Diquark Effective Theory
Colored Clusters in Hadron

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Contents

- Introduction
- Diquarks
- Chiral Diquark Effective Theory
- Conclusion
Scales in Nuclei and Hadrons

- **Atoms = nucleus + electrons**
  - size $\sim 10^5$ fm
  - binding energy $\sim 10^{-5}$ MeV/electron

- **Nuclei = protons + neutrons**
  - size $\sim$ a few - 10 fm
  - binding energy $\sim$ a few - 10 MeV/nucleon
  - excitation energy $\sim 0.1$ - 10 MeV

- **Hadrons = quarks**
  - size $\sim 0.5$ - 1 fm
  - binding energy $?$ confined
  - excitation energy $\sim 140$ MeV (pion production)
  - $\sim 100$ - 1000 MeV (hadron spectrum)
Nucleon is a colorless cluster

Why do nucleons not melt away in nuclei, when nucleons overlap with each other significantly?

Nuclear Force is Fine-Tuned:
Both the LR attraction and SR repulsion are of the hadronic energy scale

\[ V_{NN} (central) \]

\[ \sim 100 - 1000 \text{ MeV} \]
Is Kaon a colorless cluster?

- Strong attraction of $K^\text{bar}$ in nuclei
  $\Lambda^*(1405)$ as $K^\text{bar} N$ “molecular bound state”.

- $K^- \text{pp nucleus } \sim (\Lambda^*p + p \Lambda^*)$ dibaryon
  Deeply bound $K^\text{bar}$ nuclear state?

- Can $K^\text{bar}$ be a robust cluster in nuclei?
  If yes, why? Is $K^\text{bar}$ still color singlet?
  If not, what is different from nucleon?
Hidden-Charm Multiquarks

- **X(3872)** found in 2003 by Belle (KEK)
  \[ \text{not reproduced by lattice QCD using only } q-q\bar{q} \text{ operators.} \]

- **Z(3900), Z(4430)** etc.: charged hidden charm states

### X(3872)
- **Belle**
  - Mass: 3899 MeV
  - Width: 46 MeV
  - Reference: PRL 110 (2013) 252001

### Zc⁺(4430)
- **Belle**
  - Mass: 4433 MeV
  - Width: 45 MeV
  - Reference: PRL 100 (2008) 142001

### Zc⁺(3900)
- **BES III**
  - Mass: 3899 MeV
  - Width: 46 MeV

\[ Z_c^+(4430) \text{ and } Z_c^+(3900) \text{ were observed by Belle and BES III.} \]
Hidden-Charm Multiquarks

$P_c \rightarrow J/\psi + p$ (c$\bar{c}$uud)
LHCb (PRL 115 (2015) 07201) found two penta-quark states with hidden c$\bar{c}$.
Hidden-Charm Multiquarks

$P_c \to J/\psi+p (c\bar{c}uud)$
LHCb (PRL 115 (2015)) found two penta-quark states with hidden $c\bar{c}$.

$P_c(4450) (3/2^+)$
$P_c(4380) (3/2^+)$

LHCb: arXiv:1904.03947
Above the threshold
$q\bar{q}$ creation and rearrangement of multiquarks

• What governs them?
• Constituent quarks, mesons, diquarks...
• Are heavy quarks useful to know it?

by A. Hosaka

April 20, 2015

Makoto Oka (ASRC, JAEA)
Color

- Quark has 3 colors (RBG) forming color-singlet hadrons

  **meson** $q$-$q^{\text{bar}}$
  $$3 \otimes \bar{3} = 1 \oplus 8$$

  **diquark** $q$-$q$
  $$3 \otimes 3 = \bar{3} \oplus 6$$
  *not white*

  **baryon** $q$-$q$-$q$
  $$3 \otimes 3 \otimes 3 = (\bar{3} \oplus 6) \otimes 3 = (1 \oplus 8) \oplus (8 \oplus 10)$$

  $$3 \otimes 3 = \bar{3}$$
  $$\bar{3} \otimes 3 = 1$$
Color

- more quarks

$q^2 \bar{q}^2$ (tetraquarks):

$$3 \otimes 3 \otimes \bar{3} \otimes \bar{3} = (2 \times 1) \oplus (4 \times 8) \oplus 10 \oplus \overline{10} \oplus 27$$

