Diquark Effective Theory
Colored Clusters in Hadron

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Inha University

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Scales in Nuclei and Hadrons

- **Atoms = nucleus + electrons**
  size ~ 10^5 fm
  binding energy ~ 10^{-5} MeV/electron

10^5

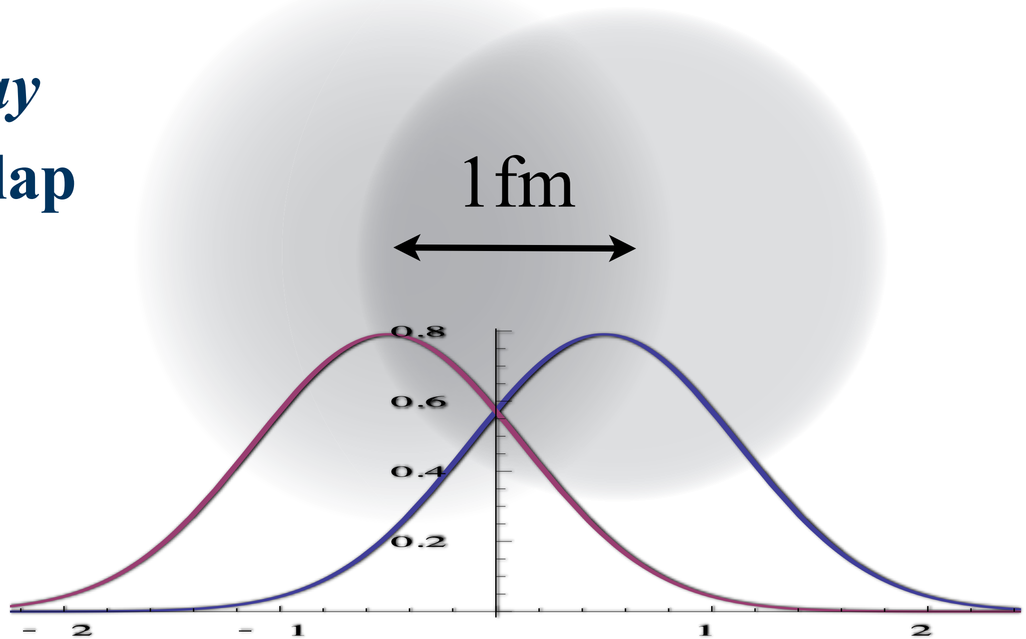
- **Nuclei = protons + neutrons**
  size ~ a few - 10 fm
  binding energy ~ a few - 10 MeV/nucleon
  excitation energy ~ 0.1 - 10 MeV

10^1

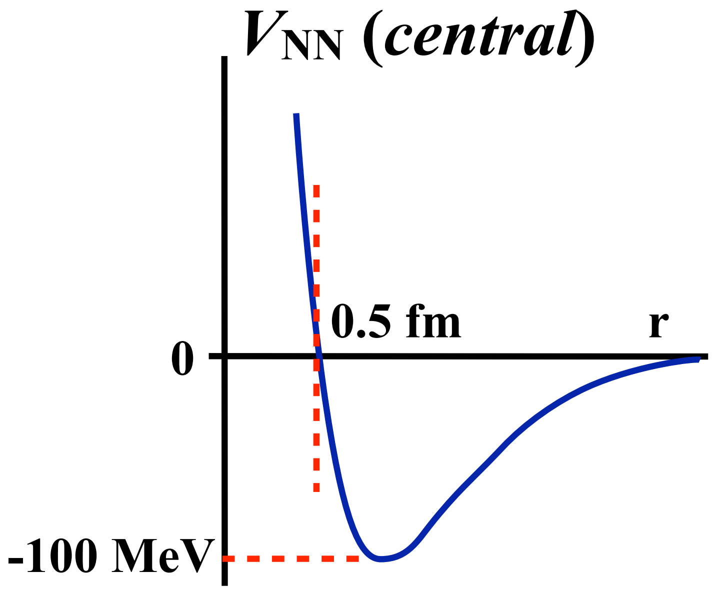
- **Hadrons = quarks**
  size ~ 0.5 - 1 fm
  binding energy ? confined
  excitation energy ~ 140 MeV (pion production)
  ~ 100 - 1000 MeV (hadron spectrum)

Nucleon is a colorless cluster

- **Why do nucleons *not melt away* in nuclei, when nucleons overlap with each other significantly?**

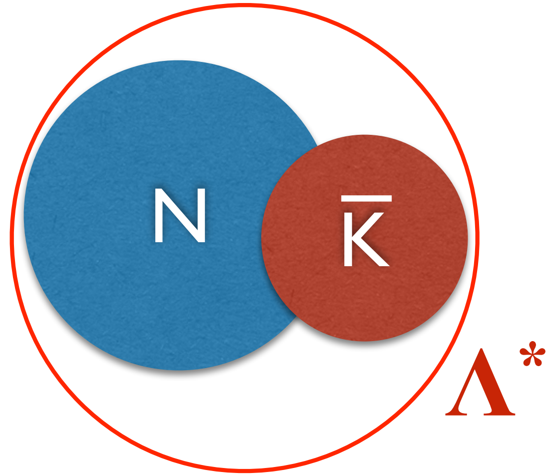


- **Nuclear Force is Fine-Tuned:**
  Both the LR attraction and SR repulsion are of the hadronic energy scale
  ~ 100 – 1000 MeV



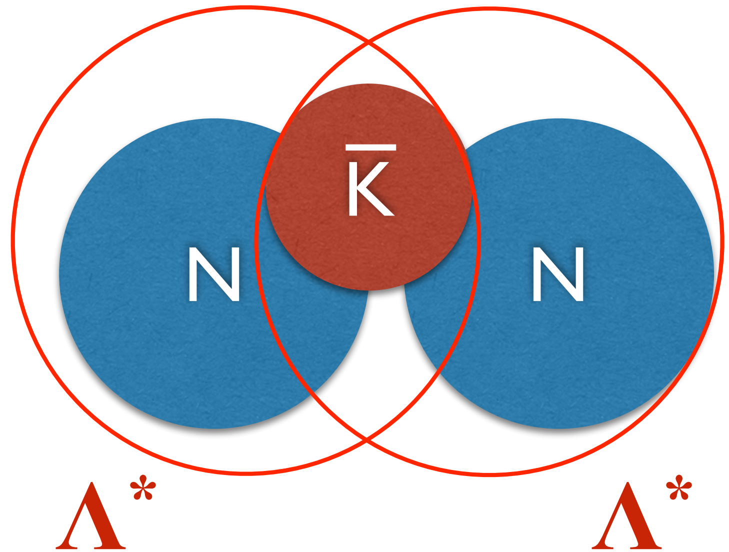
Is Kaon a colorless cluster?

- **Strong attraction of K^{bar} in nuclei**
  Λ^*(1405) as K^{bar} N "molecular bound state".



- **K$^-$ pp nucleus ~ (Λ^*p + p Λ^*) dibaryon**
  Deeply bound K^{bar} nuclear state?

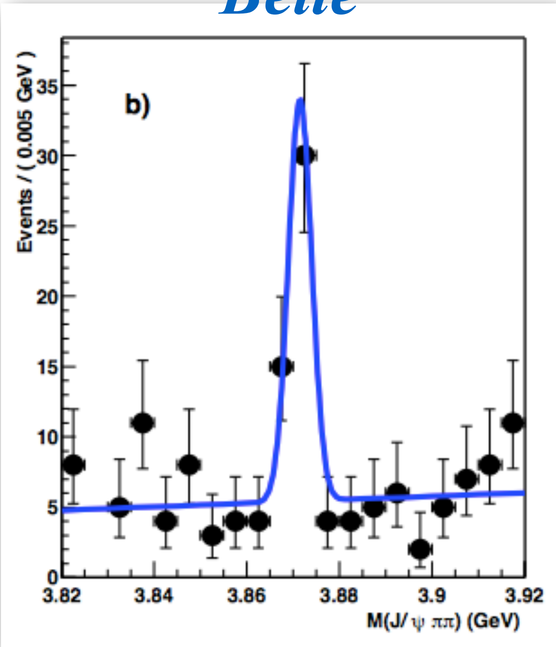
- **Can K^{bar} be a robust *cluster* in nuclei?**
  If yes, why? Is K^{bar} still color singlet?
  If not, what is different from nucleon?



Hidden-Charm Multiquarks

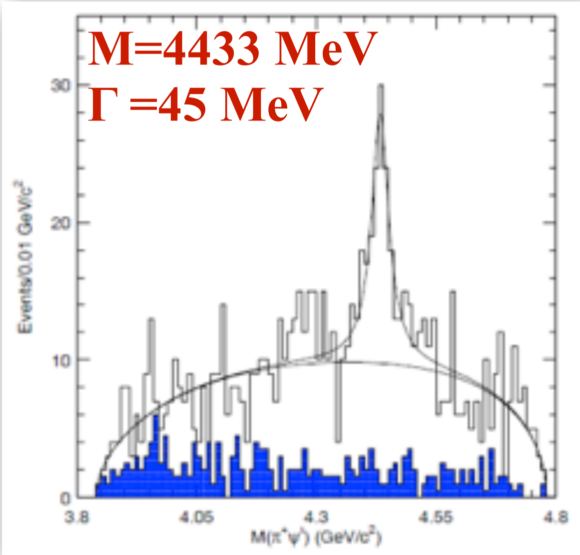
- X(3872) found in 2003 by Belle (KEK)
  → *not reproduced by lattice QCD using only q-q^{bar} operators.*
- Z(3900), Z(4430) etc. : charged hidden charm states

