the core part is governed by the valence quarks.
- The effects of the explicit SU(3) symmetry breaking are quite large.
- The **electric quadrupole moments** of the baryon decuplet were also computed.

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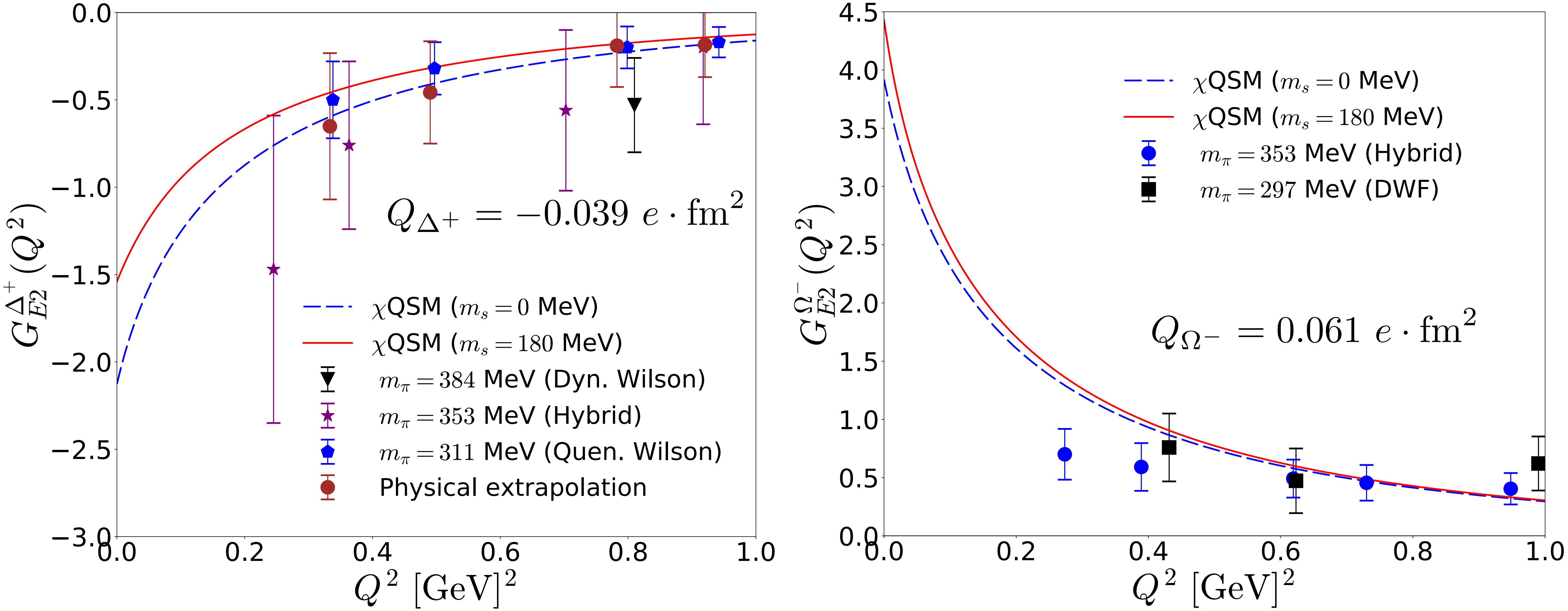

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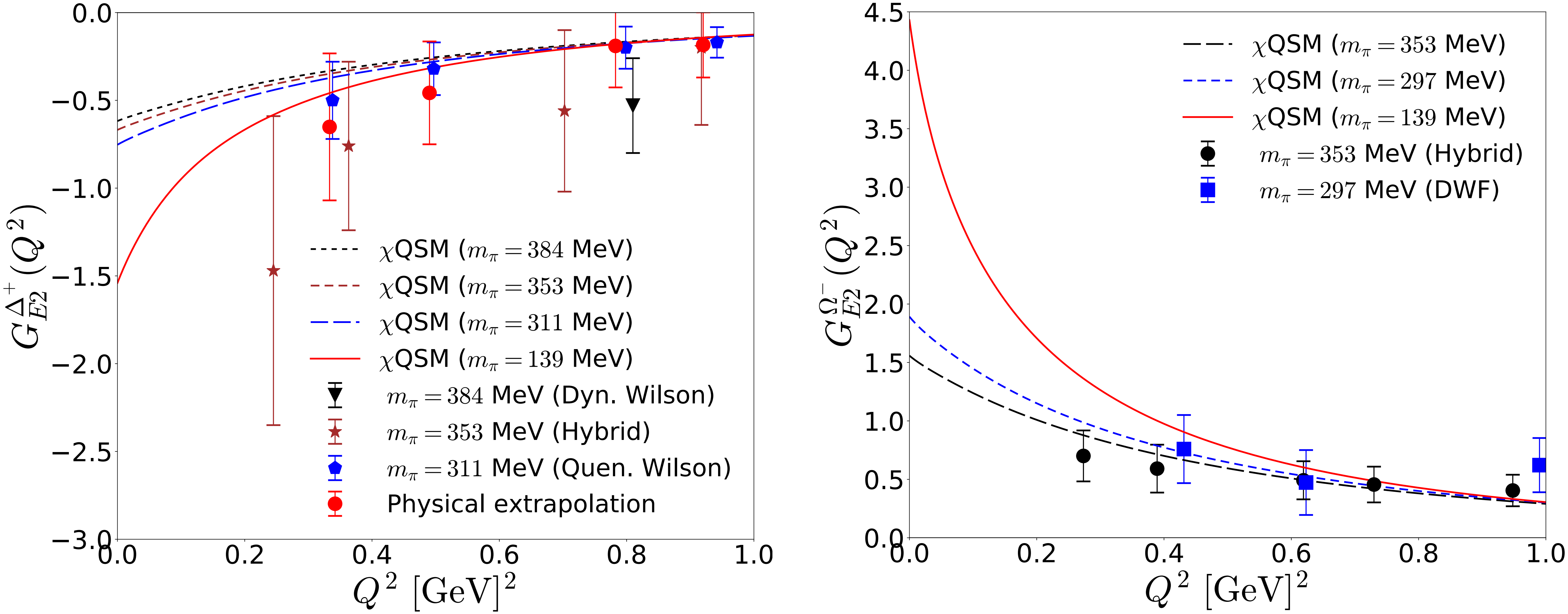

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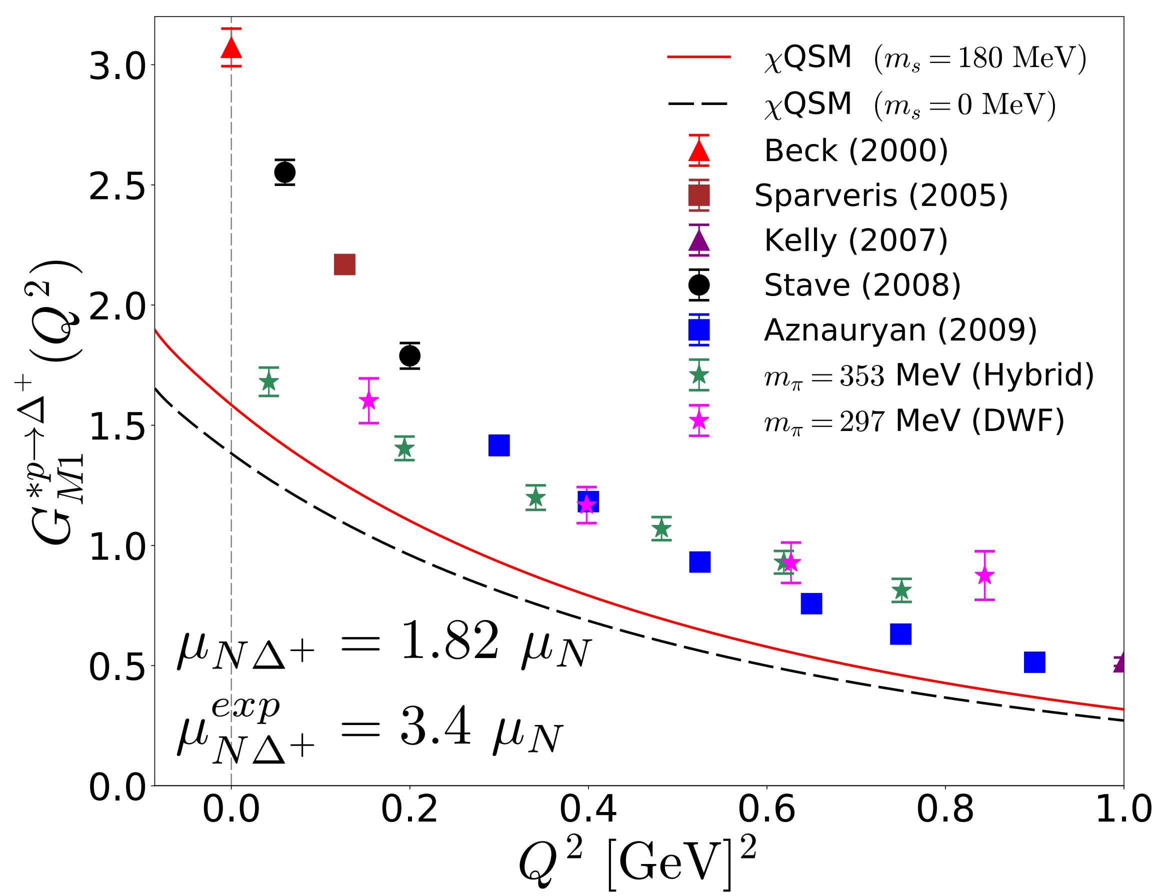
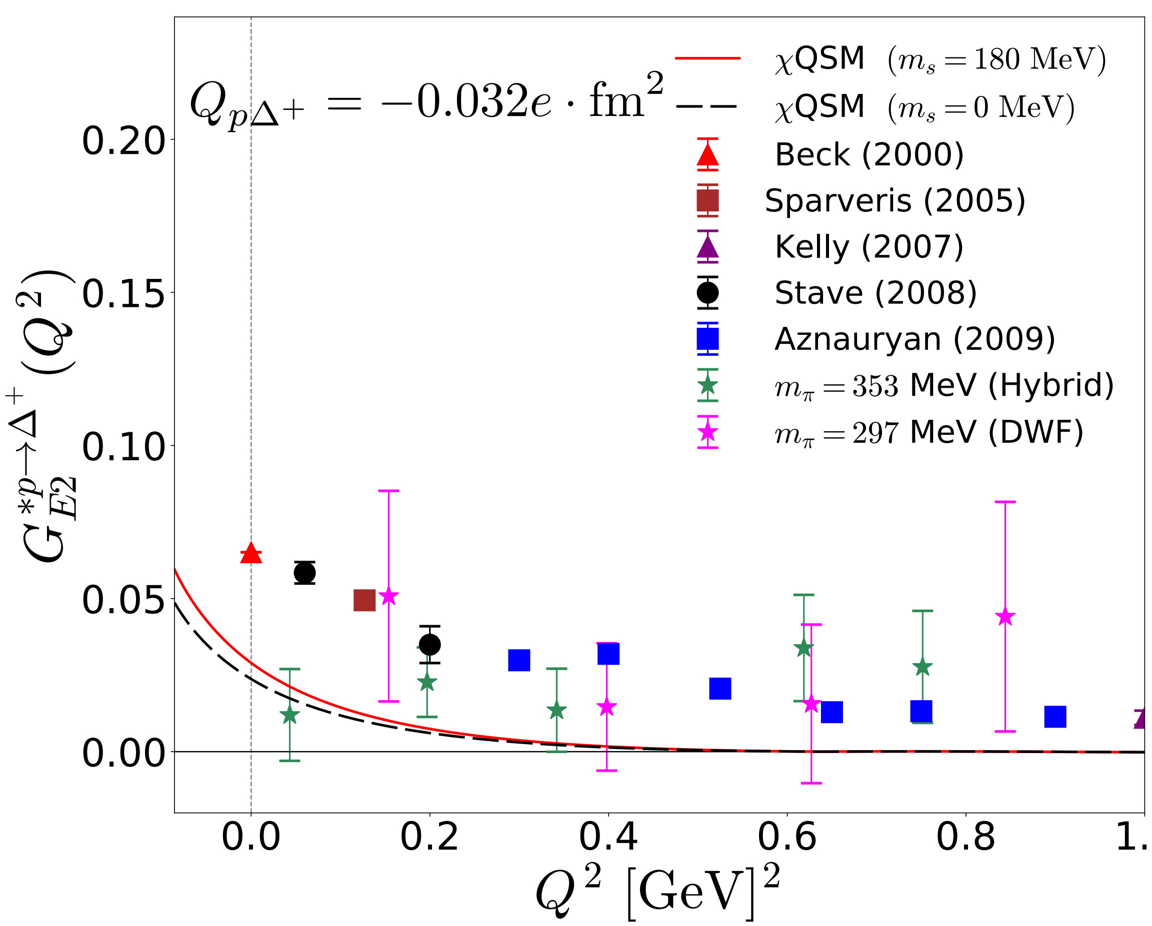
In collaboration with H.-Ch. Kim and M. Oka

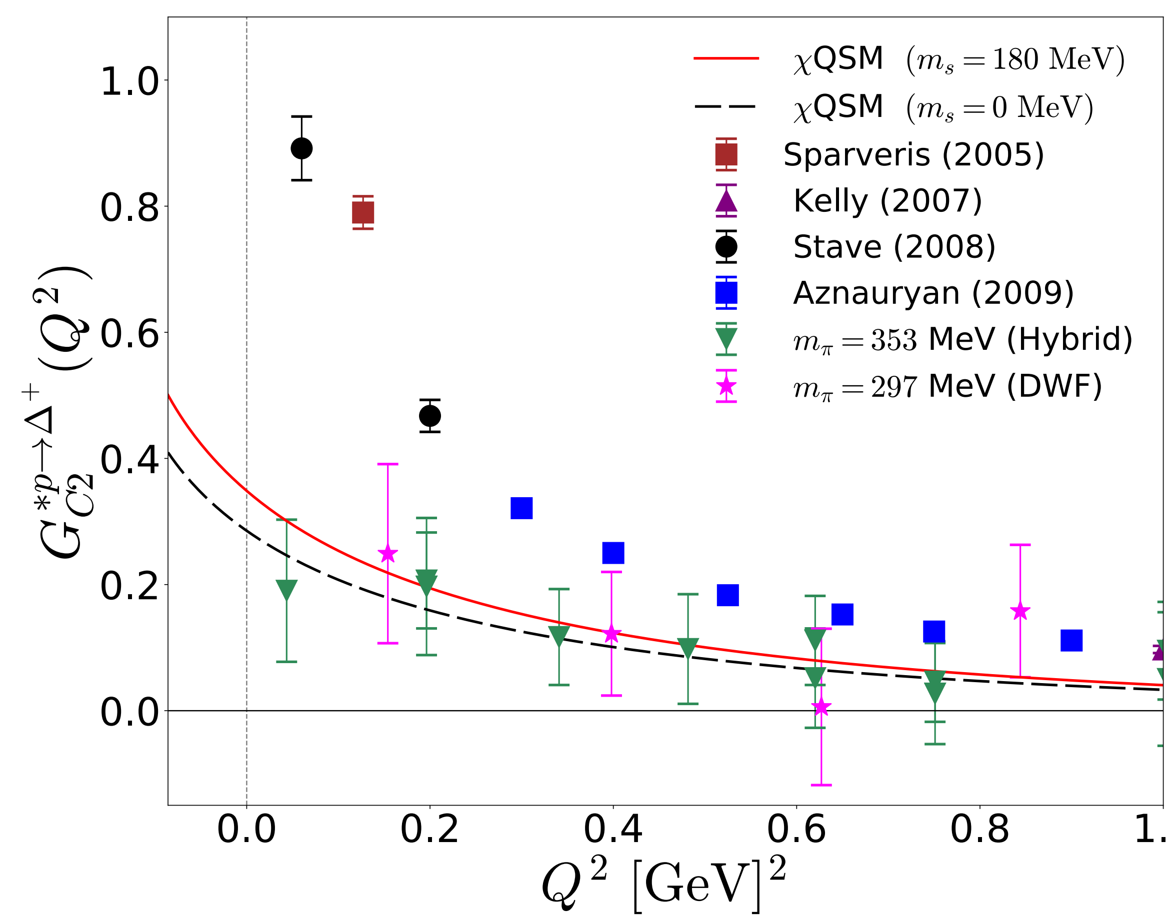
ELECTROMAGNETIC TRANSITIONS OF BARYON DECUPLET

- One-photon exchange approximation and the delta rest frame are used.
- The results of the M1 transition form factor is are underestimated in comparison with those of LQCD.
- At the pseudo-threshold,

$$G^*_{E2}(Q^2_{pt}) = \frac{M_{\Delta^+} - M_p}{2M_{\Delta^+}} G^*_{C2}(Q^2_{pt})$$







