Is ACTIVE sub-eV neutrino (ν), "Dirac or Majorana" ?, that is the question !

Inha U-HTG WS, 07/07/2022
CUP-IBS, 06/28/2022
CUBES-3, 04/11/2022
2022 CAU WS on Beyond the SM, 02/10/2022
Mini WS on Axion & Neutrino, 01/17/2022

CSK,MM,DS, arXiv://2106.11785 (PRD, in press)

C S Kim (w/ M V N Murthy and D Sahoo)
Yonsei University/Dongshin University

CONTENTS

- Introduction (nu Mass, Seesaw & 0nuBB)
- Prelude (Quantum Statistics & practical DMCT)
- Back-to-back muons (ie. $\nu - \bar{\nu}$), exception to DMCT
- Detailed Analysis & Discussion on B2B muons
- Summary
- Back-up (Details on Helicity, B2B muons, …)

INTRODUCTION

Sub-eV active neutrino mass

Seesaw mechanism

$\Delta L = 2$ processes & 0-nu-Beta-Beta

(sub-eV active) neutrinos have mass

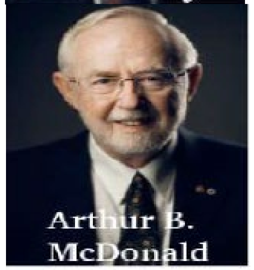
- Neutrinos are massless in SM, $m_\nu = 0$.
  All neutrinos are only left-handed (ν_L).

$$\mathscr{L}^D_{\text{mass}} = -m_\nu \left(\overline{\nu_R}\nu_L + \overline{\nu_L}\nu_R\right), \qquad m_\nu = \frac{Y_\nu \, \text{v}}{\sqrt{2}},$$

  where Y_ν = Higgs-neutrino Yukawa coupling constant,
  and v = Higgs VEV.
  No way to generate mass without right-handed neutrinos (ν_R).

- But observations of neutrino oscillation imply that **neutrinos have mass**, $m_\nu \neq 0$.

  The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita (Super-Kamiokande) and Arthur B. McDonald (Sudbury Neutrino Observatory) "for the discovery of neutrino oscillations, which shows that neutrinos have mass".



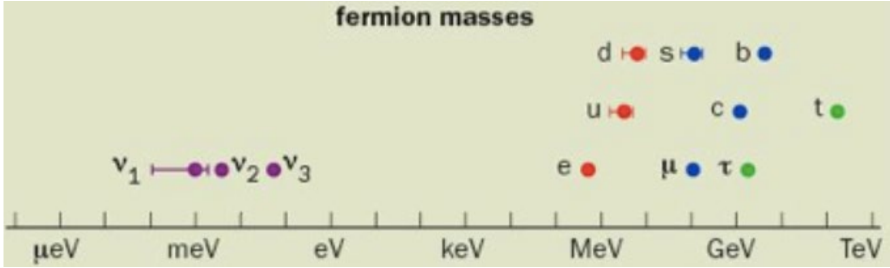
How to give neutrinos mass?

There are various suggestions as to how neutrinos can get mass.

- **Dirac mass:**
  - *Assumption:* ν_R exists.
  - *Lagrangian:*

$$\mathcal{L}^D_{\text{mass}} = -m^D_\nu \left(\overline{\nu_R}\nu_L + \overline{\nu_L}\nu_R\right).$$

  - *Disadvantage:* No reason for m^D_ν to be small.



- **Majorana mass:**
  - *Assumption:* neutrino ≡ anti-neutrino.
  - *Lagrangian:*

$$\mathcal{L}^M_{\text{mass}} = \frac{1}{2} m^M_\nu \left(\overline{\nu^C_L}\nu_L + \overline{\nu_L}\nu^C_L\right).$$

  - *Disadvantage:* $\mathcal{L}^M_{\text{mass}}$ is not invariant under $SU(2)_L \times U(1)_Y$ gauge group, so not allowed by SM.

How to give neutrinos (very small) mass ?

- **See-saw mechanism:** A simpler version of Dirac-Majorana mass, with a nice twist.

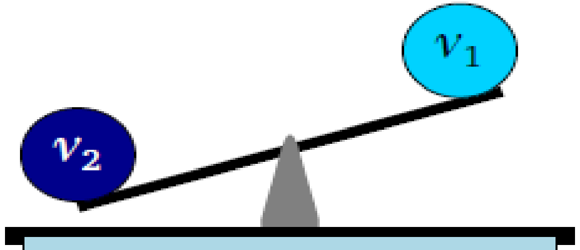
[PM,PLB67(1977)421]

  - *Assumptions:* $m_\nu^L = 0$ and $m_\nu^D \ll m_\nu^R$.
  - *Lagrangian:*
    $\mathscr{L}_{\text{mass}}^{D+M} = \frac{1}{2} m_\nu^R \left(\overline{\nu_R^C} \nu_R\right) - m_\nu^D \left(\overline{\nu_R} \nu_L\right) + \text{H.c.} = \frac{1}{2} \overline{N_L^C} M N_L + \text{H.c.}$, where
    $N_L = \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix}$ and $M = \begin{pmatrix} 0 & m_\nu^D \\ m_\nu^D & m_\nu^R \end{pmatrix}$ is the mass matrix.
  - *Mass eigenvalues:*

$$m_{2,1} = \frac{1}{2}\left(m_\nu^R \pm \sqrt{\left(m_\nu^R\right)^2 + 4\left(m_\nu^D\right)^2}\right)$$

$$\approx \frac{1}{2} m_\nu^R \left(1 \pm 1 \pm 2\left(\frac{m_\nu^D}{m_\nu^R}\right)^2\right).$$

$$\Longrightarrow m_1 \approx -\frac{\left(m_\nu^D\right)^2}{m_\nu^R} \text{ and } m_2 \approx m_\nu^R.$$



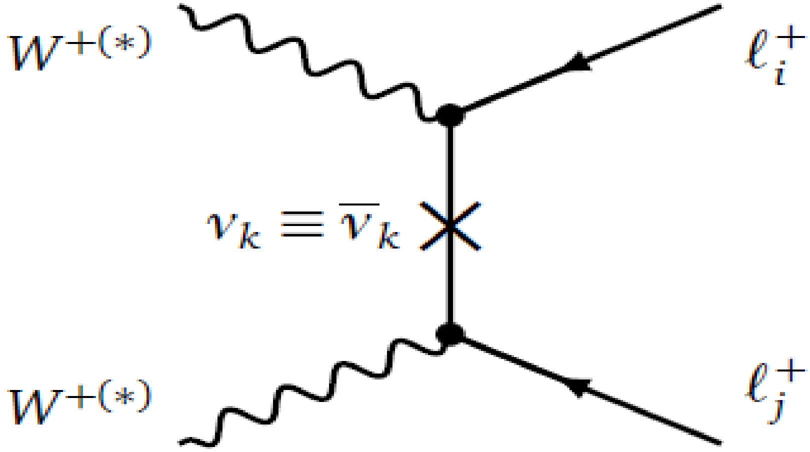
As ν_2 gets heavier, ν_1 become lighter.

  - *Advantage:* $m_1 \ll m_2$, so light neutrinos are possible.
  - *Challenges:*
    - To find the heavy ν_2 experimentally.
    - To prove that both the light ν_1 and heavy ν_2 are Majorana neutrinos.

6 / 1

Looking for Majorana neutrinos via $\Delta L = 2$ processes (1)

- Neutrinos are the only *elementary fermions* known to us that *can* have Majorana nature.
- Majorana neutrinos: $\nu \equiv \overline{\nu}$.
- Majorana neutrinos violate lepton flavor number (L), they mediate $\Delta L = 2$ processes.



$$\propto \int \frac{d^4p}{(2\pi)^4} \sum_k U_{\ell_i k} U_{\ell_j k} \frac{m_k + \not{p}}{p^2 - m_k^2}$$

- $\Delta L = 2$ processes play crucial rule to probe Majorana nature of ν's.
  - neutrinoless double-beta ($0\nu\beta\beta$) decay
  - Rare meson decays with $\Delta L = 2$
  - Collider searches at LHC

Looking for Majorana neutrinos via $\Delta L = 2$ processes (2)

- **Decay rate of any $\Delta L = 2$ process with final leptons $\ell_1^+ \ell_2^+$:**

$$\Gamma_{\Delta L=2} \propto \left| \sum_k U_{\ell_1 k} U_{\ell_2 k} \frac{m_k}{p^2 - m_k^2 + i m_k \Gamma_k} \right|^2 ,$$

where we have used the fact that $(1-\gamma^5)\, \not{p}(1-\gamma^5) = 0$.

- Light ν:

$$\Gamma_{\Delta L=2} \propto \left| \sum_k U_{\ell_1 k} U_{\ell_2 k} m_k \right|^2 = \left| m_{\ell_1 \ell_2} \right|^2 .$$

- Heavy ν:

$$\Gamma_{\Delta L=2} \propto \left| \sum_k \frac{U_{\ell_1 k} U_{\ell_2 k}}{m_k} \right|^2 .$$

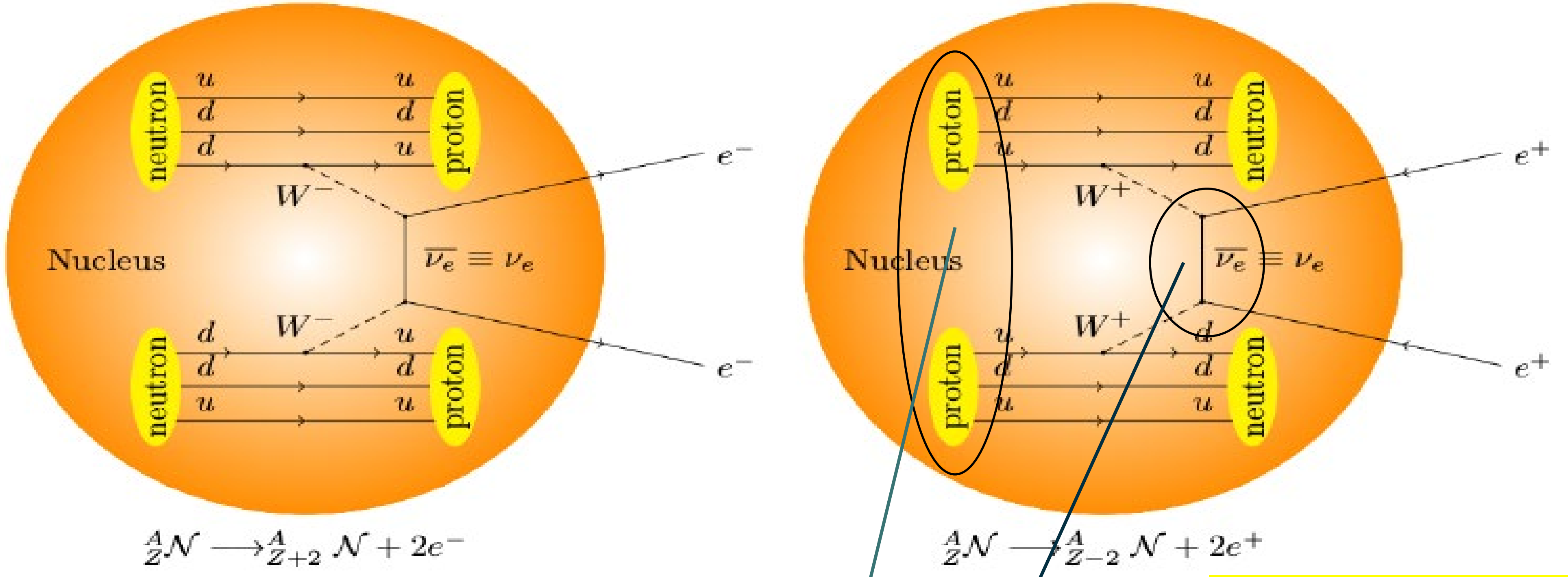
- Resonant ν:

$$\Gamma_{\Delta L=2} \propto \frac{\Gamma(N \to i)\, \Gamma(N \to f)}{m_N \Gamma_N} .$$

Neutrino-less double-beta decay $(0\nu\beta\beta)$ (1)

Lepton Number Violation (LNV)
➔ not allowed within SM

- Process:



