QCD phase structure:
Instanton, Polyakov loop, and Lee-Yang zeros

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J-PARC experiments for QCD matter

Possible hints for partial chiral restoration !?!

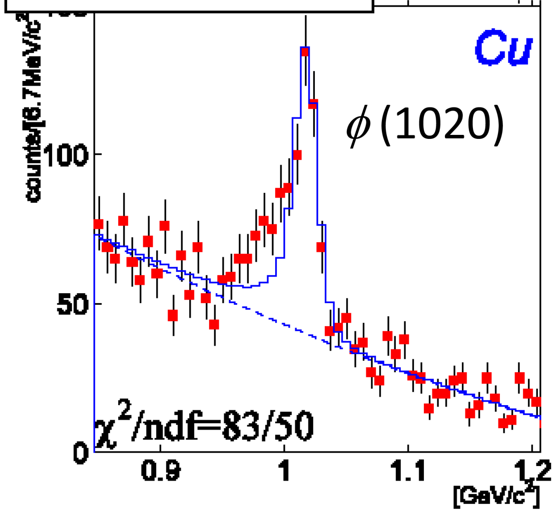
Di-lepton from pA collisions with high-E proton beam (from year 2020)

QCD properties at finite density with high statistics

Mass shift? or Width shift? of Both? or Nothing to do with the restoration?

High-E AA beam for high density baryonic matter in future

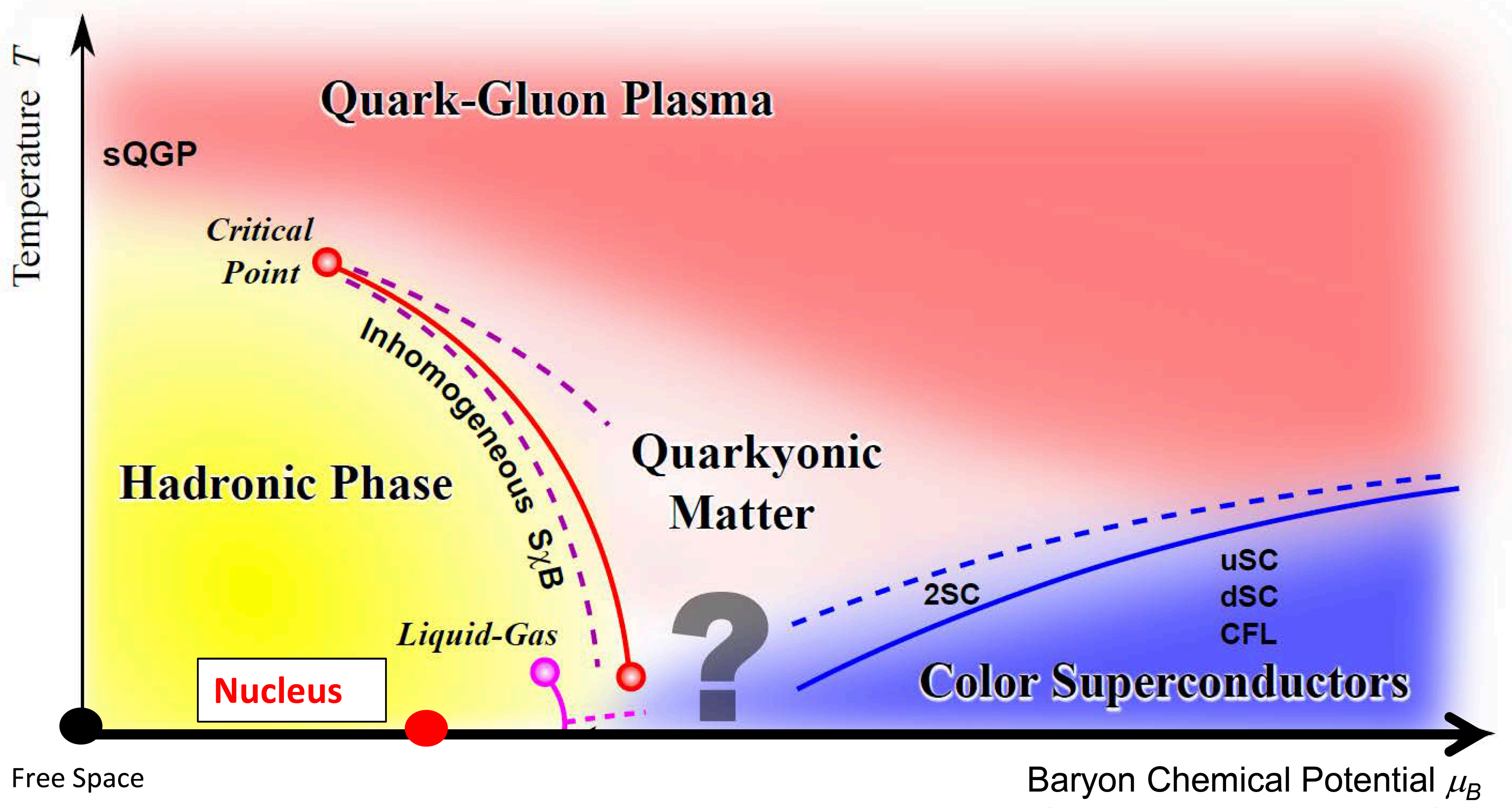
KEK E325 results



QCD at extreme conditions

QCD has nontrivial phase structure as a function of temperature and density

K. Fukushima and T. Hatsuda, Rep. Prog. Phys. 74 (2011) 014001



QCD at extreme conditions

I. Each QCD phases defined by its own order parameters

II. Behavior of order parameters governed by dynamics of symmetry

III. Symmetry and its breakdown governed by vacuum structure

Chiral symmetry ↔ Quark (chiral) condensate: Hadron or not?

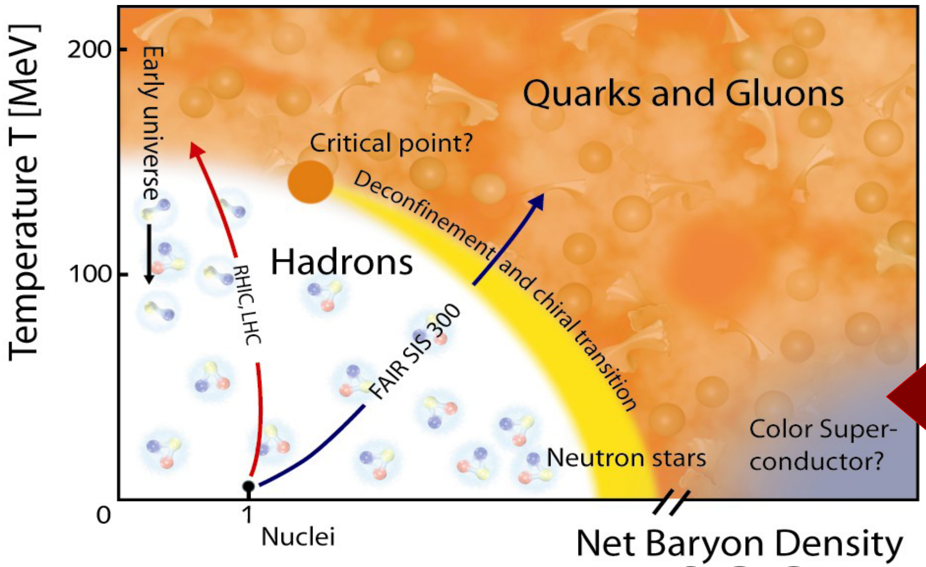
Center symmetry ↔ VEV of Polyakov loop: Confined or not?

Color symmetry ↔ Diquark condensate: Superconducting or not?