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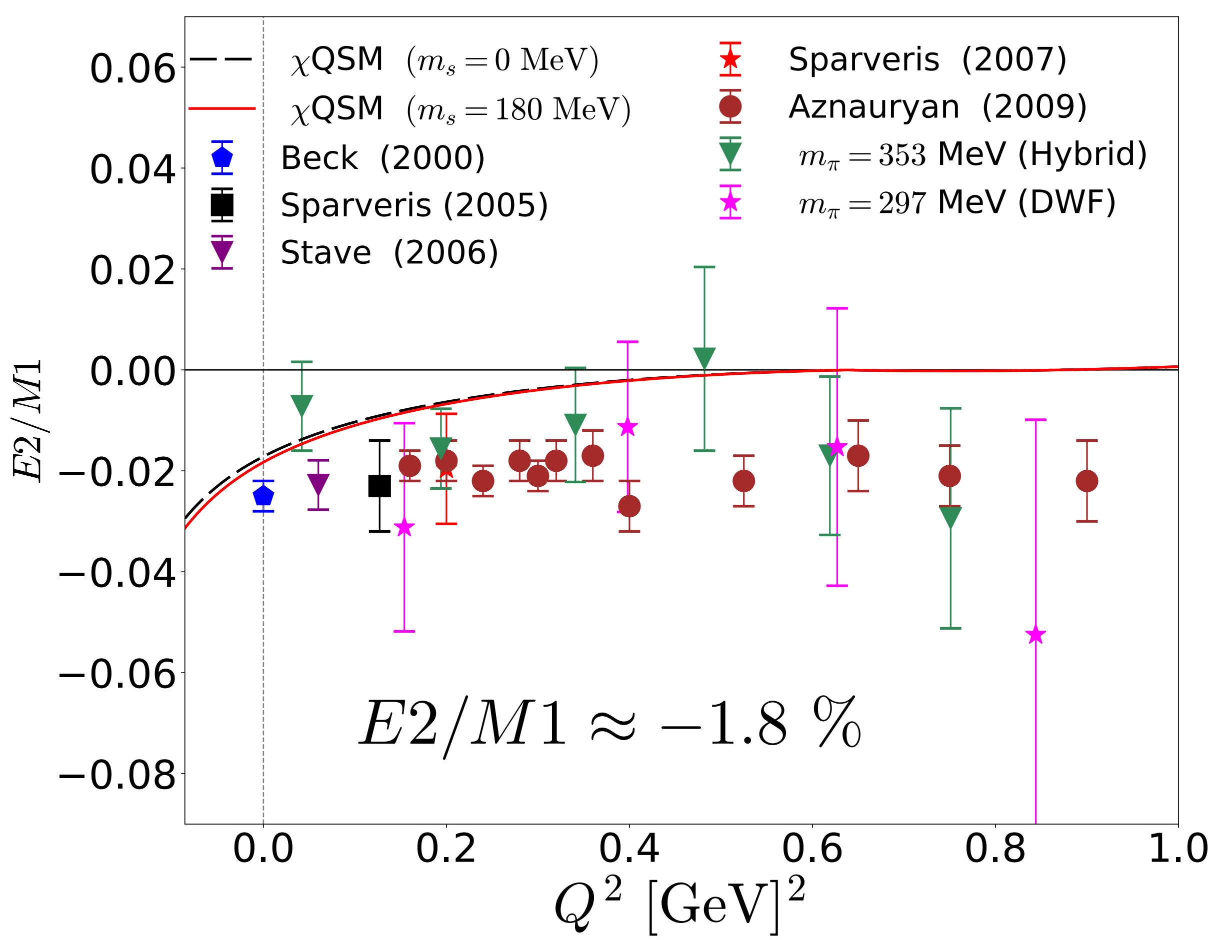
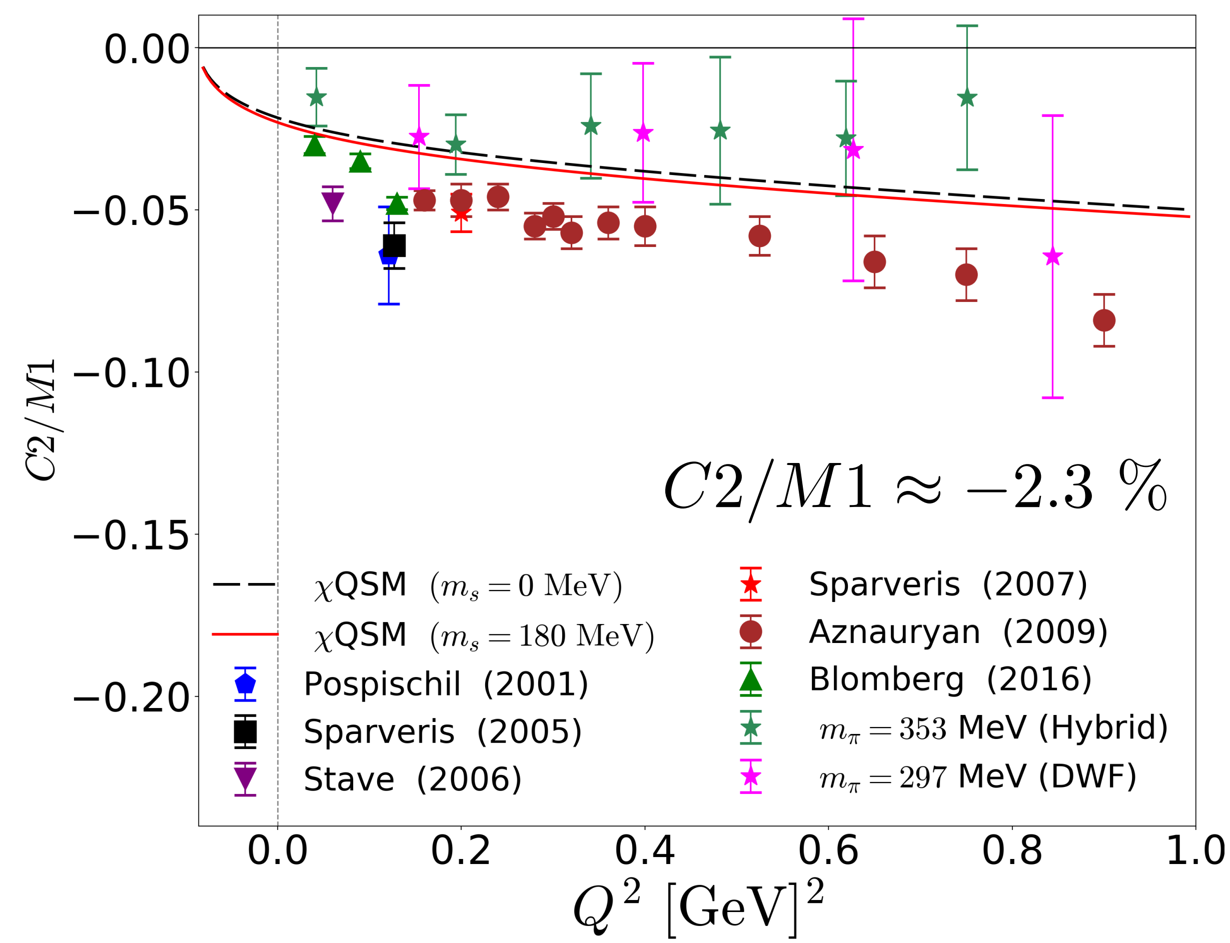
ELECTROMAGNETIC TRANSITIONS OF BARYON DECUPLET

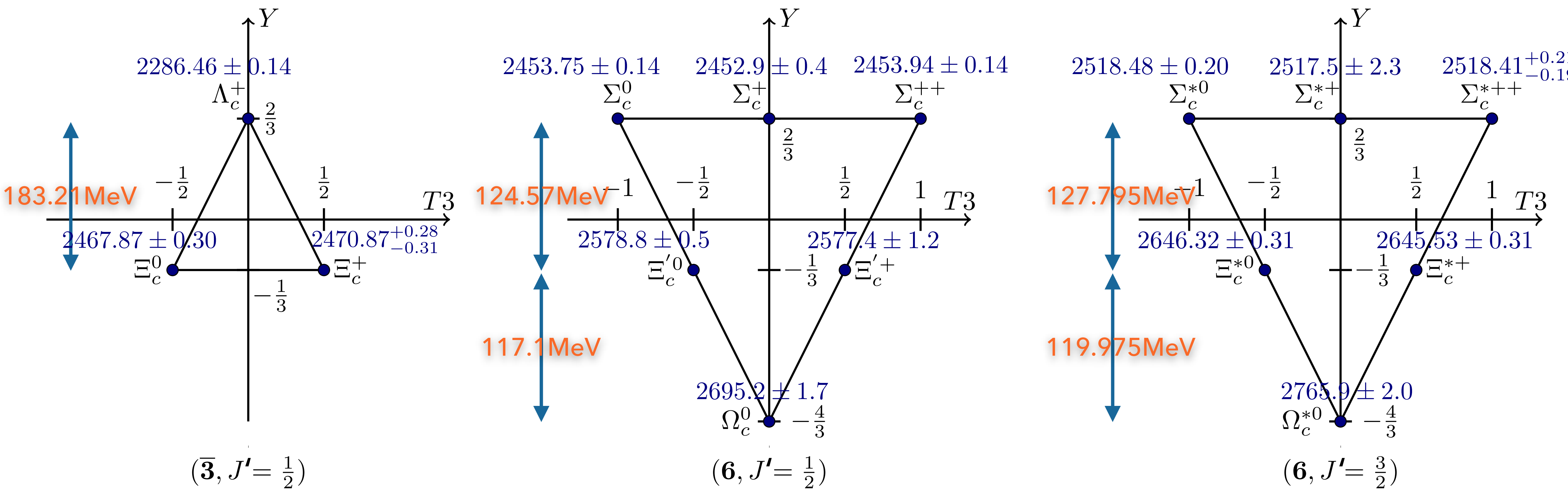
- ✓ Effects of SU(3) symmetry breaking on C2/M1 and E2/M1 improve marginally the corresponding results.
- ✓ In the case of the E2/M1, it vanishes kinematically at $Q^2 = 0.6~[\mathrm{GeV}]^2$
- ✓ The tendency of C2/M1 as a function of the momentum transfer is in good agreement with experimental data.





Heavy baryons

- Recently, interest in heavy baryons has been renewed as a series of new experimental data on them.

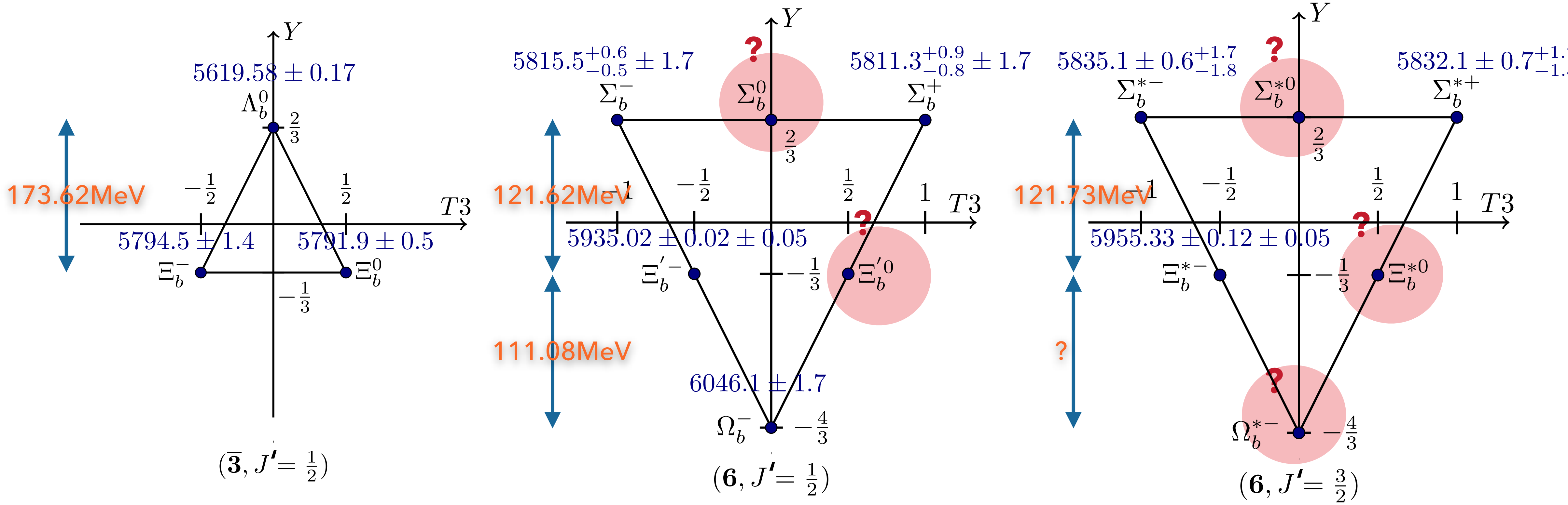


✓ As $m_Q \to \infty$, heavy quark spin is conserved, which leads to the fact that the light-quark spin is also conserved.

✓ The soliton and a heavy quark are decoupled and the heavy quark plays a role of the static color source.

C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016) and 2017 update.

- ✓ The masses of charmed baryons are well known. Thus we will first check the validity of the present approach in the charmed sector.



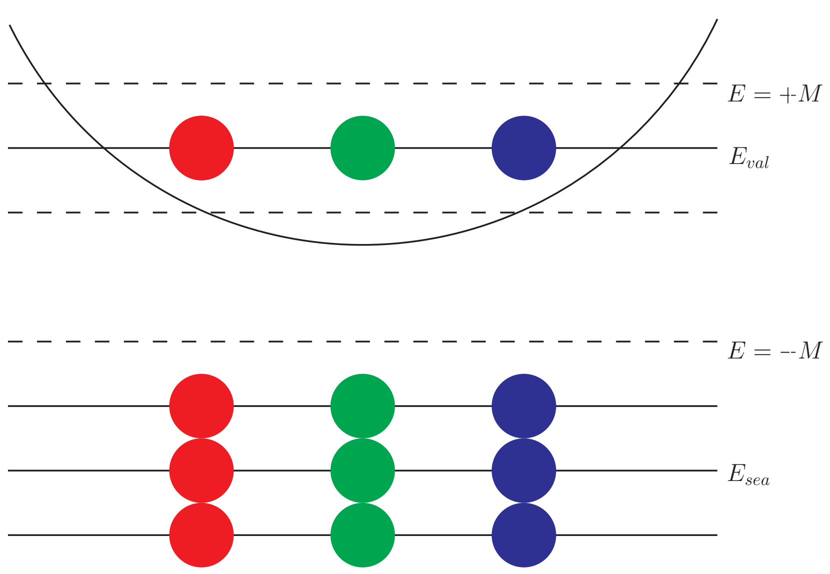
- ✓ On the other hand, some of bottom baryon masses are unknown.
- ✓ In this talk, we will show how the mass of Ω_b^{*-} is predicted.

C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016) and 2017 update.

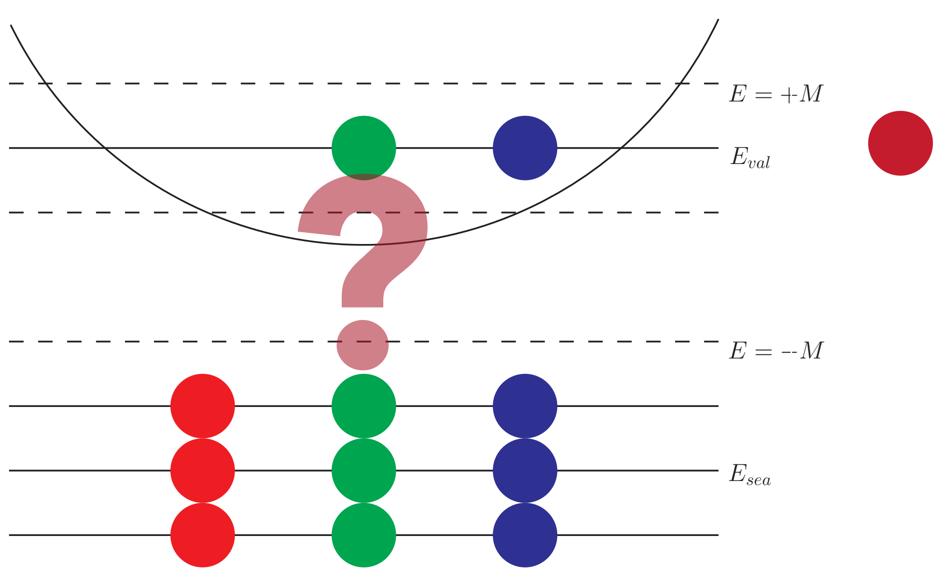
- √ A model-independent analysis has been made for the heavy baryon system, and its validity has been confirmed.

  Gh. S. Yang, H.-Ch. Kim, M. V. Polyakov and M. Praszalowicz, Phys. Rev. ***D94****, 071502 (2016)*

- √ When it comes to model calculation, the Nc-1 mean field should be tested whether it makes stabilized soliton or not.