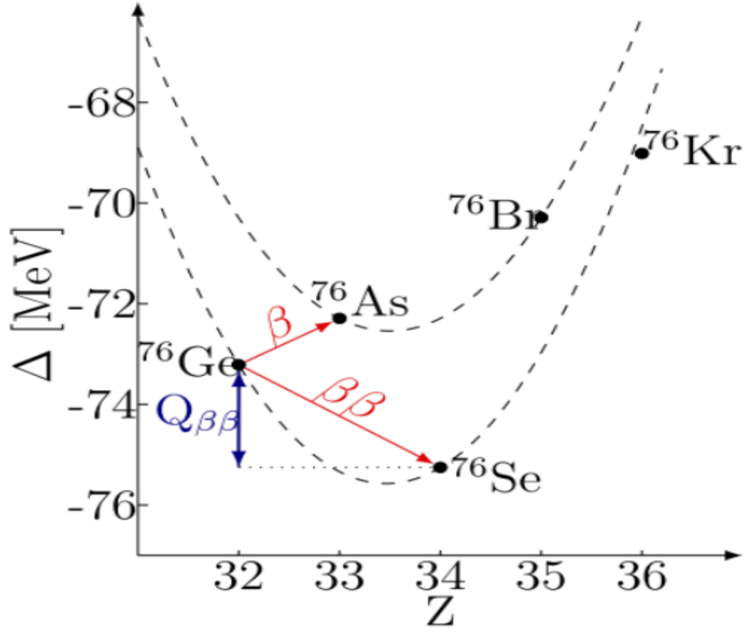
Doubly weak charged current process

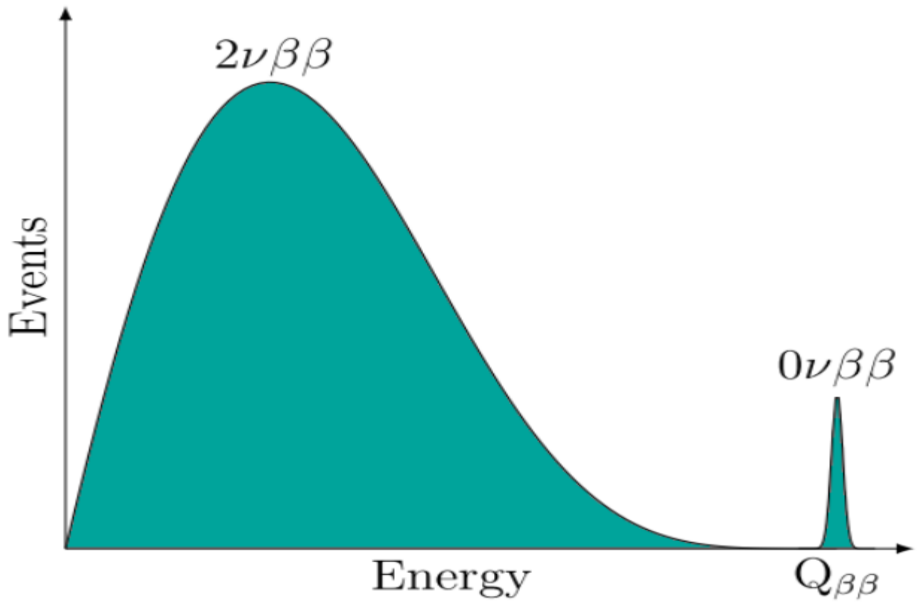
- The half-life of a nucleus decaying via $0\nu\beta\beta$ is,

$$\left[T^{0\nu}_{1/2}\right]^{-1} = G_{0\nu}\ |M_{0\nu}|\ |m_{\beta\beta}|^2,$$

Neutrino-less double-beta decay $(0\nu\beta\beta)$ (2)

- Double-beta $(2\nu\beta\beta)$ decay has been observed in 10 isotopes, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128}Te, ^{130}Te, ^{150}Nd, ^{238}U, with half-life $T_{1/2} \approx 10^{18} - 10^{24}$ years.





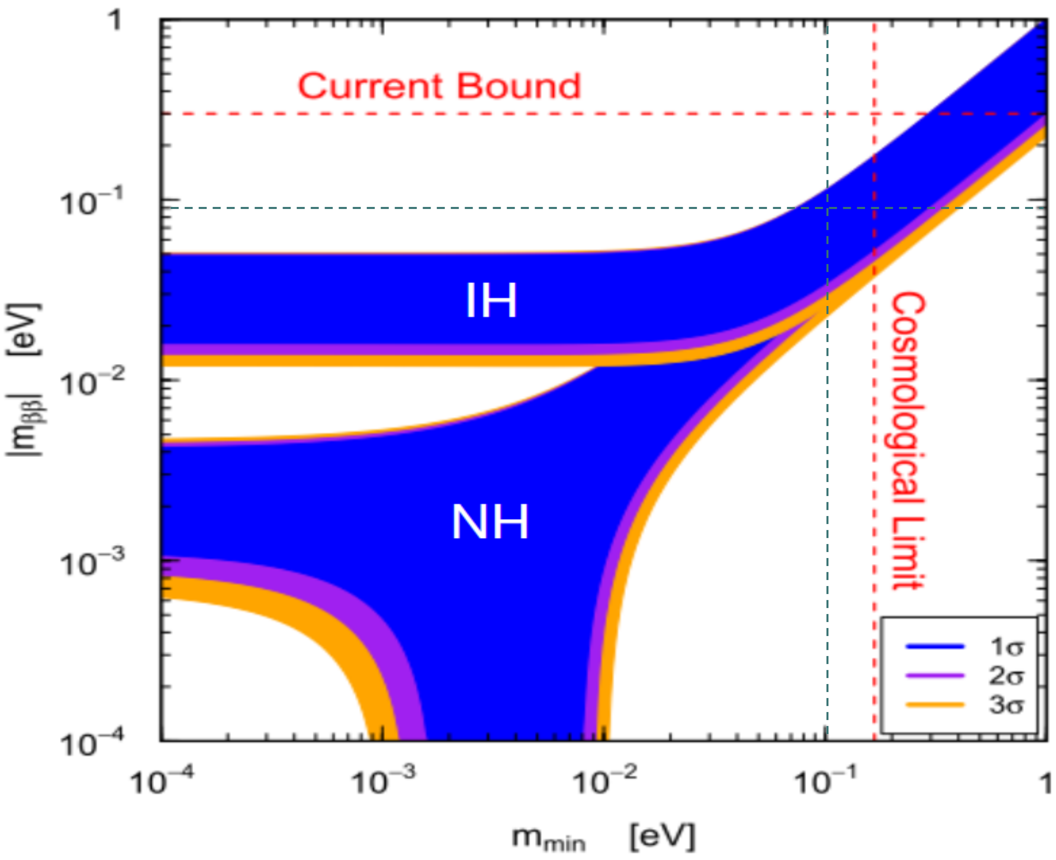
Giovanni Benato (for the GERDA collaboration), arXiv:1509.07792

- **$0\nu\beta\beta$ (forbidden in SM) is yet to be observed in any experiment.**

$$T^{0\nu}_{1/2}\left[{}^{76}\mathrm{Ge}\right] > 2.1 \times 10^{25}\ \text{years}\,(90\%\ \text{C.L.})\,.$$

M. Agostini *et al.* (GERDA Collaboration) Phys. Rev. Lett. **111**, 122503 (2013).

Neutrino-less double-beta decay $(0\nu\beta\beta)$ (3)



NH: Normal hierarchy
IH: Inverted hierarchy

S. M. Bilenky and C. Giunti
Mod. Phys. Lett. A **27**, 1230015 (2012),
arXiv:1203.5250

- **If $m_{\beta\beta} < 10^{-2}$, only NH is viable and the $T^{0\nu}_{1/2}$ will be much larger than the current experimental lower boun**

$$\left[T^{0\nu}_{1/2}\right]^{-1} = G_{0\nu}\ \left|M_{0\nu}\right|\ \left|m_{\beta\beta}\right|^2$$

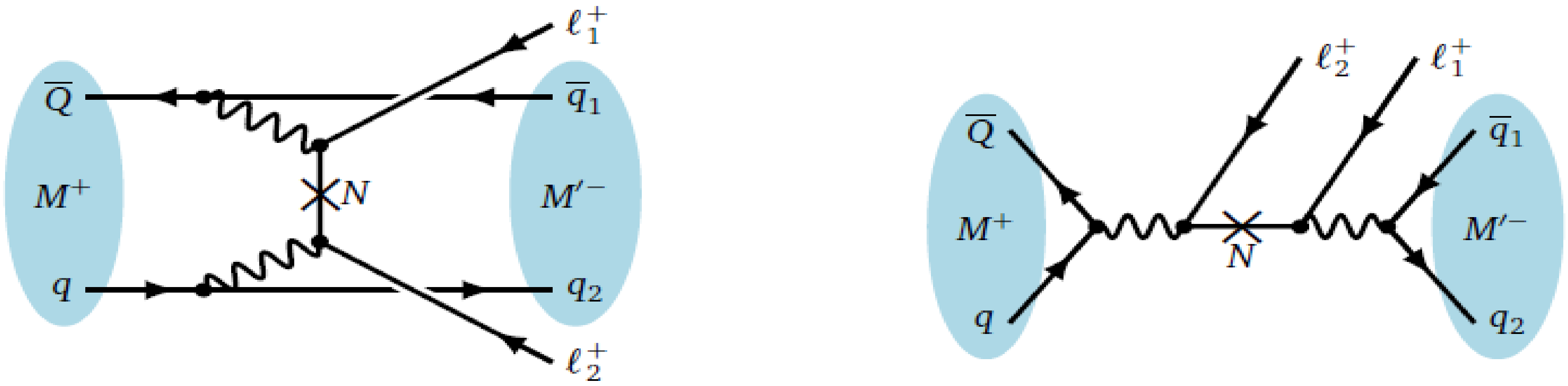
Looking for Majorana neutrinos via $\Delta L = 2$ processes (1)

(Rare meson decays for massive sterile neutrinos)

- **Processes:** $M^+ \to M'^- \ell_1^+ \ell_2^+$,
  where $M = K, D, D_s, B, B_c$ and $M' = \pi, K, D, \ldots$

  G. Cvetic, C.S. Kim, arXiv:1606.04140 (PRD **94**, 053001, 2016)

  G. Cvetic, C. Dib, S. Kang, C. S. Kim, arXiv:1005.4282 (PRD **82**, 053010, 2010)



- **No nuclear matrix element unlike** $0\nu\beta\beta$, but probes Majorana nature of massive neutrino(s) N.

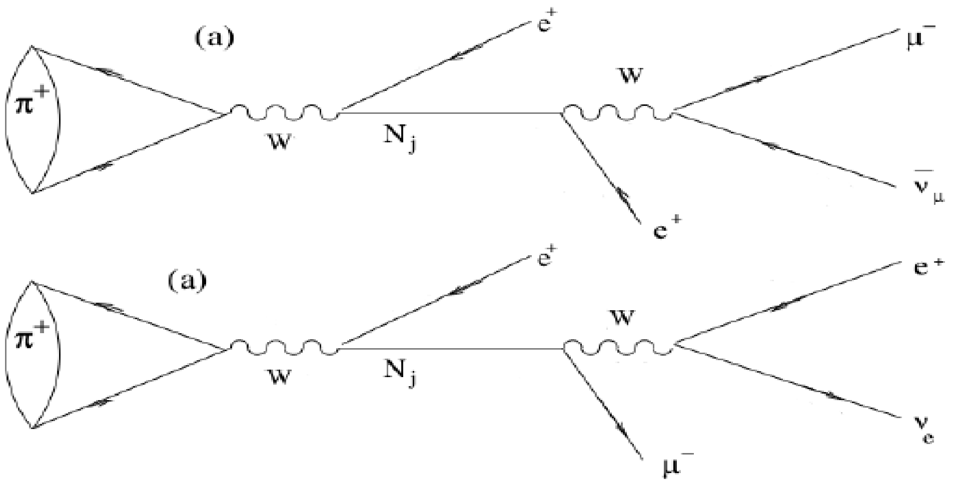
Looking for Majorana neutrinos via $\Delta L = 2$ processes (2)

(tau lepton decays & pion decays)

- **Process:**
  $\pi^{\pm} \to e^{\pm} N \to e^{\pm} e^{\pm} \mu^{\mp} \nu$

  G. Cvetič, C. S. Kim and J. Zamora-Saá,
  arXiv:1311.7554 [hep-ph]
  (J. Phys. G **41**, 075004 (2014))

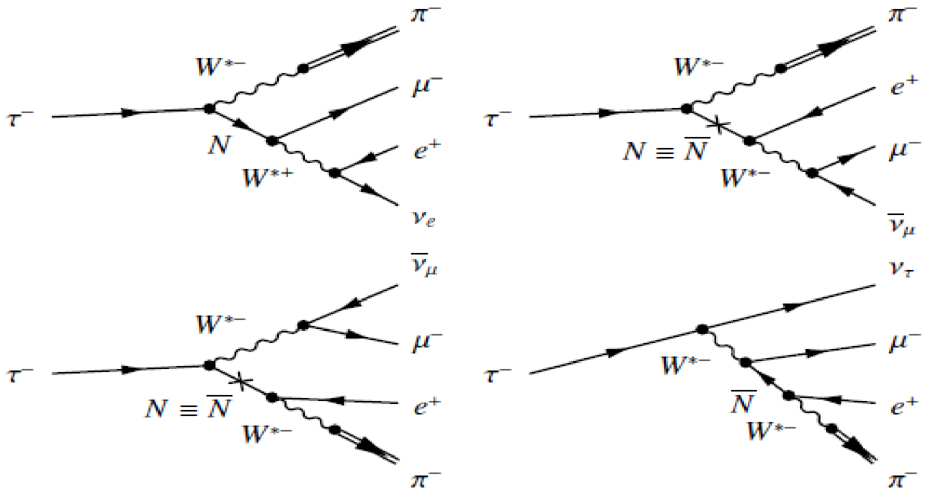
- **Mass range:**
  106 MeV $\leqslant m_N \leqslant$ 139 MeV



- **Process:** $\tau^- \to \pi^- \mu^- e^+ \nu/\overline{\nu}$

  C.S. Kim, G. L. Castro and D. Sahoo,
  arXiv:1708.00802 [hep-ph]
  (PRD **96**, 075016 (2017))

- **Mass range:**
  106 MeV $\leqslant m_N \leqslant$ 1637 MeV.