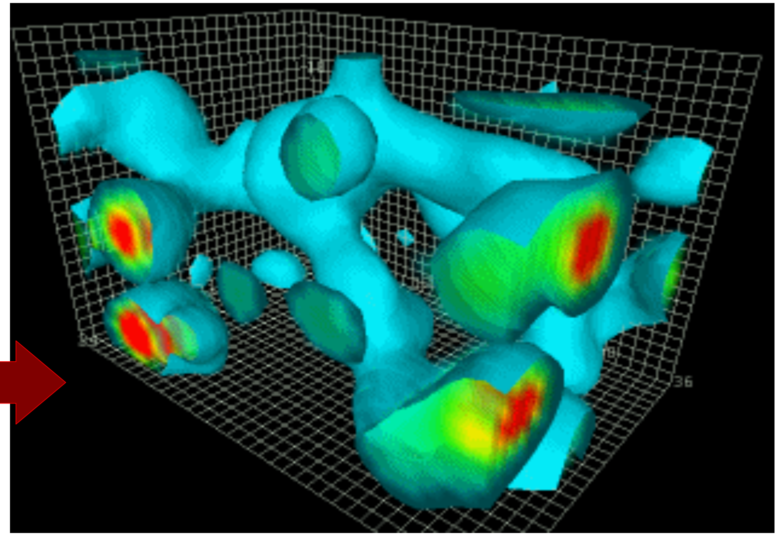
Color-flavor symmetry (locking) ↔ Diquark condensate at high density

QCD phase ↔ Symmetries of QCD ↔ QCD vacuum

Why are pA and AA collision experiments special for QCD?



Phase structure of QCD



Nontrivial QCD vacuum

Color symmetry

C/P/T-invariance

Z(N) center symmetry

Chiral symmetry

Flavor symmetry

Symmetries and their breakdown

QCD at extreme conditions

SCSB results in nonzero chiral (quark) condensate due to nonzero effective quark mass even in the chiral limit, i.e. m=0

$$-\langle\bar{\psi}\psi\rangle_{\mathrm{Mink}} = i\langle\psi^{\dagger}\psi\rangle_{\mathrm{Eucl}} = 4N_c \int \frac{d^4p}{(2\pi)^4} \frac{M(p)}{p^2 + M^2(p)}$$

Nonzero <qq> indicates hadron (Nambu-Goldstone) phase, whereas zero <qq> does non-hadronic phase, not meaning deconfinement

Thus, <qq> is an order parameter for chiral symmetry

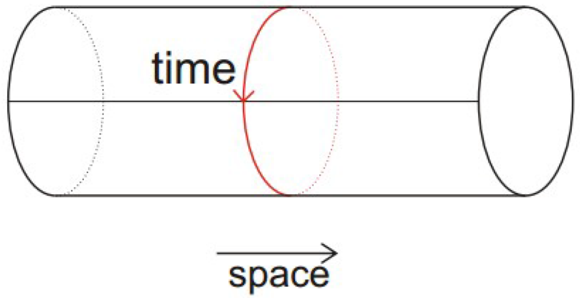
In the real world with nonzero quark current mass ~ 5 MeV, at low density, there appears crossover near T ~ 0, and it becomes 1st-order phase transition as density increases

In the vicinity of critical density, there are various and complicated phases, such as color-superconducting, quarkyonic phase, etc.

QCD at extreme conditions: Another symmetry at finite T

Dynamical (spontaneous) breakdown of center symmetry results in nonzero Polyakov-loop condensate <L>

$$
\begin{aligned}
e^{-\beta F} &= Tr[e^{-H/T}] = \sum_n \langle n| e^{-\tau \cdot H} |n\rangle_{\tau=\beta=1/T} = \\
&= \sum_n e^{-\beta E_n} = \\
&= \sum_\psi \sum_U e^{-S_{FG}} Tr\, \psi_\tau^+ U_\tau U \ldots U_0 \psi_0 = \\
&= \sum_U e^{-S_G} Tr[UU \ldots U]_{0\tau}
\end{aligned}
$$



$$
\sum_U e^{-S_G} \underbrace{Tr[UU \ldots U]_{0\tau}}_{\langle Tr\, P e^{ig\int_0^\beta A_0(\vec{x}) d\tau} \rangle_G} \rightarrow \langle L(\vec{x}) \rangle
$$

$S_{o\tau} = \psi_\tau^+ U_\tau U \ldots U_0 \psi_0$:
quark propagator $0 \rightarrow \tau$

Considering Exp[-F/T]~<L>, where F is quark free energy,
"<L>=0" means that F is infinity, so that quarks are confined

If <L> nonzero, F is finite to separate the quarks apart, i.e. deconfined

QCD at extreme conditions

Theory can help to understand HIC experiments

Equation of state of QCD matter: Lattice QCD, Effective models

Evolution of QGP: (Viscous) Hydrodynamics

Hadronization: Transport models

We want to focus on the following subjects:
Critical behaviors, transport coefficients, Effects of external B fields…

For this purpose, we want to modify the effective models in terms of temperature (as well as density)

Polyakov-loop NJL model & T-modified LIM

Medium-modified Effective models

We start from the effective Lagrangian of NJL, resulting in effective thermodynamic potential Ω, which gives EoS of QCD matter

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - \underline{m})\psi + G\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2\right)$$

We expand the four-quark interaction in terms of SBCS

$$\bar{\psi}\psi = \langle\bar{\psi}\psi\rangle_{NJL} + \delta(\bar{\psi}\psi)$$

Finite chiral condensate considered as an effective quark mass

$$M = m - 2G\langle\bar{\psi}\psi\rangle_{NJL}$$

Finally, we arrive at $\mathcal{L} = \bar{\psi}\,(i\partial\!\!\!/ - M)\,\psi - \frac{(M-m)^2}{4G}$ festing SBCS

Free quark with effective mass M

Constant potential via SBCS

Medium-modified Effective models

Employing *Matsubara formula* to convert the action S ~ [∫ d^4x Lagrangian] into thermodynamic potential

$$i\int\frac{d^4k}{(2\pi)^4}\,f(k)\longrightarrow -T\sum_n\int\frac{d^3k}{(2\pi)^3}\,f(i\omega_n+\mu,\vec{k})$$