Nc Mean field



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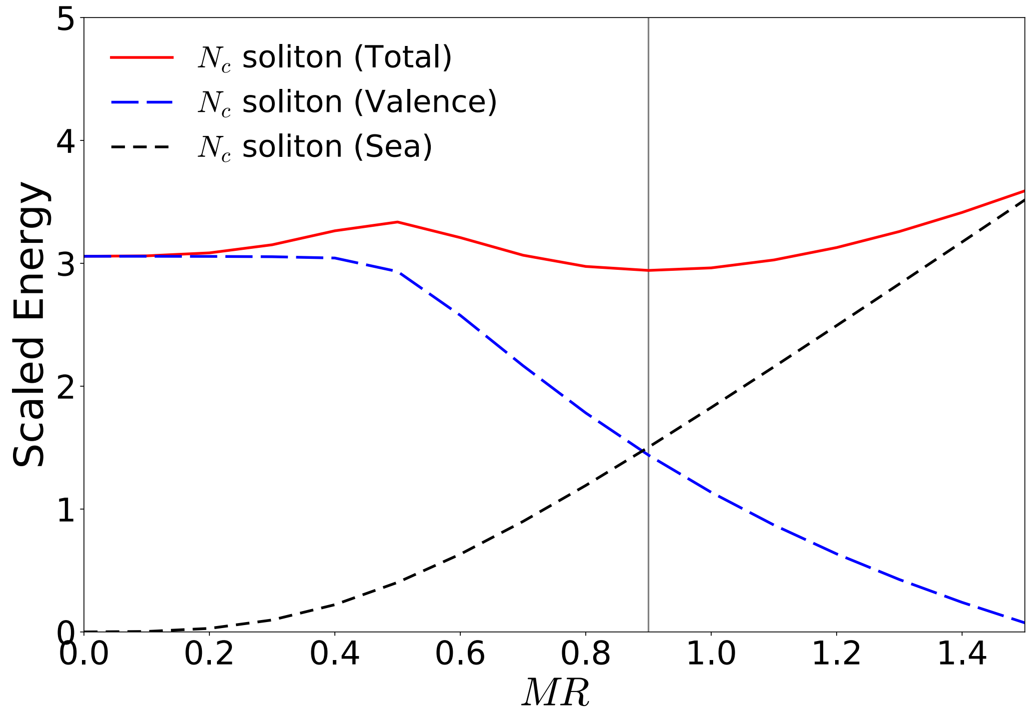
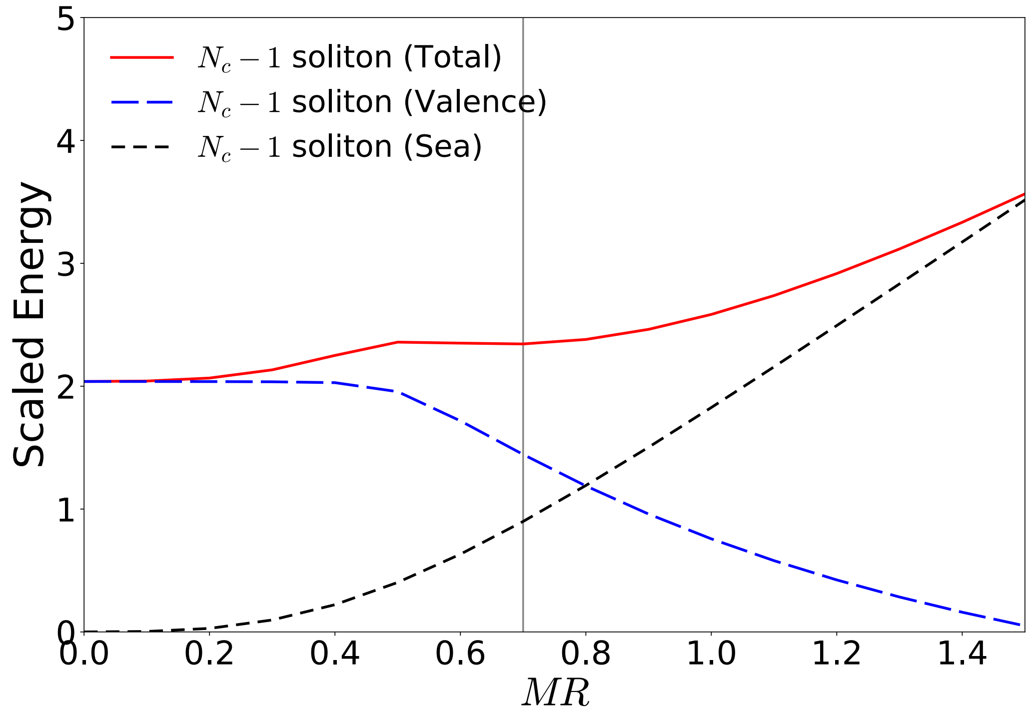
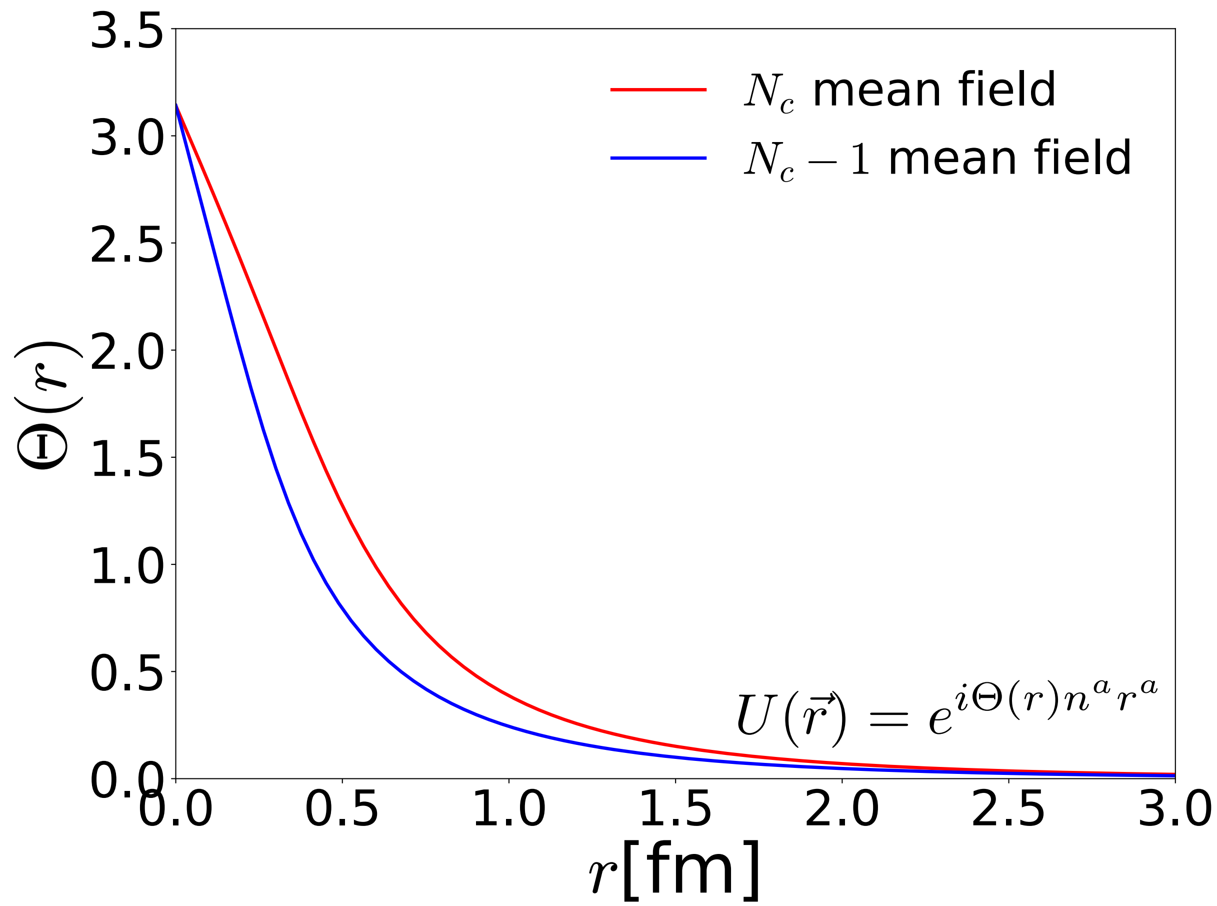
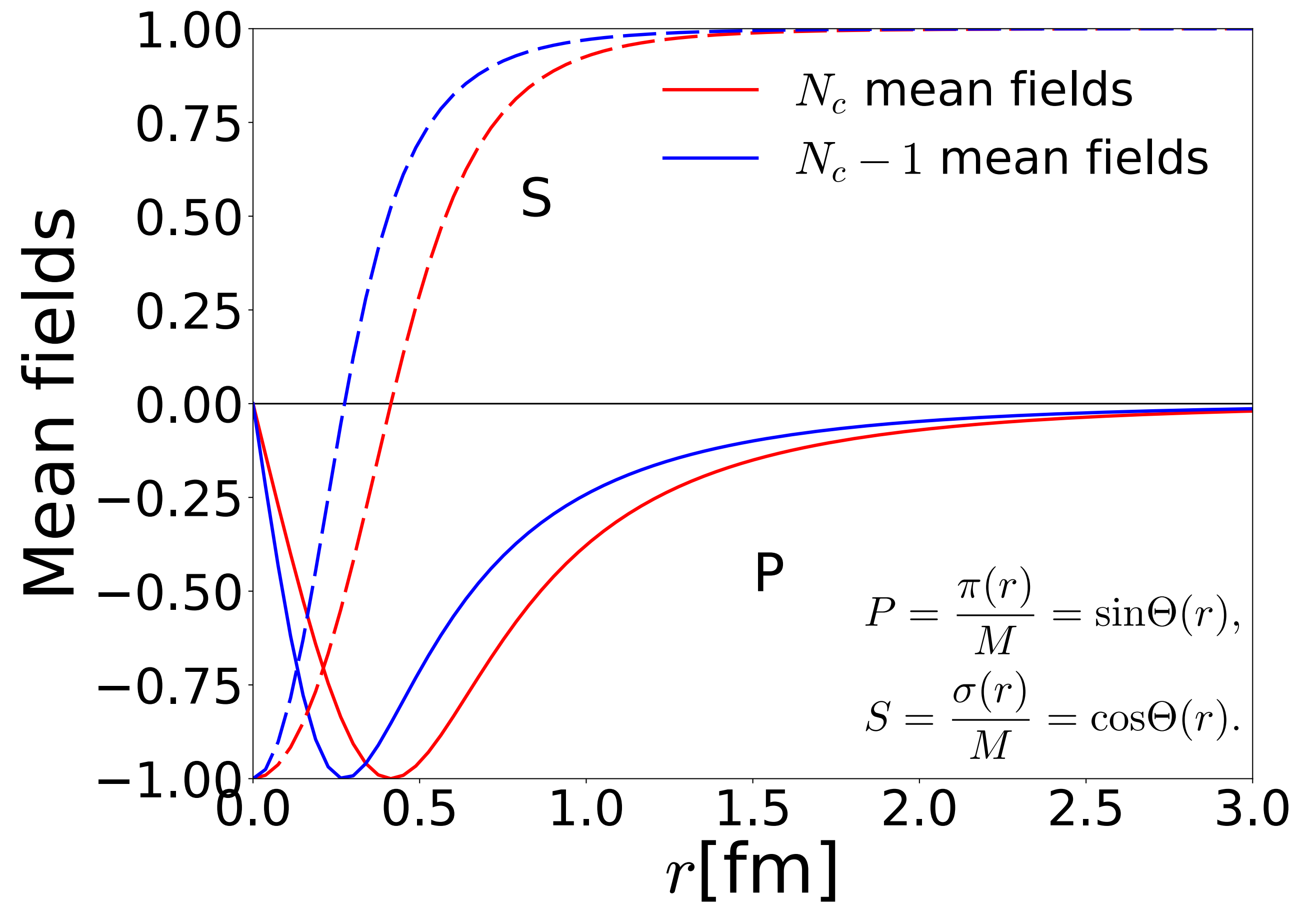
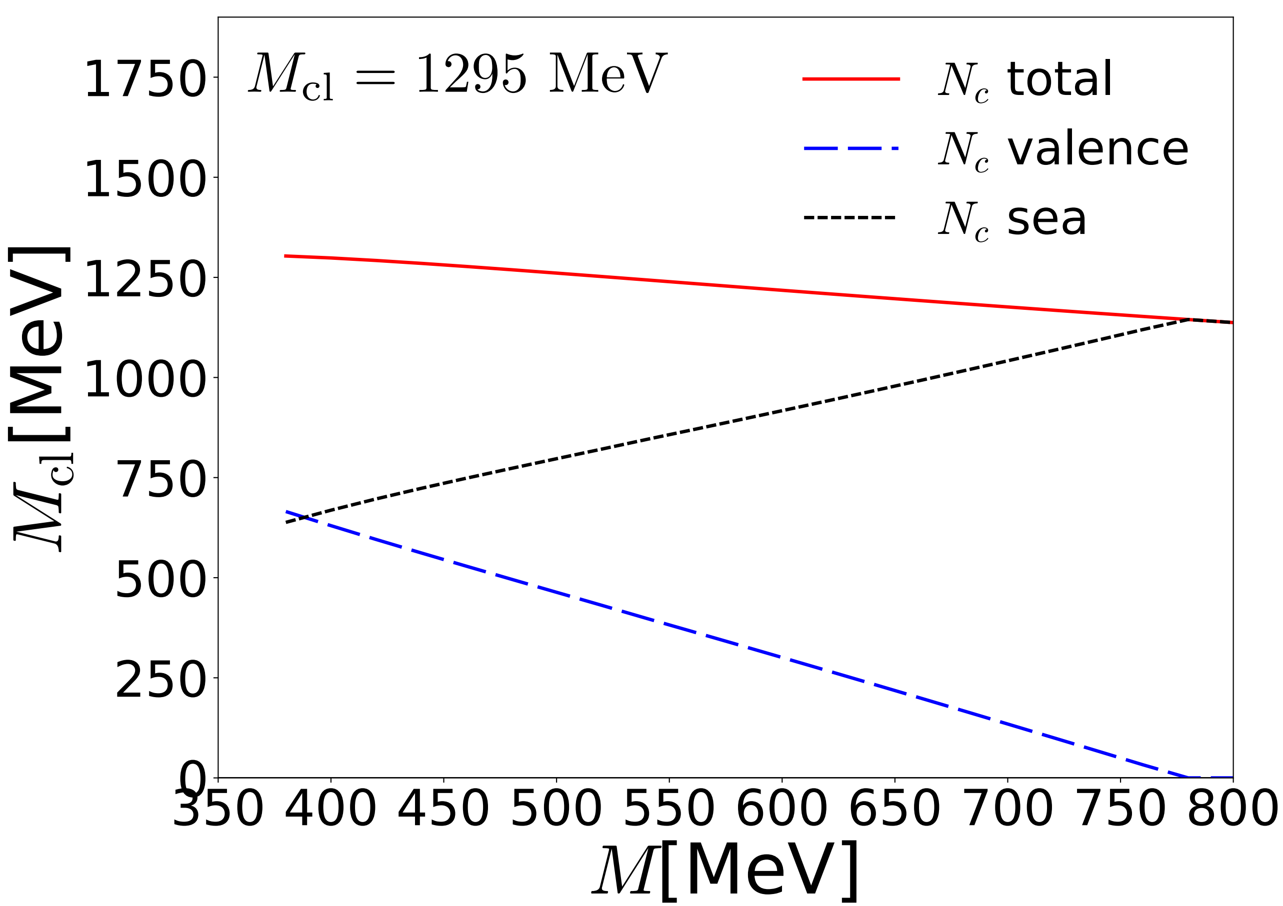