X(3872) — *Belle*



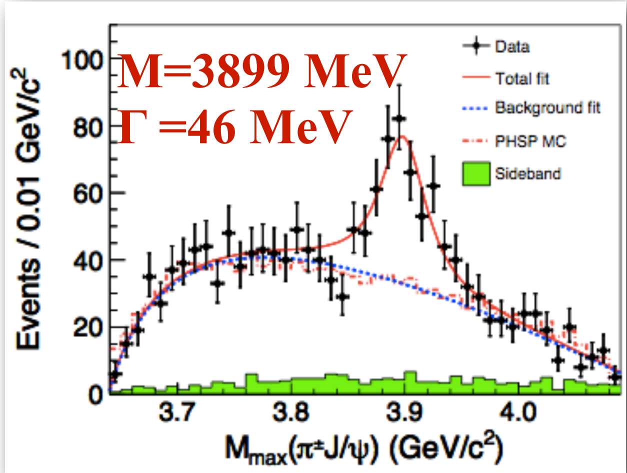
PRL 91 (2003) 262001

Z_c^+(4430) — *Belle*



PRL 100 (2008) 142001

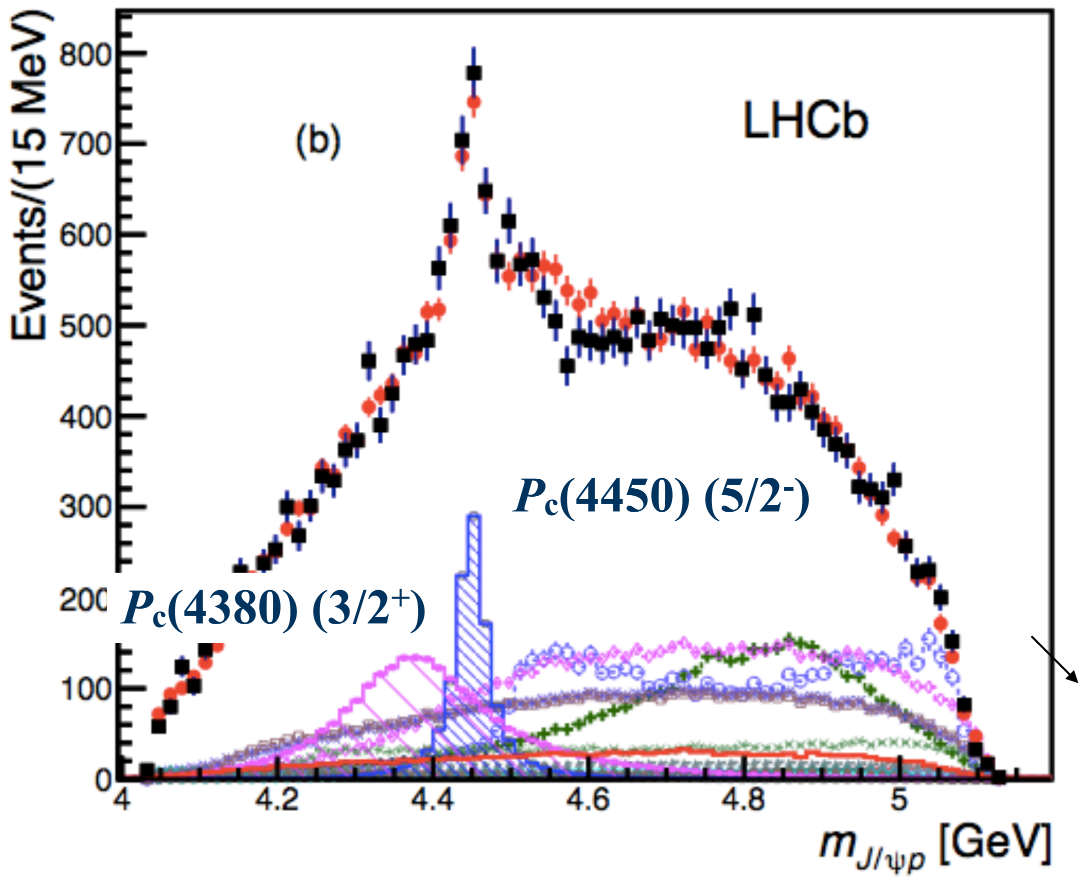
Z_c^+(3900) — *BES III*



PRL 110 (2013) 252001

Hidden-Charm Multiquarks

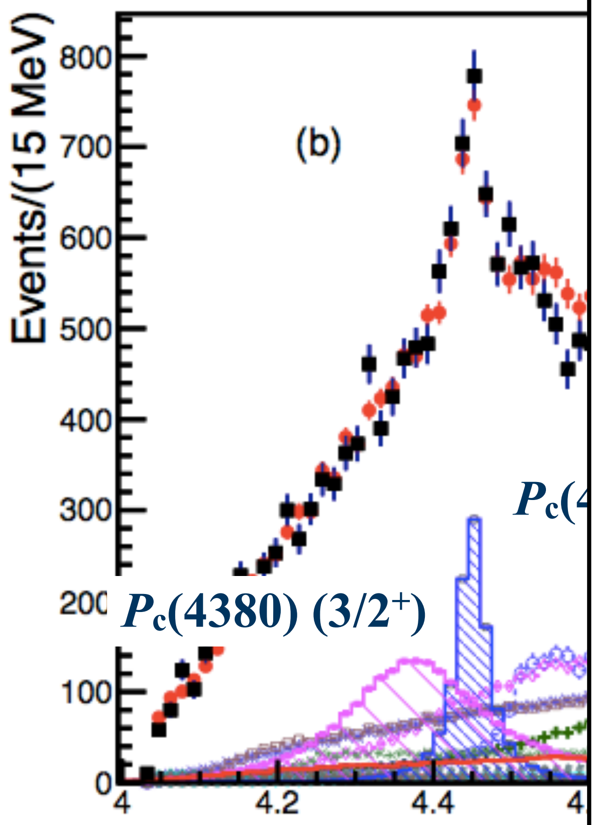
- $P_c \rightarrow J/\psi + p$ ($c\bar{c}uud$)
  LHCb (*PRL 115 (2015) 07201*) found two penta-quark states with hidden $c\bar{c}$.

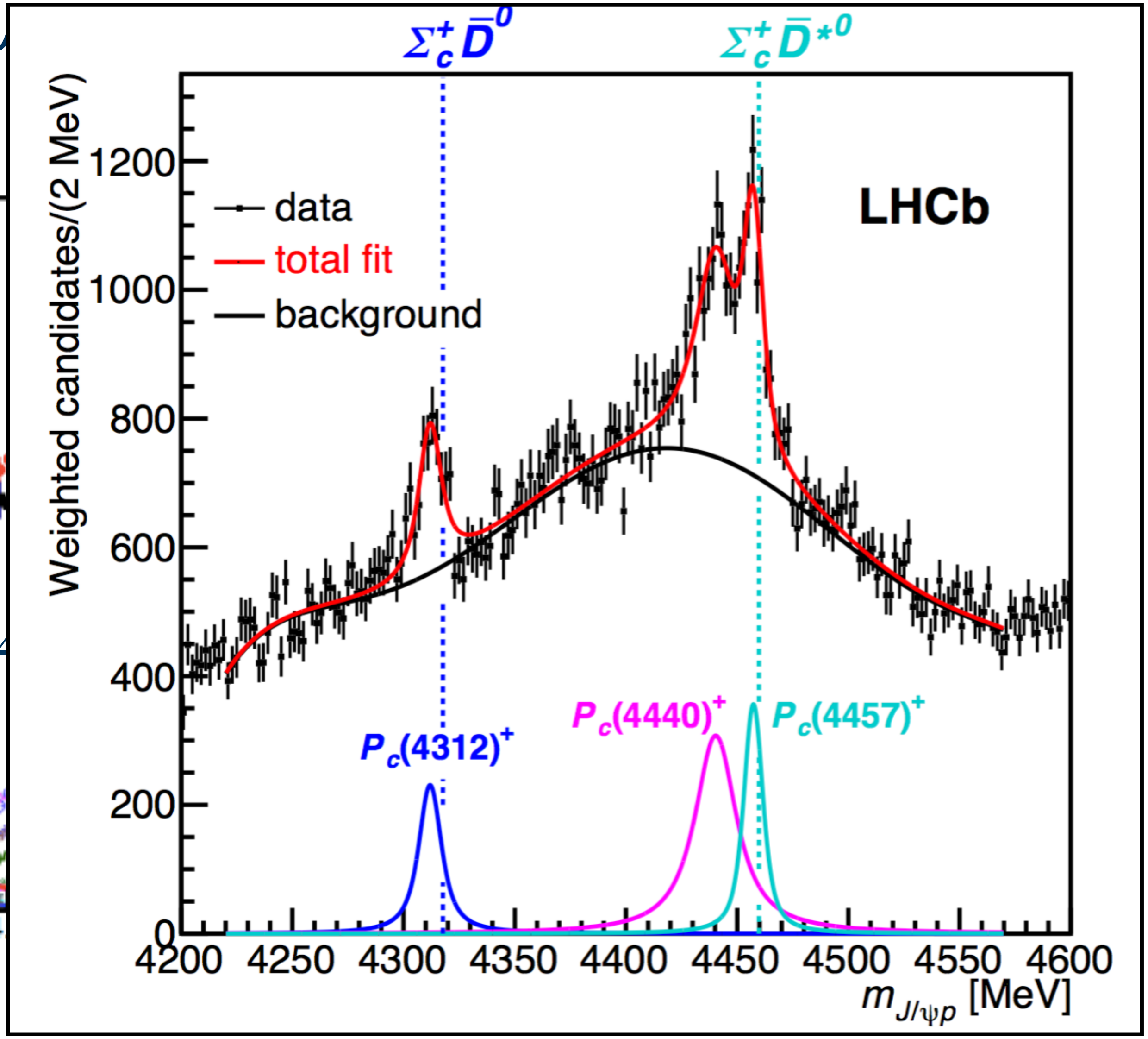


Hidden-Charm Multiquarks

- $P_c \to J/\psi + p$ ($c\bar{c}uud$)
  LHCb (*PRL 115 (2015)*
  hidden $c\bar{c}$.