with fermionic Matsubara frequencies $\omega_n=(2n+1)\pi T$

We arrive at an effective thermodynamic potential

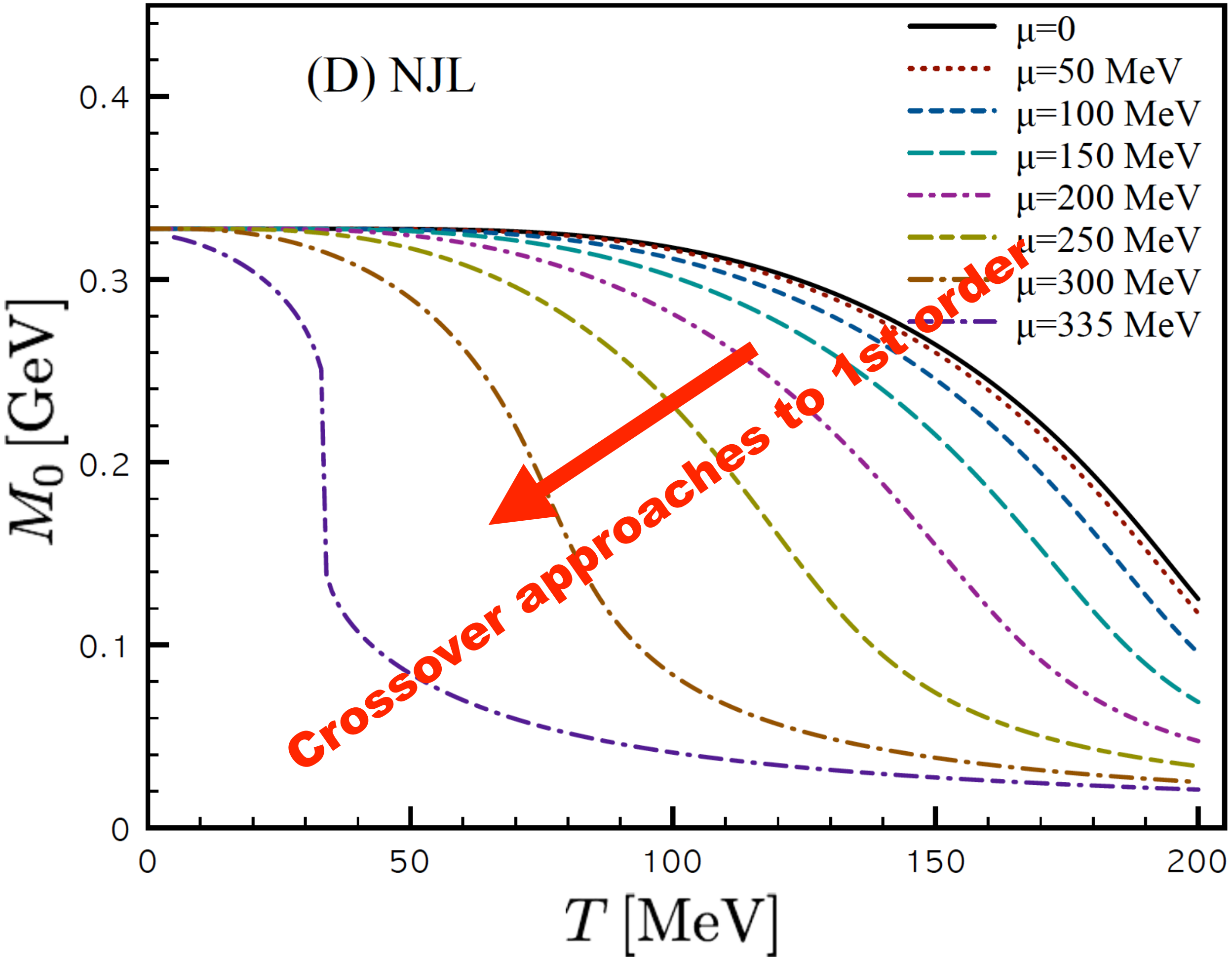
$$\Omega_{\rm NJL}=\frac{(M_0-m_q)^2}{4G}-2N_cN_f\int_0^\Lambda\frac{d^3\boldsymbol{k}}{(2\pi)^3}\left\{E_{\boldsymbol{k}0}+T\ln\left[\left(1+e^{-\frac{E_{\boldsymbol{k}0}-\mu}{T}}\right)\left(1+e^{-\frac{E_{\boldsymbol{k}0}+\mu}{T}}\right)\right]\right\}$$

Computing gap equation, giving phase diagram for SBCS

$$\frac{\partial\Omega_{\rm NJL}}{\partial M_0}=\frac{M_0-m_q}{2G}-2N_cN_f\int_0^\Lambda\frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{M_0}{E_{\boldsymbol{k}0}}\left[1-\frac{e^{-\frac{E_{\boldsymbol{k}0}-\mu}{T}}}{1+e^{-\frac{E_{\boldsymbol{k}0}-\mu}{T}}}-\frac{e^{-\frac{E_{\boldsymbol{k}0}+\mu}{T}}}{1+e^{-\frac{E_{\boldsymbol{k}0}+\mu}{T}}}\right]=0$$

Medium-modified Effective models

QCD phase diagram as a function of T and μvia NJL model



Medium-modified Effective models

Modified (augmented) NJL with Polyakov loop, i.e pNJL

Identifying the imaginary quark chemical potential as Polyakov line,

$$\begin{aligned}\Omega/V = {} & V_{\mathrm{glue}}[L] + \frac{1}{2G}(M - m_q)^2 \\ & - 2N_c N_f \int \frac{d^3 p}{(2\pi)^3} \Big\{ E_p + T \frac{1}{N_c} \\ & \times \mathrm{Tr_c} \ln\big[1 + L e^{-(E_p - \mu)/T}\big] \\ & + T \frac{1}{N_c} \mathrm{Tr_c} \ln\big[1 + L^\dagger e^{-(E_p + \mu)/T}\big] \Big\},\end{aligned}$$

$$L(\vec{x}) = \mathcal{T} \exp\left[-i \int^{\beta} dx_4 \, A_4(x_4, \vec{x}) \right]$$

$$\begin{aligned} & V_{\mathrm{glue}}[L] \cdot a^3/T \\ & = -2(d-1) e^{-\sigma a/T} |\mathrm{Tr_c}\, L|^2 \\ & \quad - \ln\big[-|\mathrm{Tr_c}\, L|^4 + 8\, \mathrm{Re}(\mathrm{Tr_c}\, L)^3 \\ & \quad - 18 |\mathrm{Tr_c}\, L|^2 + 27 \big] \end{aligned}$$

Gluonic thermodynamic potential constructed by Z(Nc) symmetry and lattice QCD information

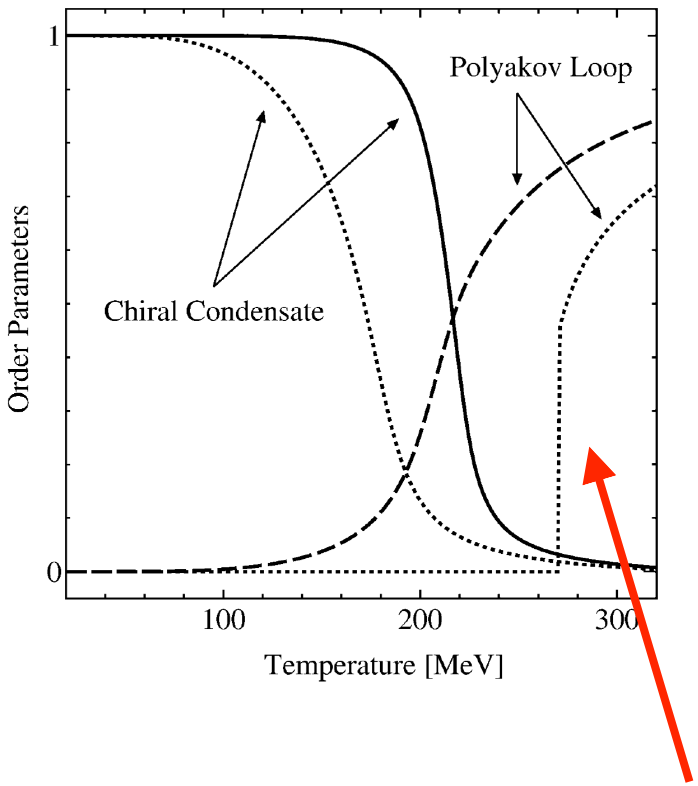
$$\Omega^{\phi}_{\mathrm{eff}} = -T^4 \left[\frac{b_2(T)}{2} (\phi \phi^*) + \frac{b_3}{6} (\phi^3 + \phi^{*3}) - \frac{b_4}{4} (\phi \phi^*)^2 \right]$$

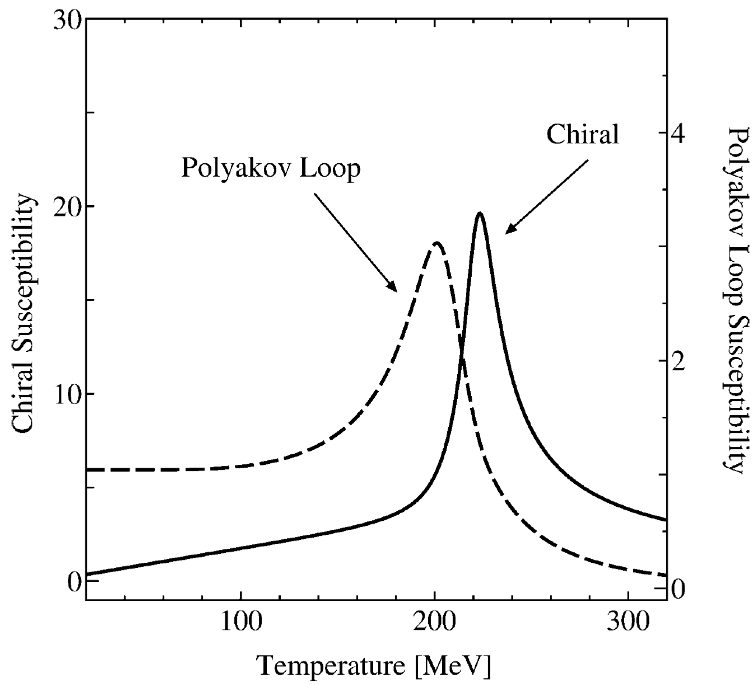
$$b_2(T) = a_0 + a_1 \left[\frac{T_0}{T} \right] + a_2 \left[\frac{T_0}{T} \right]^2 + a_3 \left[\frac{T_0}{T} \right]^3$$

Medium-modified Effective models

Realization of simultaneous crossover of chiral and de-confinement phase transitions

Two phase transitions!?!?!