$q^4 \bar{q}^4$ (pentaquarks):

$$3^4 \otimes \bar{3} = (3 \times 1) \oplus \ldots$$

$q^6$ (dibaryons):

$$3^6 = (5 \times 1) \oplus \ldots$$
Color

- more quarks

$q^2-q^\text{bar}^2$ (tetraquarks):

$3 \otimes 3 \otimes \bar{3} \otimes \bar{3} = (2 \times 1) \oplus (4 \times 8) \oplus \ldots$

$q^6$ (dibaryons):

$3^4 \otimes \bar{3} = (3 \times 1) \oplus \ldots$

$3^6 = (5 \times 1) \oplus \ldots$

Multiquarks are colorful!

To explore confinement dynamics for exotic colorful components.
Diquark

- The simplest *colorful cluster* in hadrons is the diquark.
- Spin dependent force in the (magnetic) gluon exchange
  
  **Color-Magnetic Interaction**

  \[ \Delta_{CM} \equiv \langle - \sum_{i<j} (\vec{\lambda}_i \cdot \vec{\lambda}_j)(\vec{\sigma}_i \cdot \vec{\sigma}_j) \rangle. \]

  **Scalar diquark**: \(0^+\) color \(3^{\text{bar}}\) \(\Delta_{CM} = -8\)
  
  **Axial-vector**: \(1^+\) color \(3^{\text{bar}}\) \(\Delta_{CM} = +8/3\)

- Can be related to the Hyperfine splitting of the baryon

  \[ M(1^+)-M(0^+) = (2/3) \ [M(\Delta)-M(N)] \sim 200 \ \text{MeV} \]

  \[ \Delta_{CM} \quad +8/3 \quad -8 \quad +8 \quad -8 \]
Diquark

Diquarks $D_q (=qq)$ as elements of hadrons

$D_q D_q^{\text{bar}} = qq q^{\text{bar}} q^{\text{bar}} = \text{Tetraquark}$

$D_q Q = qq Q = HQ \text{ Baryon}$

$D_q D_q Q^{\text{bar}} = qq qq Q^{\text{bar}} = \text{Pentaquark}$

$D_q D_q D_q = qq qq qq$

$= \text{Hexaquark (Dibaryon)}$
Diquarks obey the Bose-Einstein statistics. BE condensate in dense hadronic matter is expected. => color-superconducting phase
Diquark Effective Theory

Strategy

- **Consider Diquarks** as “colorful” building blocks of hadrons and hadronic matter. In order to describe their dynamics, write down the *Diquark* effective Lagrangian.

- **Warning:** As *Diquarks* are not color singlet, we need a way to compensate the color in the confined phase. For instance, assume a background color field generated by a heavy quark.

- **Lattice QCD** helps us to fix the parameters of the effective Lagrangian.
Diquark in Lattice QCD

  quench, Landau gauge fixed
  \(m_q \sim 342\) MeV, \(M(0^+) \sim 694\) MeV, \(M(1^+) \sim 810\) MeV

- Alexandrou, de Forcrand, Lucini, PRL 97, 222002 (2006)
  From Qqq system, quench, gauge invariant
  \(M(1^+) - M(0^+) \sim 200-220\) MeV, \(R(S) \sim 1\) fm
  \(M(0^-) - M(0^+) \sim 600\) MeV

- Babich, et al., PR D76, 074021 (2007)
  quench, Landau gauge
  \(M(0^+) - 2m_q \sim -200\) MeV, \(M(1^+) - M(0^+) \sim 162\) MeV

- Yujiang Bi, et al., Chinese Physics C40 (2016) 073106
  full, Landau gauge
  \(M(0^+) - m_q \sim 310\) MeV, \(M(1^+) - M(0^+) \sim 290\) MeV


Evidence for Diquarks in Lattice QCD

C. Alexandrou,¹ Ph. de Forcrand,²,³ and B. Lucini²,⁴

\[ \Delta m/\delta m_{\Delta N} = 0.67(7), 0.73(8), \text{ and } 0.67(8) \]

\[ \delta m_{\Delta N} \sim 300\text{MeV} \longrightarrow \Delta m \sim 200 - 220\text{MeV} \]
# Diquark Effective Theory