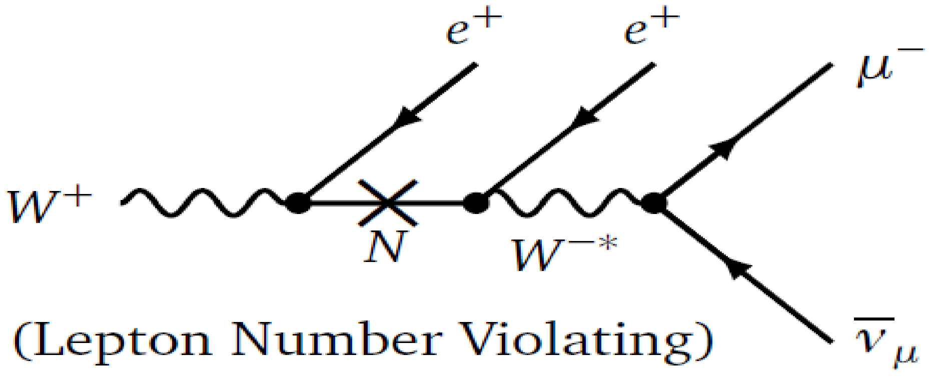


Looking for Majorana neutrinos via $\Delta L = 2$ processes (3)

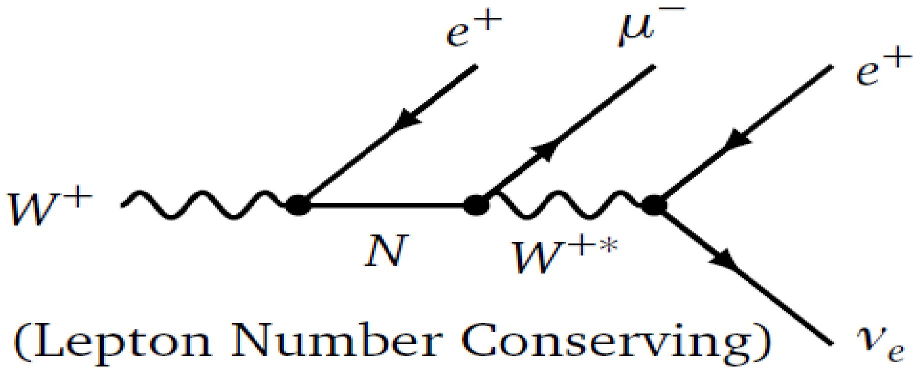
(Collider searches at LHC)

- **Processes:** $W^+ \to e^+e^+\mu^-\overline{\nu}_\mu$, $W^+ \to \mu^+\mu^+e^-\overline{\nu}_e$. Involves heavy neutrino N which can have Majorana nature as well.

  C. Dib, C.S. Kim, arXiv:1509.05981 (PRD **92**, 093009, 2015);
  C. Dib, C.S. Kim, K. Wang, J. Zhang, arXiv:1605.01123 (PRD **94**, 013005, 2016)



(Lepton Number Violating)



(Lepton Number Conserving)

- **Decay widths:**
  - LNV: $\Gamma\left(W^+ \to e^+e^+\mu^-\overline{\nu}_\mu\right) = \left|U_{Ne}\right|^4 \hat{\Gamma}$,
  - LNC: $\Gamma\left(W^+ \to e^+e^+\mu^-\overline{\nu}_\mu\right) = \left|U_{Ne}U_{N\mu}\right|^2 \hat{\Gamma}$,

  where $\hat{\Gamma} = \dfrac{G_F^3 M_W^3}{12 \times 96\sqrt{2}\pi^4} \dfrac{m_N^5}{\Gamma_N} \left(1 - \dfrac{m_N^2}{M_W^2}\right)^2 \left(1 - \dfrac{m_N^2}{2M_W^2}\right)$.

Looking for eV-scale sterile neutrino, not via Oscillation

(compared to LSND, miniBoone, …. searches light neutrino via neutrino Oscillation)

- If an eV scale sterile neutrino is present, its mixing with active flavor neutrinos would affect,
  1. muon decay $\rightarrow$ extraction of Fermi constant,
  2. leptonic decays of tau $\rightarrow$ testing unitarity of neutrino mixing matrix,
  3. semi-leptonic decays of tau & leptonic decays of pion and kaon $\rightarrow$ additional tests of unitarity of neutrino mixing matrix,
  4. invisible width of the Z boson & number of light active neutrinos, $\rightarrow$ extract individual active-sterile mixing parameters.

- Our analysis, taking precision measurements into account, supports the hypothesis that there are no such light sterile neutrinos.

C. S. Kim, G. L. Castro and D. Sahoo,
arXiv:1809.02265 [hep-ph]
(PRD **98** 11, 115021 (2018))

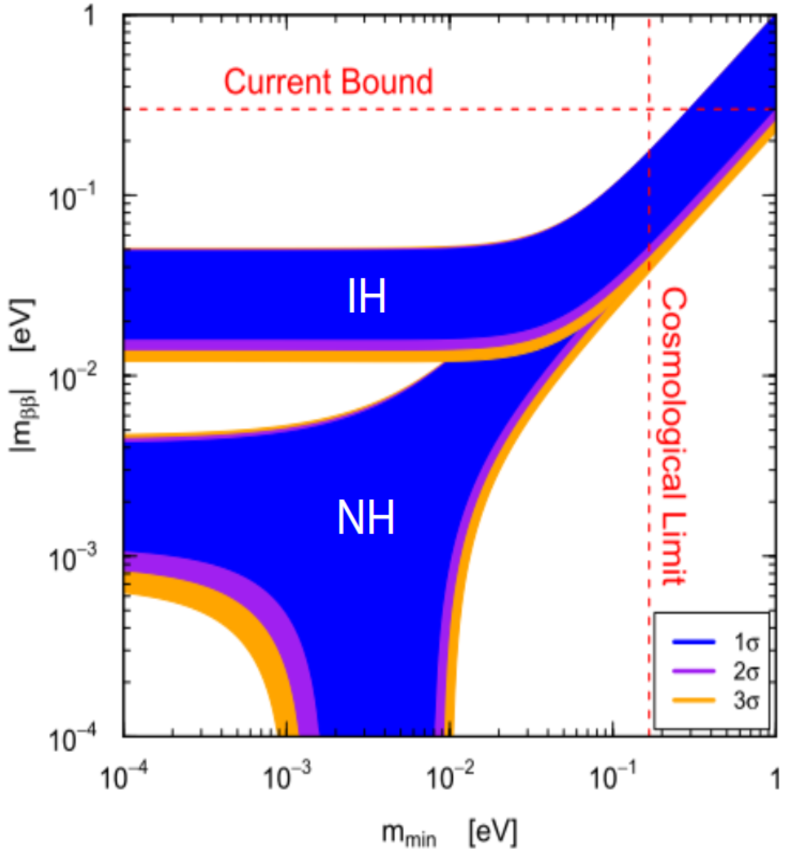
PRELUDE

Neutrino Casimir Force

Fermi-Dirac Statistics for Fermion (nu)

practical Dirac-Majorana Confusion Theorem (DMCT)

Neutrino-less Double Beta Decay 0nuBB ($\Delta L = 2$ process)



NH: Normal hierarchy
IH: Inverted hierarchy

S. M. Bilenky and C. Giunti
Mod. Phys. Lett. A **27**, 1230015 (2012),
arXiv:1203.5250

Lepton Number Violation (LNV)
➔ not allowed within SM

The half-life of a nucleus decaying via $0\nu\beta\beta$ is,

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G_{0\nu} \; |M_{0\nu}| \; |m_{\beta\beta}|^2$$

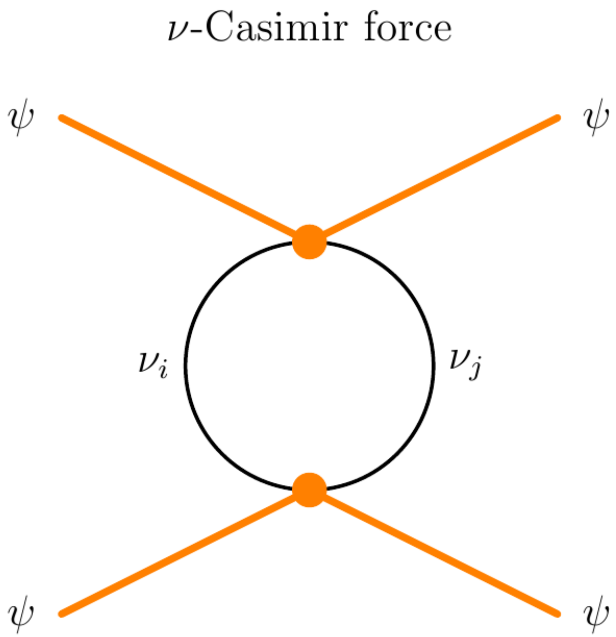
Possibility of very small mass($m_{\nu_e} \sim m_{\beta\beta}$)
May fail to observe !!

- If $m_{\beta\beta} < 10^{-2}$, only NH is viable and the $T_{1/2}^{0\nu}$ will be much larger than the current experimental lower bound.

Alternative to 0nuBB (1) – Neutrino Casimir force

Principle: Exchange of pair of neutrinos can give rise to long-range quantum force (aka neutrino Casimir force or the neutrino exchange force) between macroscopic objects, and the effective potential can differentiate Dirac and Majorana neutrinos.

G Feinberg, J Sucher, PRD166(1968)
X Xu, B Yu, 2112.03060

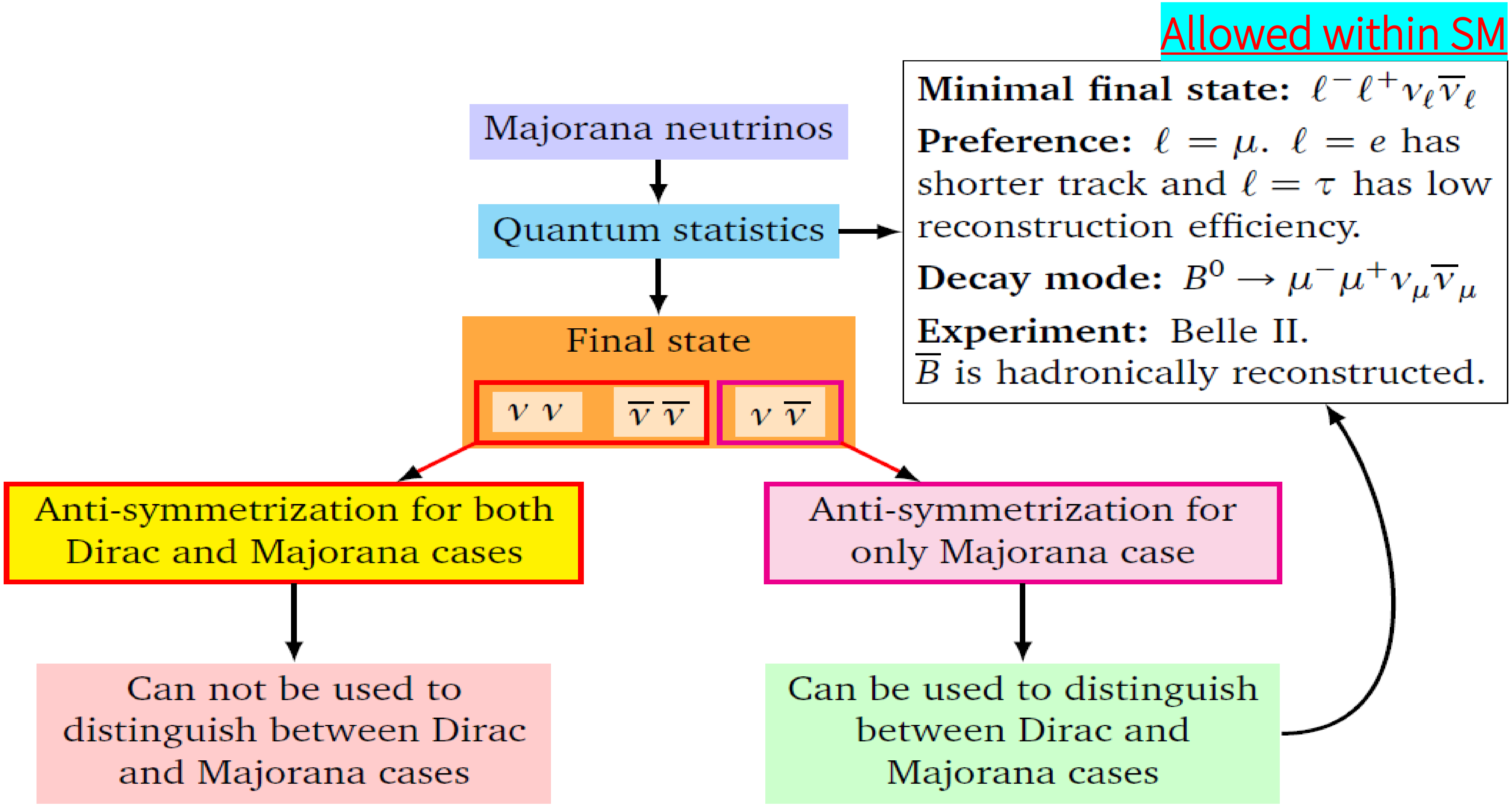


Issue: The potential (and hence the force) is proportional to product of the tiny neutrino masses in the loop.