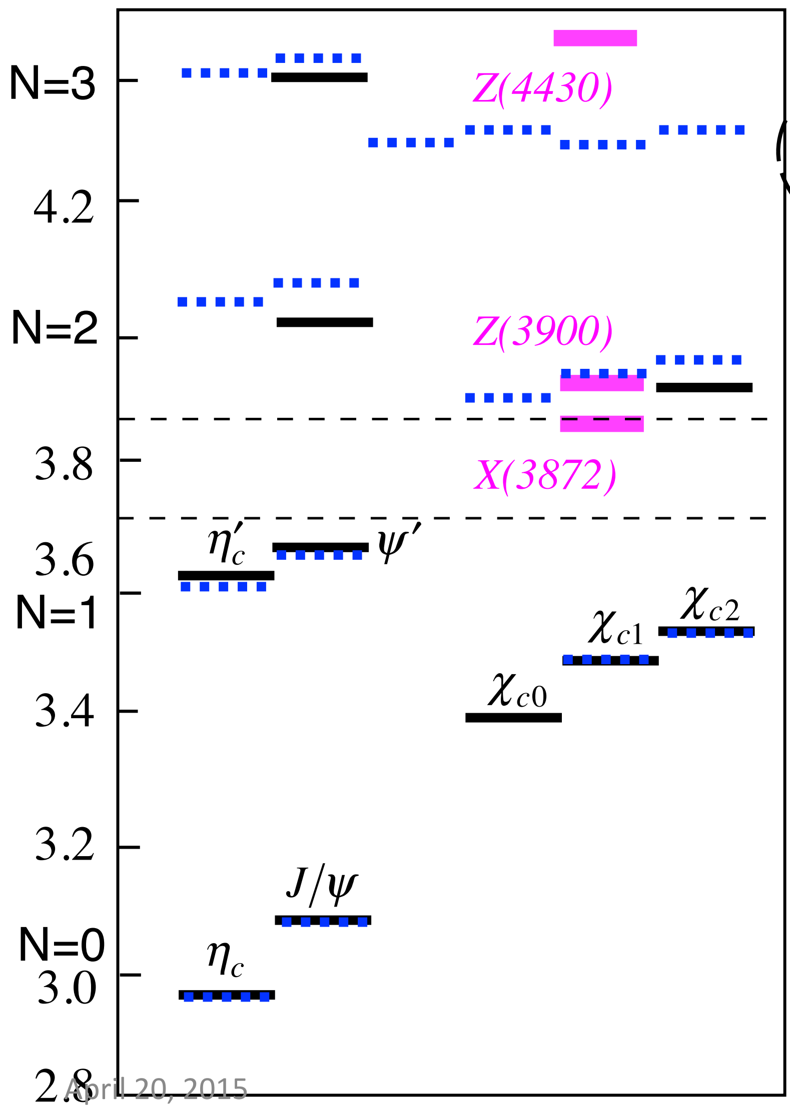
LHCb: arXiv:1904.03947

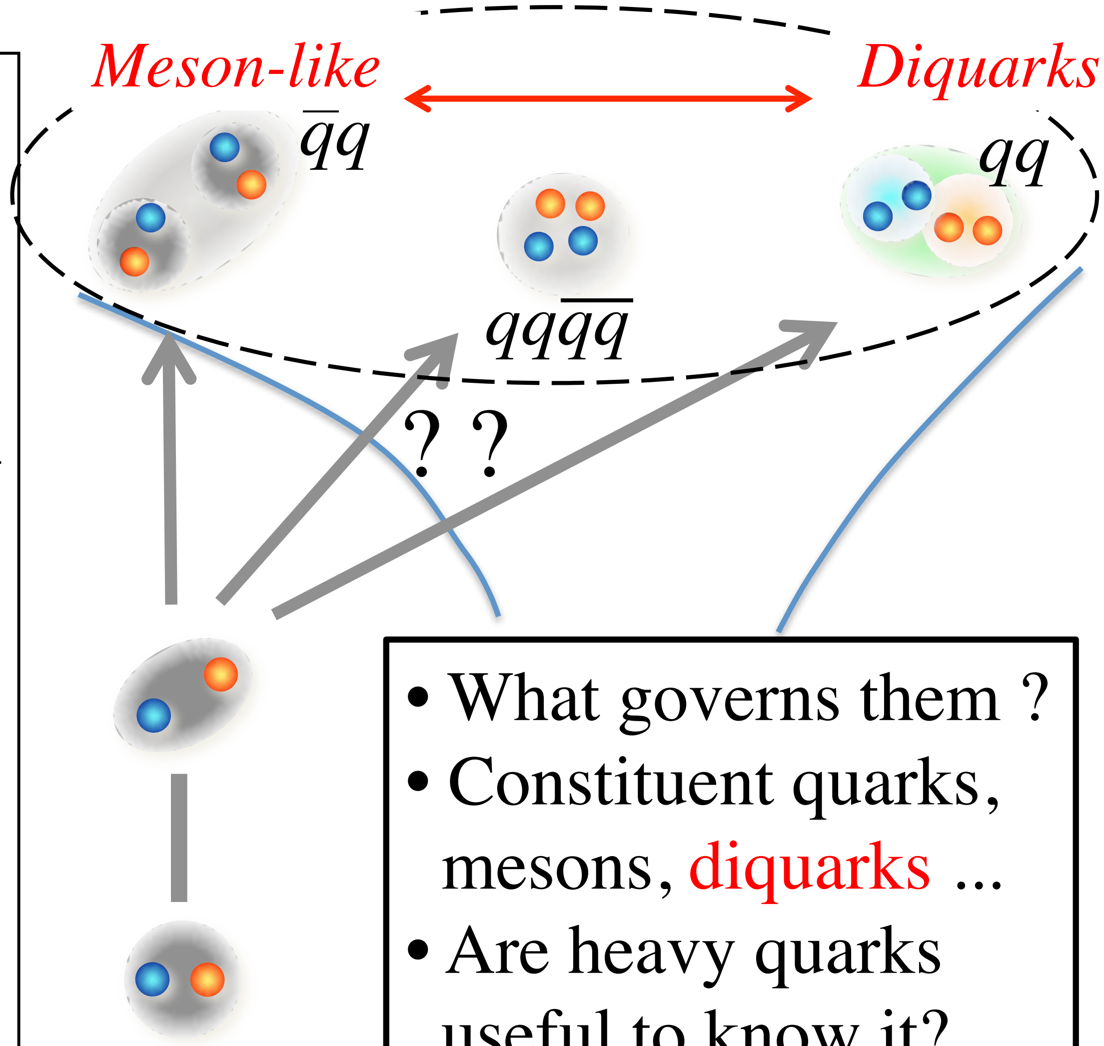




Above the threshold

$q\bar{q}$ creation and rearrangement of multiquarks





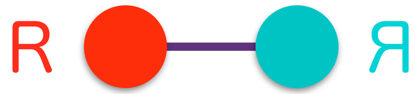
- What governs them ?
- Constituent quarks, mesons, diquarks ...
- Are heavy quarks useful to know it?

by A. Hosaka

Color

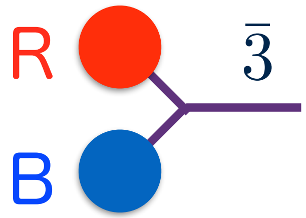
- **Quark has 3 colors (RBG) forming color-singlet hadrons**

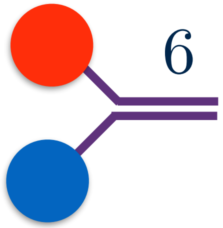
meson **q-q^{bar}** $3 \otimes \bar{3} = 1 \oplus 8$



diquark **q-q** $3 \otimes 3 = \bar{3} \oplus 6$

not white

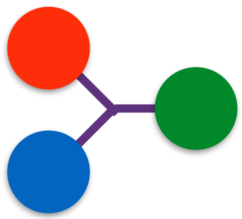




baryon **q-q-q**

$$3 \otimes 3 \otimes 3 = (\bar{3} \oplus 6) \otimes 3 = (1 \oplus 8) \oplus (8 \oplus 10)$$

$3 \otimes 3 = \bar{3}$



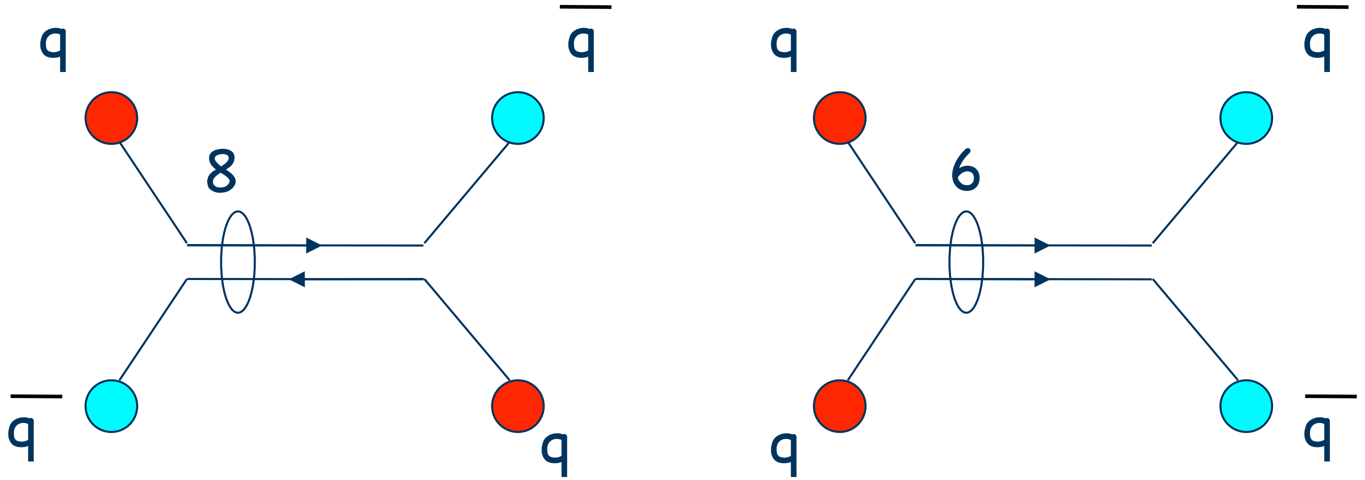
$\bar{3} \otimes 3 = 1$

Color

- **more quarks**

q^2-q^{bar2} (tetraquarks):

$$3 \otimes 3 \otimes \bar{3} \otimes \bar{3} = (2 \times 1) \oplus (4 \times 8) \oplus 10 \oplus \overline{10} \oplus 27$$



- **q^4-q^{bar} (pentaquarks):** $3^4 \otimes \bar{3} = (3 \times 1) \oplus \ldots$

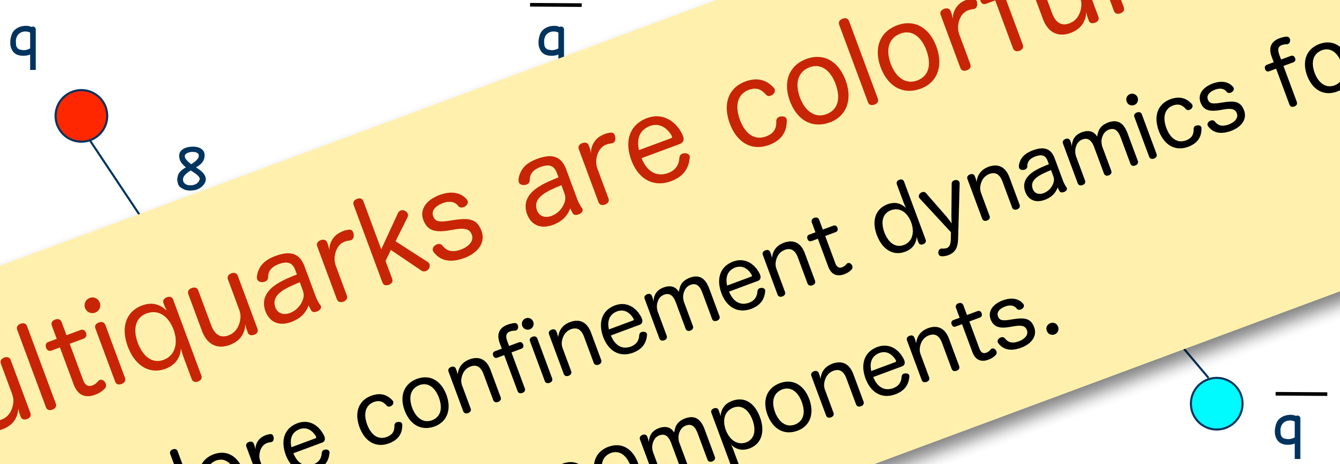
q^6 (dibaryons) : $3^6 = (5 \times 1) \oplus \ldots$

Color

- **more quarks**

q^2-q^{bar2} (tetraquarks):

$$3 \otimes 3 \otimes \bar{3} \otimes \bar{3} = (2 \times 1) \oplus (4 \times 8) \oplus \ldots$$



Multiquarks are colorful !

To explore confinement dynamics for exotic colorful components.