## Color-Flavor-Spin Structures: Pauli principle

<table>
<thead>
<tr>
<th></th>
<th>$J^\pi$</th>
<th>color</th>
<th>flavor</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(q^TCq)_A^5$</td>
<td>0$^-$</td>
<td>3</td>
<td>3</td>
<td>$^3P_0$</td>
</tr>
<tr>
<td>$(q^TC\gamma^5q)_A^3$</td>
<td>0$^+$</td>
<td>3</td>
<td>3</td>
<td>$^1S_0$</td>
</tr>
<tr>
<td>$(q^TC\gamma^\mu\gamma^5q)_A^3$</td>
<td>1$^-$</td>
<td>3</td>
<td>3</td>
<td>$^3P_1$</td>
</tr>
<tr>
<td>$(q^TC\gamma^\mu q)_S^3$</td>
<td>1$^+$</td>
<td>3</td>
<td>6</td>
<td>$^3S_1$</td>
</tr>
<tr>
<td>$(q^TC\sigma^{\mu\nu}q)_S^3$</td>
<td>1$^+$, 1$^-$</td>
<td>3</td>
<td>6</td>
<td>$^3D_1$, $^1P_1$</td>
</tr>
<tr>
<td>$(q^TCq)_S^6$</td>
<td>0$^-$</td>
<td>6</td>
<td>6</td>
<td>$^3P_0$</td>
</tr>
<tr>
<td>$(q^TC\gamma^5q)_S^6$</td>
<td>0$^+$</td>
<td>6</td>
<td>6</td>
<td>$^1S_0$</td>
</tr>
<tr>
<td>$(q^TC\gamma^\mu\gamma^5q)_S^6$</td>
<td>1$^-$</td>
<td>6</td>
<td>6</td>
<td>$^3P_1$</td>
</tr>
<tr>
<td>$(q^TC\gamma^\mu q)_A^6$</td>
<td>1$^+$</td>
<td>6</td>
<td>$\bar{3}$</td>
<td>$^3S_1$</td>
</tr>
<tr>
<td>$(q^TC\sigma^{\mu\nu}q)_A^6$</td>
<td>1$^+$, 1$^-$</td>
<td>6</td>
<td>$\bar{3}$</td>
<td>$^3D_1$, $^1P_1$</td>
</tr>
</tbody>
</table>

- **Scalar**
  - “good” diquark
- **Axial-vector**
  - “bad” diquark
- Color 6

$C \equiv i\gamma^0\gamma^2 = -C^{-1} = -C^T$
Multiquark exotic states

- Double charm tetraquark meson
  \[ T_{cc} (cu^\text{bar}d^\text{bar}, 1^+, I=0) = [cc]_{1+} [u^\text{bar}d^\text{bar}]_{0+} \]

- The lowest strong-decay threshold is DD* (L=0).
- If the scalar diquark is light enough to make \( T_{cc} \) bound below DD* threshold, \( T_{cc} \) will be a stable tetra-quark resonance.


- New possible color correlations with the production rates
Multiquark exotic states

- Double charm tetraquark meson
  - The lowest strong-decay threshold is DD* (L=0).
  - If the scalar diquark is light enough to make $T_{cc}$ bound below DD* threshold, $T_{cc}$ will be a stable tetra-quark resonance.

New possible color correlations with the production rates

Hyodo, Liu, Oka, Sudoh, Yasui, PLB721 (2013) 56-60

Multiquark exotic states

$\Lambda c, \Sigma c, \Sigma c^*(cqq)$

$qq$ flavor SU(3)

$\bar{c}c$ flavor SU(3)

$6_c(1^S_0) \otimes 6_c(1^S_0)$

$\bar{3}_c(3^S_1)$

$\Sigma c 1/2^+ \Sigma c^* 3/2^+$

$\bar{3}_c(3^S_1) \otimes \bar{3}_c(3^S_1)$

$200$ MeV

$6_c(1^S_0) \otimes 6_c(3^S_1)$

$T_{cc}^1 \left[ 6_c \right]$

$\bar{c}c$

$q=q,u,d,s$

$0^+$

$0^+ 1^+ 2^+$

$25$ MeV

$75$ MeV

$125$ MeV

$1^+$

$D^*D^*$ threshold

$T_{cc}^1 [3_c \overline{b}ar]$

$T_{cc}^1 [\bar{3}_c]$

$1^+$

color magnetic interaction

mixing suppressed

suppressed
Chiral Diquarks

- **Chiral symmetry SU(3)$_R$ x SU(3)$_L$**

  \[ q^a_{iR} = P_R q^a_i, \quad q^a_{iL} = P_L q^a_i \]

  \[ P_{R,L} \equiv \frac{1 \pm \gamma_5}{2}, \quad [P_{R,L}, C] = 0, \quad P^T_{R,L} = P_{R,L} \]