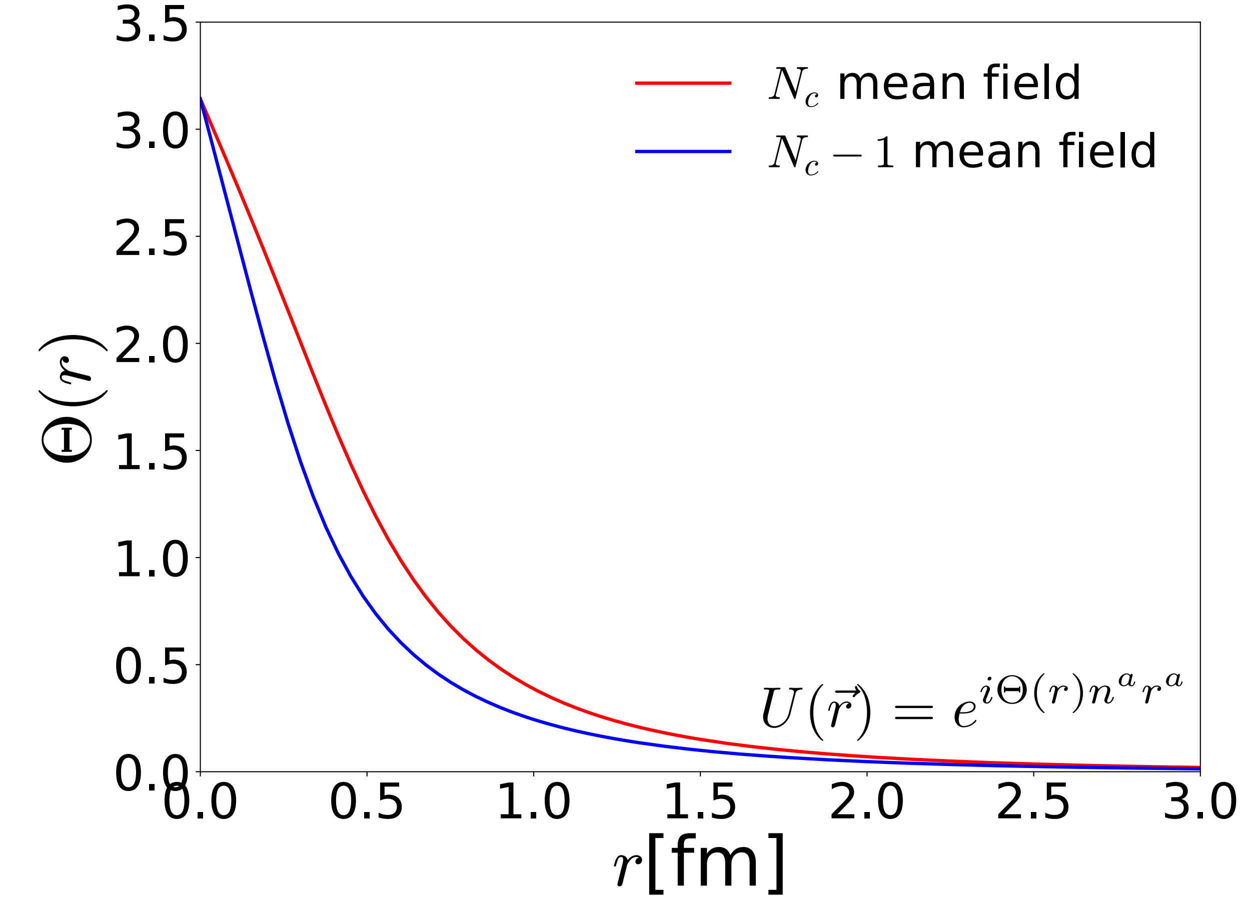


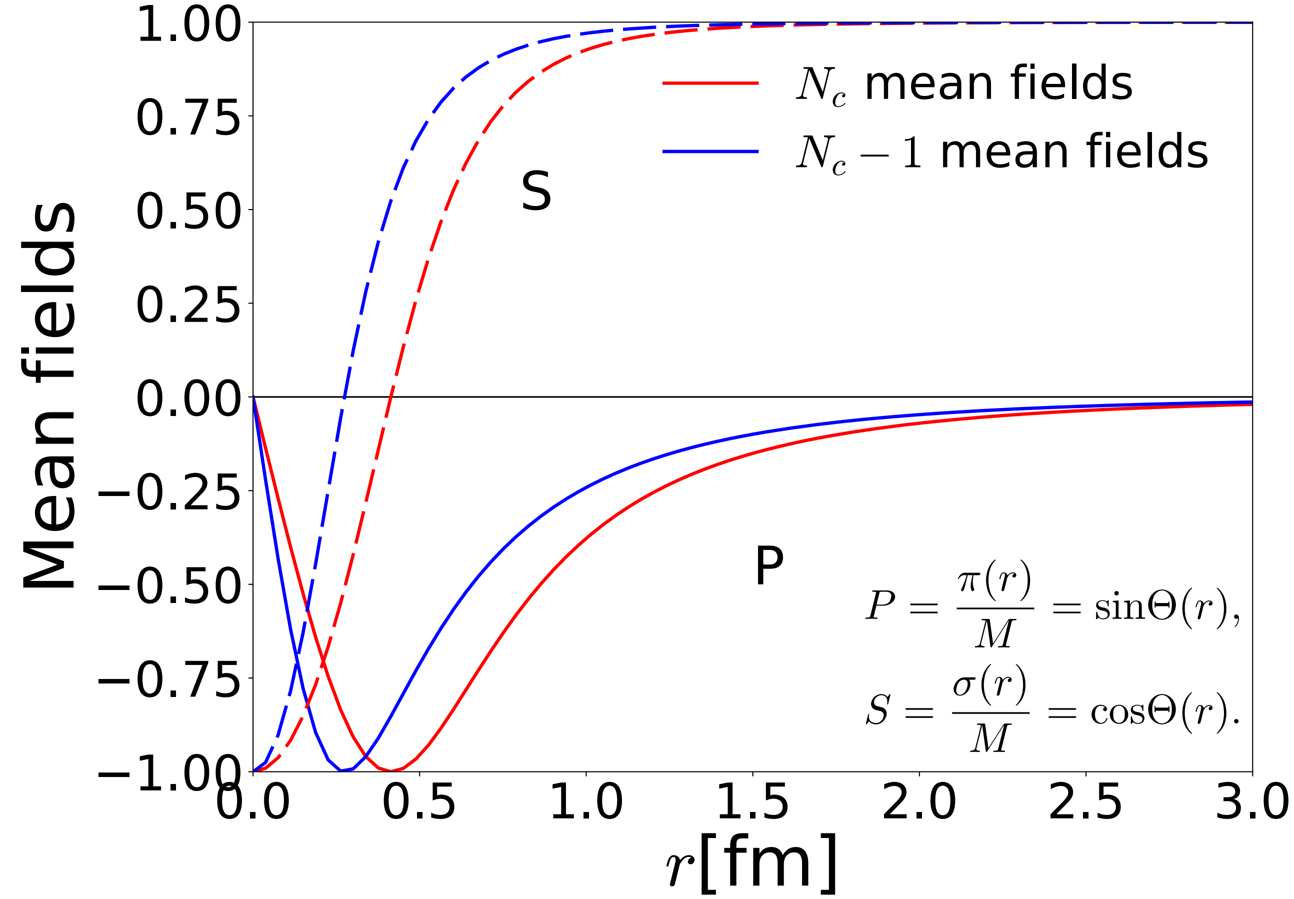


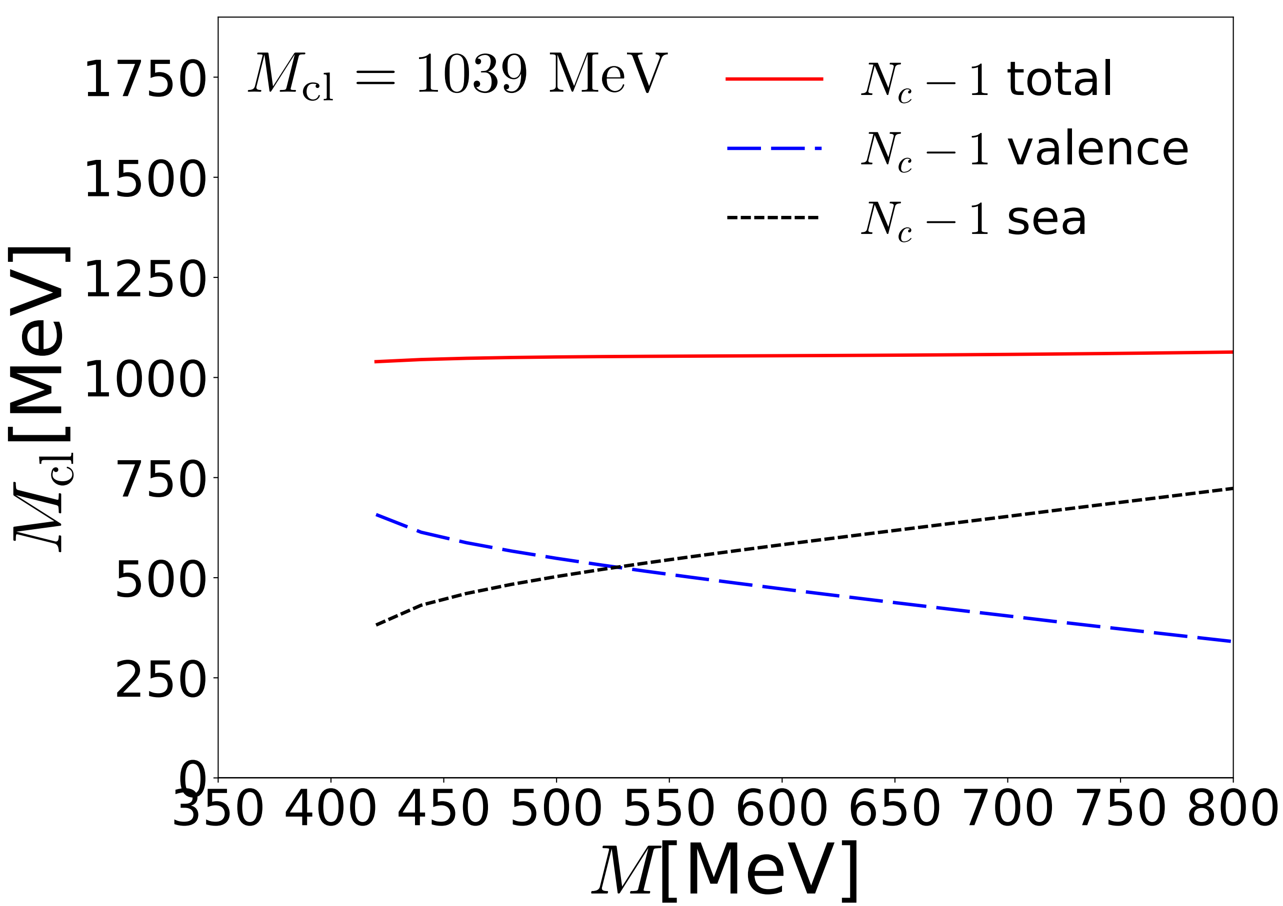




- The mean fields with Nc-1 valence quarks are quit different from those with Nc valence quarks.
- The self-consistent solutions exist only for the Nc and Nc-1 solitons with M=420 MeV.
- A solution for Nc-2 soliton appears around M=600 MeV.







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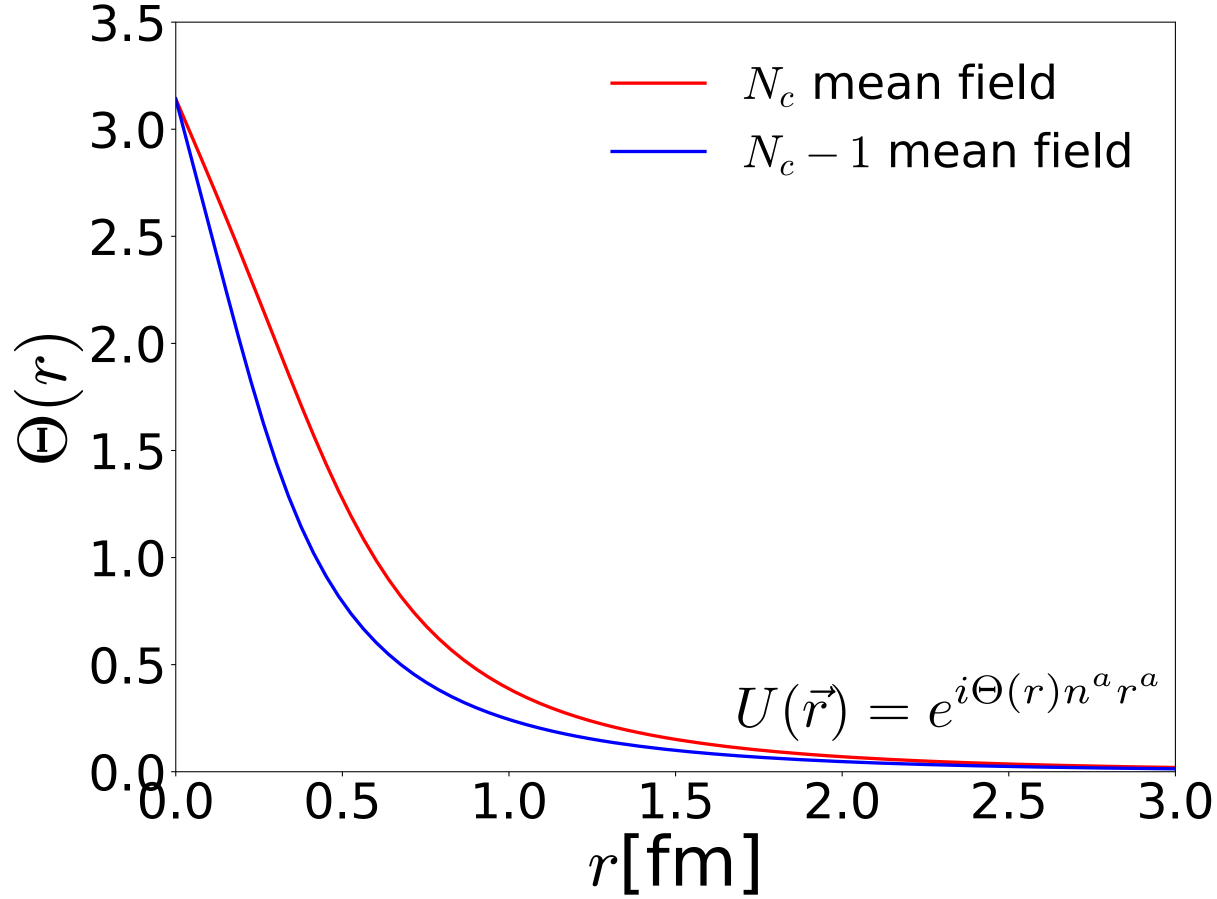