- q

$$3^4 \otimes \bar{3} = (3 \times 1) \oplus \ldots$$

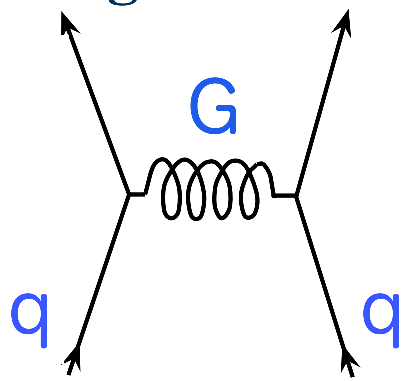
q⁶

$$3^6 = (5 \times 1) \oplus \ldots$$

Diquark

- The simplest *colorful cluster* in hadrons is the **diquark**.
- Spin dependent force in the (magnetic) gluon exchange

  Color-Magnetic Interaction



$$\Delta_{\rm CM} \equiv \langle -\sum_{i<j} (\vec{\lambda}_i \cdot \vec{\lambda}_j)(\vec{\sigma}_i \cdot \vec{\sigma}_j) \rangle$$

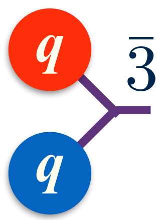
  Scalar diquark: 0^+ color $\bar{3}$ $\Delta_{\rm CM} = -8$

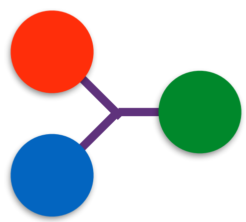
  Axial-vector: 1^+ color $\bar{3}$ $\Delta_{\rm CM} = +8/3$

- Can be related to the Hyperfine splitting of the baryon

$$M(1^+)-M(0^+) = (2/3)\,[M(\Delta)- M(N)] \sim 200 \text{ MeV}$$

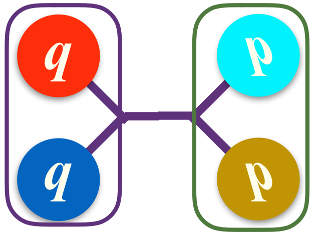
$\Delta_{\rm CM}$ +8/3 -8 +8 -8





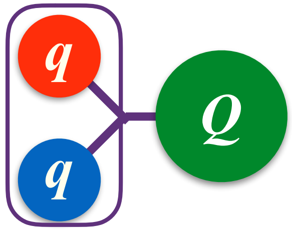
Diquark

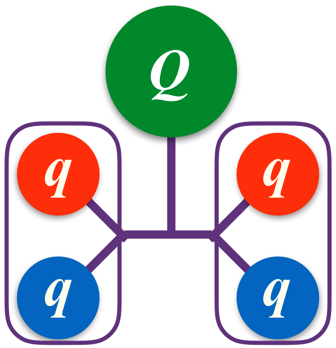
- **Diquarks D_q (=qq) as elements of hadrons**



$D_q D_q^{bar}$ = qq $q^{bar}q^{bar}$ = Tetraquark

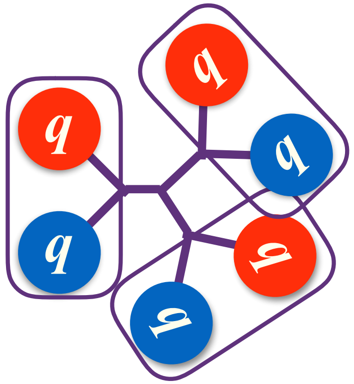
D_q Q = qq Q = HQ Baryon





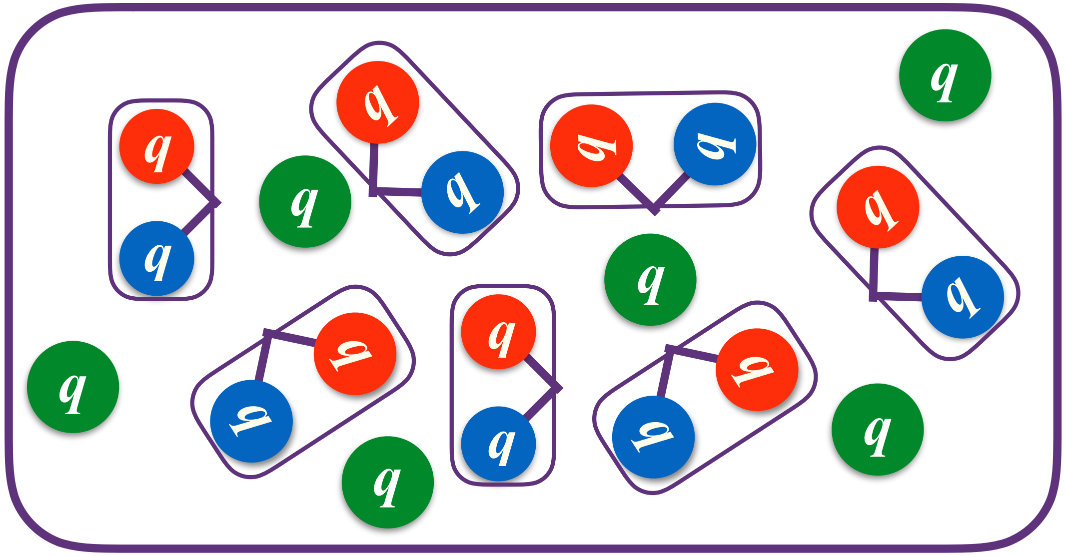
$D_q D_q Q^{bar}$ = qq qq Q^{bar} = Pentaquark

$D_q D_q D_q$ = qq qq qq
= Hexaquark
(Dibaryon)



Diquark

- Diquarks obey the Bose-Einstein statistics.
  BE condensate in dense hadronic matter is expected.
  => color-superconducting phase



Diquark Effective Theory

Strategy

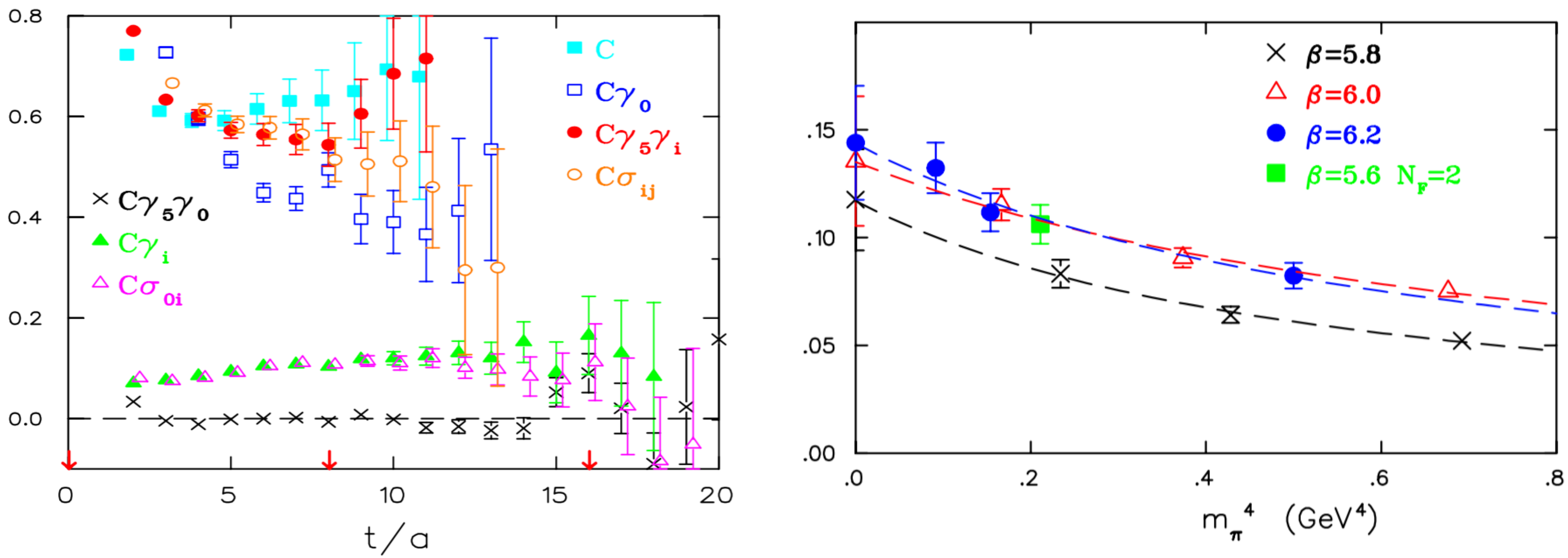
- Consider *Diquarks* as “colorful” building blocks of hadrons and hadronic matter. In order to describe their dynamics, write down the *Diquark* effective Lagrangian.
- Warning: As *Diquarks* are not color singlet, we need a way to compensate the color in the confined phase. For instance, assume a background color field generated by a heavy quark.
- Lattice QCD helps us to fix the parameters of the effective Lagrangian.

Diquark in Lattice QCD

- **Hess, Karsch, Laermann, Wetzorke, PR D58, 111502 (1998)**
  quench, Landau gauge fixed
  $m_q \sim 342$ MeV, $M(0^+) \sim 694$ MeV, $M(1^+) \sim 810$ MeV
- **Alexandrou, de Forcrand, Lucini, PRL 97, 222002 (2006)**
  From Qqq system, quench, gauge invariant
  $M(1^+) - M(0^+) \sim 200\text{-}220$ MeV, $R(S) \sim 1$ fm
  $M(0^-) - M(0^+) \sim 600$ MeV
- **Babich, et al., PR D76, 074021 (2007)**
  quench, Landau gauge
  $M(0^+) - 2m_q \sim -200$ MeV, $M(1^+) - M(0^+) \sim 162$ MeV
- **Yujiang Bi, et al., Chinese Physics C40 (2016) 073106**
  full, Landau gauge
  $M(0^+) - m_q \sim 310$ MeV, $M(1^+) - M(0^+) \sim 290$ MeV

Evidence for Diquarks in Lattice QCD

C. Alexandrou,[1] Ph. de Forcrand,[2,3] and B. Lucini[2,4]



$$\Delta m/\delta m_{\Delta N} = 0.67(7),\ 0.73(8),\ \text{and}\ 0.67(8)$$

$$\delta m_{\Delta N} \sim 300\text{MeV} \longrightarrow \Delta m \sim 200 - 220\text{MeV}$$

Diquark Effective Theory

- **Color-Flavor-Spin Structures: Pauli principle**

$C \equiv i\gamma^0\gamma^2 = -C^{-1} = -C^T$

	J^π	color	flavor	
$(q^T C q)_A^{\bar{3}}$	0^-	$\bar{3}$	$\bar{3}$	$^3P_0$
$(q^T C\gamma^5 q)_A^{\bar{3}}$	0^+	$\bar{3}$	$\bar{3}$	$^1S_0$
$(q^T C\gamma^\mu\gamma^5 q)_A^{\bar{3}}$	1^-	$\bar{3}$	$\bar{3}$	$^3P_1$
$(q^T C\gamma^\mu q)_S^{\bar{3}}$	1^+	$\bar{3}$	6	$^3S_1$
$(q^T C\sigma^{\mu\nu} q)_S^{\bar{3}}$	$1^+, 1^-$	$\bar{3}$	6	$^3D_1, {}^1P_1$
$(q^T C q)_S^6$	0^-	6	6	$^3P_0$
$(q^T C\gamma^5 q)_S^6$	0^+	6	6	$^1S_0$
$(q^T C\gamma^\mu\gamma^5 q)_S^6$	1^-	6	6	$^3P_1$
$(q^T C\gamma^\mu q)_A^6$	1^+	6	$\bar{3}$	$^3S_1$
$(q^T C\sigma^{\mu\nu} q)_A^6$	$1^+, 1^-$	6	$\bar{3}$	$^3D_1, {}^1P_1$

scalar "good" diquark

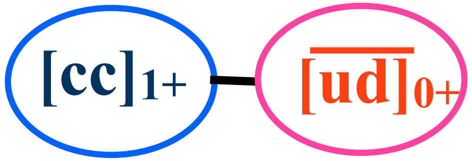
axial-vector "bad" diquark

color 6

Multiquark exotic states

- **Double charm tetraquark meson**

  $T_{cc}\ (ccu^{bar}d^{bar}, 1^{+}, I=0) = [cc]_{1+}\ [u^{bar}d^{bar}]_{0+}$



  - The lowest strong-decay threshold is DD* (L=0).