Due to quark-L interaction, <L> shows crossover, rather than 1st order in pure-glue theory

Medium-modified Effective models

T-modified LIM:(mLIM) Instanton parameters are modified with trivial-holonomy caloron solution (Not dyon, vortex, or something)

Caloron is an instanton solution for periodic in Euclidean time, i.e temperature, but no confinement

Distribution func. via trivial-holonomy (Harrington-Shepard) caloron

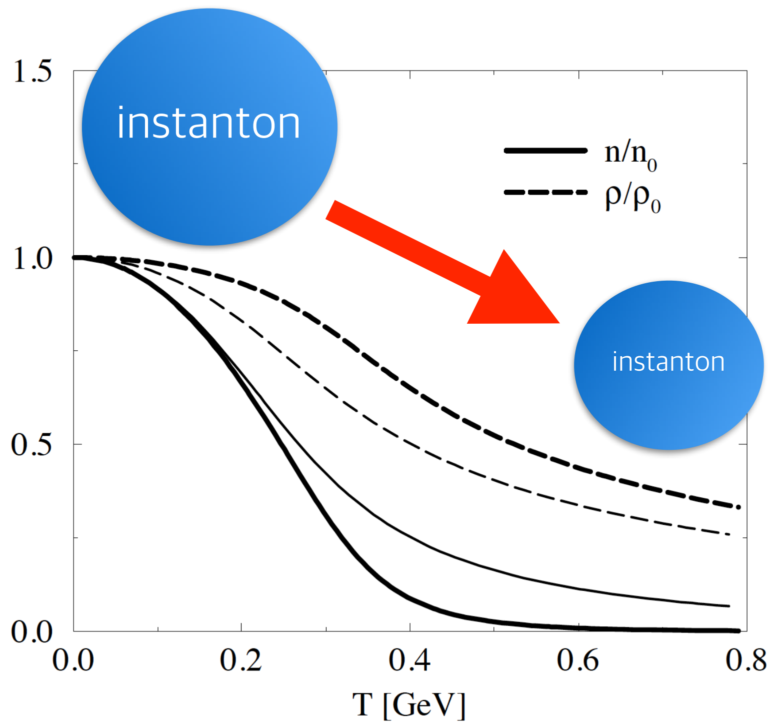
$$d(\rho,T) = \mathcal{C}\,\rho^{b-5}\exp\left[-\mathcal{F}(T)\rho^2\right],\quad \mathcal{F}(T) = \frac{1}{2}A_{N_c}T^2 + \left[\frac{1}{4}A_{N_c}^2T^4 + \nu\bar{\beta}\gamma n\right]^{\frac{1}{2}}$$

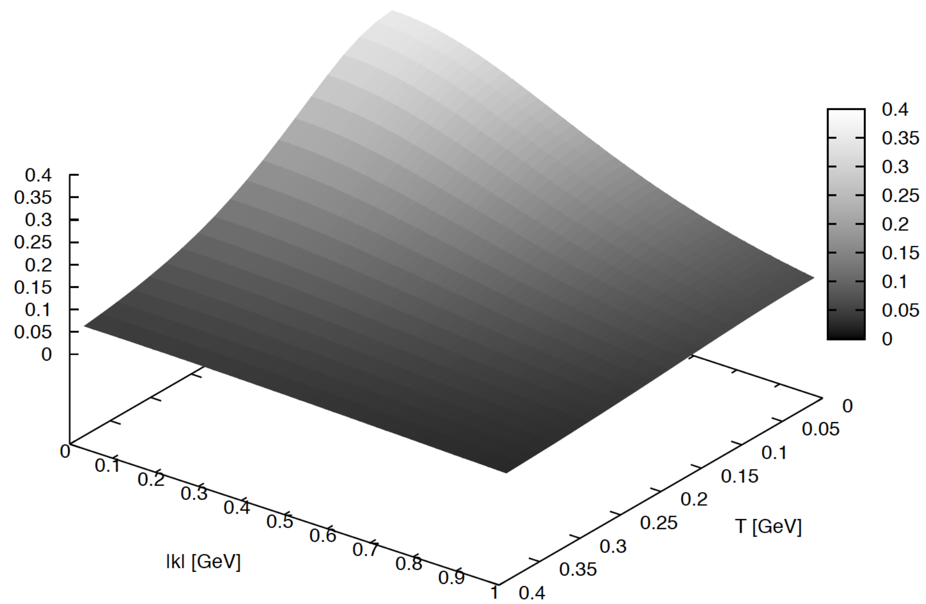
$$A_{N_c} = \frac{1}{3}\left[\frac{11}{6}N_c - 1\right]\pi^2,\quad \gamma = \frac{27}{4}\left[\frac{N_c}{N_c^2-1}\right]\pi^2,\quad b = \frac{11N_c - 2N_f}{3}.$$

Using this, we modify the two instanton parameters as functions of T

Medium-modified Effective models

mLIM parameters (left) and effective quark mass M (right)





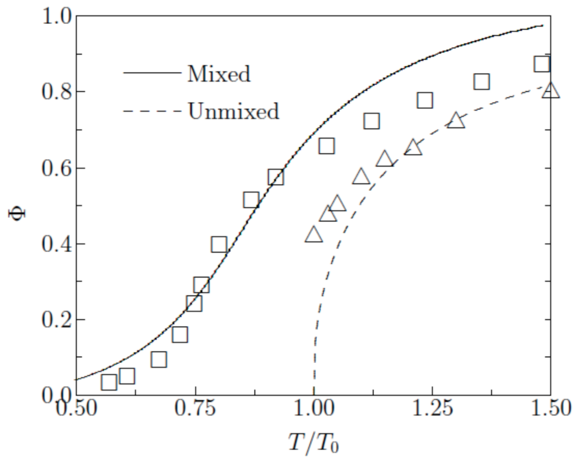
Hence, effective quark mass plays the role of UV regulator

Medium-modified Effective models

Finally, we arrive at an effective thermodynamic potential via instanton and Polyakov loop

S.i.N., J.Phys. G37 (2010) 075002

$$\begin{aligned}\Omega_{\rm eff} &= \Omega^{q+\Phi}_{\rm eff} + \Omega^{\Phi}_{\rm eff} = 2\sigma^2 - 2N_f\Bigg[N_c\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}E_{\boldsymbol{k},T} \\ &+ T\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\ln\left[1+N_c\left(\Phi+\bar{\Phi}\,e^{-\frac{E_{\boldsymbol{k},T}}{T}}\right)e^{-\frac{E_{\boldsymbol{k},T}}{T}}+e^{-\frac{3E_{\boldsymbol{k},T}}{T}}\right] \\ &+ T\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\ln\left[1+N_c\left(\bar{\Phi}+\Phi\,e^{-\frac{E_{\boldsymbol{k},T}}{T}}\right)e^{-\frac{E_{\boldsymbol{k},T}}{T}}+e^{-\frac{3E_{\boldsymbol{k},T}}{T}}\right]\Bigg] \\ &- T^4\left[\frac{b_2(T)}{2}(\Phi\,\bar{\Phi})+\frac{b_3}{6}(\Phi^3+\bar{\Phi}^3)-\frac{b_4}{4}(\Phi\,\bar{\Phi})^2\right],\end{aligned}$$