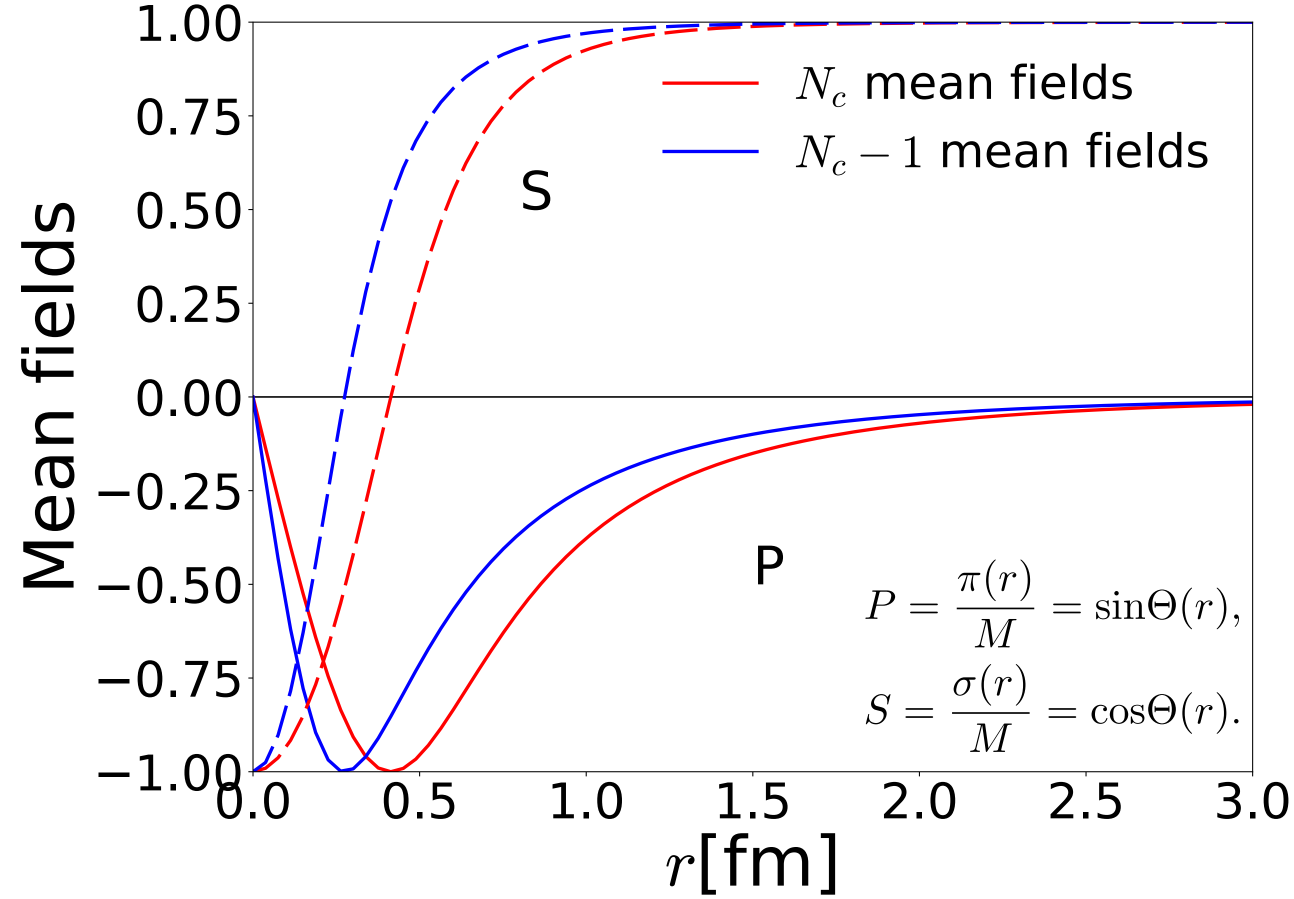

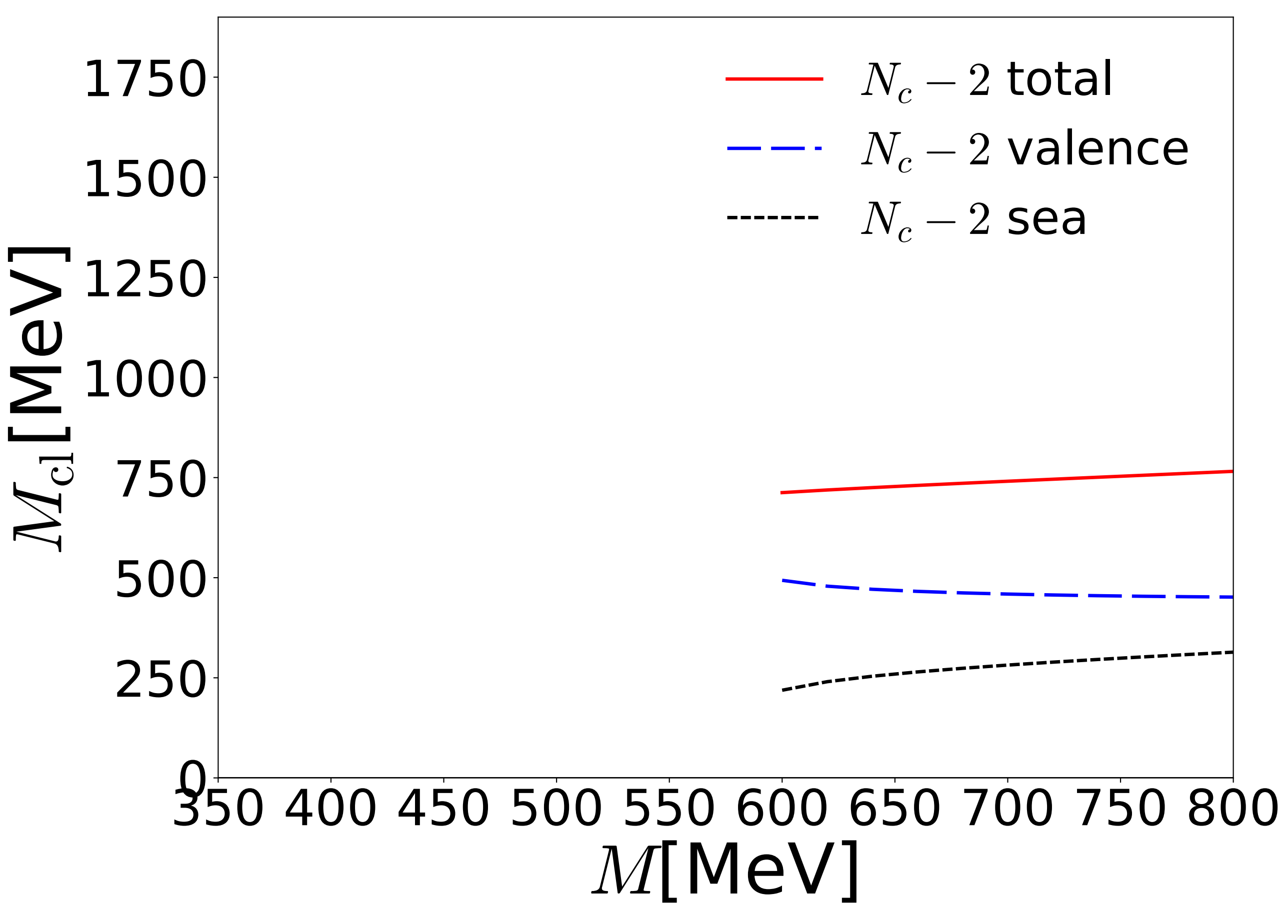

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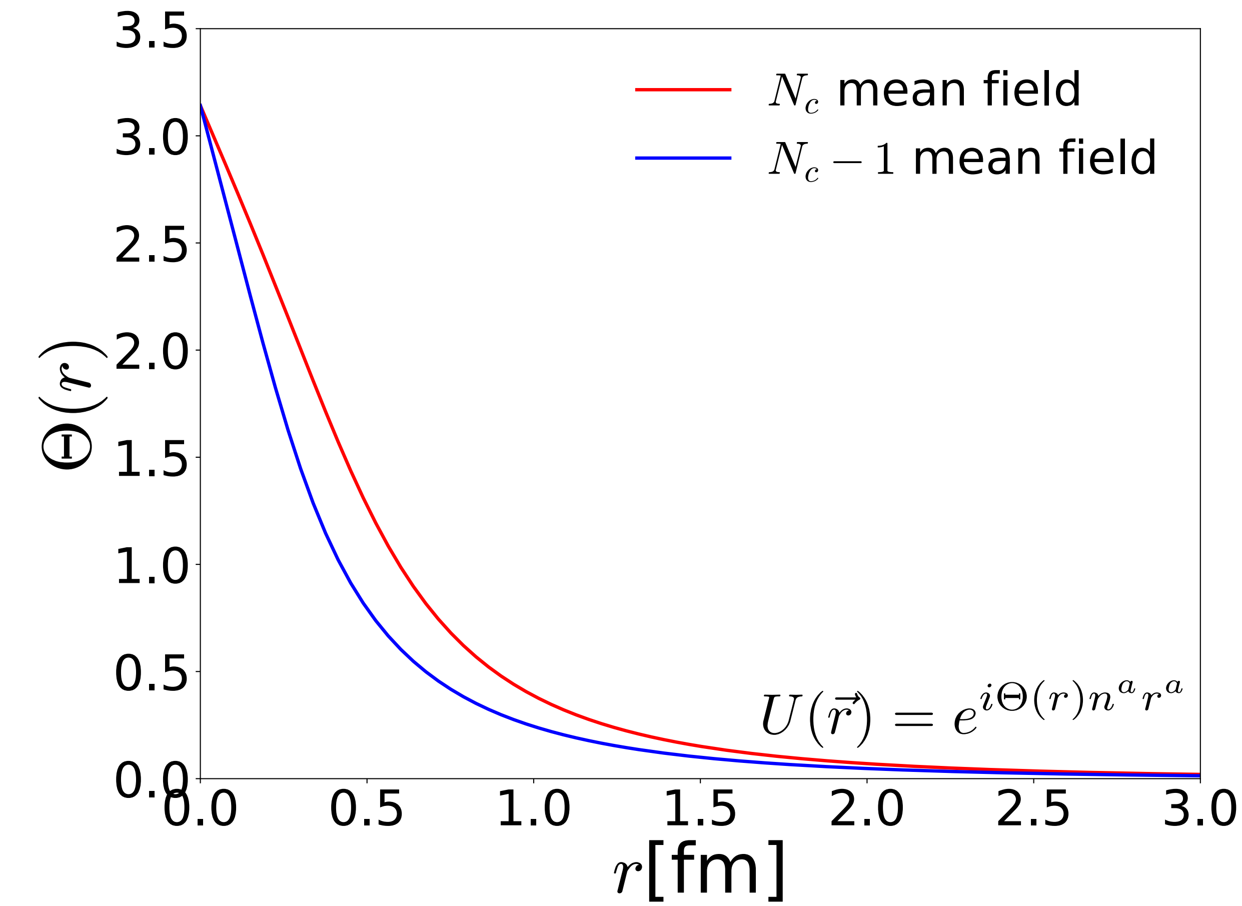
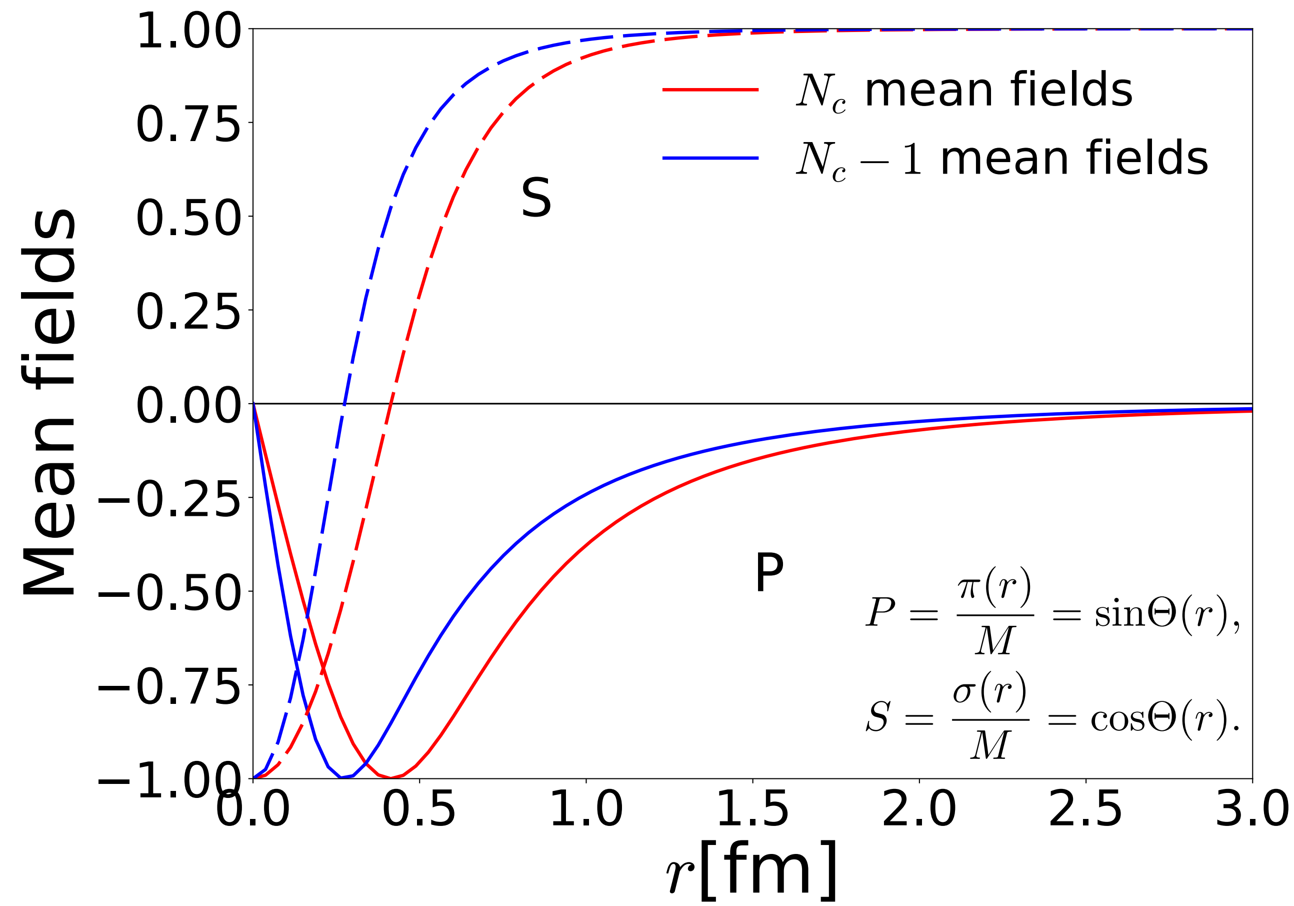
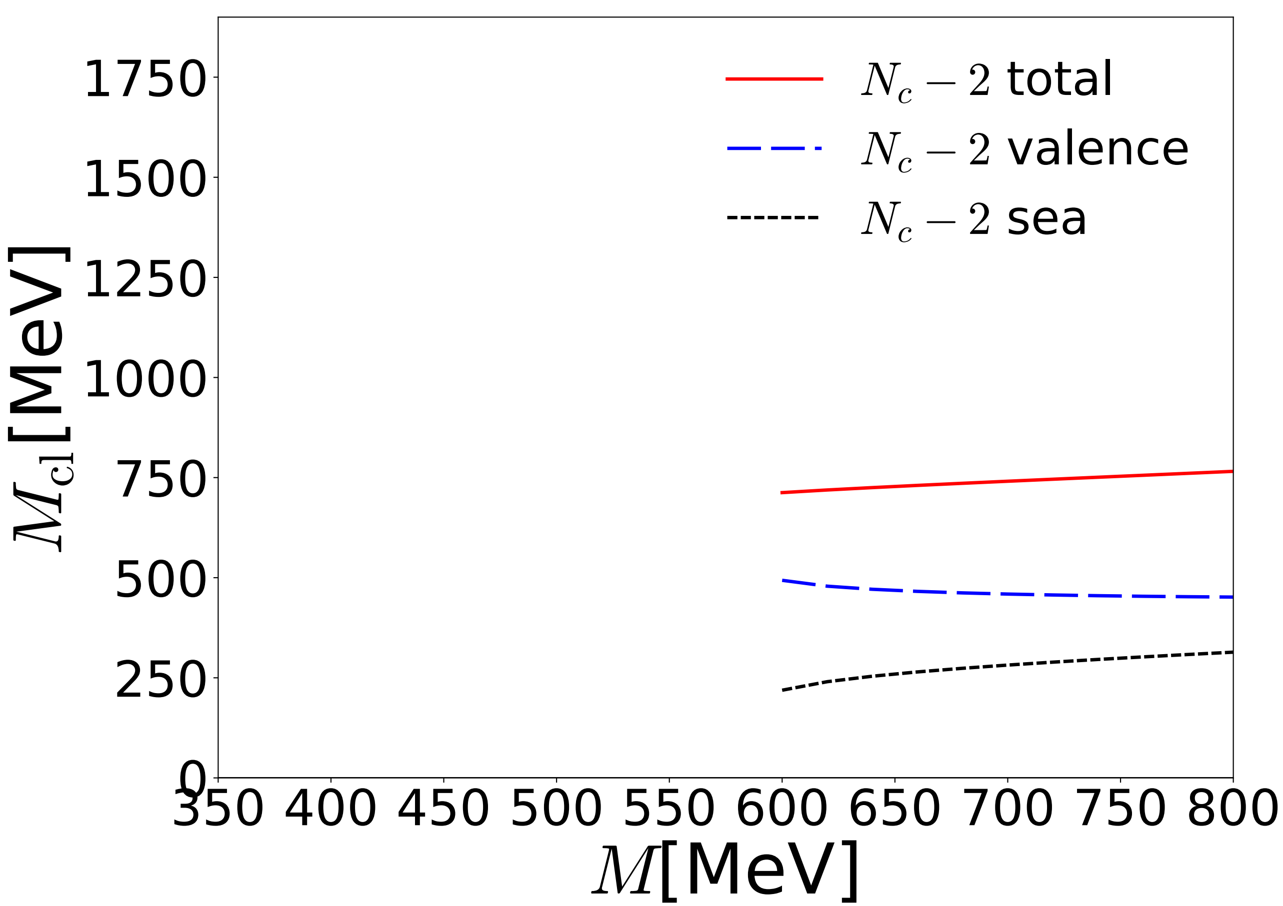






✓ The mean fields with Nc-1 valence

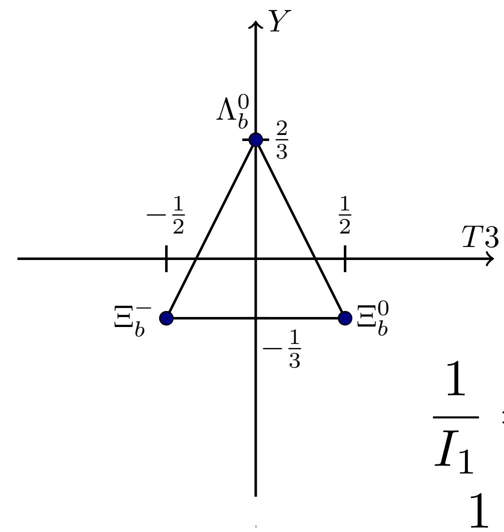
	$N_c - 1$ mean field	N_c mean field
I_1[fm]	1.10	1.11
I_2[fm]	0.52	0.53
K_1[fm]	0.69	0.43
K_2[fm]	0.37	0.27
M_{cl} [MeV]	1039	1295

around M=600 MeV.

$$H = H_{\rm sym} + H_{\rm sb} + H_{\rm hf}$$

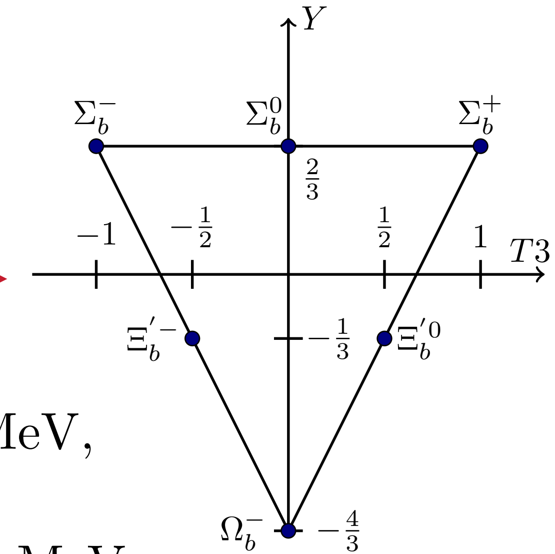
✓ The rotational energy yields the energy difference between the $\overline{\mathbf{3}}$ and the $\mathbf{6}$.

$$\overline{\mathbf{3}} \to (p,\, q) = (0,\, 1) \to J = 0 \ \ \text{antitriplet}$$
$$\mathbf{6} \to (p,\, q) = (2,\, 0) \to J = 1 \ \ \text{sextet.}$$



$(\overline{\mathbf{3}}, J = \frac{1}{2})$

$$\frac{1}{I_1} = 172 \text{ MeV},$$
$$\frac{1}{I_1^{\rm exp}} \sim 177 \text{ MeV}.$$



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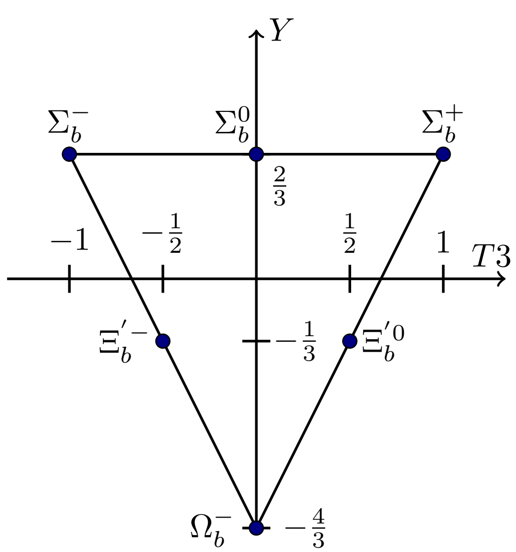
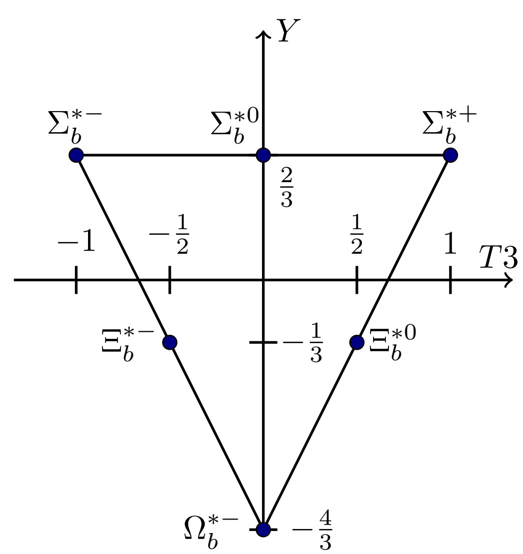


$(\mathbf{6}, J = \frac{1}{2})$

$$\frac{\kappa}{m_Q M_{\rm sol}}$$

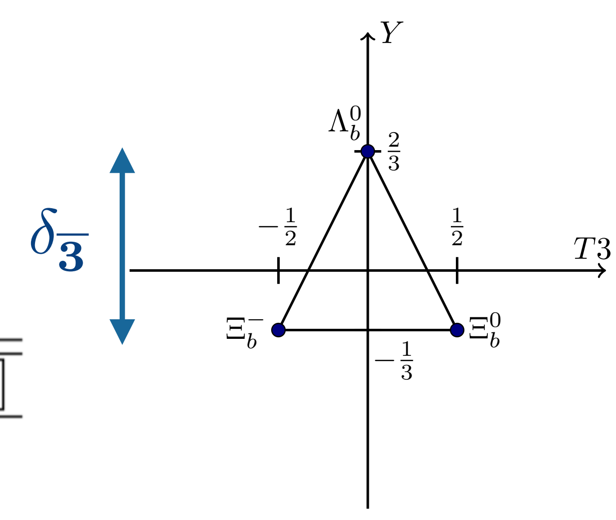


$(\mathbf{6}, J = \frac{3}{2})$

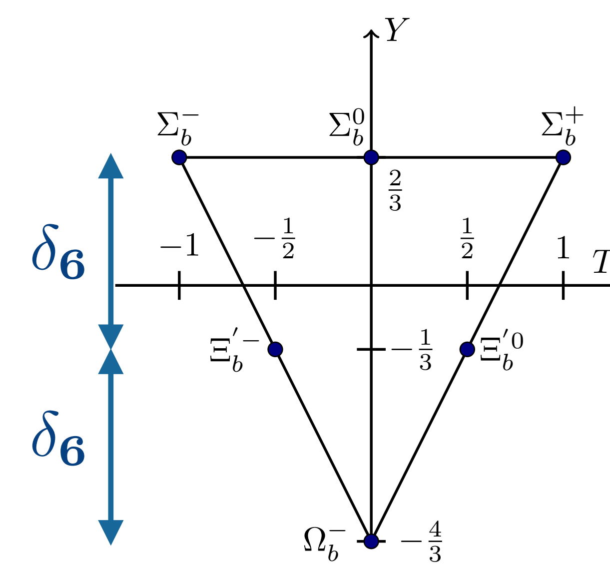
✓ We have to introduce the hyperfine interaction to lift the degeneracy in the sextet representations.

✓ Mass correction from the flavor SU(3) symmetry breaking.

	This work	Model Independent [21]	Experiment [23]
$\delta_{\overline{\mathbf{3}}}$	-180.3	-203.8	$\sim$ -182.9
$\delta_{\mathbf{6}}$	-139.7	-135.2	$\sim$ -122.4



$(\overline{\mathbf{3}}, J = \frac{1}{2})$



$(\mathbf{6}, J = \frac{1}{2})$

MASS SPECTRA OF HEAVY BARYONS

- The soliton mass is overestimated. Thus, instead of using it, we employ the experimental center mass, so we predict the all masses of $\overline{\mathbf{3}}$ and $\mathbf{6}$.

$\mathcal{R}_J^Q$	B_c	This work	Experiment [23]
$\overline{\mathbf{3}}_{1/2}^c$	Λ_c	2278.4	2286.5±0.1
$\overline{\mathbf{3}}_{1/2}^c$	Ξ_c	2458.6	2469.4±0.3
$\mathbf{6}_{1/2}^c$	Σ_c	2438.6	2453.5±0.1
$\mathbf{6}_{1/2}^c$	Ξ_c'	2578.3	2576.8±2.1
$\mathbf{6}_{1/2}^c$	Ω_c	2718.1	2695.2±1.7
$\mathbf{6}_{3/2}^c$	Σ_c^*	2506.7	2518.1±0.8
$\mathbf{6}_{3/2}^c$	Ξ_c^*	2646.4	2645.9±0.4
$\mathbf{6}_{3/2}^c$	Ω_c^*	2786.2	2765.9±2.0

$\mathcal{R}_J^Q$	B_b	This work	Experiment [23]
$\overline{\mathbf{3}}_{1/2}^b$	Λ_b	5608.2	5619.5±0.2
$\overline{\mathbf{3}}_{1/2}^b$	Ξ_b	5788.5	5793.1±0.7
$\mathbf{6}_{1/2}^b$	Σ_b	5800.3	5813.4±1.3
$\mathbf{6}_{1/2}^b$	Ξ_b'	5940.1	5935.0±0.05
$\mathbf{6}_{1/2}^b$	Ω_b	6079.8	6048.0±1.9
$\mathbf{6}_{3/2}^b$	Σ_b^*	5820.6	5833.6±1.3
$\mathbf{6}_{3/2}^b$	Ξ_b^*	5960.3	5955.3±0.1
$\mathbf{6}_{3/2}^b$	Ω_b^*	6100.1	-

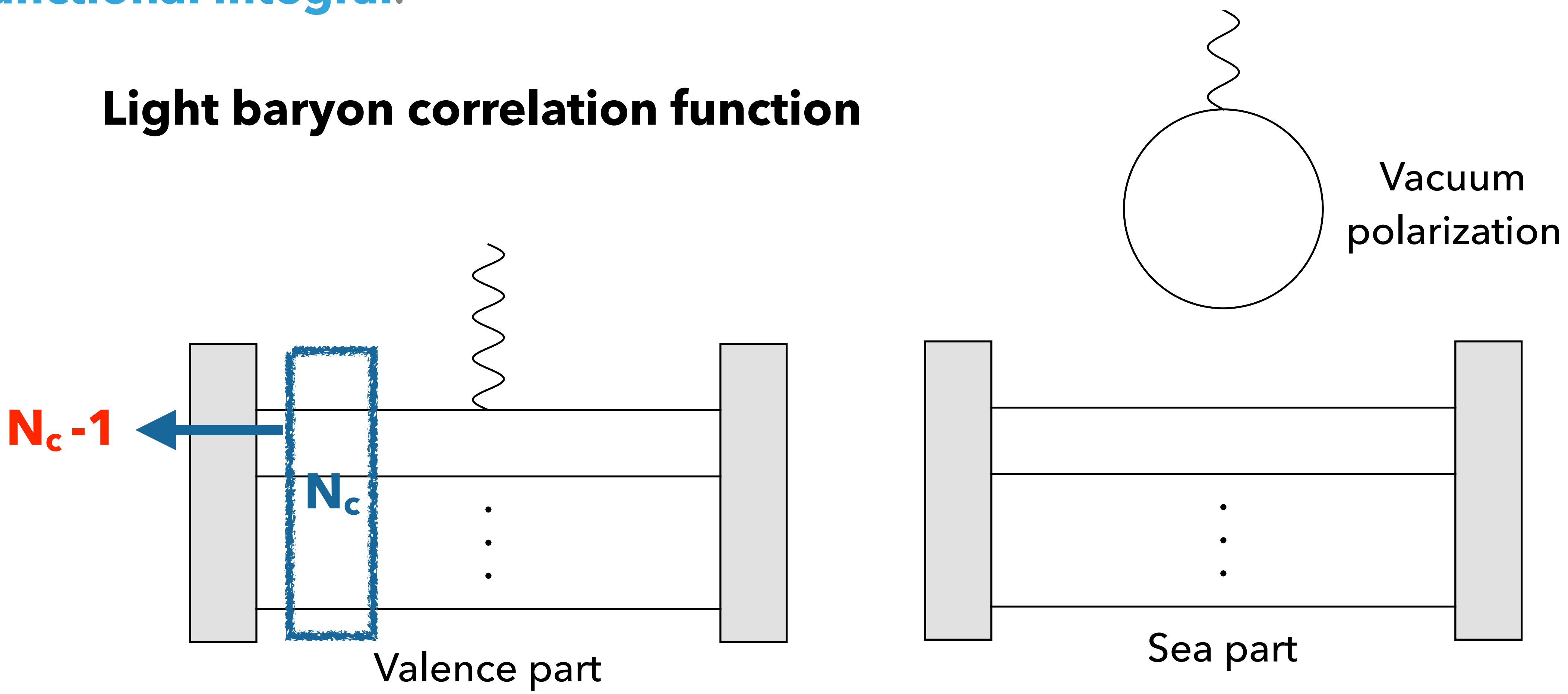
The results obtatained are in good agreement with the experiment result, and even we predict mass of Ω_b^* .