** thermal fluctuation, van der Waals force

Status: Experimental study is still awaited.

Alternative to 0nuBB (2) – Quantum Statistics



practical Dirac-Majorana Confusion Theorem (1)

Consider the SM allowed decay, e.g. $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$

Amplitude for Dirac case $\mathscr{M}^D = \mathscr{M}(p_1, p_2),$

For Majorana case $\mathscr{M}^M = \frac{1}{\sqrt{2}}\left(\mathscr{M}(p_1, p_2) - \mathscr{M}(p_2, p_1)\right).$

required to know 4-momenta of p_1 and p_2, to be useful

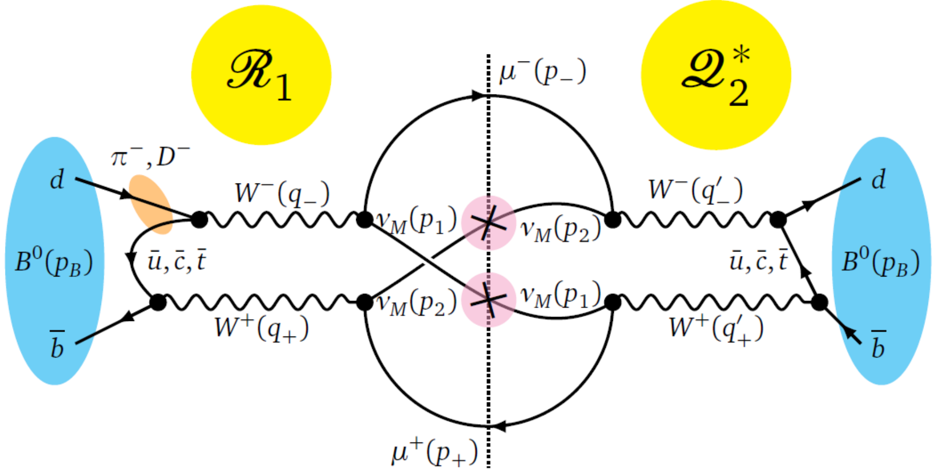
Difference between D and M

$$|\mathscr{M}^D|^2 - |\mathscr{M}^M|^2 = \frac{1}{2}\Big(\underbrace{|\mathscr{M}(p_1, p_2)|^2}_{\text{Direct term}} - \underbrace{|\mathscr{M}(p_2, p_1)|^2}_{\text{Exchange term}}\Big) + \underbrace{\mathrm{Re}\left(\mathscr{M}(p_1, p_2)^* \mathscr{M}(p_2, p_1)\right)}_{\text{Interference term}}.$$

Dirac-Majorana Confusion Theorem (2)

Interference term

$$\mathrm{Re}\left(\mathcal{M}(p_1,p_2)^*\,\mathcal{M}(p_2,p_1)\right) \propto m_\nu^2.$$



In general
(useful if p_1 and/or p_2 are known)

$$\underbrace{|\mathcal{M}(p_1,p_2)|^2}_{\text{Direct term}} \neq \underbrace{|\mathcal{M}(p_2,p_1)|^2}_{\text{Exchange term}}.$$

(= , only for very special BSM case)

However, after integration
(required if momenta p_1 and p_2 are unobservable)

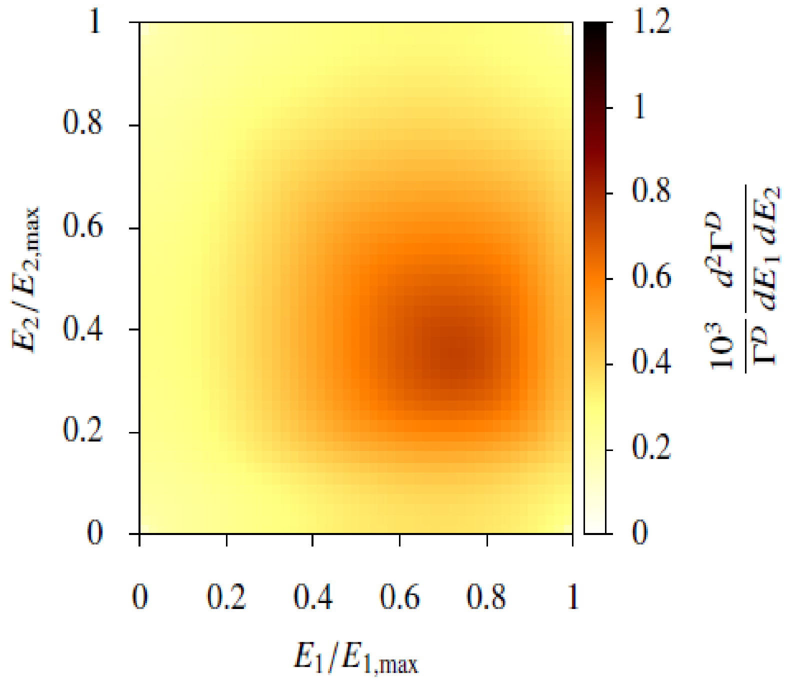
$$\iint \underbrace{|\mathcal{M}(p_1,p_2)|^2}_{\text{Direct term}} \,\mathrm{d}^4p_1\,\mathrm{d}^4p_2 = \iint \underbrace{|\mathcal{M}(p_2,p_1)|^2}_{\text{Exchange term}} \,\mathrm{d}^4p_1\,\mathrm{d}^4p_2,$$

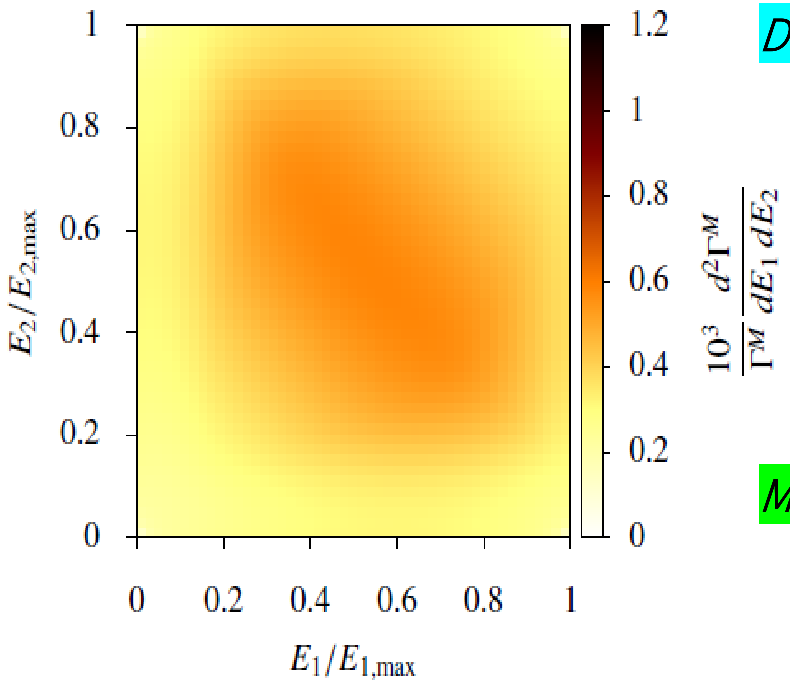
Different distribution, but the same total rate (DMCT)

$$\mathscr{M}^D = \mathscr{M}(p_1, p_2), \qquad \mathscr{M}^M = \frac{1}{\sqrt{2}}\left(\mathscr{M}(p_1, p_2) - \mathscr{M}(p_2, p_1)\right).$$

Dirac case

Majorana case





D=in general, no reason to be symmetric

M=always, must be symmetric

$$\underbrace{|\mathscr{M}(p_1, p_2)|^2}_{\text{Direct term}} \neq \underbrace{|\mathscr{M}(p_2, p_1)|^2}_{\text{Exchange term}}.$$

$$\iint \left(|\mathscr{M}^D|^2 - |\mathscr{M}^M|^2\right) d^4p_1\, d^4p_2 \quad \propto m_\nu^2.$$

Dirac-Majorana Confusion Theorem (3)

Therefore,
(if momenta p_1 and p_2 are unobservable)

$$\iint \left(|\mathscr{M}^D|^2 - |\mathscr{M}^M|^2 \right) d^4p_1\, d^4p_2$$

$$= 2 \iint \underbrace{\mathrm{Re}\left(\mathscr{M}(p_1, p_2)^* \mathscr{M}(p_2, p_1) \right)}_{\text{Interference term}} d^4p_1\, d^4p_2$$

In general,

$$\propto m_\nu^2.$$

Practical Dirac-Majorana confusion theorem: By looking at the total decay rate or any other kinematic test of a process allowed in the SM, it is practically impossible to distinguish between the Dirac and Majorana neutrinos in the limit neutrino mass goes to zero.

Weak neutral current process in SM

B. Kayser, Phys. Rev. D **26**, 1662 (1982).

No general proof independent of process or observable

History trying to overcome DMCT, but only confirming

All for weak neutral current process in SM

$$\gamma^* \to \nu\bar{\nu}$$ [B Kayser, PRD26(1982)]

$$Z \to \nu\bar{\nu}$$ [RE Shrock, eConf(1982)]

$$e^+e^- \to \nu\bar{\nu}$$ [E Ma, JT Pantaleone, PRD40(1989)]

$$K^+ \to \pi^+\nu\bar{\nu}$$ [JF Nieves, PB Pal, PRD32(1985)] **

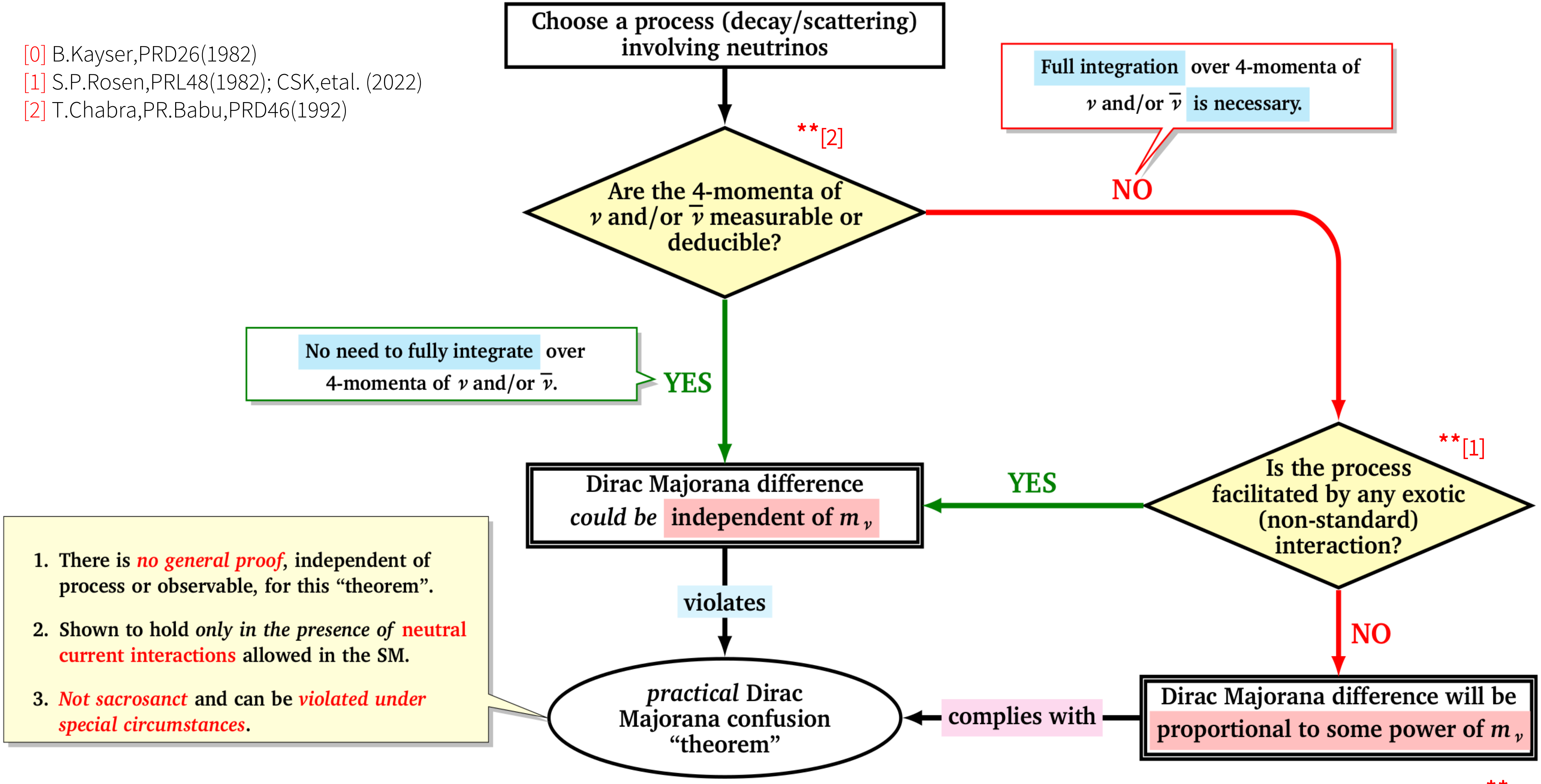
$$e^+e^- \to \nu\bar{\nu}\gamma$$ [T Chabra, PR Babu, PRD46(1992)] **