  - If the scalar diquark is light enough to make T_{cc} bound below DD* threshold, T_{cc} will be a stable tetra-quark resonance.

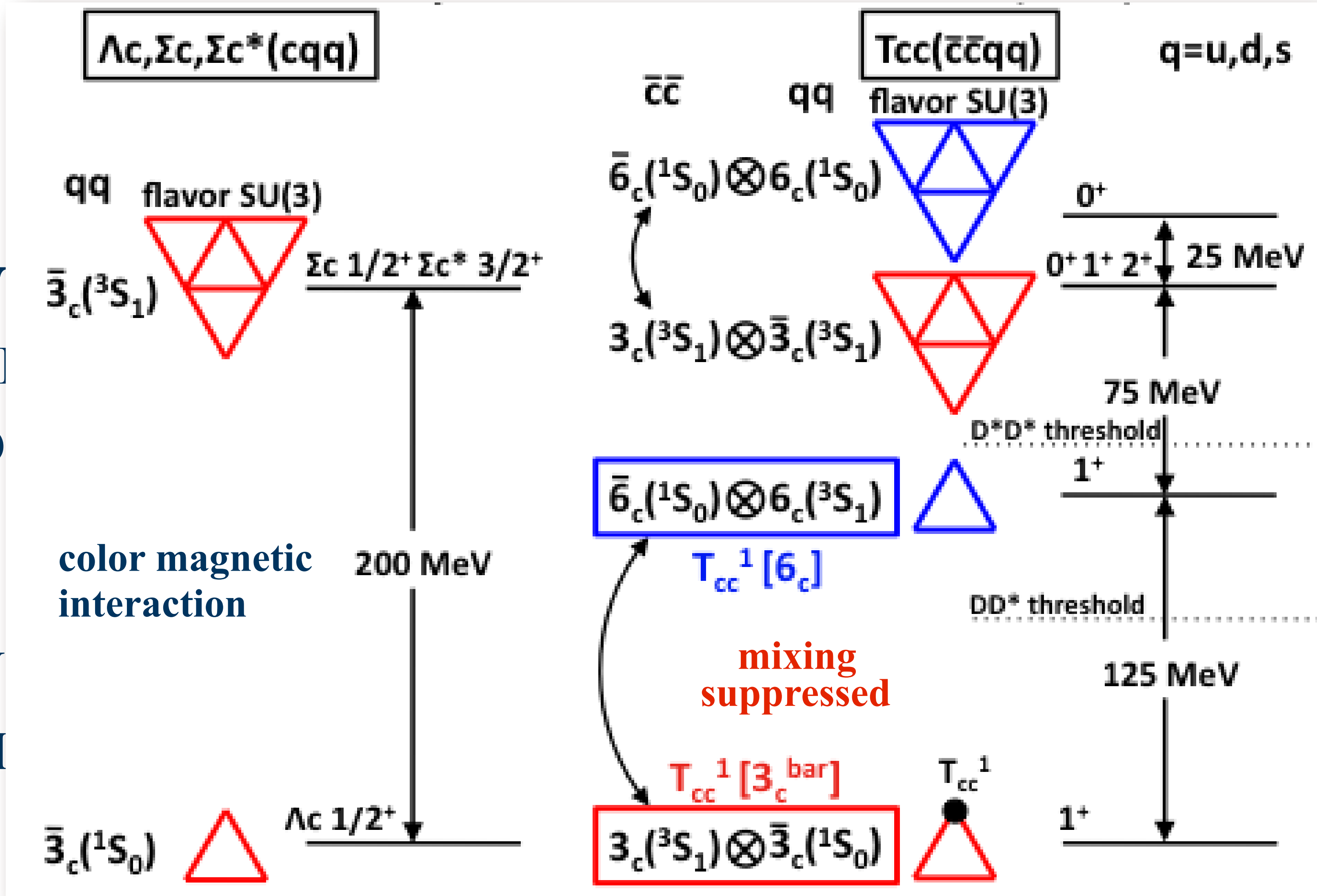
  S. Zouzou, et al., Z. Phys. C30 (1986)457
  H.J. Lipkin, Phys. Lett. B172 (1986) 242

- **New possible color correlations with the production rates**

  Hyodo, Liu, Oka, Sudoh, Yasui, PLB721 (2013) 56-60.

Multiquark exotic states

- **Double charm tetraquark meson**



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Chiral Diquarks

- **Chiral symmetry SU(3)$_R$ x SU(3)$_L$**

$$q^a_{iR} = P_R\, q^a_i, \quad q^a_{iL} = P_L\, q^a_i$$

$$P_{R,L} \equiv \frac{1 \pm \gamma_5}{2}, \qquad [P_{R,L}, C] = 0, \quad P^T_{R,L} = P_{R,L}$$

- **Scalar chiral diquarks (color 3$^{\text{bar}}$)**

$d^a_{iR} \equiv \epsilon_{ijk}(q^T_{jR}\, C\, q_{kR})^{\bar{3}}$ Right scalar diquark, chiral $(\bar{3},1)$, color $\bar{3}$

$d^a_{iL} \equiv \epsilon_{ijk}(q^T_{jL}\, C\, q_{kL})^{\bar{3}}$ Left scalar diquark, chiral $(1, \bar{3})$, color $\bar{3}$

- **Parity eigenstates: 0$^+$, 0$^-$ diquarks**

$$S^a_i = d^a_{iR} - d^a_{iL} = \epsilon_{ijk}(q^T_j\, C\gamma_5\, q_k)^{\bar{3}}$$

$$P^a_i = d^a_{iR} + d^a_{iL} = \epsilon_{ijk}(q^T_j\, C\, q_k)^{\bar{3}}$$

$$(\bar{3}, 1) + (1, \bar{3})$$

Chiral Diquarks

- **$SU(3)_R$ x $SU(3)_L$ transform for scalar diquarks**

$$q_R \to U_R\, q_R = (U_R)_{ij} q_{jR}, \quad U_R \in SU(3)_R$$
$$q_L \to U_L\, q_L = (U_L)_{ij} q_{jL}, \quad U_L \in SU(3)_L$$

$$d_R \to d_R\, U_R^\dagger \quad (\bar{3}, 1), \qquad d_R^\dagger \to U_R\, d_R^\dagger \quad (3, 1)$$
$$d_L \to d_L\, U_L^\dagger \quad (1, \bar{3}), \qquad d_L^\dagger \to U_L\, d_L^\dagger \quad (1, 3)$$

$$d_R^a (d_R^a)^\dagger \equiv d_{iR}^a (d_{iR}^a)^\dagger, \quad d_L^a (d_L^a)^\dagger \equiv d_{iL}^a (d_{iL}^a)^\dagger, \quad \text{chiral invariant, color singlet}$$

$d_R^a (d_R^a)^\dagger + d_L^a (d_L^a)^\dagger =$ Lorentz scalar, color singlet, chiral invariant

- **Strategy upgraded:**

  Write down general forms of the chiral invariant Lagrangian for the chiral diquarks with a background chiral meson field.

Chiral Diquark Effective Theory

- **Chiral meson field: Σ_S (scalar nonet), Σ_P (pseudo-scalar nonet)**

$$\Sigma = \Sigma_S + i\Sigma_P \to U_L \Sigma U_R^\dagger \quad (\bar{3}, 3)$$
$$\Sigma_S = \Phi_S / f_\pi, \quad \Sigma_P = \Phi_P / f_\pi \quad \langle \Sigma_S \rangle = 1, \quad \langle \Sigma_P \rangle = 0$$
$$\mathcal{P} : \quad d_R \leftrightarrow d_L, \quad \Sigma \to \Sigma^\dagger$$

- **The effective Lagrangian (of the linear sigma model)**

$$\mathcal{L} = -D_\mu d_R \left(D_\mu d_R\right)^\dagger - D_\mu d_L \left(D_\mu d_L\right)^\dagger$$
$$-m_0^2 (d_R d_R^\dagger + d_L d_L^\dagger) - m_1^2 (d_R \Sigma^\dagger d_L^\dagger + d_L \Sigma d_R^\dagger)$$

$$D_\mu = \partial_\mu + ig\, T^\alpha G_\mu^{\alpha\,\mathrm{ext}}$$

Chiral Diquark Effective Theory

$$\mathcal{L} = -D_\mu d_R (D_\mu d_R)^\dagger - D_\mu d_L (D_\mu d_L)^\dagger$$
$$- m_0^2(d_R d_R^\dagger + d_L d_L^\dagger) - m_1^2(d_R \Sigma^\dagger d_L^\dagger + d_L \Sigma d_R^\dagger)$$

chiral invariant mass **CSB mass**

- **For the chiral invariant vacuum, $\langle\Sigma\rangle = \langle\Sigma_s\rangle = 0$ ($\langle\sigma\rangle = 0$), the diquark mass is given by m_0.**
- **For normal vacuum Σ=1, the masses of 0^+ and 0^- diquarks are given by**

$$M^2 = \begin{pmatrix} m_0^2 & m_1^2 \\ m_1^2 & m_0^2 \end{pmatrix} \longrightarrow M = \sqrt{m_0^2 \pm m_1^2}$$

$$\sqrt{m_0^2 - m_1^2} \to S = d_R - d_L(0^+) \qquad \sqrt{m_0^2 + m_1^2} \to P = d_R + d_L(0^-)$$

$$\Delta_{M^2} = M_P^2 - M_S^2 = 2m_1^2$$

Chiral Diquark Effective Theory

- Σ_P (pseudo-scalar nonet) couplings

$$\Sigma = \Sigma_S + i\Sigma_P \to U_L \Sigma U_R^\dagger \quad (\bar{3}, 3) \qquad\qquad \Sigma_P = \frac{1}{f_\pi}\lambda^a \Phi_P^a$$

$$V = (-i)m_1^2(d_R\Sigma_P d_L^\dagger - d_L \Sigma_P d_R^\dagger)$$
$$\to (-i)\frac{m_1^2}{f_\pi}(d_R\lambda^a d_L^\dagger - d_L\lambda^a d_R^\dagger)\,\Phi_P^a = (-i)\frac{m_1^2}{2f_\pi}(P\lambda^a S^\dagger - S\lambda^a P^\dagger)\,\Phi_P^a$$

- "Goldberger-Treiman" relation

$$g_{pSP} = \frac{m_{S1}^2}{2f_\pi} = \frac{M_P^2 - M_S^2}{f_\pi} = \frac{\Delta_M^2}{f_\pi}$$