- **Scalar chiral diquarks (color $3^{\text{bar}}$)**

  \[ d^a_{iR} \equiv \epsilon_{ijk}(q^T_{jR} C q^a_{kR}) \]

  \[ \text{Right scalar diquark, chiral } (\bar{3},1), \text{ color } \bar{3} \]

  \[ d^a_{iL} \equiv \epsilon_{ijk}(q^T_{jL} C q^a_{kL}) \]

  \[ \text{Left scalar diquark, chiral } (1, \bar{3}), \text{ color } \bar{3} \]

- **Parity eigenstates: $0^+$, $0^-$ diquarks**

  \[ S^a_i = d^a_{iR} - d^a_{iL} = \epsilon_{ijk}(q^T_{jL} C \gamma_5 q^a_k) \]

  \[ (\bar{3}, 1) + (1, \bar{3}) \]

  \[ P^a_i = d^a_{iR} + d^a_{iL} = \epsilon_{ijk}(q^T_{jL} C q^a_k) \]
Chiral Diquarks

- **SU(3)_R x SU(3)_L transform for scalar diquarks**

  \[
  q_R \rightarrow U_R q_R = (U_R)_{ij} q_{jR}, \quad U_R \in SU(3)_R \\
  q_L \rightarrow U_L q_L = (U_L)_{ij} q_{jL}, \quad U_L \in SU(3)_L
  \]

  \[
  d_R \rightarrow d_R U_R^\dagger \quad (\bar{3}, 1), \quad d_R^\dagger \rightarrow U_R d_R^\dagger \quad (3, 1) \\
  d_L \rightarrow d_L U_L^\dagger \quad (1, \bar{3}), \quad d_L^\dagger \rightarrow U_L d_L^\dagger \quad (1, 3)
  \]

  \[
  d_R^\alpha(d_R^\alpha)^\dagger \equiv d_{iR}^\alpha(d_{iR}^\alpha)^\dagger, \quad d_L^\alpha(d_L^\alpha)^\dagger \equiv d_{iL}^\alpha(d_{iL}^\alpha)^\dagger, \quad \text{chiral invariant, color singlet} \\
  d_R^\alpha(d_R^\alpha)^\dagger + d_L^\alpha(d_L^\alpha)^\dagger = \text{Lorentz scalar, color singlet, chiral invariant}
  \]

- **Strategy upgraded:**

  Write down general forms of the chiral invariant Lagrangian for the chiral diquarks with a background chiral meson field.
Chiral Diquark Effective Theory

**Chiral meson field:** $\Sigma_S$ (scalar nonet), $\Sigma_P$ (pseudo-scalar nonet)

$$\Sigma = \Sigma_S + i\Sigma_P \rightarrow U_L \Sigma U_R^\dagger \quad (\bar{3}, 3)$$

$$\Sigma_S = \Phi_S/f_\pi, \quad \Sigma_P = \Phi_P/f_\pi \quad \langle \Sigma_S \rangle = 1, \quad \langle \Sigma_P \rangle = 0$$

$\mathcal{P}$: $d_R \leftrightarrow d_L, \quad \Sigma \rightarrow \Sigma^\dagger$