$$|es> \to |gs> +\gamma\nu\bar{\nu}$$ [Y Yoshimura, PRD75(2007)], ….

$$e^-\gamma \to e^-\nu\bar{\nu}$$ [JM Berryman etal, PRD98(2018)]

** All practically impossible to measure momenta of nu-nubar ➔ Need integrate out ➔ pDMCT

[0] B.Kayser,PRD26(1982)
[1] S.P.Rosen,PRL48(1982); CSK,etal. (2022)
[2] T.Chabra,PR.Babu,PRD46(1992)



another Comment on pDMCT

** Is there smooth transition between Majorana to Dirac neutrinos under m → 0 limit ??

(a) When m = 0 both Dirac and Majorana neutrinos can be described as Weyl fermions. The reduction of neutrino degrees of freedom from 4 to 2 for m = 0 is a discrete jump, and not a continuous change. So the massless neutrino is an entirely different species than a massive one even with extremely tiny mass.

(b) Dirac neutrino and antineutrino are fully distinguishable, while Majorana neutrino and antineutrino are quantum mechanically indistinguishable. There is no smooth limit that takes indistinguishable particles and makes them distinguishable. There is no intermediate state between distinguishable and indistinguishable particles.

(c) Majorana neutrino and antineutrino pair have to obey Fermi-Dirac statistics while Dirac neutrino and antineutrino pair do not. We emphasize that statistics of particles does not depend on a parameter like mass.

BACK-TO-BACK muons (ie. B2B $\nu - \bar{\nu}$) - experimentally observable exception to DMCT

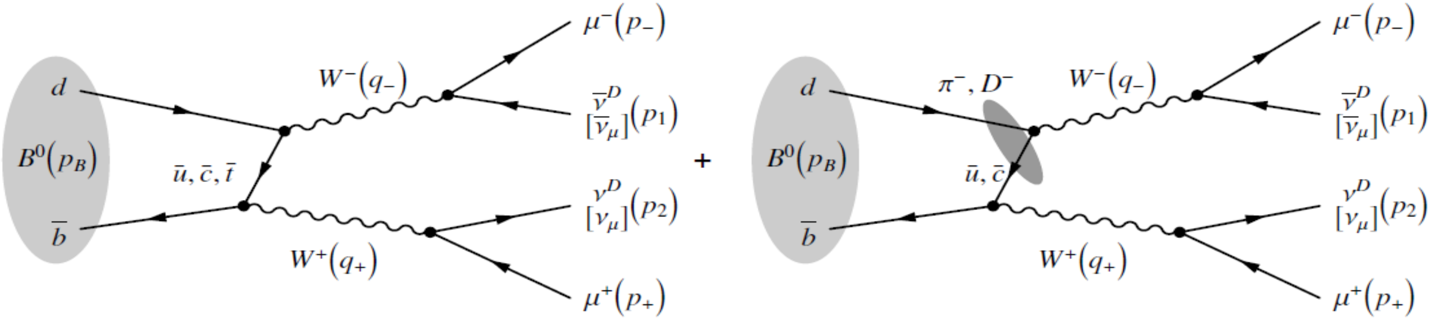
Thought experiment, exception to DMCT

Back-to-back muons (ie. B2B $\nu - \bar{\nu}$)

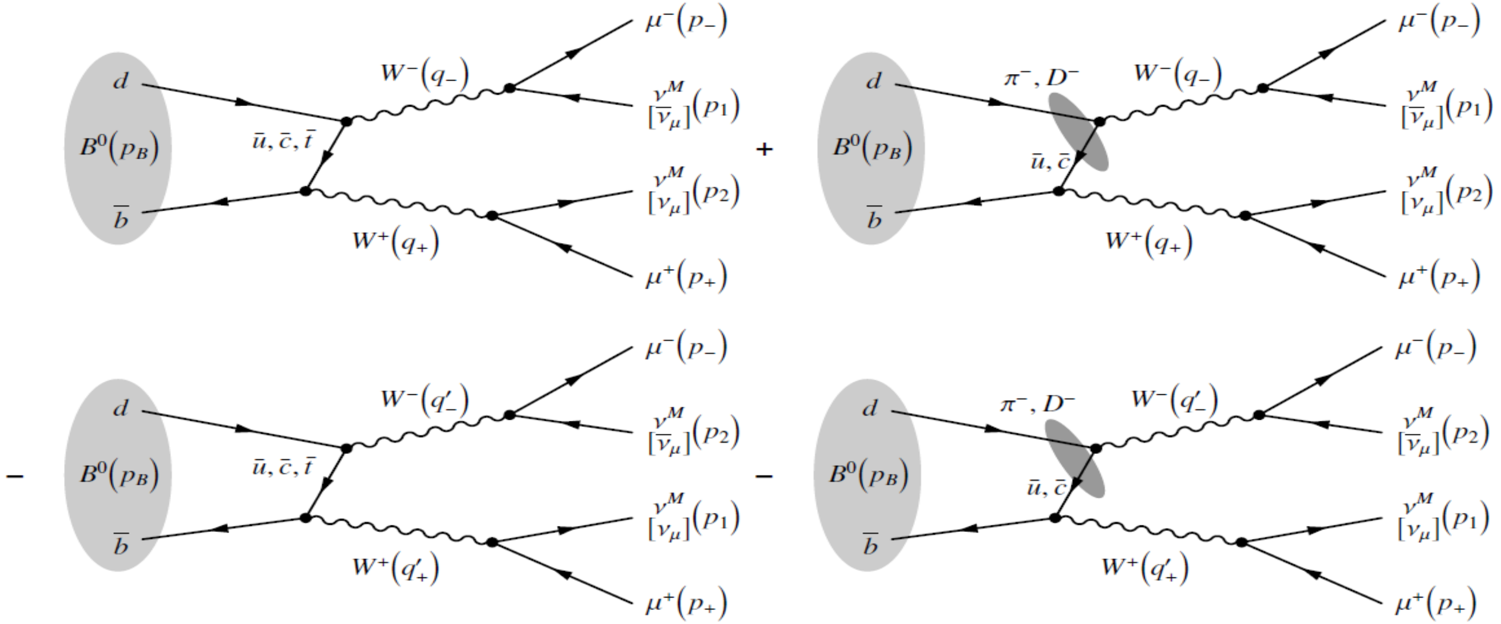
Helicity consideration

Study on $B^0(p_B) \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2)$,

Doubly weak charged current process in SM



(a) For Dirac neutrinos: $\nu_\mu \equiv \nu^D$, $\bar{\nu}_\mu \equiv \bar{\nu}^D$.



(b) For Majorana neutrinos: $\nu_\mu = \bar{\nu}_\mu \equiv \nu^M$.

Thought experiment, exception to DMCT

CONSIDER the SM allowed decay $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$,

At a special case when nu and nu-bar are collinear, $p_1 = p_2$.

For Majorana case (Anti-symmetrization of nu-nubar) : $\mathscr{M}^M_{\text{collinear}} = 0$

For Dirac Case : in general $\mathscr{M}^D_{\text{collinear}} \neq 0.$

$$\left\langle \left|\mathscr{M}^D_{\text{collinear}}\right|^2 \right\rangle = 64\, G_F^4\, |\mathbb{F}_a|^2\, (p_\nu \cdot p_+)\, (p_\nu \cdot p_-)\,, \quad \text{not} \propto m_\nu^2.$$

THEREFORE, in collinear case $\iint \left(|\mathscr{M}^D|^2 - |\mathscr{M}^M|^2\right) \mathrm{d}^4 p_1\, \mathrm{d}^4 p_2$ not $\propto m_\nu^2.$

Back-to-back muons, (easily measurable exception to DMCT)

$$B^0(p_B) \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$$

In the rest frame of parent B meson,
IF muon- and muon+ are back-to back, ie. flying with 3 momenta of equal magnitude but opposite direction
➔ nu and nu-bar also back-to-back

$$E_1 = E_2 = E_\nu = m_B/2 - E_\mu$$

$$m_{\nu\nu}^2 = 4E_\nu^2$$

$$m_{\mu\mu}^2 = (m_B - 2E_\nu)^2$$

$$Y_m = \sqrt{(m_B/2 - E_\nu)^2 - m_\mu^2}$$

$$Y_n = \sqrt{E_\nu^2 - m_\nu^2}$$

➔ All kinematic variables are calculable or measurable ,

Only the angle (between $\nu - \bar{\nu}$ and $\mu_- - \mu_+$) UNKNOWN

$$\frac{d\Gamma}{dE_\mu^2 d\sin\theta} \to \frac{d\Gamma}{dE_\nu^2 d\sin\theta}$$

Need not integrate out nu-nubar full phase space,
only unmeasurable angle (θ) integrate out ➔ Overcoming DMCT constraint

** Helicity & Chirality

and the *smaller components* are

$$\frac{m_\nu}{2E_\nu} \approx 0$$

$$\frac{m_\mu}{2E_\mu} \sim 0.04$$

$$u^{(+)}_{C,L}(p) = +\frac{m}{2E}\,\chi^{(+)}(\vec{p}),$$
$$u^{(-)}_{C,R}(p) = -\frac{m}{2E}\,\chi^{(-)}(\vec{p}),$$
$$v^{(+)}_{C,R}(p) = -\frac{m}{2E}\,\chi^{(-)}(\vec{p}),$$
$$v^{(-)}_{C,L}(p) = +\frac{m}{2E}\,\chi^{(+)}(\vec{p}).$$

In simple terms, these equations state that for a fermionic particle in ultra-relativistic case:

(i) positive helicity state is mostly right-handed, and

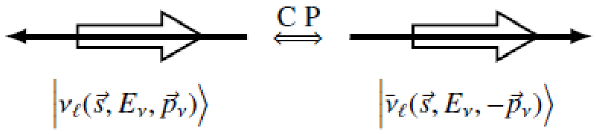
(ii) negative helicity state is mostly left-handed.