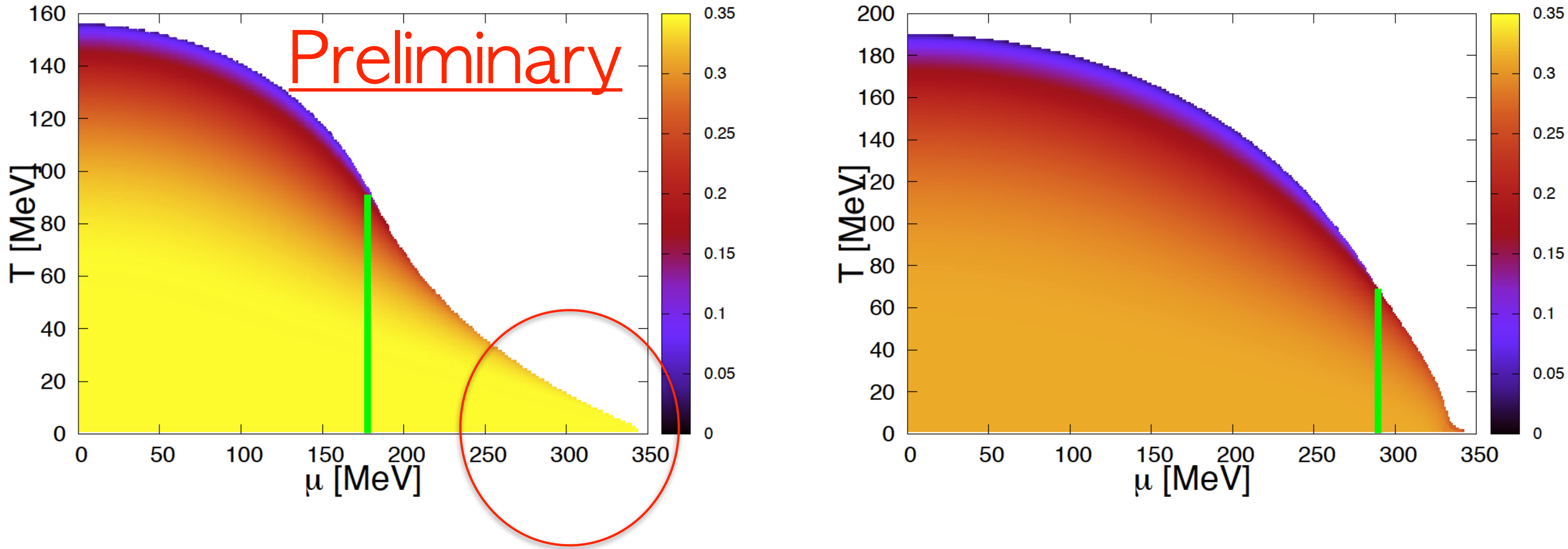


Basically, we have similar results with pNJL results

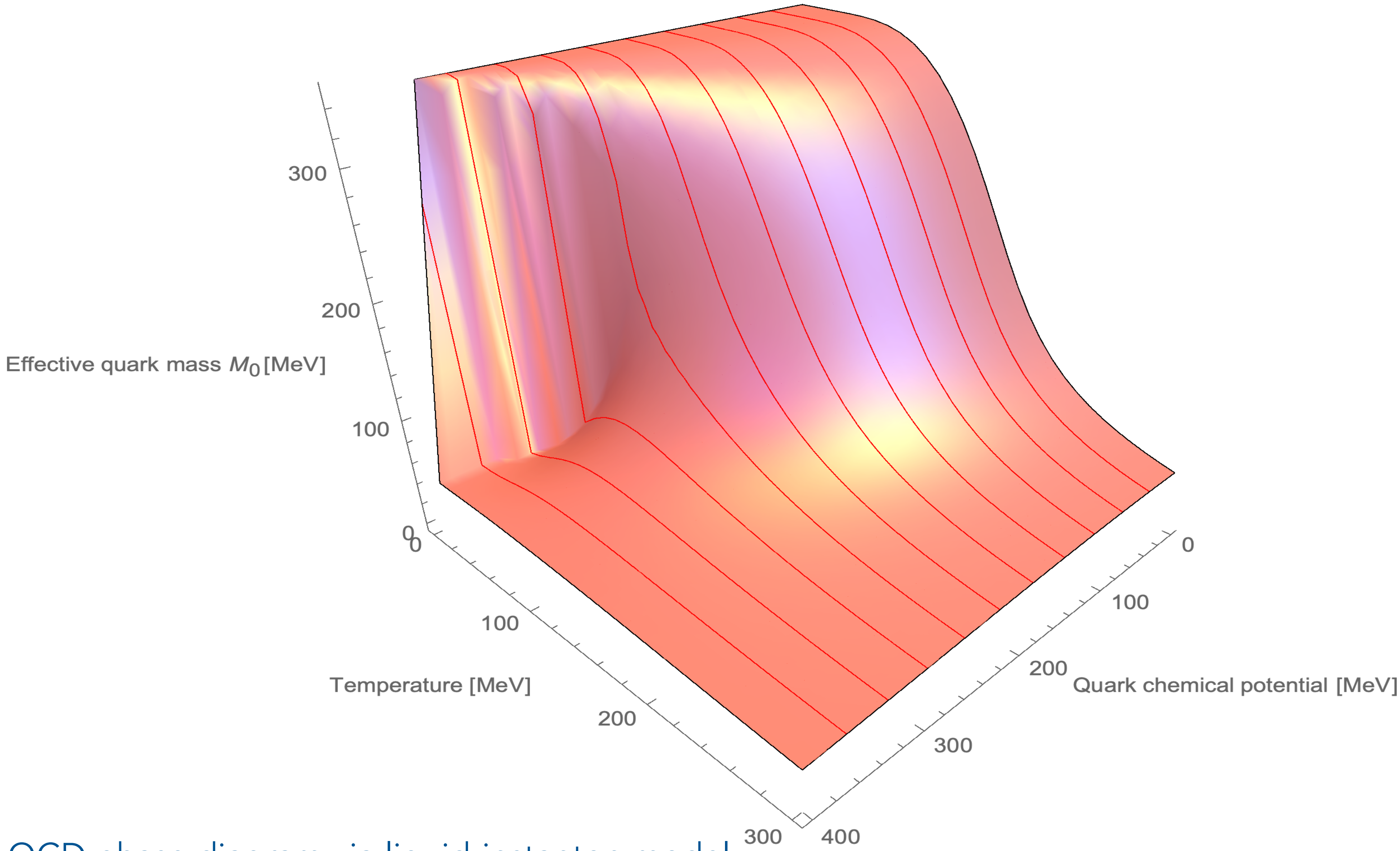
In detail, positions for critical T and ρ, structure of phase shift, etc. are different quantitatively

Medium-modified Effective models

We plot the phase diagrams via mLIM (left) and NJL (right)



The effects of T-dependent model parameters are obvious!



QCD phase diagram via liquid-instanton model

Lee-Yang zeros (LYZ) for QCD phase transition

A thermodynamic quantity as a function of complex fugacity z (ξ)

$$f(z) = \lim_{N\to\infty} \frac{1}{N} \log Z_N(z) \text{ exists for all } z > 0.$$

$$Z_{\mathrm{GC}}(\mu, T, V) = \sum_{n=-N_{\max}}^{N_{\max}} Z_C(n, T, V)\xi^n .$$