**The effective Lagrangian** (of the linear sigma model)

$$\mathcal{L} = -D_\mu d_R \left( D_\mu d_R \right)^\dagger - D_\mu d_L \left( D_\mu d_L \right)^\dagger$$

$$-m_0^2(d_Rd_R^\dagger + d_Ld_L^\dagger) - m_1^2(d_R\Sigma^\dagger d_L^\dagger + d_L\Sigma d_R^\dagger)$$

$$D_\mu = \partial_\mu + ig T^\alpha G^\alpha_{\mu}^{\text{ext}}$$
Chiral Diquark Effective Theory

\[ \mathcal{L} = -D_\mu d_R (D_\mu d_R)^\dagger - D_\mu d_L (D_\mu d_L)^\dagger - m_0^2 (d_R d_R^\dagger + d_L d_L^\dagger) - m_1^2 (d_R \Sigma^\dagger d_L^\dagger + d_L \Sigma d_R^\dagger) \]

chiral invariant mass CSB mass

- For the chiral invariant vacuum, \( \langle \Sigma \rangle = \langle \Sigma_s \rangle = 0 \) (\( \langle \sigma \rangle = 0 \)), the diquark mass is given by \( m_0 \).

- For normal vacuum \( \Sigma = 1 \), the masses of \( 0^+ \) and \( 0^- \) diquarks are given by

\[
M^2 = \begin{pmatrix} m_0^2 & m_1^2 \\ m_1^2 & m_0^2 \end{pmatrix} \longrightarrow M = \sqrt{m_0^2 \pm m_1^2}
\]

\[
\sqrt{m_0^2 - m_1^2} \rightarrow S = d_R - d_L (0^+) \quad \sqrt{m_0^2 + m_1^2} \rightarrow P = d_R + d_L (0^-)
\]

\[
\Delta M^2 = M_P^2 - M_S^2 = 2m_1^2
\]
Chiral Diquark Effective Theory

\( \Sigma_P \) (pseudo-scalar nonet) couplings

\[
\Sigma = \Sigma_S + i \Sigma_P \rightarrow U_L \Sigma U_R^{\dagger} \quad (\bar{3}, 3) \quad \Sigma_P = \frac{1}{f_\pi} \lambda^a \Phi_P^a
\]

\[
V = (-i) m_1^2 (\bar{d}_R \Sigma_P d_L^{\dagger} - \bar{d}_L \Sigma_P d_R^{\dagger})
\rightarrow (-i) \frac{m_1^2}{f_\pi} (\bar{d}_R \lambda^a d_L^{\dagger} - \bar{d}_L \lambda^a d_R^{\dagger}) \Phi_P^a = (-i) \frac{m_1^2}{2 f_\pi} (P \lambda^a S^{\dagger} - S \lambda^a P^{\dagger}) \Phi_P^a
\]

“Goldberger-Treiman” relation

\[
g_{PS}^{SP} = \frac{m_{S1}^2}{2 f_\pi} = \frac{M_P^2 - M_S^2}{f_\pi} = \frac{\Delta_M^2}{f_\pi}
\]
Chiral Diquark Effective Theory

Vector + Axial-vector (3,3) Diquarks

\[ d_{ij}^{\mu_a} \equiv \epsilon_{abc}(q_{iL}^{bT} C\gamma^\mu q_{jR}^c) = \epsilon_{abc}(q_{jR}^{bT} C\gamma^\mu q_{iL}^c) \quad \text{chiral (3,3) vector diquark} \]

\[ d_{V[ij]}^{\mu_a} = d_{ij}^{\mu_a} - d_{ji}^{\mu_a} = \epsilon_{abc}(q_{i}^{bT} C\gamma^\mu \gamma^5 q_{j}^c) \quad \text{Vector } 1^- \text{ diquark, flavor } 3 \]

\[ d_{A[ij]}^{\mu_a} = d_{ij}^{\mu_a} + d_{ji}^{\mu_a} = \epsilon_{abc}(q_{i}^{bT} C\gamma^\mu q_{j}^c) \quad \text{Axial-vector } 1^+ \text{ diquark, flavor } 6 \]

\[ d^\mu \rightarrow U_L d^\mu U_R^{T}, \quad (3,3) \quad d^{\mu\dagger} \rightarrow U_R^{T\dagger} d^\mu U_L^{\dagger} \quad (\bar{3}, \bar{3}) \]

\[ \mathcal{L} = -\frac{1}{2} \text{Tr}[F^{\mu\nu} F_{\mu\nu}^{\dagger}] - m_0^2 \text{Tr}[d^\mu d_{\mu}^{\dagger}] - m_1^2 \text{Tr}[\Sigma^{\dagger} d^\mu \Sigma^T d_{\mu}^{\dagger T}] \]