Chiral Diquark Effective Theory

- **Vector + Axial-vector (3,3) Diquarks**

$$d^{\mu a}_{ij} \equiv \epsilon_{abc}(q^{bT}_{iL}\, C\gamma^{\mu}\, q^{c}_{jR}) = \epsilon_{abc}(q^{bT}_{jR}\, C\gamma^{\mu}\, q^{c}_{iL}) \quad \text{chiral (3,3) vector diquark}$$

$$d^{\mu a}_{V[ij]} = d^{\mu a}_{ij} - d^{\mu a}_{ji} = \epsilon_{abc}(q^{bT}_{i}\, C\gamma^{\mu}\gamma^{5}\, q^{c}_{j}) \quad \text{Vector } 1^{-} \text{ diquark, flavor } \bar{3}$$

$$d^{\mu a}_{A\{ij\}} = d^{\mu a}_{ij} + d^{\mu a}_{ji} = \epsilon_{abc}(q^{bT}_{i}\, C\gamma^{\mu}\, q^{c}_{j}) \quad \text{Axial-vector } 1^{+} \text{ diquark, flavor } 6$$

$$d^{\mu} \longrightarrow U_L d^{\mu} U_R^{T}, \quad (3,3) \qquad d^{\mu\dagger} \longrightarrow U_R^{T\dagger} d^{\mu} U_L^{\dagger} \quad (\bar{3},\bar{3})$$

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}[F^{\mu\nu}F^{\dagger}_{\mu\nu}] - m_0^2\mathrm{Tr}[d^{\mu}d^{\dagger}_{\mu}] - m_1^2\mathrm{Tr}[\Sigma^{\dagger}d^{\mu}\Sigma^{T}d^{\dagger T}_{\mu}]$$

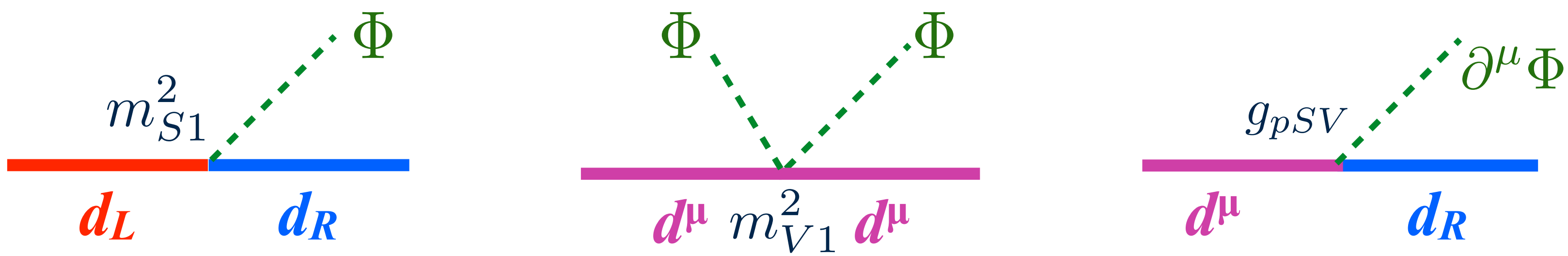
$$F^{\mu\nu} = D^{\mu}d^{\nu} - D^{\nu}d^{\mu}$$

Chiral Diquark Effective Theory

- **Full (Scalar + Vector) Diquark Effective Theory**

$$\begin{aligned}\mathcal{L} = &-D_\mu d_R (D_\mu d_R)^\dagger - D_\mu d_L (D_\mu d_L)^\dagger \\ &-m_{S0}^2 (d_R d_R^\dagger + d_L d_L^\dagger) - m_{S1}^2 (d_R \Sigma^\dagger d_L^\dagger + d_L \Sigma d_R^\dagger) \\ &-\frac{1}{2}\mathrm{Tr}[F^{\mu\nu} F_{\mu\nu}^\dagger] - m_{V0}^2 \mathrm{Tr}[d^\mu d_\mu^\dagger] - m_{V1}^2 \mathrm{Tr}[\Sigma^\dagger d^\mu \Sigma^T d_\mu^{\dagger T}] \\ &-g_{pSV} \mathrm{Tr}[d^\mu \epsilon_R d_R^\dagger \partial_\mu \Sigma^\dagger + d^\mu (\partial_\mu \Sigma)^T \epsilon_L d_L^\dagger + (\mathrm{c.c.})]\end{aligned}$$

- **Expected Couplings of PS meson**



$\Xi_c \leftrightarrow \Lambda_c \bar{K}$

$\Sigma_c \to \Lambda_c \pi$

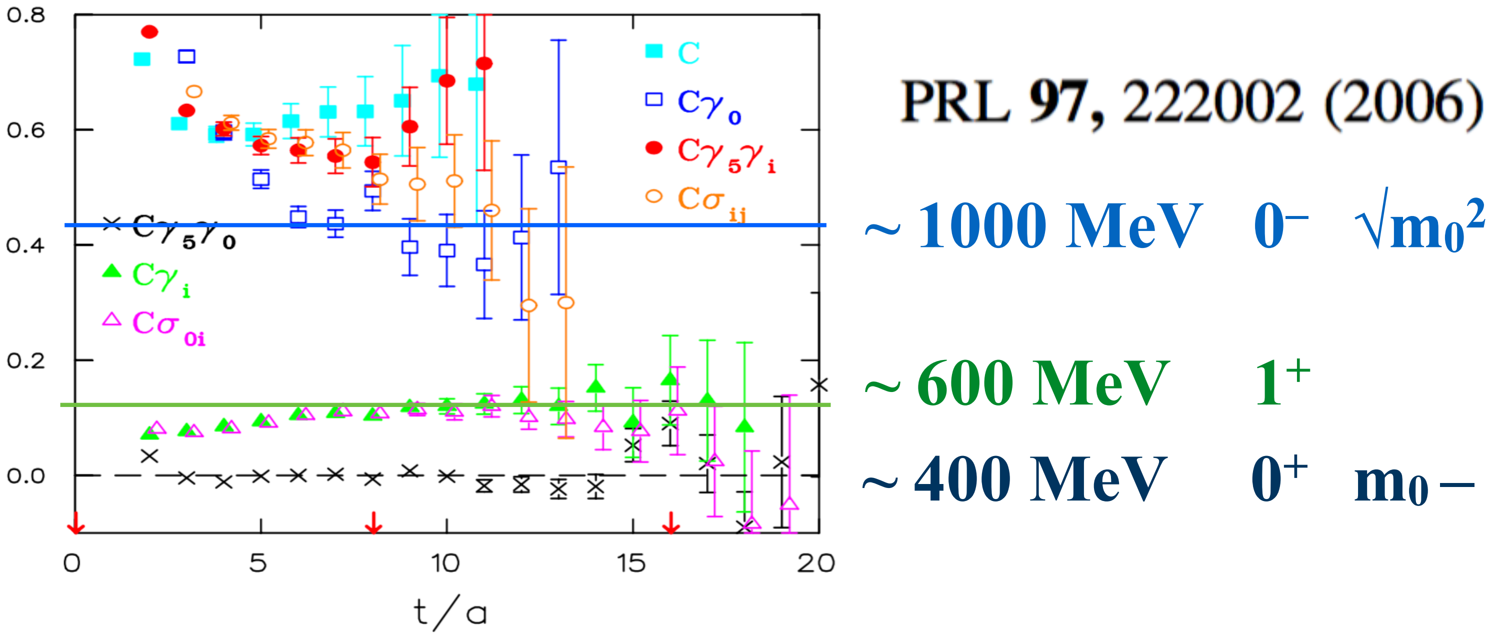
Chiral Diquark Effective Theory

- **D.K.Hong, Y.J. Sohn, I. Zahed, PL B596 (2004) 191.**
  D.K. Hong, C. Song, IJMP A27 (2012) 1250051.
  Non-linear chiral Diquark effective theory (for pentaquark/ tetraquarks)

- **Y. Kawakami, M. Harada,**
  PR D97 (2018) 114024, PR D99 (2019) 094016.
  Chiral effective theory of Single Heavy Baryons (HQ symmetry)

Chiral Diquark Effective Theory

C. Alexandrou et al., Quenched QCD, PRL 97, 222002 (2006)



~ 1000 MeV 0^- $\sqrt{m_0^2 + m_1^2}$

~ 600 MeV 1^+

~ 400 MeV 0^+ $m_0 - m_1$

$m_0 \sim 760$ MeV, $m_1 \sim 640$ MeV (SU(3) chiral limit)

Under restoration of chiral symmetry, $\langle\Sigma\rangle = \langle\Sigma_s\rangle \to 0$,

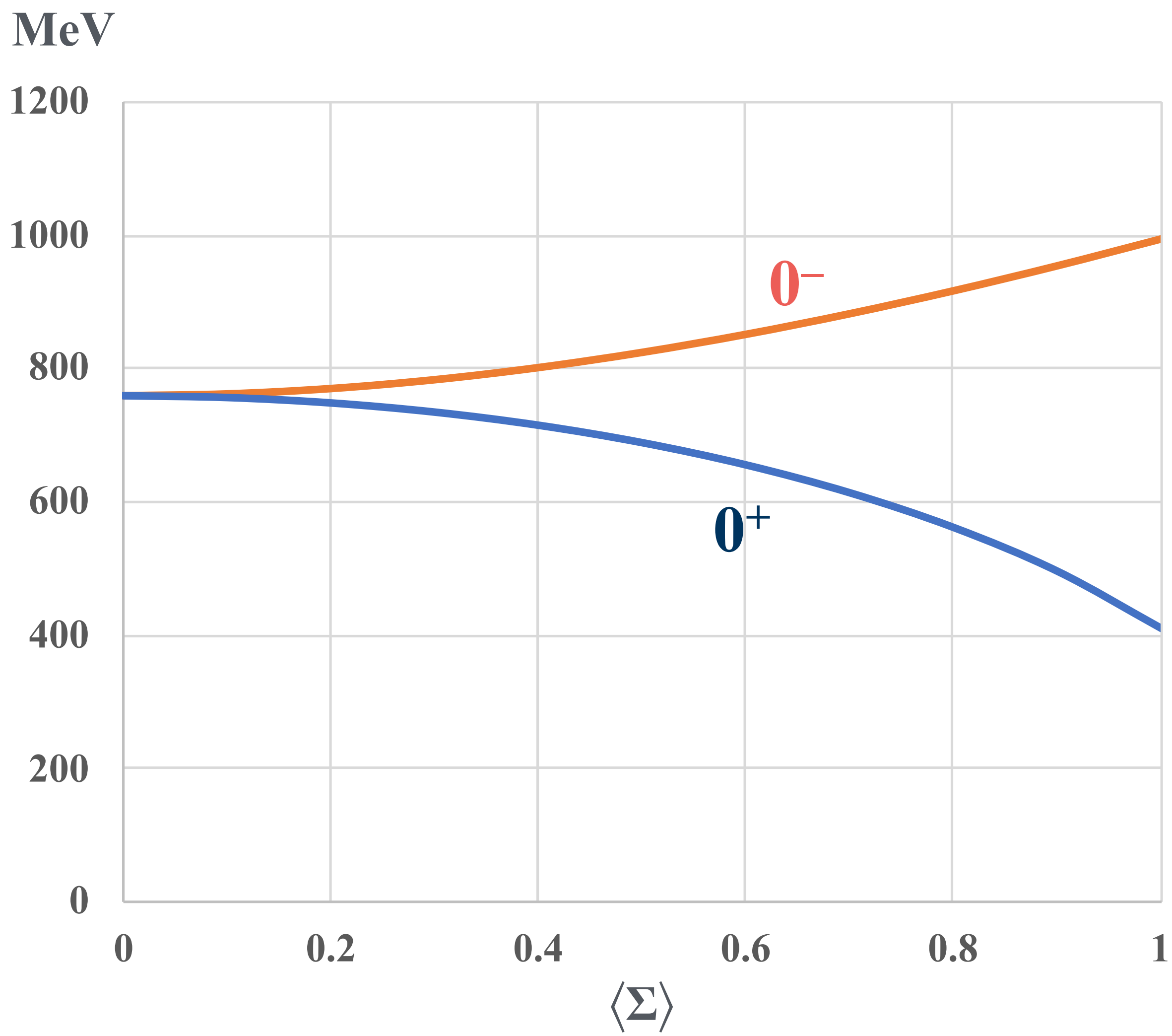
the mass of the Scalar diquark and Pseudoscalar diquark

will be degenerate to be $m_0 \sim 760$ MeV.