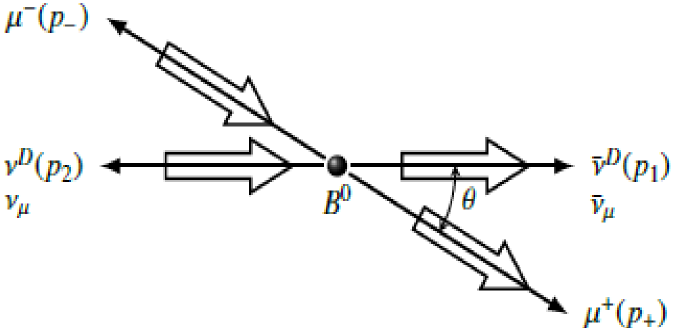
Similarly, for a fermionic anti-particle in ultra-relativistic case:

(i) positive helicity state is mostly left-handed, and

(ii) negative helicity state is mostly right-handed.

$$\mathrm{C\,P}\,|\nu_\ell(\vec{s}, E_\nu, \vec{p}_\nu)\rangle = \eta_P\,|\bar{\nu}_\ell(\vec{s}, E_\nu, -\vec{p}_\nu)\rangle\,,$$

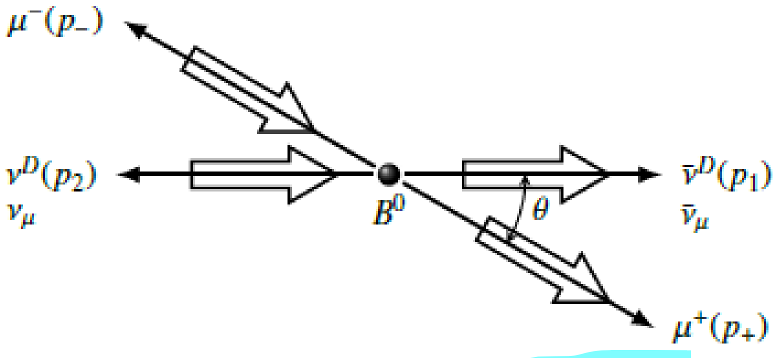




$$B^0(p_B) \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$$

** Compare to $\pi^\pm \to \mu^\pm \nu$
to test parity violation & CP conservation

Helicity Configuration of back-to-back muons in rest frame of B in decay $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, (1)

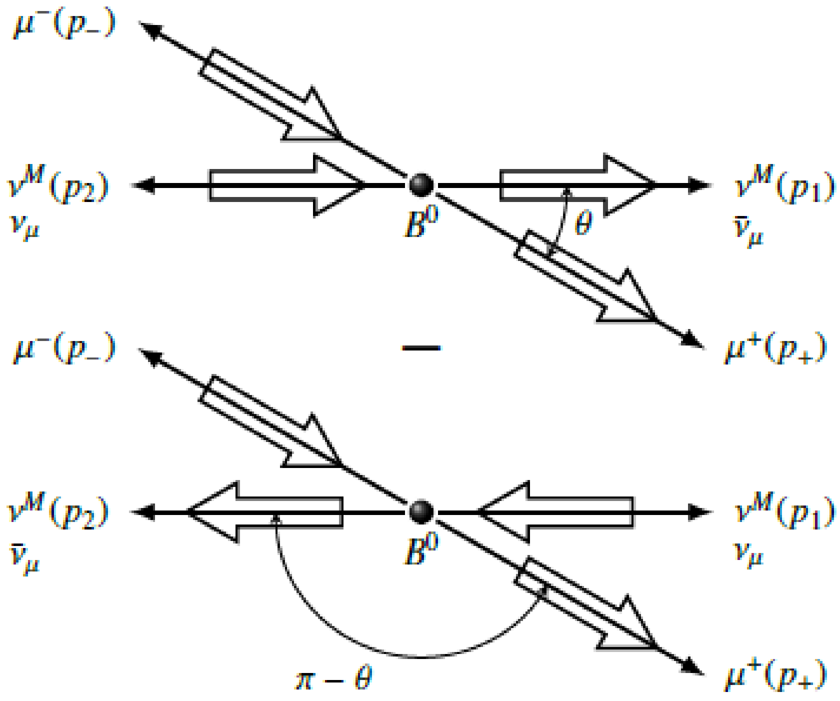


(a) Helicity configuration involving Dirac neutrinos, $\nu_\mu \equiv \nu^D$, $\bar{\nu}_\mu \equiv \bar{\nu}^D$.

$$\left|\mathcal{M}^D_{\leftrightarrow}\right|^2 \propto \underbrace{\left(1-\cos\theta\right)^2}_{\text{Direct term}}.$$

$$\left|\mathcal{M}^M_{\leftrightarrow}\right|^2 \propto \frac{1}{2}\Big[\underbrace{\left(1-\cos\theta\right)^2}_{\text{Direct term}} + \underbrace{\left(1-\cos(\pi-\theta)\right)^2}_{\text{Exchange term}} - \underbrace{O\left(m_\nu^2\right)}_{\text{Interference term}}\Big]$$

$$\simeq 1+\cos^2\theta.$$



(b) Helicity configuration involving Majorana neutrinos, $\nu_\mu = \bar{\nu}_\mu \equiv \nu^M$.

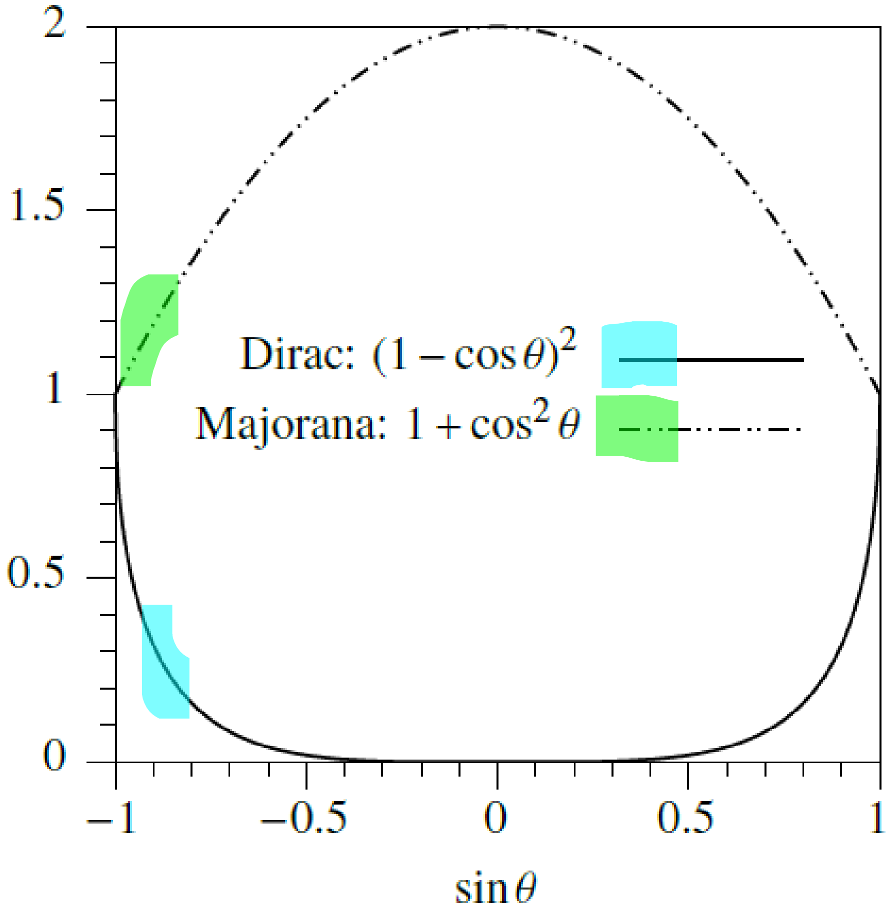
The antisymmetrization for Majorana case gives the exchange term (via $p_1 \leftrightarrow p_2$ exchange) and is not associated with any helicity flip, as shown

Helicity Configuration of back-to-back muons in rest frame of B in decay $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, (2)

$$\left|\mathcal{M}^D_{\leftrightarrow}\right|^2 \propto \underbrace{(1-\cos\theta)^2}_{\text{Direct term}}.$$

$$\left|\mathcal{M}^M_{\leftrightarrow}\right|^2 \propto \frac{1}{2}\Big[\underbrace{(1-\cos\theta)^2}_{\text{Direct term}} + \underbrace{(1-\cos(\pi-\theta))^2}_{\text{Exchange term}} - \underbrace{O(m_\nu^2)}_{\text{Interference term}}\Big]$$

$$\simeq 1+\cos^2\theta.$$



** Presently nu nu-bar totally missing, the angle θ is completely unknown, therefore, need to integrate out. ➔ BR(M) >> BR(D)

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons

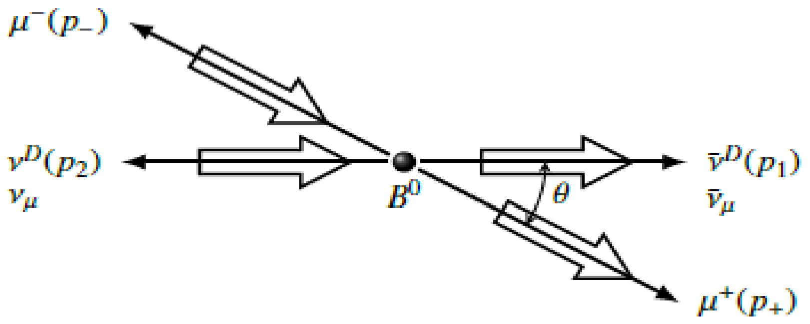
(Comments on Measurement of angular distribution)

θ = angle between (mu mu - nu nu) ➔ angle between (μ^+ and $\bar{\nu}$)

** Presently nu nu-bar totally missing

With futuristic detector (ie. **neutrino near detector**), ν from μ^- and $\bar{\nu}$ from μ^+ can be detected !

(i) Assuming Dirac neutrino, $\nu \neq \bar{\nu}$, the angle θ uniquely decided



(a) Helicity configuration involving Dirac neutrinos, $\nu_\mu \equiv \nu^D$, $\bar{\nu}_\mu \equiv \bar{\nu}^D$.

$$|\mathscr{M}^D_{\leftrightarrow}|^2 \propto \underbrace{(1 - \cos\theta)^2}_{\text{Direct term}} .$$

(ii) Assuming Majorana neutrino, $\nu \equiv \bar{\nu}$, the angle not uniquely decided ➔ need symmetrized

$$|\mathscr{M}^M_{\leftrightarrow}|^2 \propto \frac{1}{2}\Big[\underbrace{(1 - \cos\theta)^2}_{\text{Direct term}} + \underbrace{(1 - \cos(\pi - \theta))^2}_{\text{Exchange term}} - \underbrace{O(m_\nu^2)}_{\text{Interference term}}\Big] \simeq 1 + \cos^2\theta.$$

DETAILED ANALYSIS & DISCUSSION ON B2B MUONS

[CSK,MM,DS, arXiv://2106.11785 (PRD, in press)]

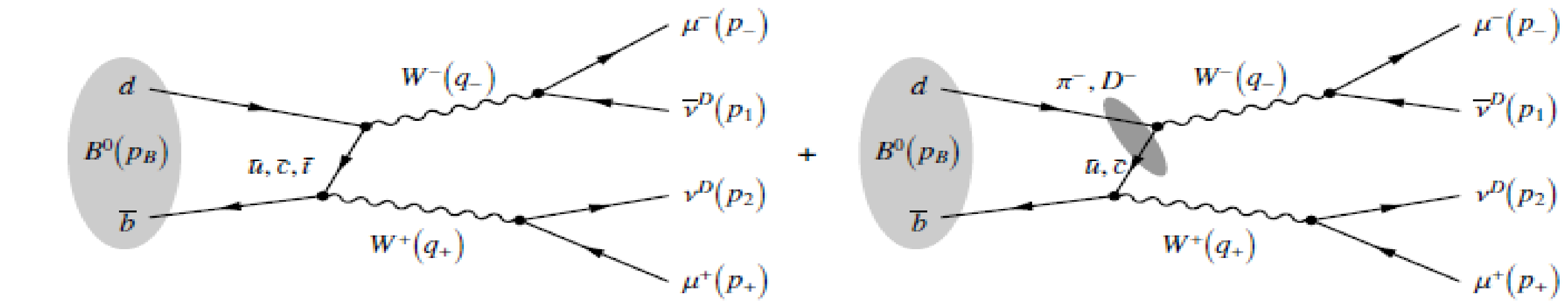
Detailed study of $B^0(p_B) \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$

Detailed study of $B^0(p_B) \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$ w/ B2B muons