\[ F^{\mu\nu} = D^\mu d^\nu - D^\nu d^\mu \]
Chiral Diquark Effective Theory

- **Full (Scalar + Vector) Diquark Effective Theory**

\[
\mathcal{L} = -D_\mu d_R (D_\mu d_R)^\dagger - D_\mu d_L (D_\mu d_L)^\dagger \\
- m^2_{S0} (d_R d_R^\dagger + d_L d_L^\dagger) - m^2_{S1} (d_R \Sigma^\dagger d_L^\dagger + d_L \Sigma d_R^\dagger) \\
- \frac{1}{2} \text{Tr} [F_{\mu\nu} F_{\mu\nu}^\dagger] - m^2_{V0} \text{Tr} [d_\mu d_\mu^\dagger] - m^2_{V1} \text{Tr} [\Sigma^\dagger d_\mu^\dagger \Sigma d_\mu^T] \\
- g_{pSV} \text{Tr} [d_\mu \epsilon_R d_R^\dagger \partial_\mu \Sigma^\dagger + d_\mu^\dagger (\partial_\mu \Sigma)^T \epsilon_L d_L^\dagger + (\text{c.c.})]
\]

- **Expected Couplings of PS meson**

\[\Xi_c \leftrightarrow \Lambda_c \bar{K}\]

\[\Sigma_c \rightarrow \Lambda_c \pi\]
Chiral Diquark Effective Theory

  Non-linear chiral Diquark effective theory (for pentaquark/tetraquarks)

- Y. Kawakami, M. Harada,
  Chiral effective theory of Single Heavy Baryons (HQ symmetry)
Chiral Diquark Effective Theory

C. Alexandrou et al., Quenched QCD, PRL 97, 222002 (2006)

PRL 97, 222002 (2006)

$\sim 1000 \text{ MeV} \quad 0^- \quad \sqrt{m_0^2 + m_1^2}$

$\sim 600 \text{ MeV} \quad 1^+$

$\sim 400 \text{ MeV} \quad 0^+ \quad m_0 - m_1$

$m_0 \sim 760 \text{ MeV}, \quad m_1 \sim 640 \text{ MeV} \quad (\text{SU}(3) \text{ chiral limit})$

Under restoration of chiral symmetry, $\langle \Sigma \rangle = \langle \Sigma_s \rangle \rightarrow 0$, the mass of the Scalar diquark and Pseudoscalar diquark will be degenerate to be $m_0 \sim 760 \text{ MeV}$. 
\[ \langle \Sigma \rangle = \langle \Sigma_s \rangle \rightarrow 0 \]

\[ m_0 \sim 760 \text{ MeV}, \ m_1 \sim 640 \text{ MeV} \ (\text{SU(3) chiral limit}) \]

\[ m_0 \sim 760 \text{ MeV} \]
Diquark mass differences from unquenched lattice QCD

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Zhaofeng Liu(刘朝峰)$^{2;3}$  Hao-Xue Qiao(乔豪学)$^1$  Yi-Bo Yang(杨一波)$^3$

$\begin{align*}
\text{c005, } J_c^5 \text{ and } J_c^{05} \\
\vdots & : aM_0 = 0.6700 \\
\bigstar & : aM_0 = 0.0670 \\
\times & : aM_0 = 0.0135
\end{align*}$

Makoto Oka (ASRC, JAEA)
Diquark mass differences from unquenched lattice QCD

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Zhaofeng Liu (刘朝峰)\textsuperscript{2,3} Hao-Xue Qiao (乔豪学)\textsuperscript{1} Yi-Bo Yang (杨一玻)\textsuperscript{3}

Table 8. Diquark masses and mass difference for various valence quark masses on ensemble c005. The first line is a linear extrapolation in $am_q$ to the chiral limit with the lowest four data points. $a^{-1}=1.75$ GeV