If $Z_N(z) \neq 0$ for a region R in z, then f(z) is analytic

Thus, only in thermodynamic limit (N→∞), Z_N can be zero

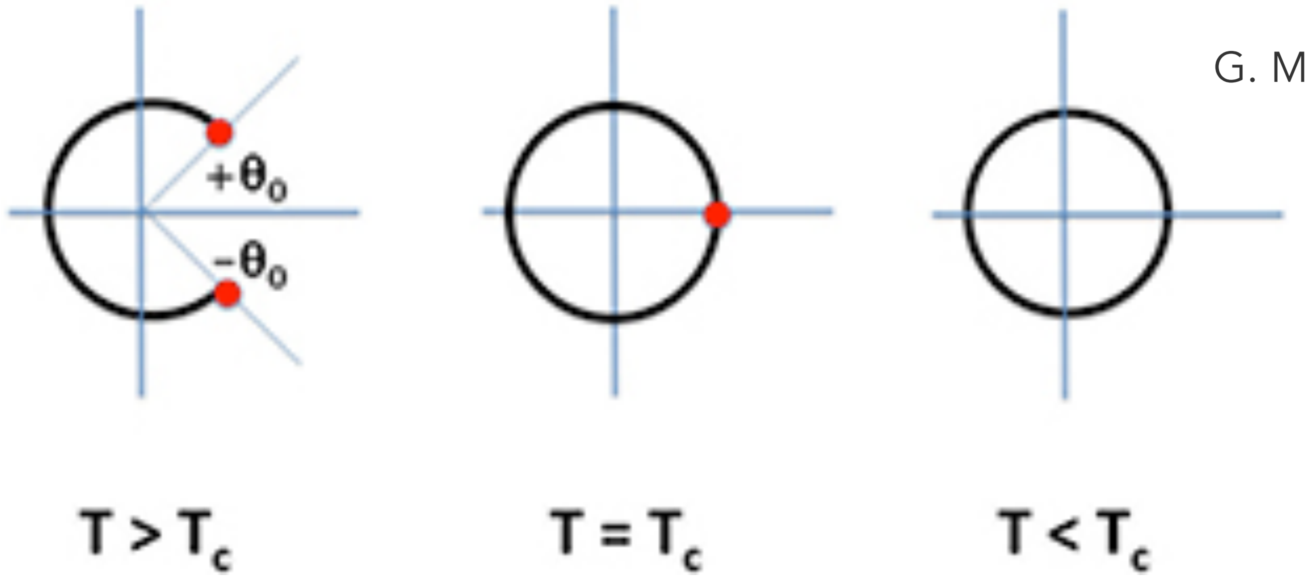
If z for $Z_N(z)=0$ stays away from real z axis, there are no singularities

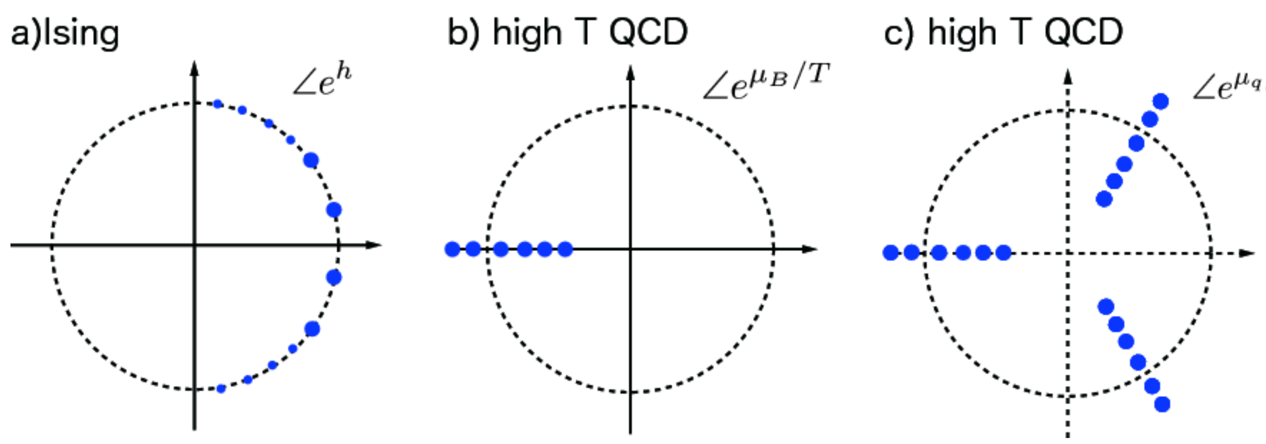
QCD phase transition characterized by **singularities** (susceptibilities)

These singularities corresponds to $Z_{GC}=0$ (T.D.Lee and C.N.Yang)

Lee-Yang zeros (LYZ) for QCD phase transition

Schematic LYZs on complex z plane

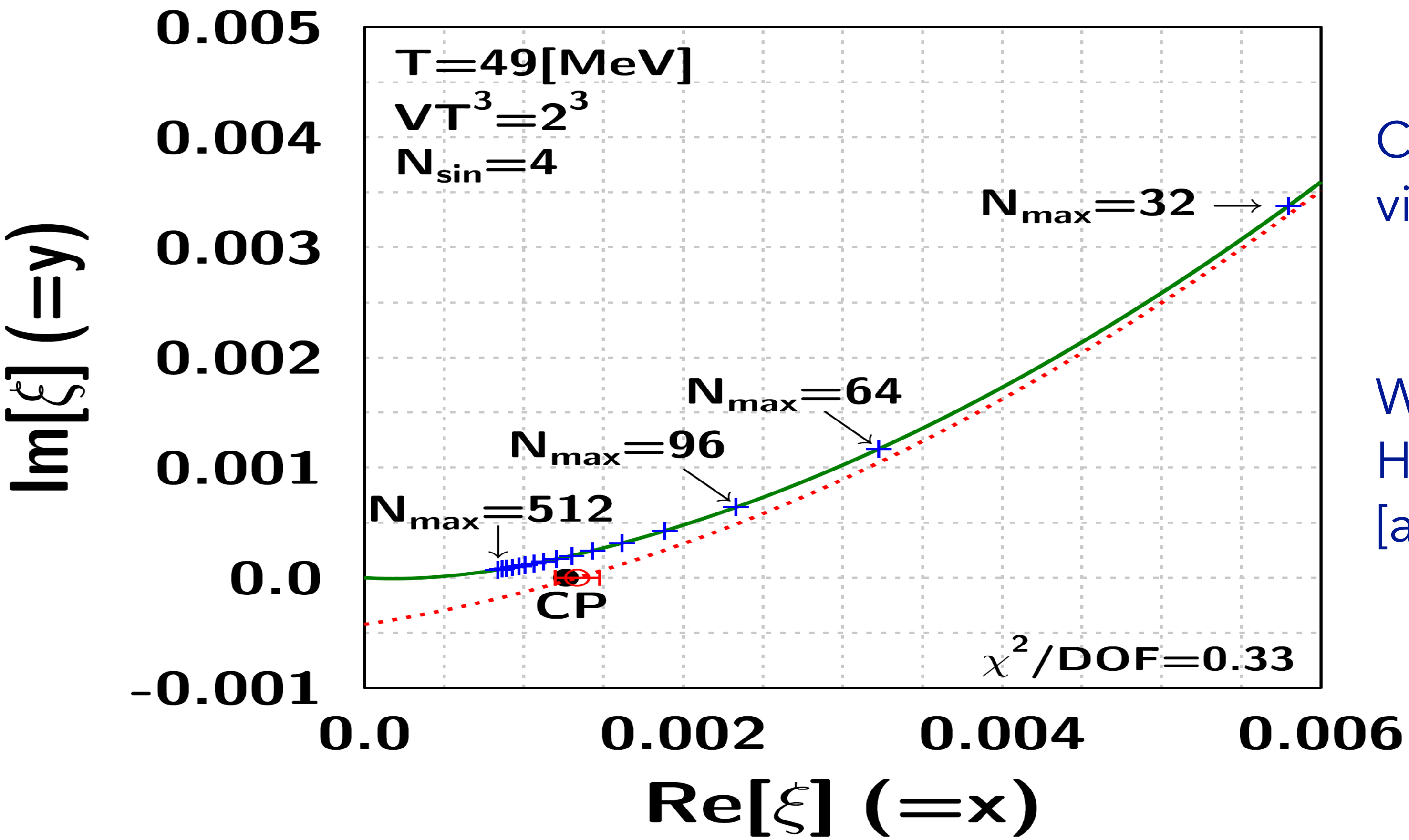




Lee-Yang zeros (LYZ) for QCD phase transition

GC partition function as a finite polynomial with fugacity

$$Z_{\mathrm{GC}}(\mu, T, V) = \sum_{n=-N_{\max}}^{N_{\max}} Z_C(n, T, V)\xi^n .$$