[14]Gh. S. Yang, H.-Ch. Kim, M. V. Polyakov and M. Praszalowicz, Phys. Rev. **D94**, 071502 (2016)

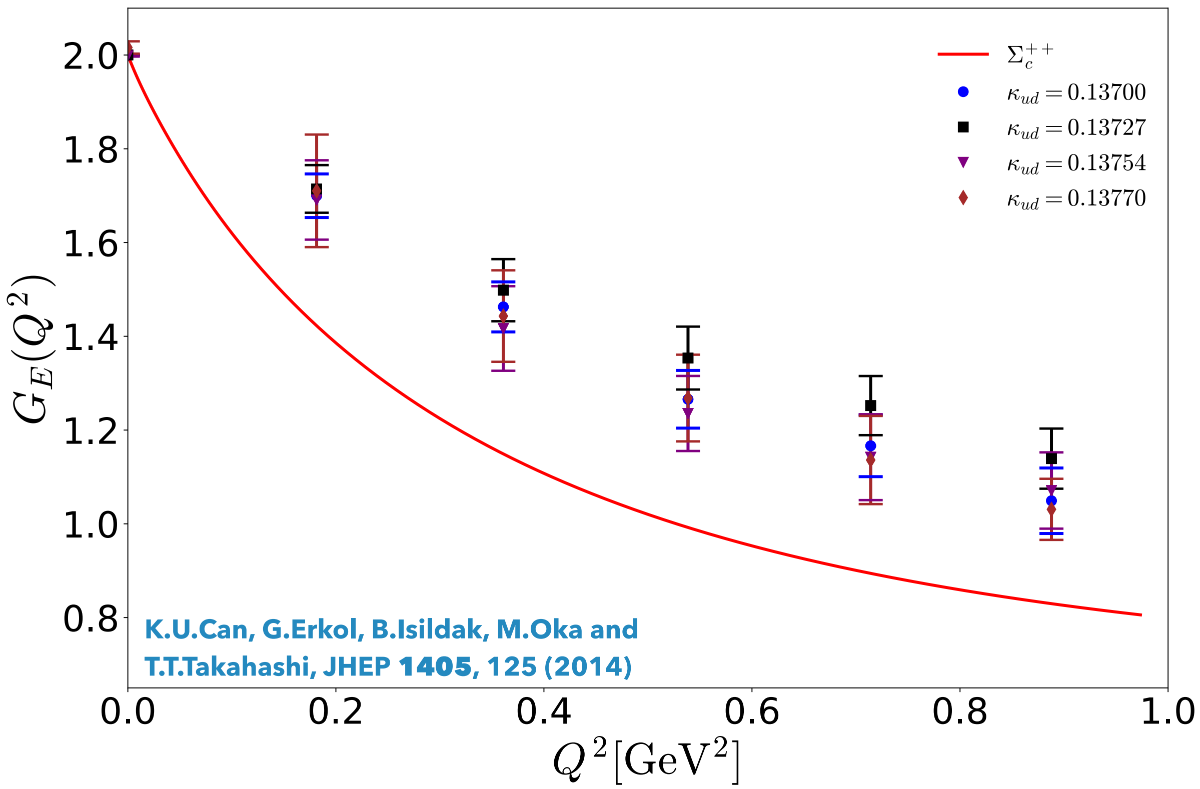
- Electromagnetic current

$$J_\mu(x) = \overline{\psi}(x)\gamma_\mu \hat{\mathcal{Q}}\psi(x) + e_Q \overline{\Psi}\gamma_\mu \Psi \qquad \hat{\mathcal{Q}} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} = \frac{1}{2}\left(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right)$$

- The matrix elements of the electromagnetic form factors are found by using the **functional integral**.



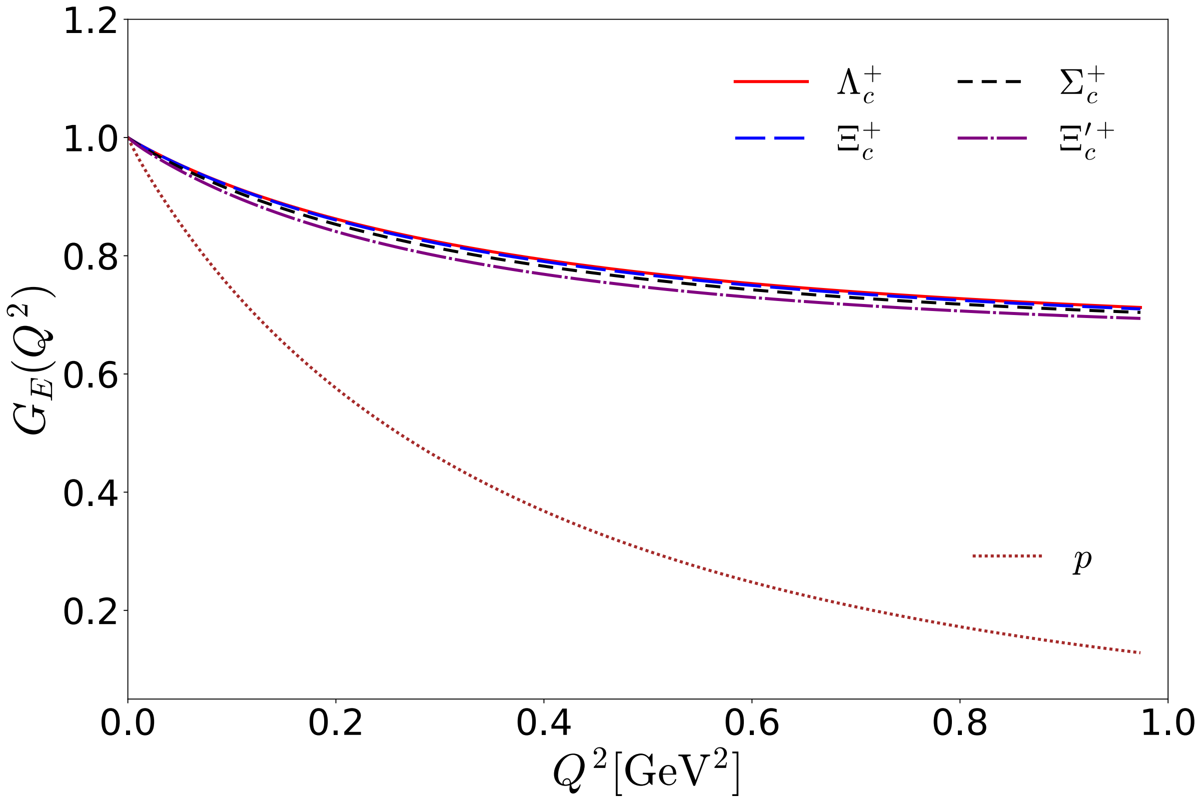
- The correlation function is changed **only for the valence part** in the singly heavy baryon sector.



Electric form factor

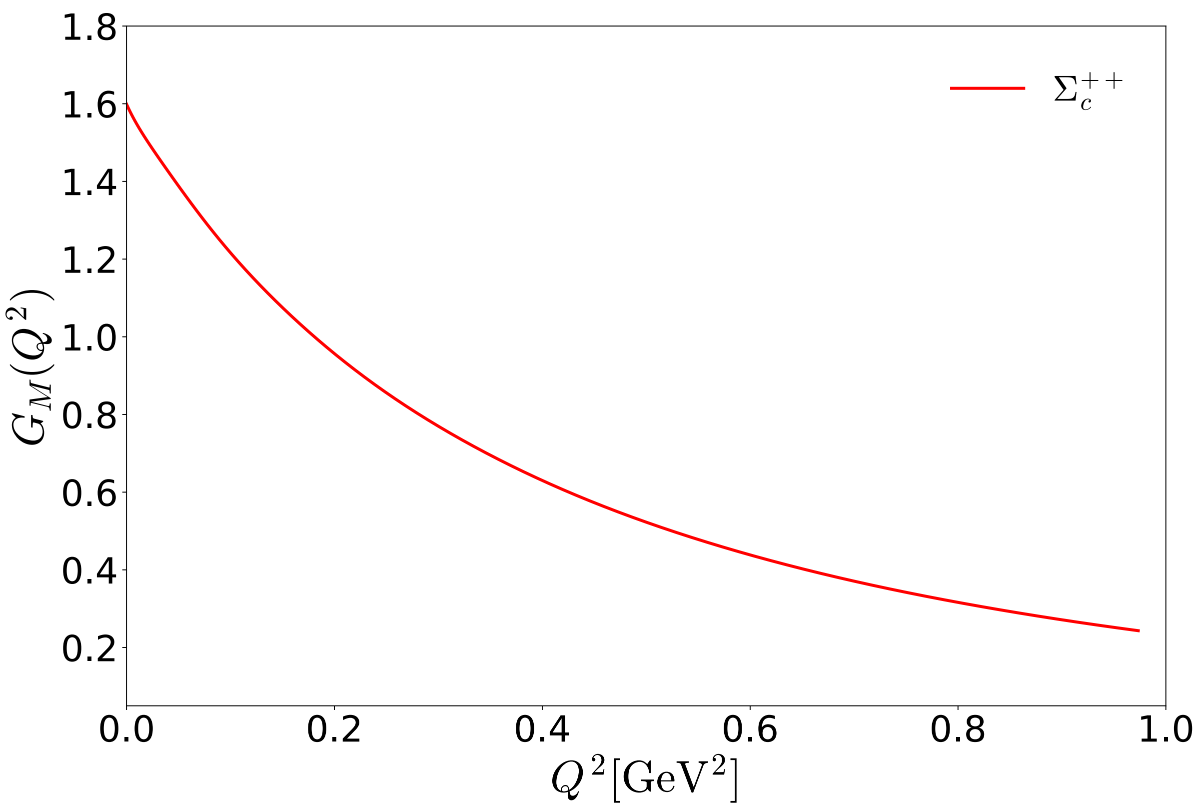
$$G_E^B(q^2) = \int d^3z j_0(|\boldsymbol{q}||\boldsymbol{z}|)\mathcal{G}_E^B(\boldsymbol{z}) + G_E^Q(q^2),$$

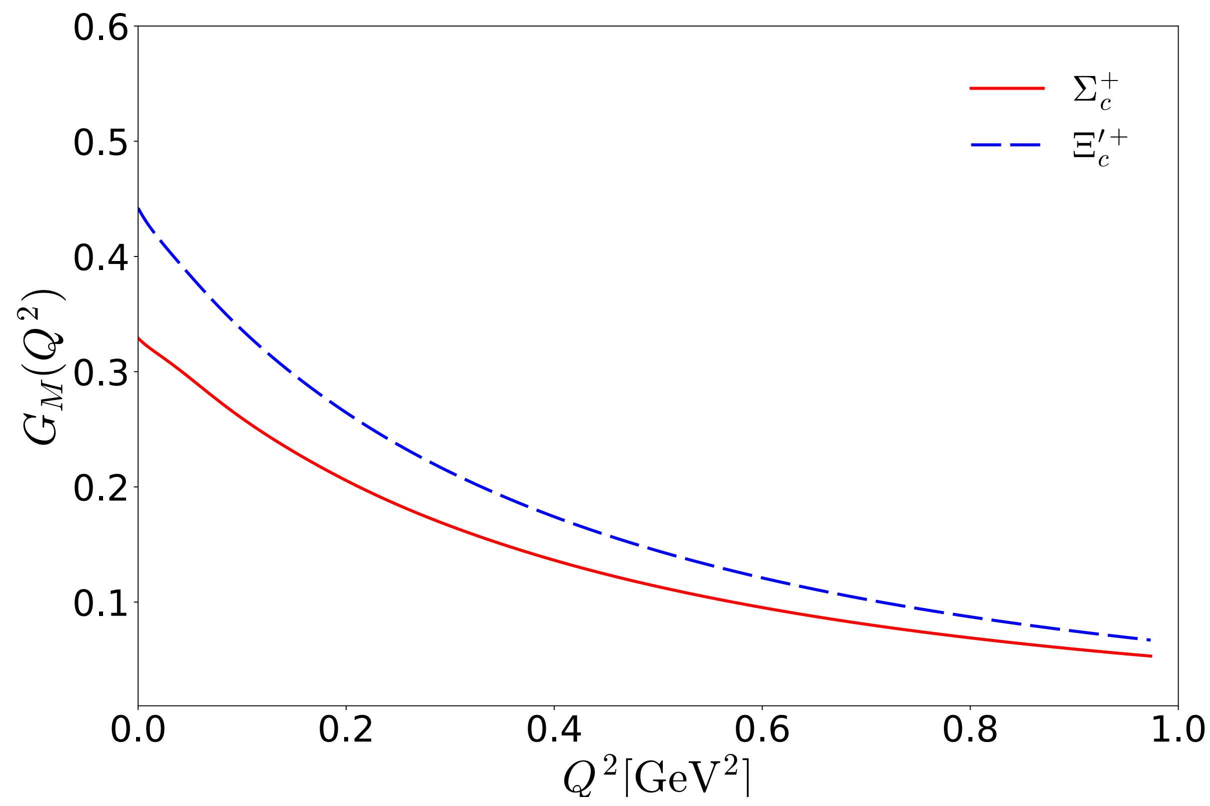
- The electric form of the lattice QCD calculation decrease quite slowly as Q^2 increases. (**The pion mass is not a physical mass in the lattice calculation**)



- **The heavy baryons are electrically compact objects.**

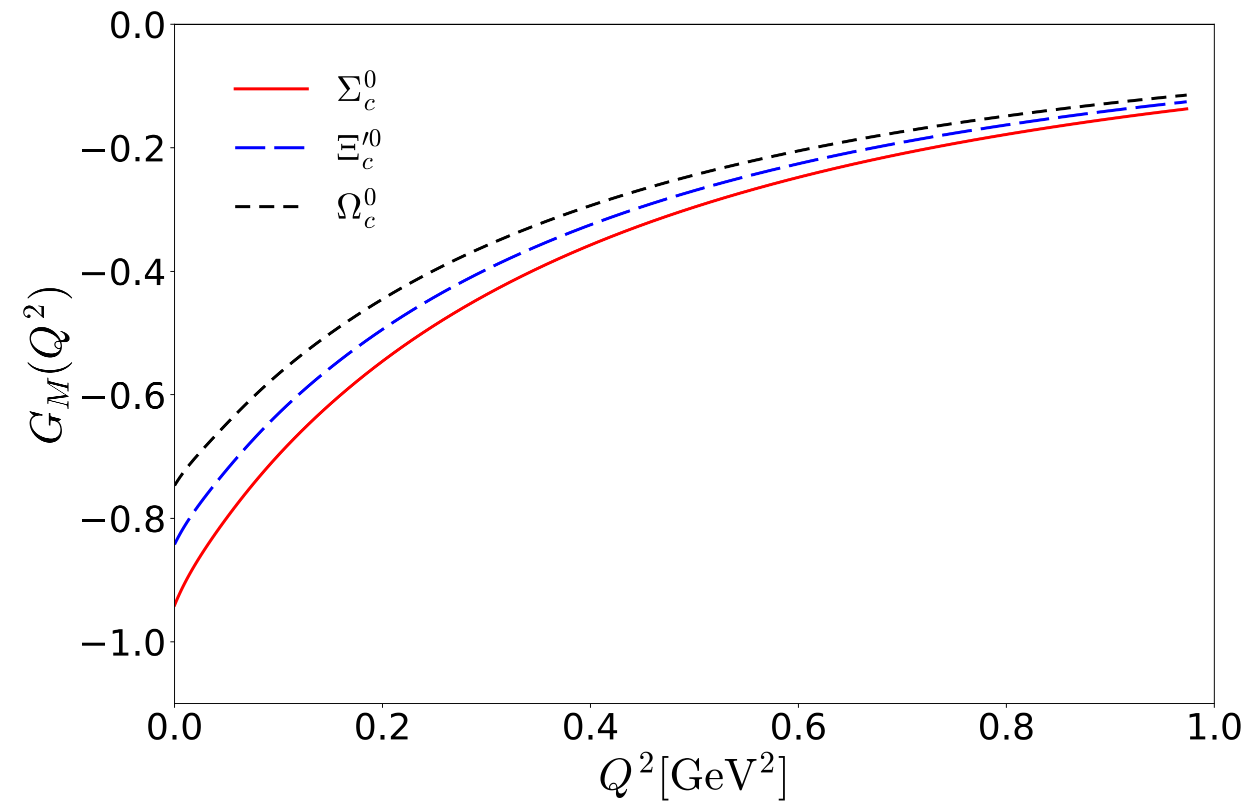
J. Y. Kim and H. C. Kim, Phys. Rev. D **97**, no.11, 114009(2018).