<table>
<thead>
<tr>
<th>$am_q$</th>
<th>$aM_{q+}(J^c_{\pi}^{95})$</th>
<th>$aM_{1+}(J^c_1)$</th>
<th>$a(M_{1+} - M_{0+})$</th>
<th>$aM_{0-}(J^c_1)$</th>
<th>$aM_{1-}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.4142(63)</td>
<td>0.584(21)</td>
<td>0.166(22)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.01350</td>
<td>0.4534(70)</td>
<td>0.611(29)</td>
<td>0.158(31)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.02430</td>
<td>0.4875(52)</td>
<td>0.635(18)</td>
<td>0.148(19)</td>
<td>0.796(52)</td>
<td>-</td>
</tr>
<tr>
<td>0.04890</td>
<td>0.5692(37)</td>
<td>0.694(10)</td>
<td>0.1248(98)</td>
<td>0.862(23)</td>
<td>0.987(53)</td>
</tr>
<tr>
<td>0.06700</td>
<td>0.6166(48)</td>
<td>0.7300(85)</td>
<td>0.1134(93)</td>
<td>0.904(18)</td>
<td>1.003(41)</td>
</tr>
<tr>
<td>0.15000</td>
<td>0.8293(70)</td>
<td>0.8907(68)</td>
<td>0.0614(89)</td>
<td>1.056(29)</td>
<td>1.140(24)</td>
</tr>
<tr>
<td>0.33000</td>
<td>1.1830(30)</td>
<td>1.2334(55)</td>
<td>0.0504(45)</td>
<td>1.378(17)</td>
<td>1.454(21)</td>
</tr>
<tr>
<td>0.67000</td>
<td>1.8265(39)</td>
<td>1.8604(68)</td>
<td>0.0339(62)</td>
<td>1.976(12)</td>
<td>2.025(16)</td>
</tr>
</tbody>
</table>

$M(0^+) \sim 720 \text{ MeV}$

$M(1^+) - M(0^+) \sim 290 \text{ MeV}$  \hspace{1cm} $M(1^+) \sim 1010 \text{ MeV}$

$M(0^-) - M(0^+) \sim 540 \text{ MeV}$  \hspace{1cm} $M(0^-) \sim 1260 \text{ MeV}$  \hspace{1cm} $m_0 = 1030 \text{ MeV}, m_1 = 730 \text{ MeV}$

$M(1^-) - M(1^+) \sim 510 \text{ MeV}$  \hspace{1cm} $M(1^-) \sim 1520 \text{ MeV}$

Makoto Oka (ASRC, JAEA)
$m_0 = 1030$ MeV, $m_1 = 730$ MeV
Diquarks in Heavy Baryons

From Heavy baryon spectroscopy
Λ_Q/Σ_Q with S(0^+)/A(1^+) diquarks

Diquarks
S(0^+) ud (S=0, I=0)
A(1^+) (uu, ud, dd) (S=1, I=1)
Diquarks in Heavy Baryons (P-wave)

by A. Hosaka

$\lambda$ mode

L=0

L=1

$\rho$ mode

L=0

L=1

P-wave

single quark mode

S-wave

m_Q=m_q

m_Q \neq m_q

ground states

$3^3\rho_{0,1,2}$

$6^1\rho_1$

$A_{L=0}^\perp Q$

$V_{L=0}^\perp Q$

$P_{L=0}^\perp Q$

$1/2^+$

$1/2^-$

$3/2^+$

$3/2^-$

$5/2^+$

$5/2^-$

$1/2^+$

$1/2^-$

$3/2^+$

$3/2^-$

$\Sigma$

$\Lambda$

ground states
Diquarks in Heavy Baryons (P-wave)

Heavy baryons are good probes of diquarks in hadrons or hadronic matter!

by A. Hosaka
Conclusion

- We construct a chiral effective theory of Diquarks. Scalar and Pseudo-Scalar Diquarks are paired in $(\bar{3}, 1) + (1, \bar{3})$. Vector and Axial-Vector Diquarks are in $(3, 3)$ representation.

- For the SP sector, chiral invariant mass term and SB mass term are available. Using the LQCD data of the Diquarks, we may determined the chiral invariant mass $\sim 700$ MeV. We obtain the GT relation for the PS meson-Diquark coupling.

- For the VA sector, we get a flavor 6 axial-vector and $3^{\text{bar}}$ vector. Chiral invariant and $\langle \Sigma \rangle^2$ mass terms are allowed. (No linear term)

- Future directions, perspectives
  Heavy baryon spectroscopy
  Exotic states, such as tetra quark, diquark matter, . .