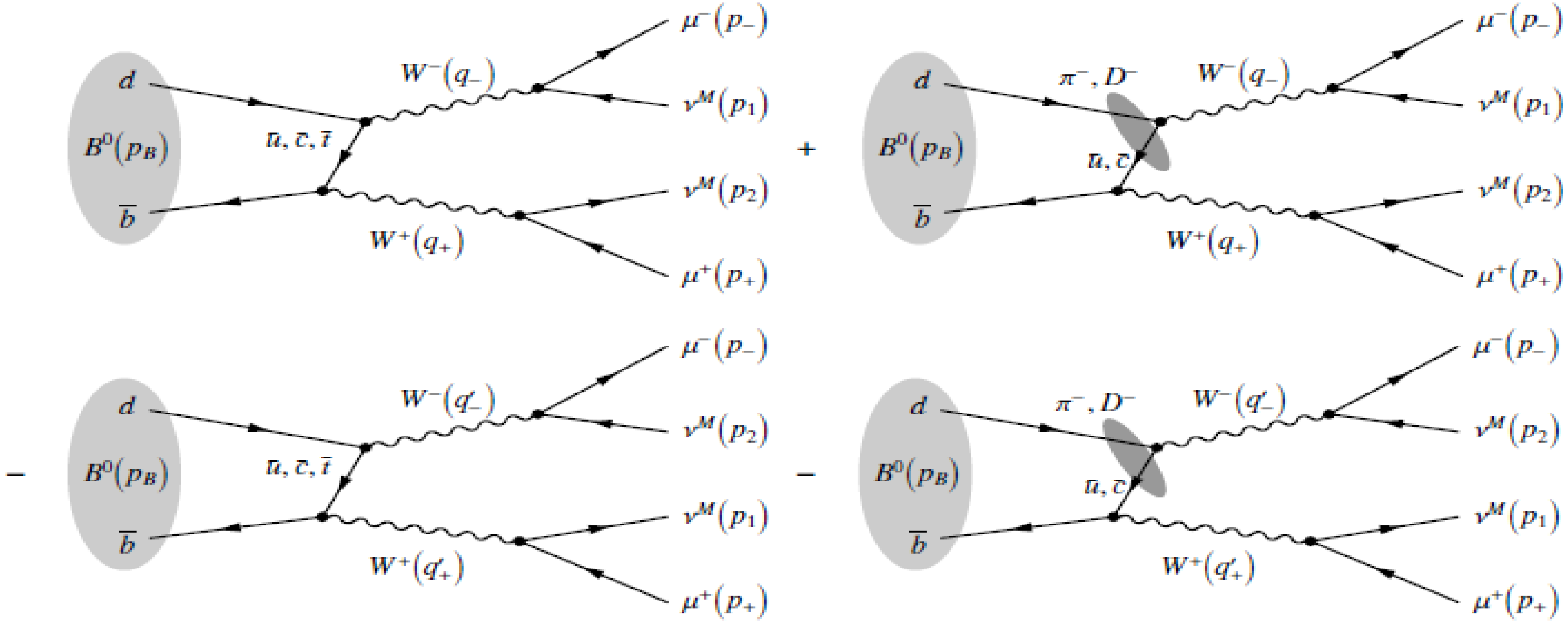
Discussions on $B^0(p_B) \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$ w/ B2B muons

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, (1)

Doubly weak charged current process in SM



(a) For Dirac neutrinos: $\nu_\mu \equiv \nu^D$, $\bar{\nu}_\mu \equiv \bar{\nu}^D$.



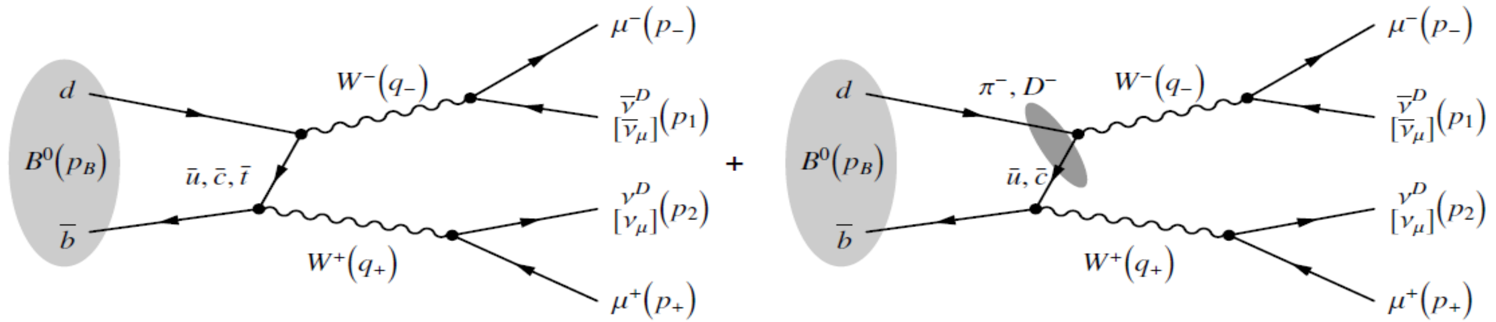
(b) For Majorana neutrinos: $\nu_\mu = \bar{\nu}_\mu \equiv \nu^M$.

$$\mathscr{M}^D = \frac{G_F^2}{2} H^{\alpha\beta} L_{\alpha\beta} \equiv \mathscr{Q}_{12} + \mathscr{R}_{12},$$

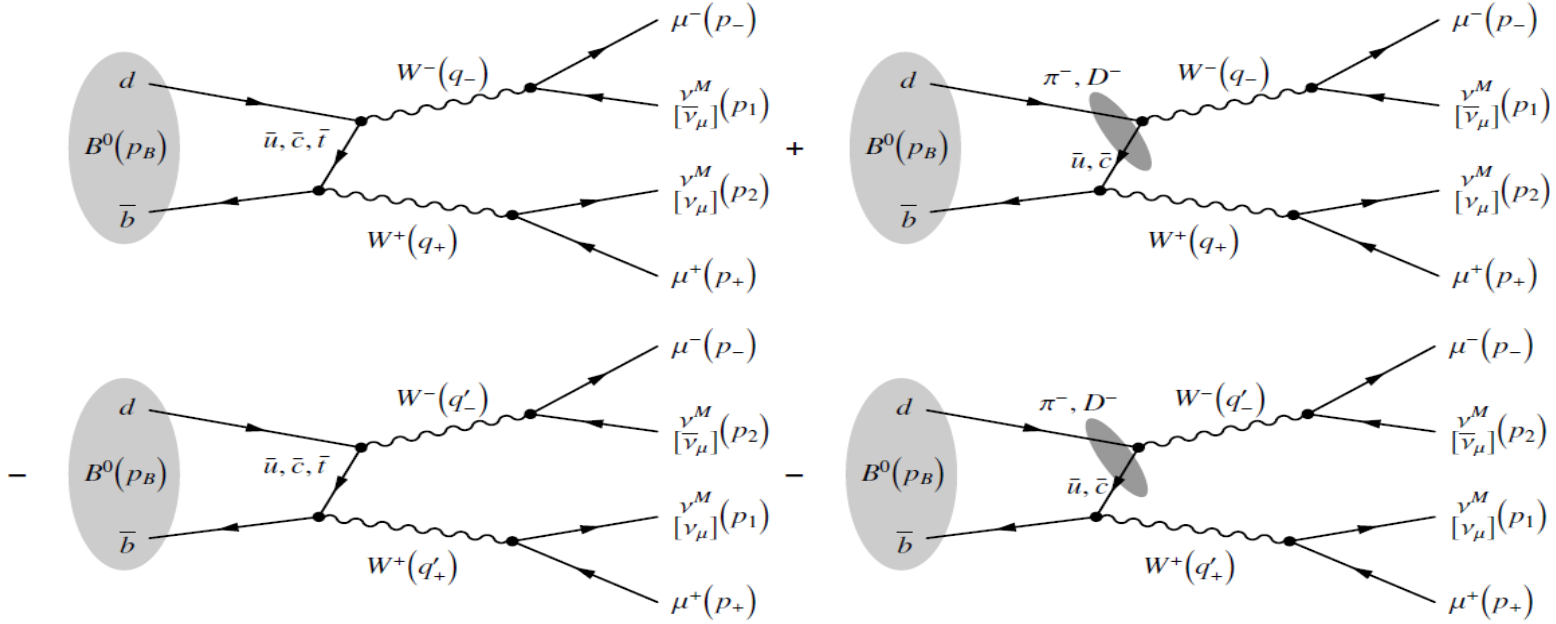
$$\mathscr{M}^M = \frac{G_F^2}{2\sqrt{2}} \left(H^{\alpha\beta} L_{\alpha\beta} - H'^{\alpha\beta} L'_{\alpha\beta} \right) \equiv \frac{1}{\sqrt{2}} \left(\mathscr{Q}_{12} - \mathscr{Q}_{21} + \mathscr{R}_{12} - \mathscr{R}_{21} \right),$$

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, (1)

Doubly weak charged current process in SM



(a) For Dirac neutrinos: $\nu_\mu \equiv \nu^D$, $\bar{\nu}_\mu \equiv \bar{\nu}^D$.



(b) For Majorana neutrinos: $\nu_\mu = \bar{\nu}_\mu \equiv \nu^M$.

$$\mathscr{M}^D = \frac{G_F^2}{2} H^{\alpha\beta} L_{\alpha\beta} \equiv \mathscr{Q}_{12} + \mathscr{R}_{12},$$

$$\mathscr{M}^M = \frac{G_F^2}{2\sqrt{2}} \left(H^{\alpha\beta} L_{\alpha\beta} - H'^{\alpha\beta} L'_{\alpha\beta} \right)$$

$$\equiv \frac{1}{\sqrt{2}} \left(\mathscr{Q}_{12} - \mathscr{Q}_{21} + \mathscr{R}_{12} - \mathscr{R}_{21} \right),$$

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, (2)

$$L_{\alpha\beta} = \left[\bar{u}(p_-)\gamma_\alpha\left(1-\gamma^5\right)v(p_1)\right]\left[\bar{u}(p_2)\gamma_\beta\left(1-\gamma^5\right)v(p_+)\right] \qquad L'_{\alpha\beta} = \left[\bar{u}(p_-)\gamma_\alpha\left(1-\gamma^5\right)v(p_2)\right]\left[\bar{u}(p_1)\gamma_\beta\left(1-\gamma^5\right)v(p_+)\right]$$

$$\mathcal{R}_{12} = \frac{G_F^2}{2}\mathbf{H}^{\alpha\beta}L_{\alpha\beta}, \quad \mathbf{H}^{\alpha\beta} \equiv V_{ub}^*V_{ud}\mathbf{V}_\pi^{\alpha\beta} + V_{cb}^*V_{cd}\mathbf{V}_D^{\alpha\beta} = \left(\mathbf{F}_+q_+^\beta + \mathbf{F}_-q_-^\beta\right)q_-^\alpha, \qquad \mathbf{V}_R^{(\prime)\alpha\beta} = \frac{f_R}{q_-^{(\prime)2} - m_R^2 + im_R\Gamma_R}q_-^{(\prime)\alpha}\left(F_{R+}^{(\prime)}q_+^{(\prime)\beta} + F_{R-}^{(\prime)}q_-^{(\prime)\beta}\right),$$

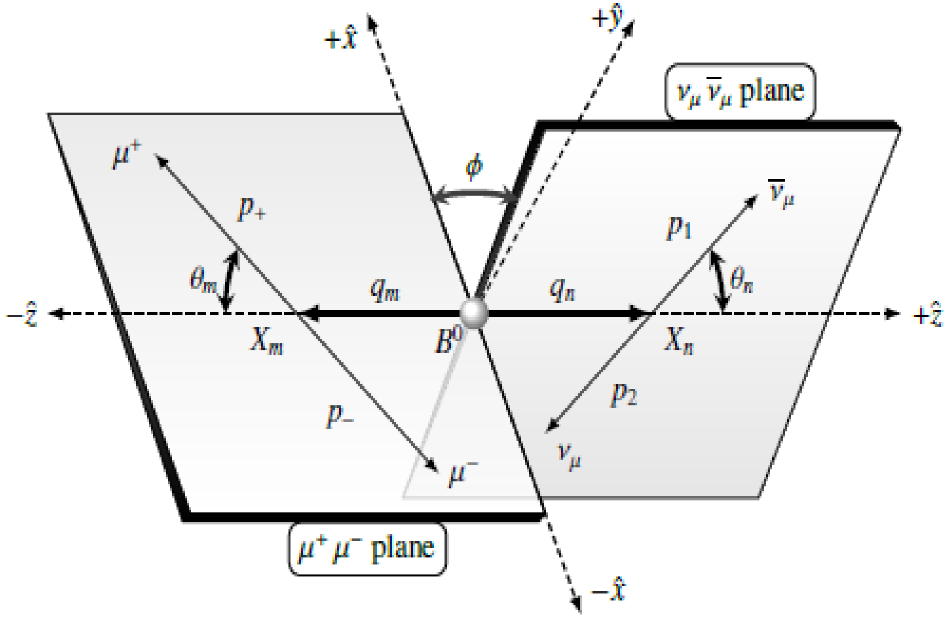
$$\mathcal{R}_{21} \equiv \frac{G_F^2}{2}\mathbf{H}'^{\alpha\beta}L'_{\alpha\beta}, \quad \mathbf{H}'^{\alpha\beta} \equiv V_{ub}^*V_{ud}\mathbf{V}_\pi'^{\alpha\beta} + V_{cb}^*V_{cd}\mathbf{V}_D'^{\alpha\beta} = \left(\mathbf{F}'_+q_+'^\beta + \mathbf{F}'_-q_-'^\beta\right)q_-'^\alpha,$$

$$\mathcal{Q}_{12} = \frac{G_F^2}{2}\left(\sum_{Q=u,c,t}V_{Qb}^*V_{Qd}\mathbb{V}_Q^{\alpha\beta}\right)L_{\alpha\beta} = \frac{G_F^2}{2}\mathbb{H}^{\alpha\beta}L_{\alpha\beta}, \qquad \mathbb{V}_Q^{(\prime)\alpha\beta} = F_a^{(\prime)Q}g^{\alpha\beta} + F_b^{(\prime)Q}p_B^\alpha p_B^\beta + i\,F_c^{(\prime)Q}\epsilon^{\alpha\beta\mu\nu}q_{+\mu}^{(\prime)}q_{-\nu}^{(\prime)},$$

$$\mathbb{H}^{\alpha\beta} = \mathbb{F}_a\,g^{\alpha\beta} + \mathbb{F}_b\,p_B^\alpha p_B^\beta + i\,\mathbb{F}_c\,\epsilon^{\alpha\beta\mu\nu}q_{+\mu}q_{-\nu}, \qquad \mathbb{F}_i \equiv \mathbb{F}_i\left(q_+^2, q_-^2\right) = \sum_{Q=u,c,t}V_{Qb}^*V_{Qd}\,F_i^Q\left(q_+^2, q_-^2\right),$$

$$\mathcal{Q}_{21} = \frac{G_F^2}{2}\mathbb{H}'^{\alpha\beta}L'_{\alpha\beta} \qquad \mathbb{H}'^{\alpha\beta} = \mathbb{F}'_a\,g^{\alpha\beta} + \mathbb{F}'_b\,p_B^\alpha p_B^\beta + i\,\mathbb{F}'_c\,\epsilon^{\alpha\beta\mu\nu}q'_{+\mu}q'_{-\nu},$$