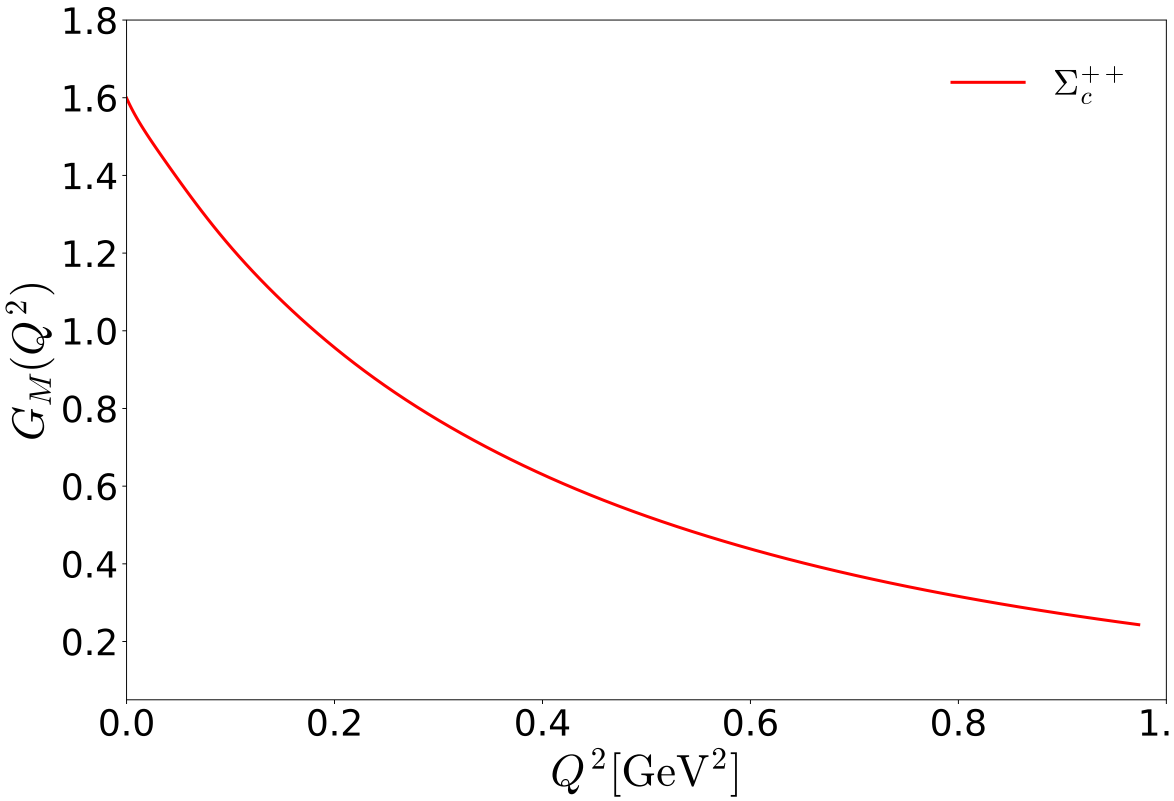


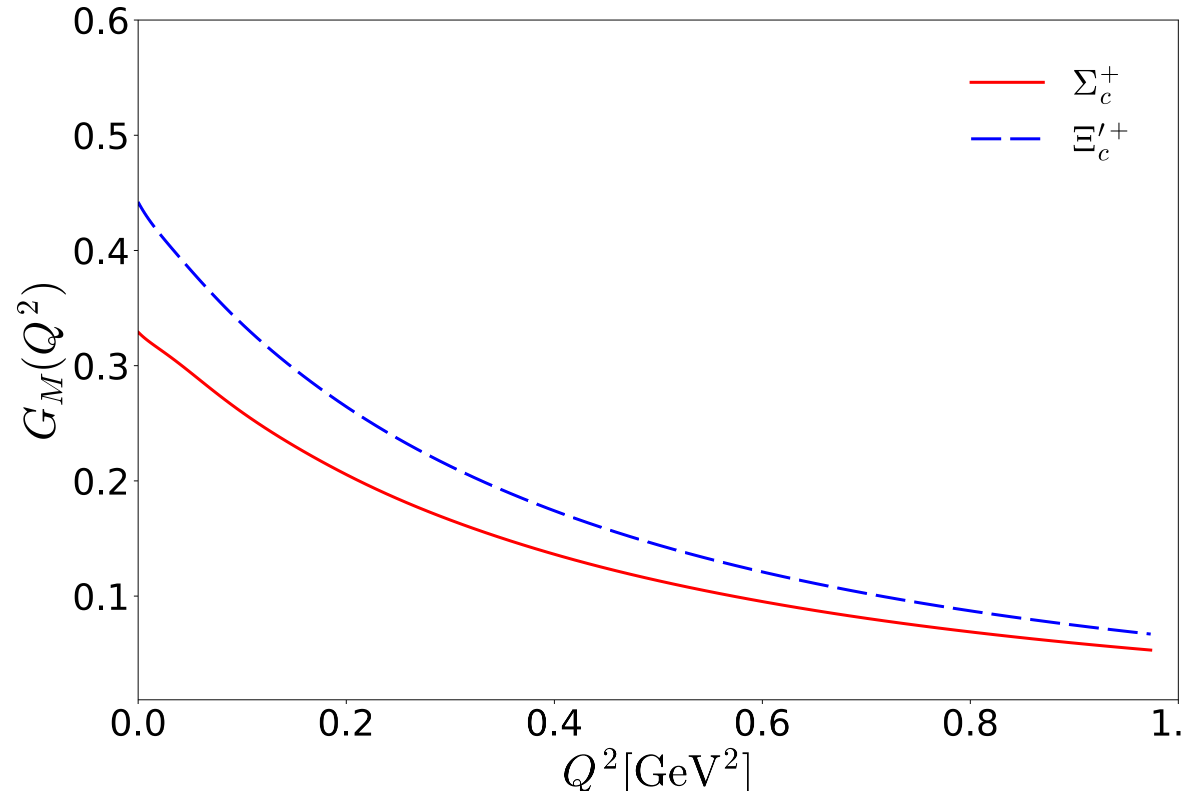
Magnetic form factor

$$G_M^B(q^2) = \frac{M_N}{|\boldsymbol{q}|}\int d^3z \frac{j_1(|\boldsymbol{q}||\boldsymbol{z}|)}{|\boldsymbol{z}|}\mathcal{G}_M^B(\boldsymbol{z}),$$

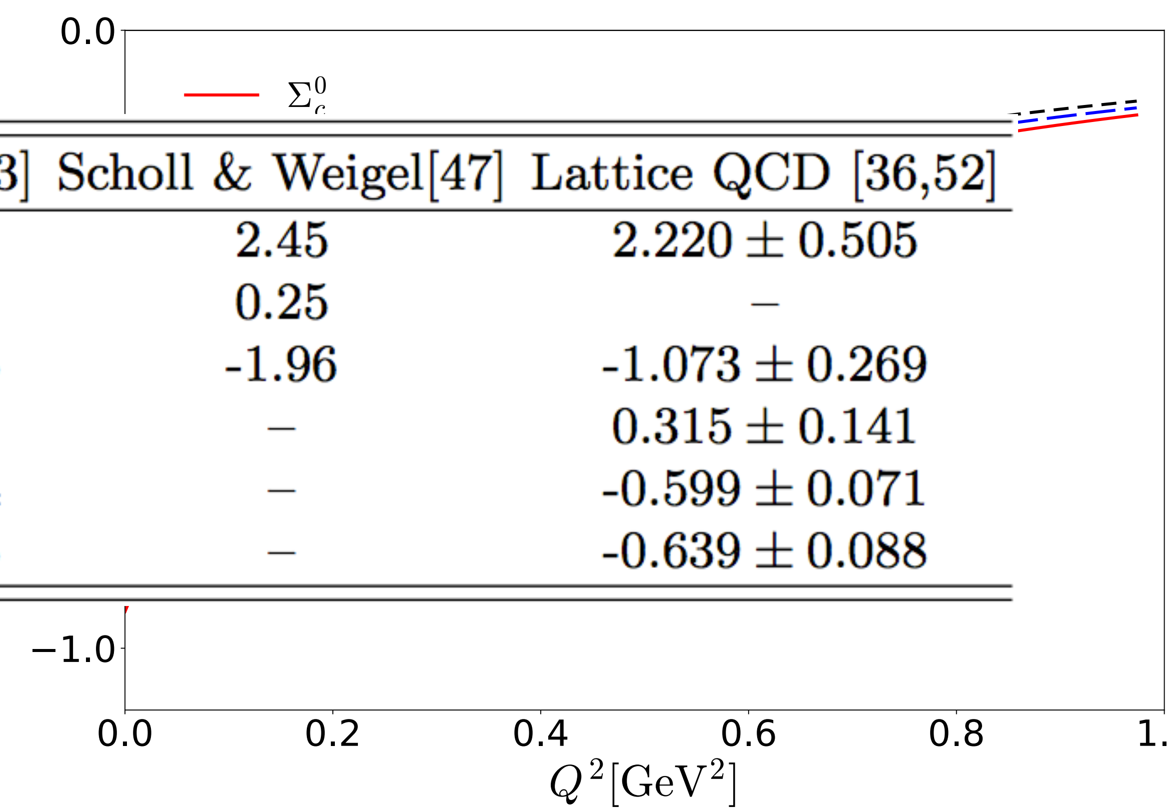


J. Y. Kim and H. C. Kim, Phys. Rev. D **97**, *no.*11, 114009(2018).





Baryon	$\mu_{B_c}^{(m_s=0\,\mathrm{MeV})}$	$\mu_{B_c}^{(m_s=174\,\mathrm{MeV})}$	Yang et al.[33]	Scholl & Weigel[47]	Lattice QCD [36,52]
Σ_c^{++}	1.58	1.60	2.15 ± 0.1	2.45	2.220 ± 0.505
Σ_c^{+}	0.39	0.33	0.46 ± 0.03	0.25	–
Σ_c^{0}	-0.79	-0.94	-1.24 ± 0.05	-1.96	-1.073 ± 0.269
$\Xi_c'^{+}$	0.39	0.44	0.60 ± 0.02	–	0.315 ± 0.141
$\Xi_c'^{0}$	-0.79	-0.84	-1.05 ± 0.04	–	-0.599 ± 0.071
Ω_c^{0}	-0.79	-0.75	-0.85 ± 0.05	–	-0.639 ± 0.088



*J. Y. Kim and H. C. Kim, Phys. Rev. D***97***, no.11, 114009(2018).*

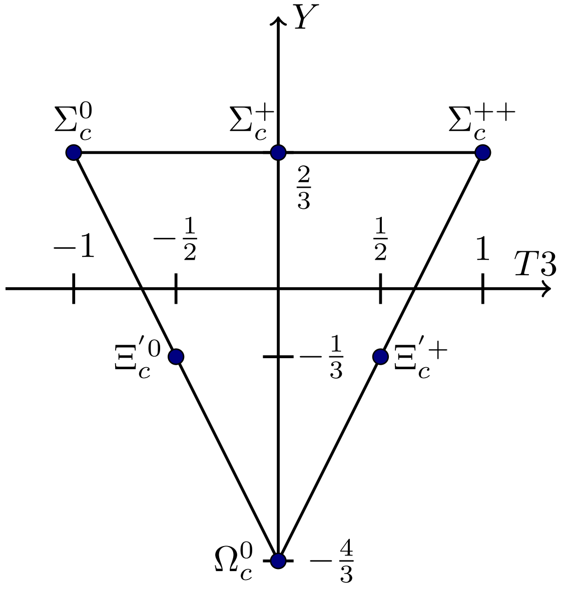
- ✓ The electric form factors of the spin-3/2 heavy baryons are exactly same as those of the spin-1/2 heavy baryons within the present frame work.

- ✓ The magnetic form factors are as follows:

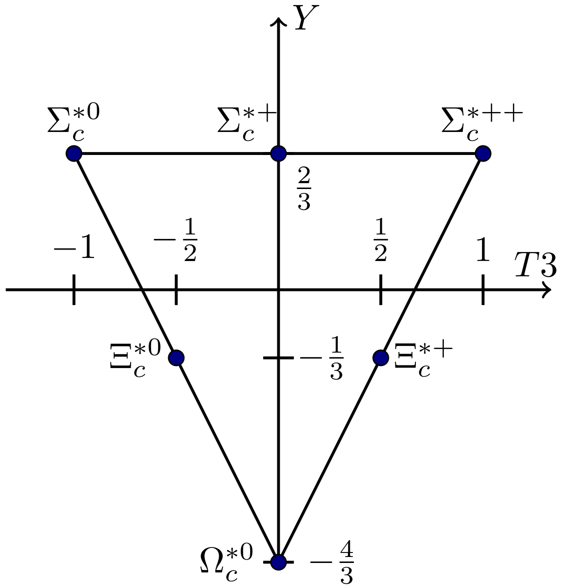
$$G_{M1}(Q^2)[6_1^{3/2}, B_c] = \frac{3}{2} G_{M1}(Q^2)[6_1^{1/2}, B_c]$$

G. S. Yang and H. C. Kim, Phys. Lett. B **781**, 601(2018).

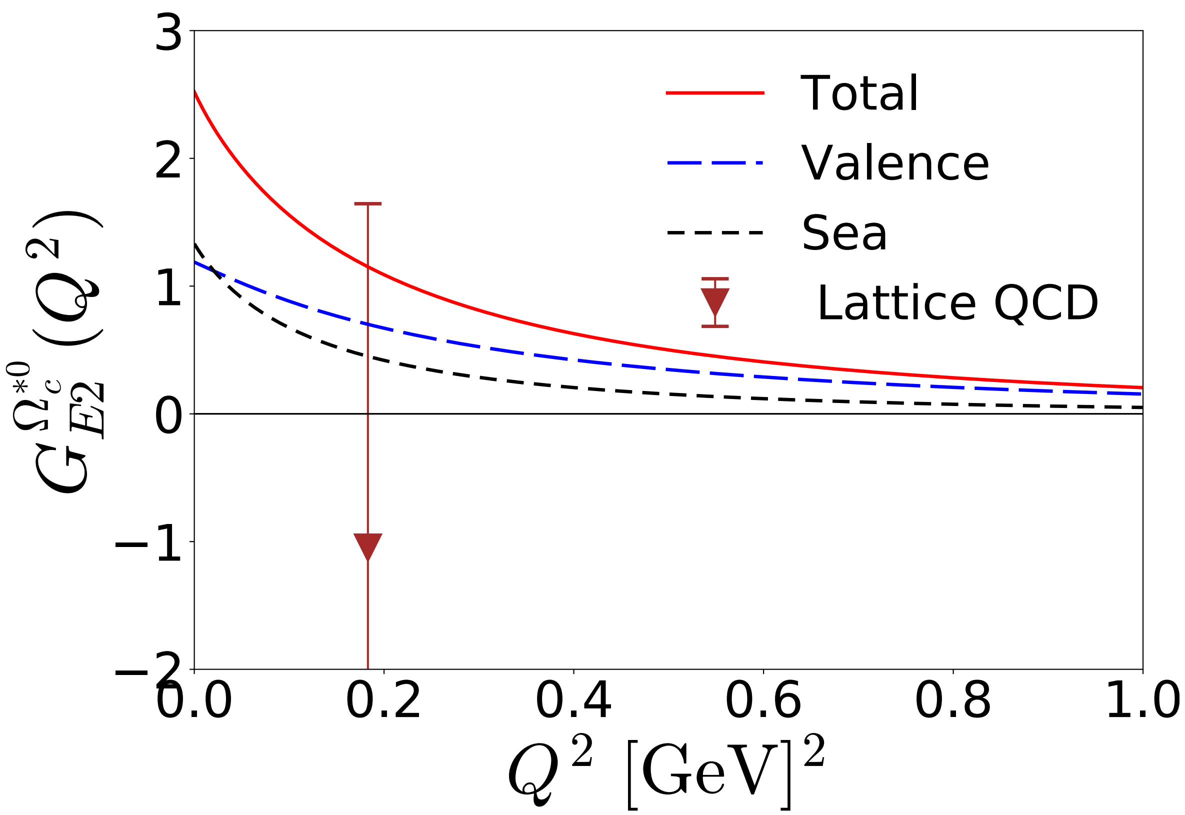
- ✓ There is no heavy quark contribution to the E2 form factor in the present approach.

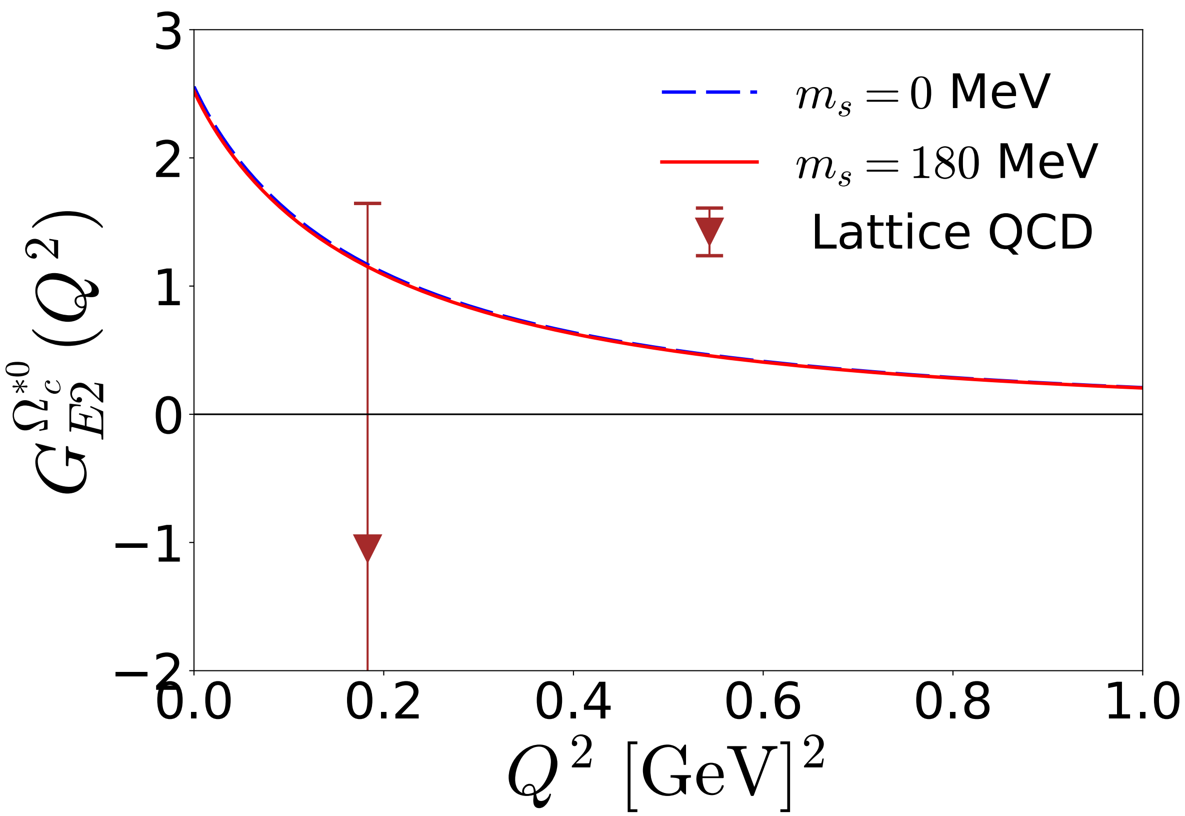


$(\mathbf{6}, J = \frac{1}{2})$



$(\mathbf{6}, J = \frac{3}{2})$





Q_B	Σ_c^{*++}	Σ_c^{*+}	Σ_c^{*0}	Ξ_c^{*+}	Ξ_c^{*0}	Ω_c^{*0}
$m_s = 180$	−0.0261	−0.0048	0.0166	−0.0072	0.0147	0.0128
$m_s = 0$	−0.0259	−0.0065	0.0130	−0.0065	0.0130	0.0130
Valence	−0.0123	−0.0022	0.0078	−0.0034	0.0069	0.0060
Sea	−0.0138	−0.0025	0.0088	−0.0038	0.0078	0.0068

$$Q_{\Delta^+} = -0.039\ e \cdot \mathrm{fm}^2$$
$$Q_{\Omega^-} = 0.061\ e \cdot \mathrm{fm}^2$$

- We investigate the **electromagnetic form factors of baryon decuplet and its transition form factors.**

- The size of Δ^+ turns out electrically slightly larger than the proton.