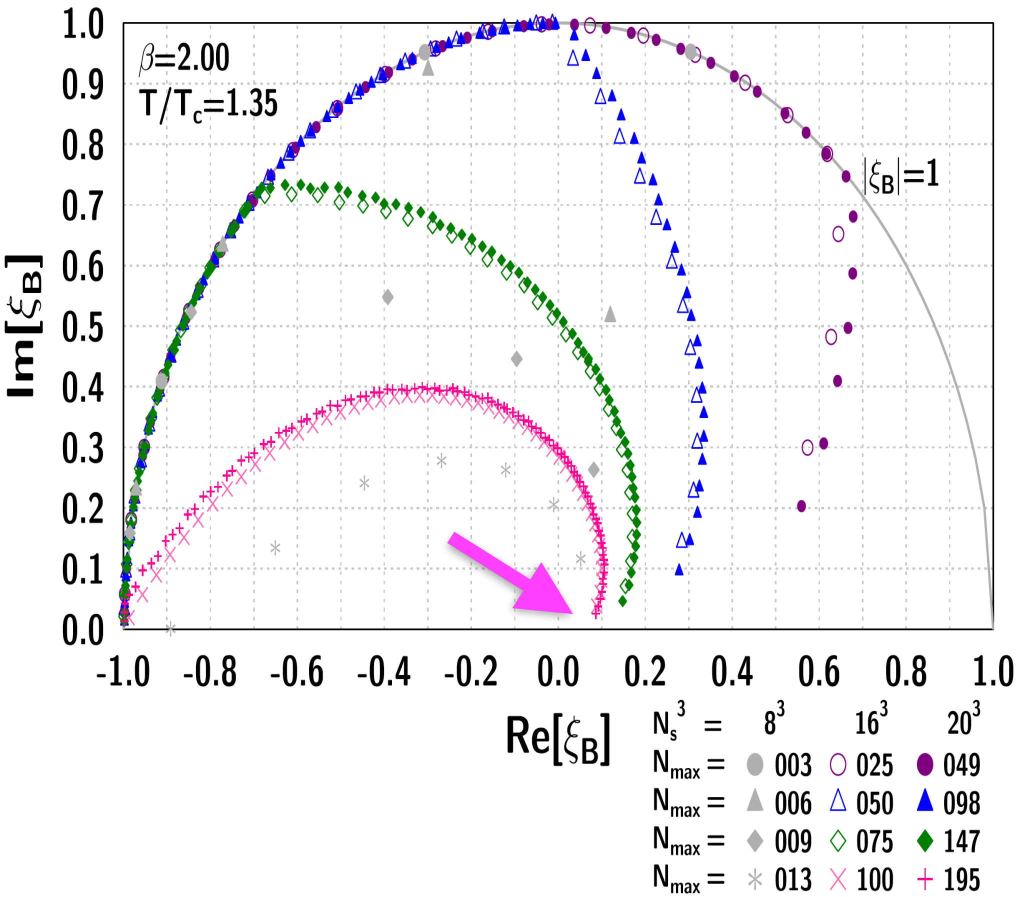
Chiral phase transition via NJL

Wakayama, Hosaka, [arXiv:1905.10956]

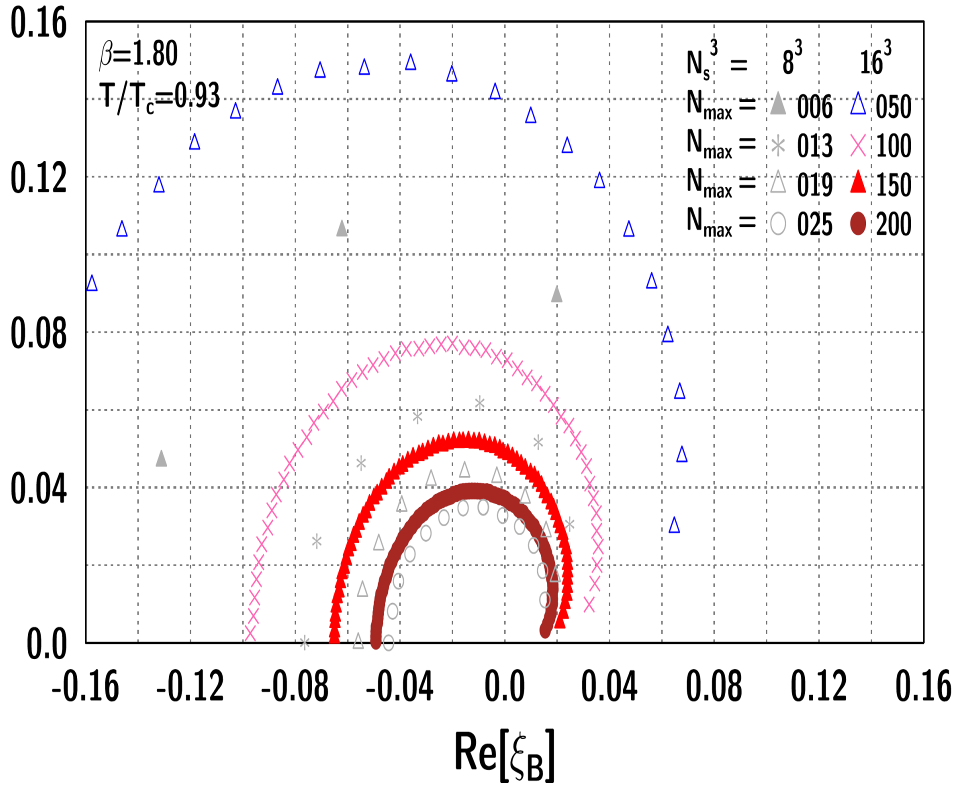
Lee-Yang zeros (LYZ) for QCD phase transition

Canonical approach LQCD (Wakayama et al.)