$\langle \Sigma \rangle = \langle \Sigma_s \rangle \rightarrow 0$

$m_0 \sim 760$ MeV, $m_1 \sim 640$ MeV (SU(3) chiral limit)

$m_0 \sim 760$ MeV



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Diquark mass differences from unquenched lattice QCD

Yujiang Bi(毕玉江)[1;1)] Hao Cai(蔡浩)[1;2)] Ying Chen(陈莹)[2] Ming Gong(宫明)[2]
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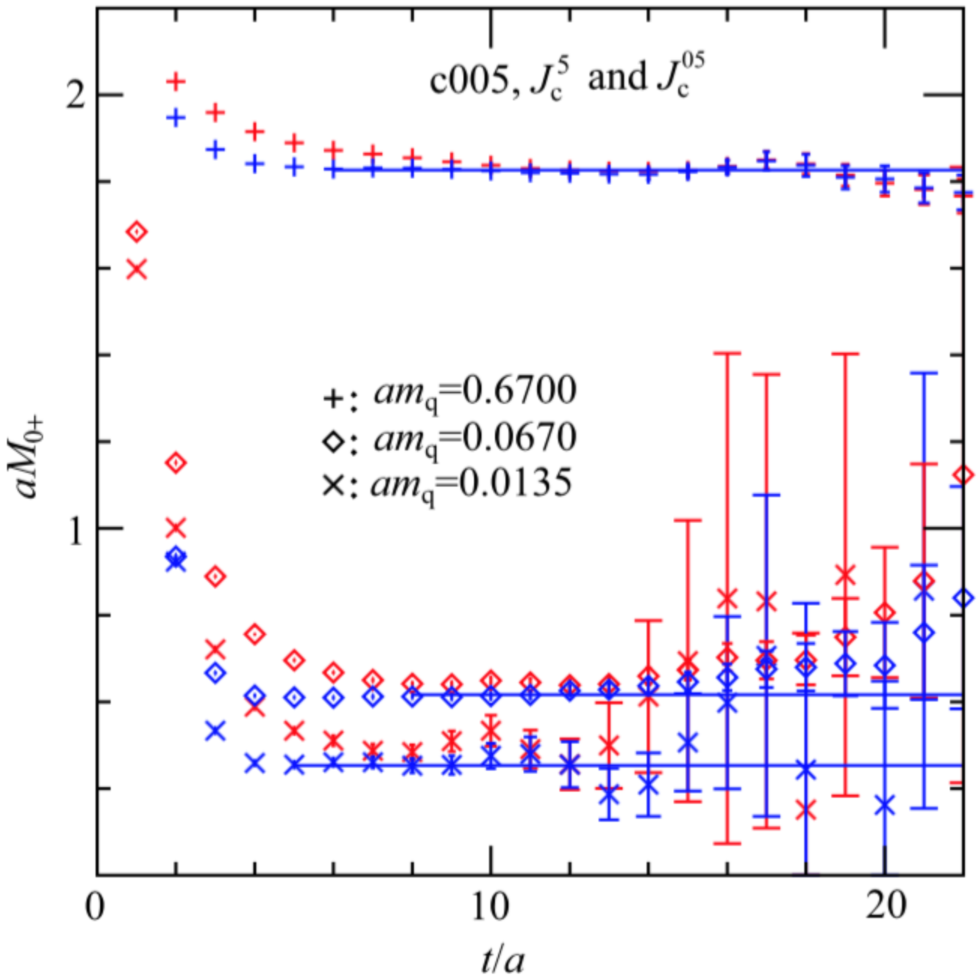

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Table 8. Diquark masses and mass difference for various valence quark masses on ensemble c005. The first line is a linear extrapolation in am_q to the chiral limit with the lowest four data points. a^{-1}=1.75 GeV

am_q	$aM_{0^+}(J_c^{05})$	$aM_{1^+}(J_c^i)$	$a(M_{1^+} - M_{0^+})$	$aM_{0^-}(J_c^I)$	aM_{1^-}
0.0	0.4142(63)	0.584(21)	0.166(22)	-	-
0.01350	0.4534(70)	0.611(29)	0.158(31)	-	-
0.02430	0.4875(52)	0.635(18)	0.148(19)	0.796(52)	-
0.04890	0.5692(37)	0.694(10)	0.1248(98)	0.862(23)	0.987(53)
0.06700	0.6166(48)	0.7300(85)	0.1134(93)	0.904(18)	1.003(41)
0.15000	0.8293(70)	0.8907(68)	0.0614(89)	1.056(29)	1.140(24)
0.33000	1.1830(30)	1.2334(55)	0.0504(45)	1.378(17)	1.454(21)
0.67000	1.8265(39)	1.8604(68)	0.0339(62)	1.976(12)	2.025(16)