- The electric quadrupole moment shows how much a particle is deformed, and it is found to be $Q_{\Delta^+} = -0.039\ e \cdot \mathrm{fm}^2$ and $Q_{\Omega^-} = 0.061\ e \cdot \mathrm{fm}^2$.

- The E2 and the C2 transition form factors satisfy the constraint condition at the pseudo-threshold, and corresponding amplitudes are very small in comparison with the M1 amplitude as much as ~2%.

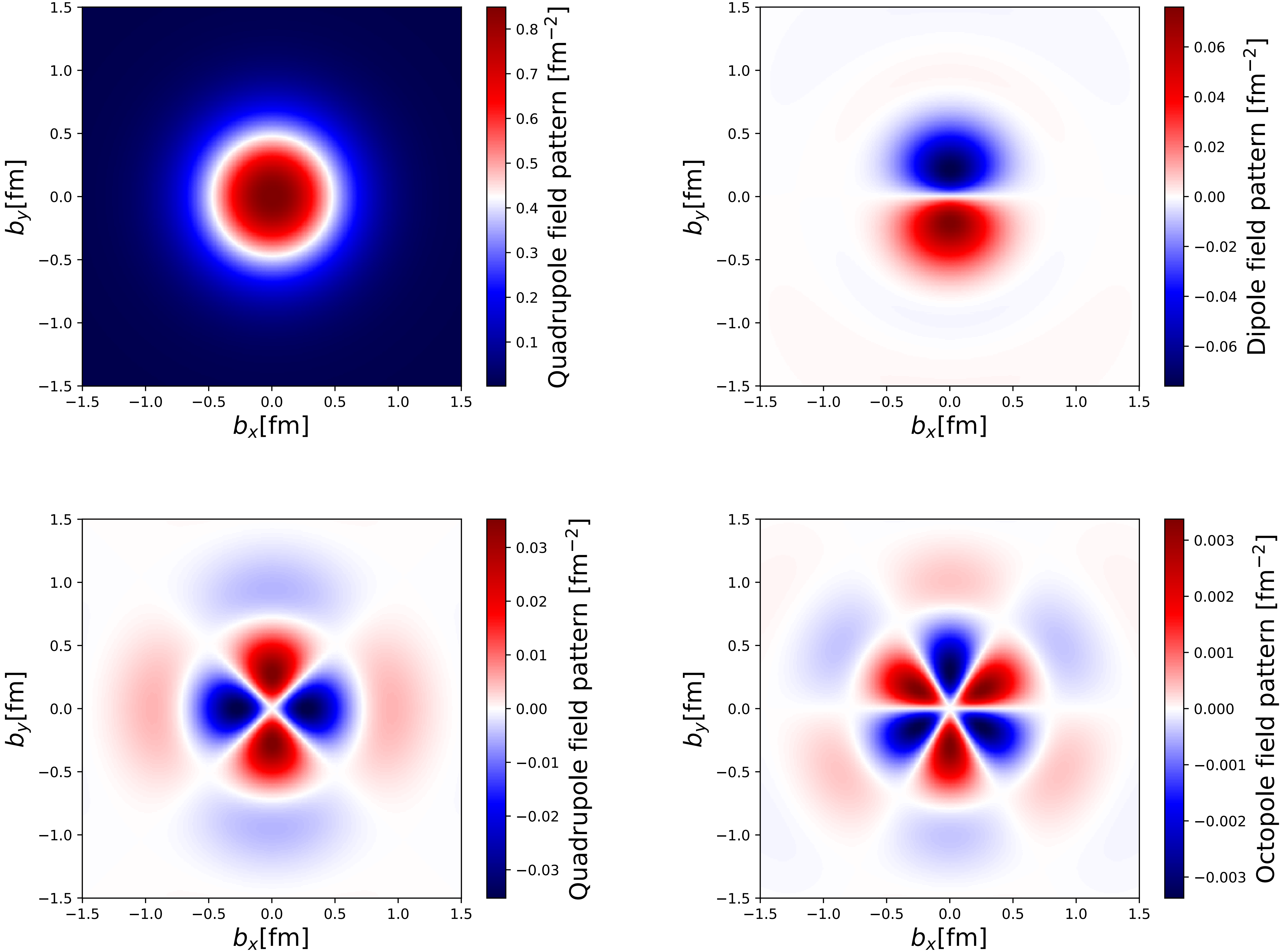
- The mean-field approach succeed in a description of the mass splittings in the heavy baryon sector and even predicts the mass of Ω_b^{*-}.

- **The heavy baryons are electrically compact objects.**

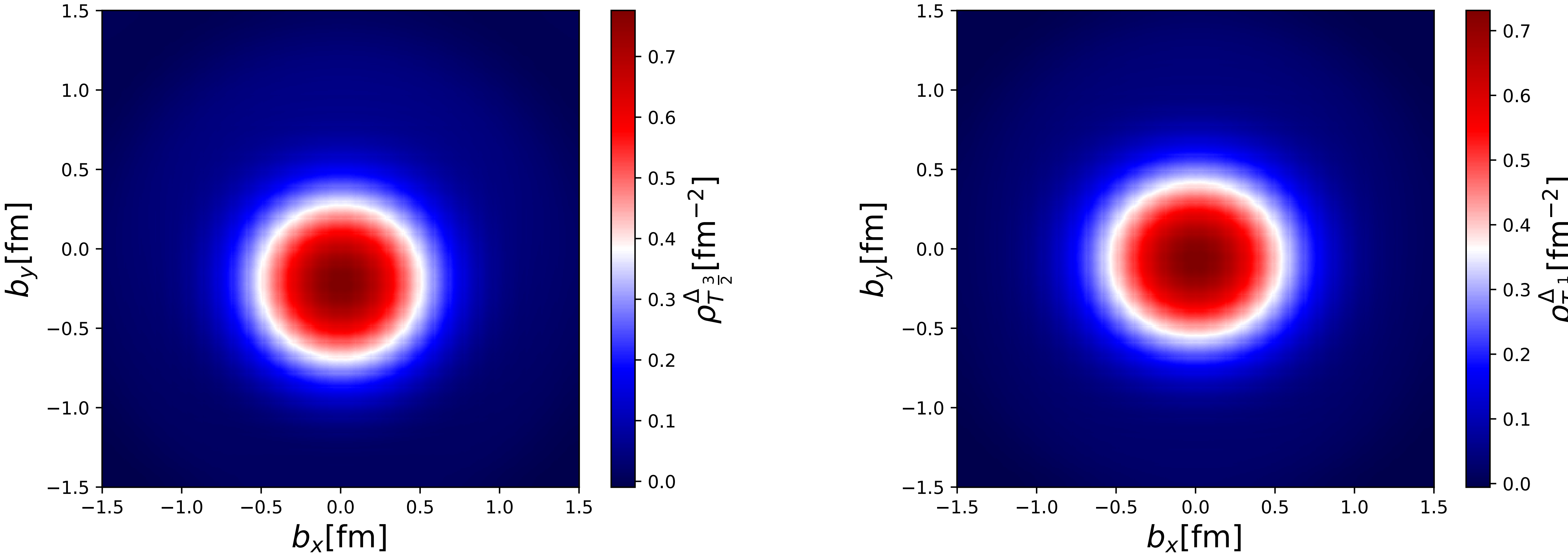
- The electric quadrupole moments of the heavy baryons are small, compared with those of the baryon decuplet.

- The heavy baryon transition form factors are under investigation, and the corresponding result will soon come out.

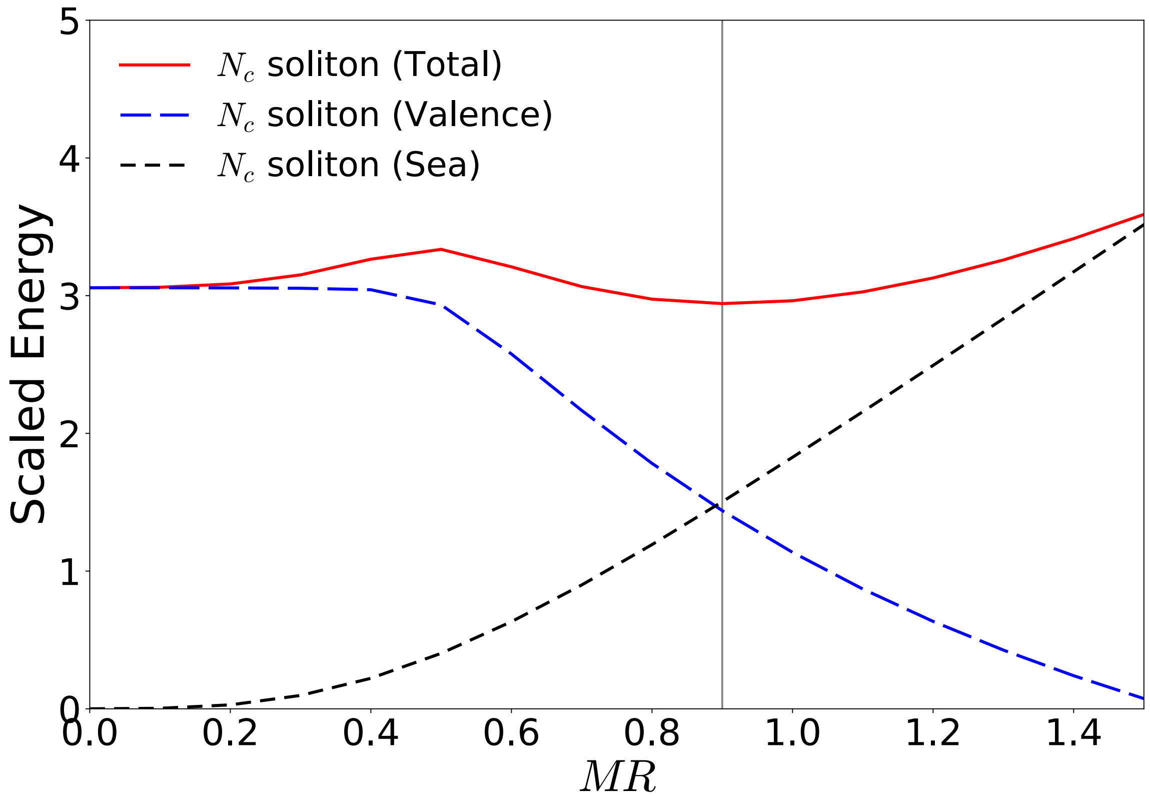
THANK YOU VERY MUCH!

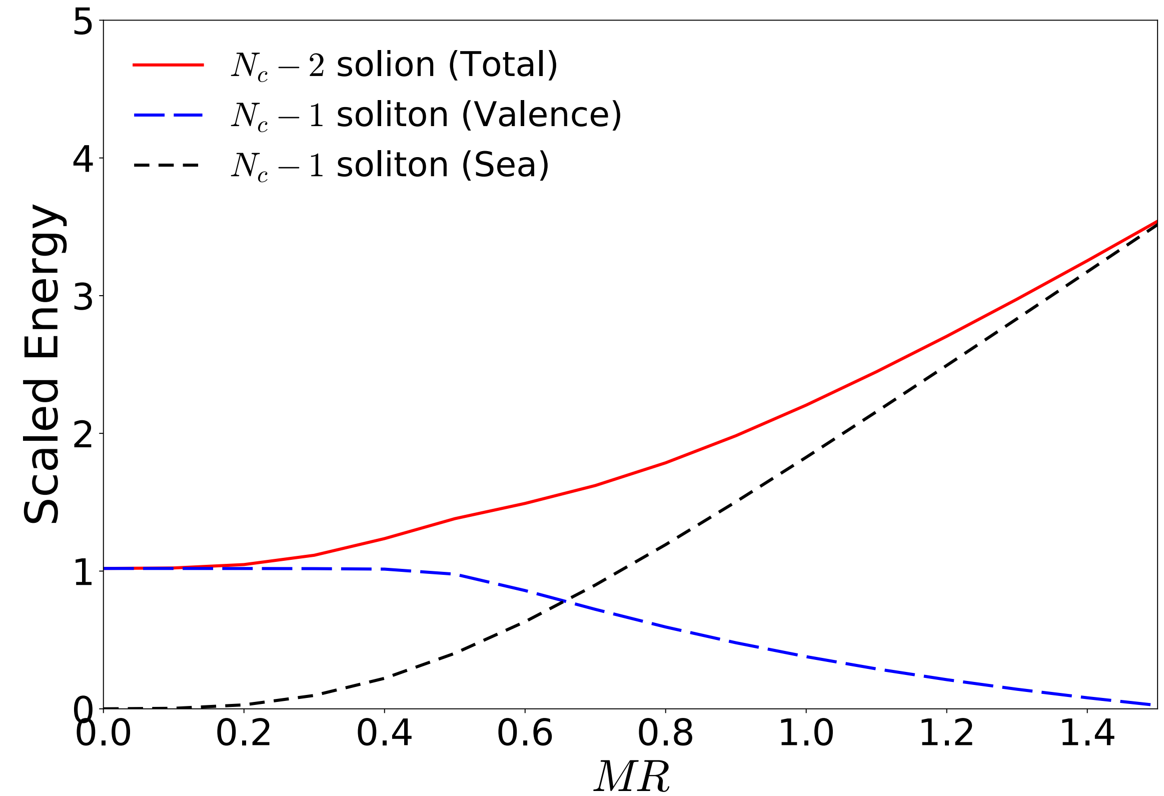


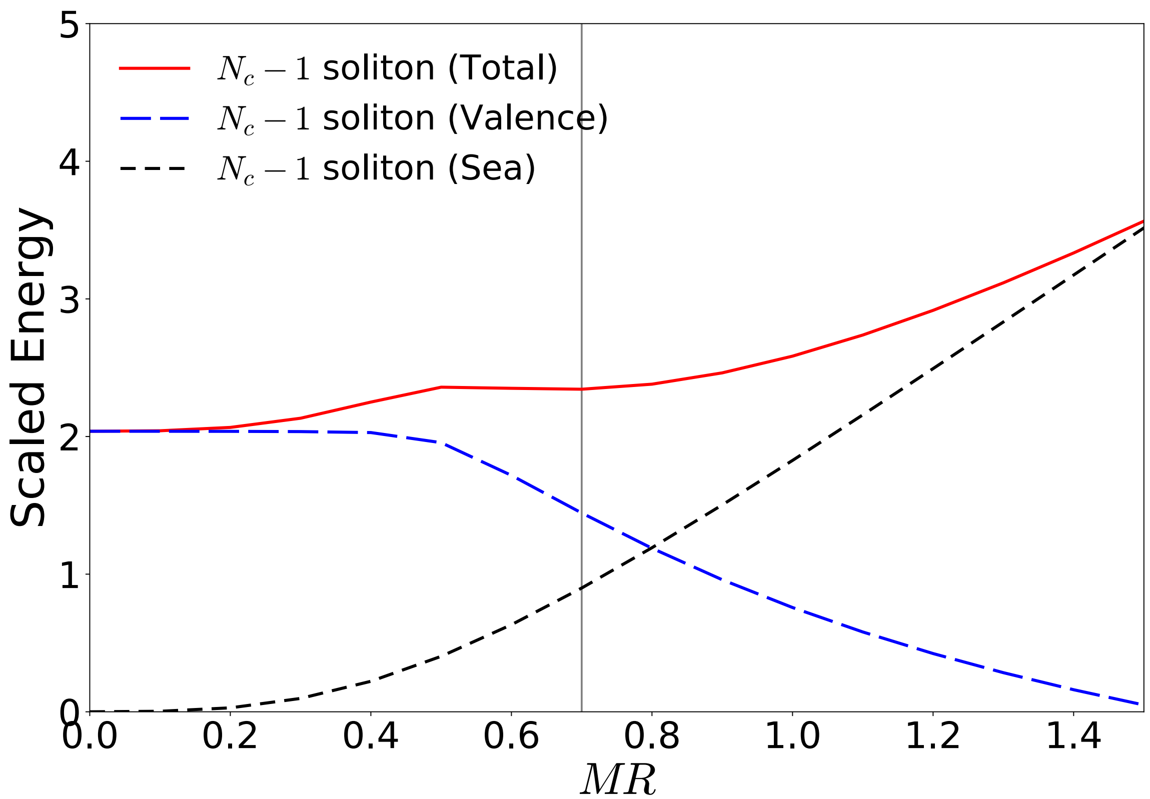
Quadrupole field pattern [fm^{-2}]
Dipole field pattern [fm^{-2}]
Quadrupole field pattern [fm^{-2}]
Octopole field pattern [fm^{-2}]
b_x[fm]
b_y[fm]



b_y[fm]
b_x[fm]
$\rho^{\Delta}_{T\frac{3}{2}}$[fm^{-2}]
b_y[fm]
b_x[fm]
$\rho^{\Delta}_{T\frac{1}{2}}$[fm^{-2}]







$$U(\vec{r}) = e^{i\Theta(r)n^a r^a}$$