Roberge-Weiss phase transition

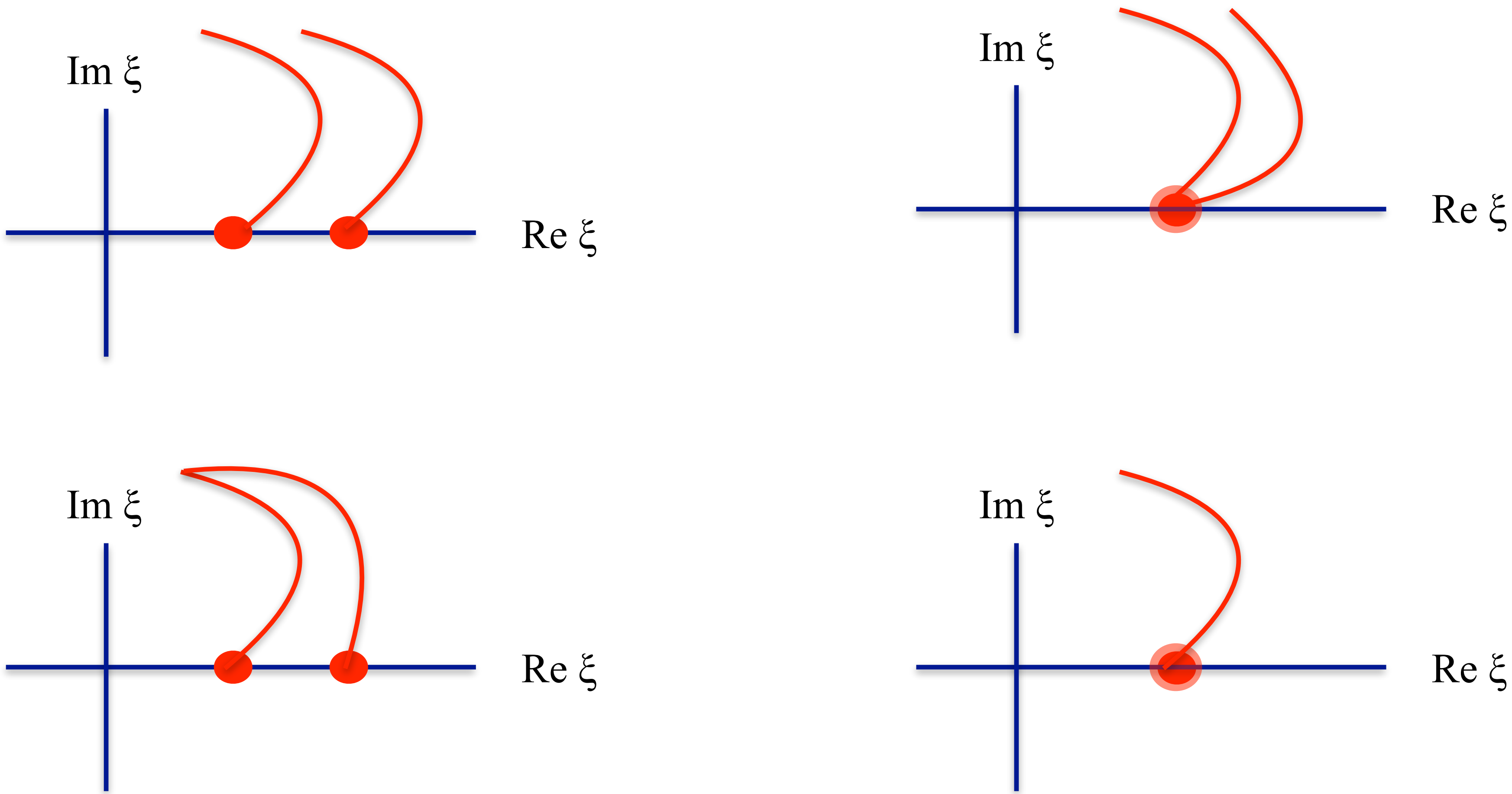


Confined phase near Tc



Lee-Yang zeros (LYZ) for QCD phase transition

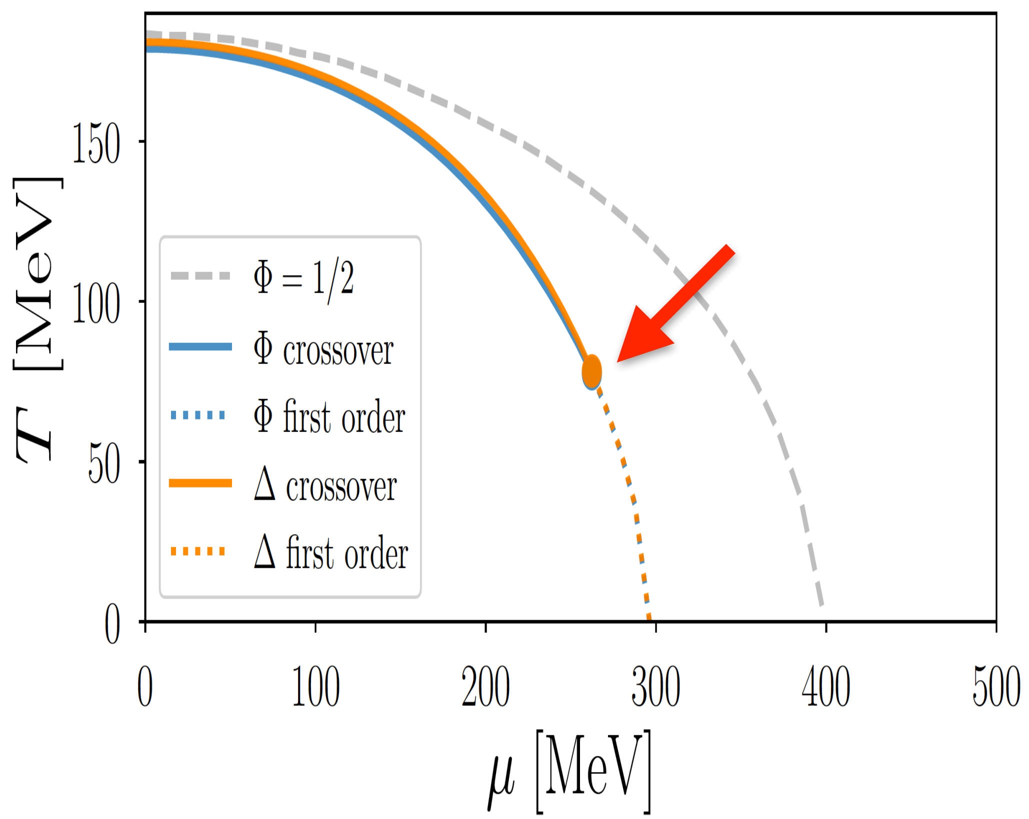
Then, what happens for two phase transitions??? (pNJL)



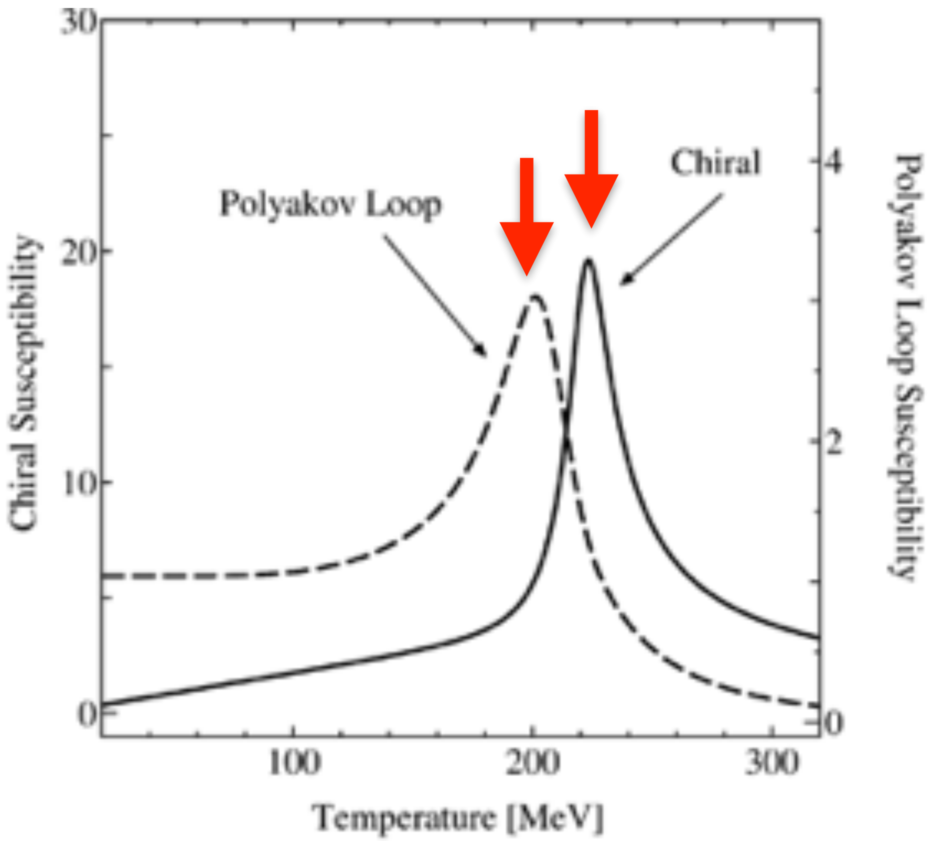
Lee-Yang zeros (LYZ) for QCD phase transition

Then, what happens for two phase transitions??? (pNJL)

A. Folkestadand J.O. Andersen
(Pisarski-Skokov chiral matrix model)



K. Fukushima (pNJL)



Summary

QCD has rich phase structures corresponding to symmetries

Polyakov-loop models enable us to study rich QCD phases

Instanton explains QCD phase transition in natural ways

Instanton + Polyakov loop shows different QCD phase diagram

What happens for LYZs for pNJL (or mLIM?)

Lattice QCD will show two zeros? or single overlapped zeros?

Canonical approach does work? Stay tuned!!!

Thank you for your attention!

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