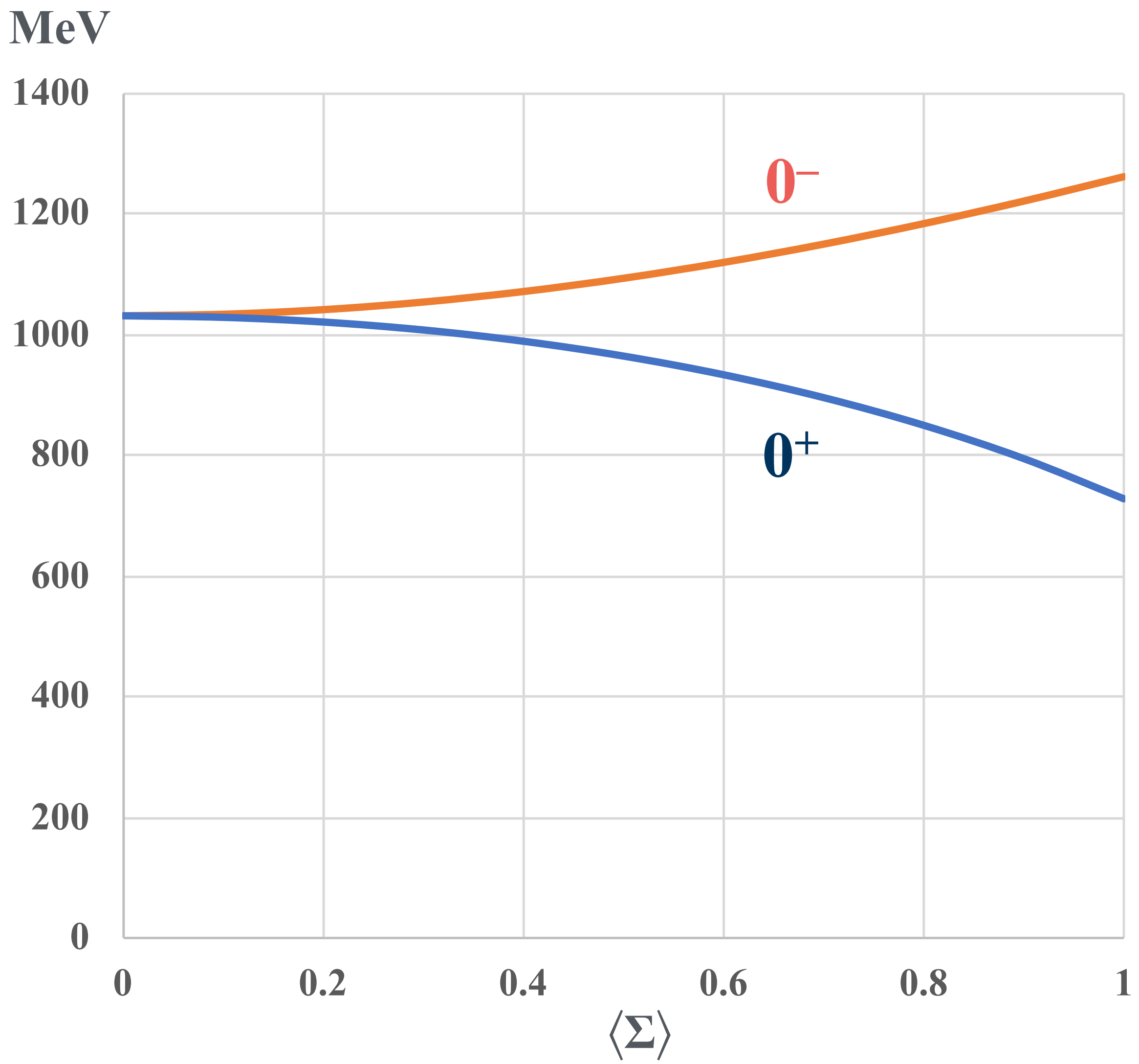
$M(0^+) \sim 720$ MeV

$M(1^+)-M(0^+) \sim 290$ MeV $M(1^+) \sim 1010$ MeV

$M(0^-)- M(0^+) \sim 540$ MeV $M(0^-) \sim 1260$ MeV m_0= 1030 MeV, m_1=730 MeV

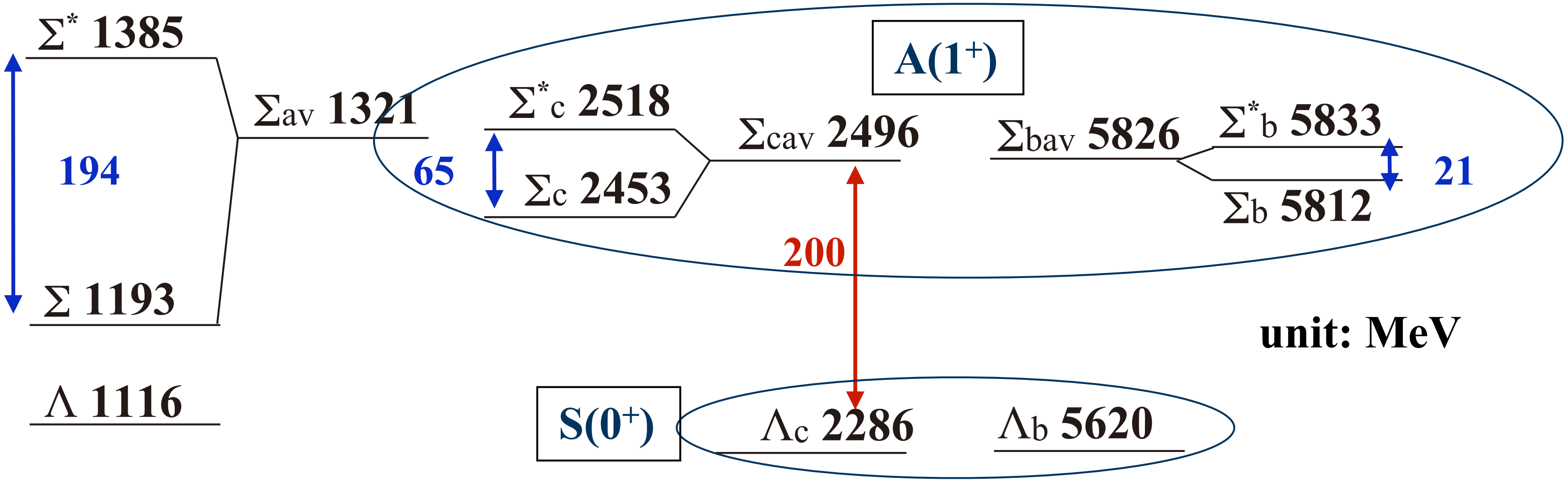
$M(1^-)- M(1^+) \sim 510$ MeV $M(1^-) \sim 1520$ MeV

m_0= 1030 MeV, m_1=730 MeV

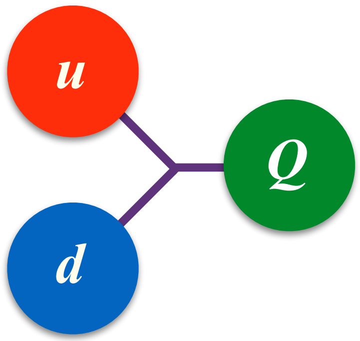


Diquarks in Heavy Baryons

- **From Heavy baryon spectroscopy**
  Λ_Q/Σ_Q with $S(0^+)/A(1^+)$ diquarks

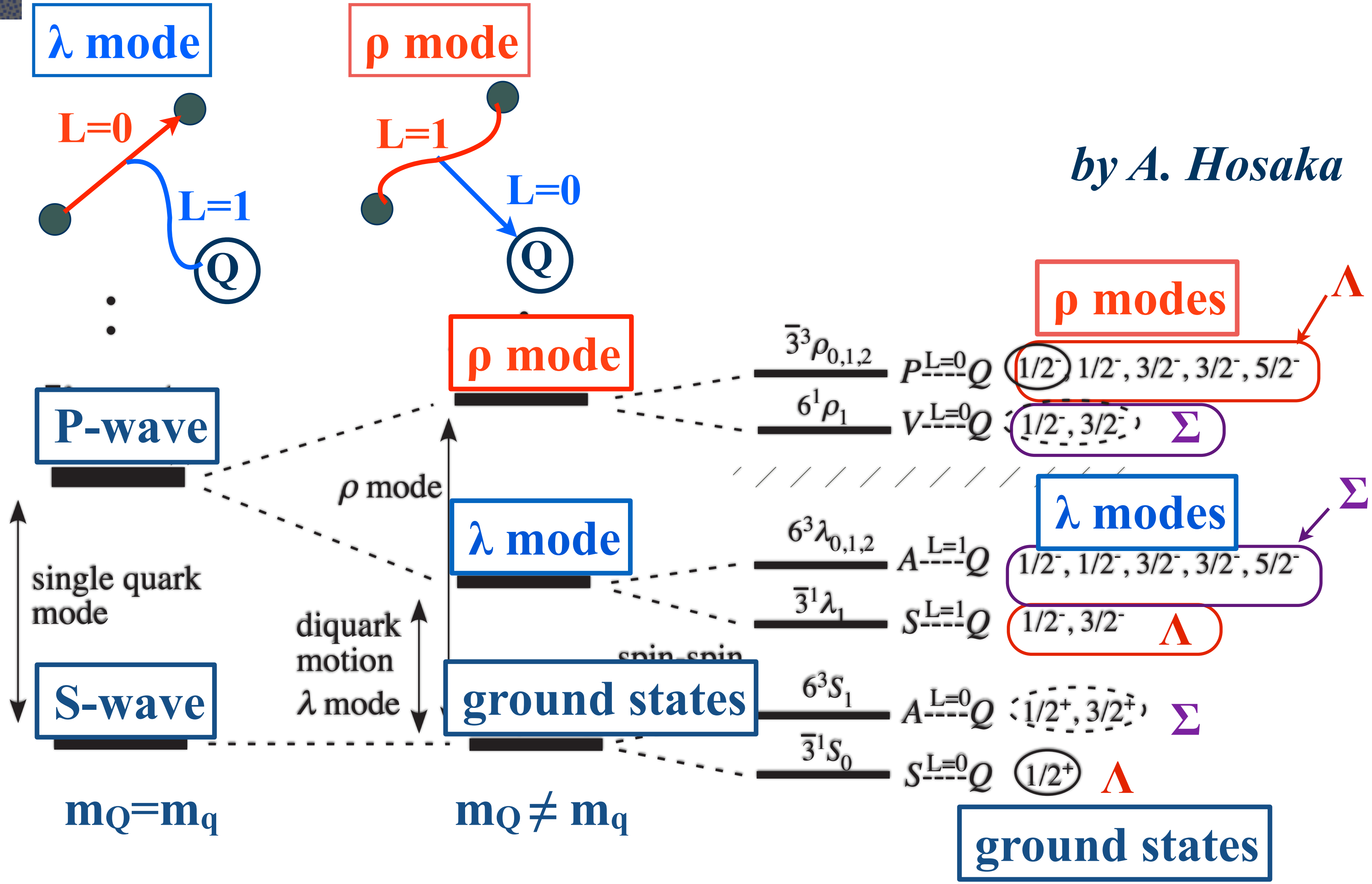


Diquarks
$S(0^+)$ *ud (S=0, I=0)*
$A(1^+)$ *(uu,ud,dd) (S=1, I=1)*



Diquarks in Heavy Baryons (P-wave)

by A. Hosaka



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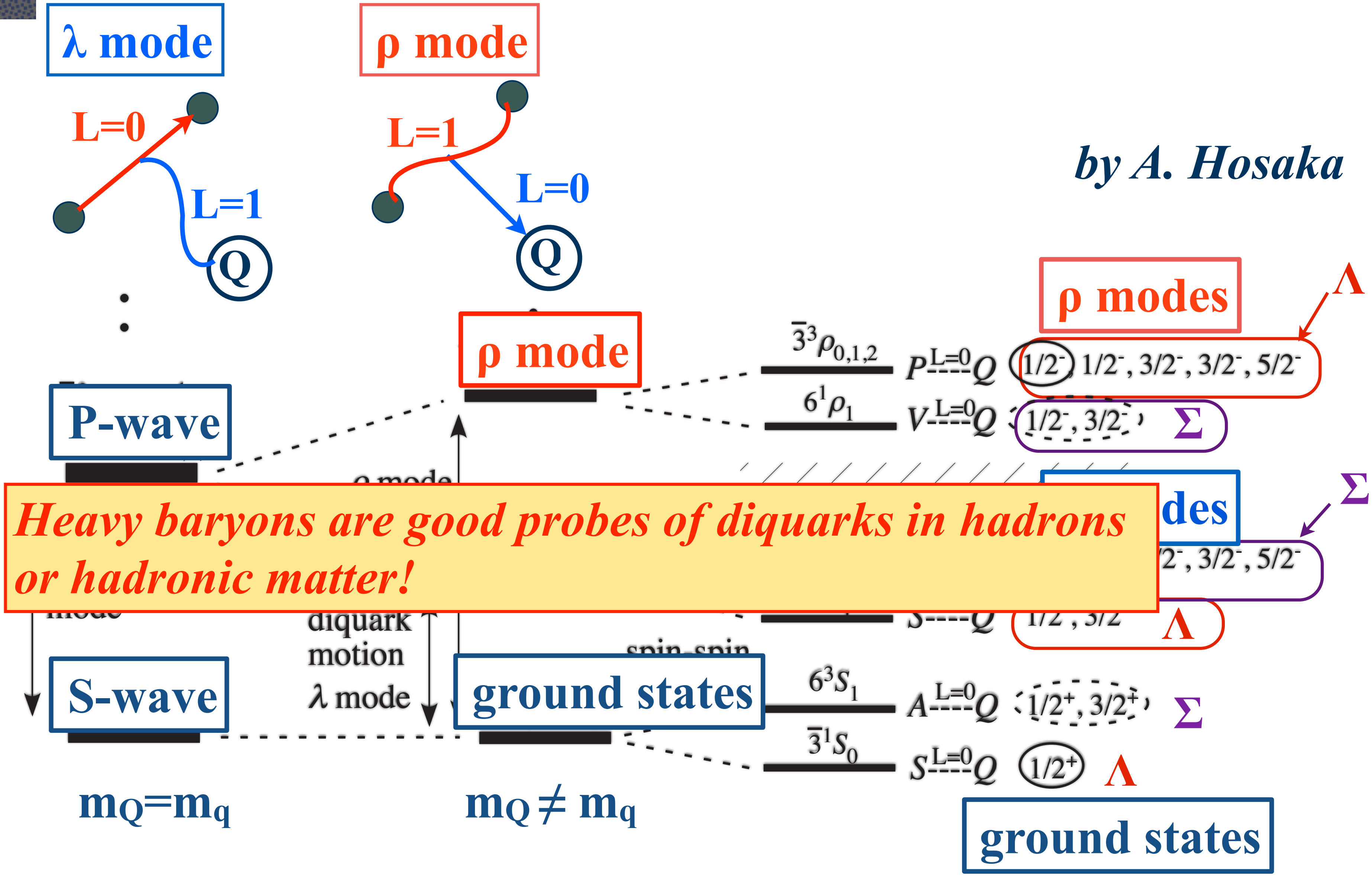

Heavy baryons are good probes of diquarks in hadrons or hadronic matter!

Conclusion

- **We construct a chiral effective theory of Diquarks.**
  Scalar and Pseudo-Scalar Diquarks are paired in $(\bar{3},1)+(1,\bar{3})$**.**
  Vector and Axial-Vector Diquarks are in (3,3) representation.
- **For the SP sector, chiral invariant mass term and SB mass term are available. Using the LQCD data of the Diquarks, we may determined the chiral invariant mass ~700 MeV. We obtain the GT relation for the PS meson-Diquark coupling.**
- **For the VA sector, we get a flavor 6 axial-vector and 3^{bar} vector. Chiral invariant and $\langle\Sigma\rangle^2$ mass terms are allowed. (No linear term)**
- **Future directions, perspectives**
  Heavy baryon spectroscopy
  Exotic states, such as tetra quark, diquark matter, . .