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, (3)



$$\frac{d^5\Gamma^{D/M}}{dm_{\mu\mu}^2\, dm_{\nu\nu}^2\, d\cos\theta_m\, d\cos\theta_n\, d\phi} = \frac{Y\, Y_m\, Y_n \left\langle \left|\mathscr{M}^{D/M}\right|^2 \right\rangle}{(4\pi)^6\, m_B^2\, m_{\mu\mu}\, m_{\nu\nu}},$$

$$Y = \frac{\sqrt{\lambda\left(m_B^2, m_{\mu\mu}^2, m_{\nu\nu}^2\right)}}{2m_B},$$

$$Y_m = \frac{\sqrt{m_{\mu\mu}^2 - 4m_\mu^2}}{2},$$

$$Y_n = \frac{\sqrt{m_{\nu\nu}^2 - 4m_\nu^2}}{2},$$

After integrating out unobservable neutrino phase space

$$\frac{d^3\Gamma^M}{dm_{\mu\mu}^2\, dm_{\nu\nu}^2\, d\cos\theta_m} - \frac{d^3\Gamma^D}{dm_{\mu\mu}^2\, dm_{\nu\nu}^2\, d\cos\theta_m} \propto m_\nu^2,$$

(confirming pDMCT)

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2),$ (3-1)

$$\frac{d^3\Gamma^{D/M}}{dm_{\mu\mu}^2\, dm_{\nu\nu}^2\, d\cos\theta_m} = \frac{Y\, Y_m\, Y_n}{(4\pi)^6 m_B^2\, m_{\mu\mu}\, m_{\nu\nu}} \int_{-1}^{1}\int_0^{2\pi} \left\langle \left|\mathscr{M}^{D/M}\right|^2 \right\rangle d\cos\theta_n\, d\phi.$$

(in case neutrinos unobservable)

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, (3-2)

$$
\begin{aligned}
&\frac{d^3\Gamma^M}{dm_{\mu\mu}^2\, dm_{\nu\nu}^2\, d\cos\theta_m} - \frac{d^3\Gamma^D}{dm_{\mu\mu}^2\, dm_{\nu\nu}^2\, d\cos\theta_m} = \frac{G_F^4\, Y\, Y_m\, Y_n}{2\,(4\,\pi)^6\, m_B^2\, m_{\mu\mu}\, m_{\nu\nu}} \int_{-1}^{1}\int_0^{2\pi} d\cos\theta_n\, d\phi \\
&\times\Big(-|\mathbb{F}_a|^2 S^M_{aa} - |\mathbb{F}_b|^2 S^M_{bb} - |\mathbb{F}_c^2|\, S^M_{cc} - |\mathbf{F}_+|^2 S^M_{pp} - |\mathbf{F}_-|^2 S^M_{mm} + |\mathbb{F}'_a|^2 S^M_{a'a'} + |\mathbb{F}'_b|^2 S^M_{b'b'} + |\mathbb{F}'_c|^2 S^M_{c'c'} + |\mathbf{F}'_+|^2 S^M_{p'p'} + |\mathbf{F}'_-|^2 S^M_{m'm'} \\
&\quad - \mathrm{Re}\left(\mathbb{F}_a\mathbb{F}_b^*\right) R^M_{ab} - \mathrm{Re}\,(\mathbb{F}_a\mathbb{F}_c^*)\, R^M_{ac} - \mathrm{Re}\,(\mathbb{F}_a\mathbf{F}_+^*)\, R^M_{ap} - \mathrm{Re}\,(\mathbb{F}_a\mathbf{F}_-^*)\, R^M_{am} - \mathrm{Re}\,(\mathbb{F}_b\mathbb{F}_c^*)\, R^M_{bc} - \mathrm{Re}\,(\mathbb{F}_b\mathbf{F}_+^*)\, R^M_{bp} - \mathrm{Re}\,(\mathbb{F}_b\mathbf{F}_-^*)\, R^M_{bm} \\
&\quad - \mathrm{Re}\,(\mathbb{F}_c\mathbf{F}_-^*)\, R^M_{cm} + \mathrm{Re}\left(\mathbb{F}'_a\mathbb{F}'^*_b\right) R^M_{a'b'} + \mathrm{Re}\,(\mathbb{F}'_a\mathbb{F}'^*_c)\, R^M_{a'c'} + \mathrm{Re}\,(\mathbb{F}'_a\mathbf{F}'^*_+)\, R^M_{a'p'} + \mathrm{Re}\,(\mathbb{F}'_a\mathbf{F}'^*_-)\, R^M_{a'm'} + \mathrm{Re}\left(\mathbb{F}'_b\mathbb{F}'^*_c\right) R^M_{b'c'} \\
&\quad + \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}'^*_+\right) R^M_{b'p'} + \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}'^*_-\right) R^M_{b'm'} + \mathrm{Re}\,(\mathbb{F}'_c\mathbf{F}'^*_-)\, R^M_{c'm'} - \mathrm{Re}\,(\mathbf{F}_+\mathbf{F}_-^*)\, R^M_{pm} + \mathrm{Re}\,(\mathbf{F}'_+\mathbf{F}'^*_-)\, R^M_{p'm'} \\
&\quad - \mathrm{Im}\left(\mathbb{F}_a\mathbb{F}_b^*\right) I^M_{ab} - \mathrm{Im}\,(\mathbb{F}_a\mathbb{F}_c^*)\, I^M_{ac} - \mathrm{Im}\,(\mathbb{F}_a\mathbf{F}_-^*)\, I^M_{am} - \mathrm{Im}\,(\mathbb{F}_b\mathbb{F}_c^*)\, I^M_{bc} - \mathrm{Im}\,(\mathbb{F}_c\mathbf{F}_+^*)\, I^M_{cp} - \mathrm{Im}\,(\mathbb{F}_c\mathbf{F}_-^*)\, I^M_{cm} \\
&\quad + \mathrm{Im}\left(\mathbb{F}'_a\mathbb{F}'^*_b\right) I^M_{a'b'} + \mathrm{Im}\,(\mathbb{F}'_a\mathbb{F}'^*_c)\, I^M_{a'c'} + \mathrm{Im}\,(\mathbb{F}'_a\mathbf{F}'^*_-)\, I^M_{a'm'} + \mathrm{Im}\left(\mathbb{F}'_b\mathbb{F}'^*_c\right) I^M_{b'c'} + \mathrm{Im}\,(\mathbb{F}'_c\mathbf{F}'^*_+)\, I^M_{c'p'} + \mathrm{Im}\,(\mathbb{F}'_c\mathbf{F}'^*_-)\, I^M_{c'm'} \\
&\quad + m_\nu^2\Big(\mathrm{Re}\,(\mathbb{F}_a\mathbb{F}'^*_a)\, R^M_{aa'} + \mathrm{Re}\left(\mathbb{F}_a\mathbb{F}'^*_b\right) R^M_{ab'} + \mathrm{Re}\,(\mathbb{F}_a\mathbf{F}'^*_+)\, R^M_{ap'} + \mathrm{Re}\,(\mathbb{F}_b\mathbb{F}'^*_a)\, R^M_{ba'} + \mathrm{Re}\left(\mathbb{F}_b\mathbb{F}'^*_b\right) R^M_{bb'} + \mathrm{Re}\,(\mathbb{F}_b\mathbb{F}'^*_c)\, R^M_{bc'} \\
&\qquad + \mathrm{Re}\,(\mathbb{F}_b\mathbf{F}'^*_+)\, R^M_{bp'} + \mathrm{Re}\,(\mathbb{F}_b\mathbf{F}'^*_-)\, R^M_{bm'} + \mathrm{Re}\left(\mathbb{F}_c\mathbb{F}'^*_b\right) R^M_{cb'} + \mathrm{Re}\,(\mathbb{F}_c\mathbb{F}'^*_c)\, R^M_{cc'} + \mathrm{Re}\,(\mathbb{F}_c\mathbf{F}'^*_+)\, R^M_{cp'} + \mathrm{Re}\,(\mathbb{F}_c\mathbf{F}'^*_-)\, R^M_{cm'} \\
&\qquad + \mathrm{Re}\,(\mathbb{F}'_a\mathbf{F}_+^*)\, R^M_{a'p} + \mathrm{Re}\,(\mathbb{F}'_a\mathbf{F}_-^*)\, R^M_{a'm} + \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}_+^*\right) R^M_{b'p} + \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}_-^*\right) R^M_{b'm} + \mathrm{Re}\,(\mathbb{F}'_c\mathbf{F}_+^*)\, R^M_{c'p} + \mathrm{Re}\,(\mathbb{F}'_c\mathbf{F}_-^*)\, R^M_{c'm} \\
&\qquad + \mathrm{Im}\,(\mathbb{F}_a\mathbb{F}'^*_c)\, I^M_{ac'} + \mathrm{Im}\,(\mathbb{F}_a\mathbf{F}'^*_-)\, I^M_{am'} + \mathrm{Im}\,(\mathbb{F}_b\mathbf{F}'^*_+)\, I^M_{bp'} + \mathrm{Im}\,(\mathbb{F}_b\mathbf{F}'^*_-)\, I^M_{bm'} + \mathrm{Im}\,(\mathbb{F}_c\mathbb{F}'^*_a)\, I^M_{ca'} + \mathrm{Im}\,(\mathbb{F}_c\mathbb{F}'^*_c)\, I^M_{cc'} \\
&\qquad + \mathrm{Im}\,(\mathbb{F}_c\mathbf{F}'^*_+)\, I^M_{cp'} + \mathrm{Im}\,(\mathbb{F}_c\mathbf{F}'^*_-)\, I^M_{cm'} + \mathrm{Im}\left(\mathbb{F}'_b\mathbf{F}_+^*\right) I^M_{b'p} + \mathrm{Im}\left(\mathbb{F}'_b\mathbf{F}_-^*\right) I^M_{b'm} + \mathrm{Im}\,(\mathbb{F}'_c\mathbf{F}_+^*)\, I^M_{c'p} + \mathrm{Im}\,(\mathbb{F}'_c\mathbf{F}_-^*)\, I^M_{c'm}\Big)\Big). \quad \propto m_\nu^2,
\end{aligned}
$$

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (1)

Kinematics of back-to-back muons at B-rest frame (ie. B2B nu-nubar)

$$E_1 = E_2 = E_\nu = m_B/2 - E_\mu$$
$$m_{\nu\nu}^2 = 4E_\nu^2$$
$$m_{\mu\mu}^2 = (m_B - 2E_\nu)^2$$
$$Y_m = \sqrt{(m_B/2 - E_\nu)^2 - m_\mu^2}$$
$$Y_n = \sqrt{E_\nu^2 - m_\nu^2}$$

$$\cos\theta_n = 0$$
$$\phi = 0$$
$$\theta_m = \pi/2 - \theta$$
$$(\cos\theta_m = \sin\theta)$$

** θ = angle between (mu mu - nu nu)

➔ Only $(E_\mu, \sin\theta)$ are independent variable ➔ $\frac{d\Gamma}{dE_\mu^2 d\sin\theta}$

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (1-1)

$$\frac{d^3\Gamma^D_{\leftrightarrow}}{dE_\mu^2\, d\sin\theta} - \frac{d^3\Gamma^M_{\leftrightarrow}}{dE_\mu^2\, d\sin\theta} = \frac{G_F^4\, \sqrt{E_\mu^2 - m_\mu^2}}{(4\,\pi)^6\; m_B\, E_\mu} \left(\frac{m_B}{2} - E_\mu\right)^2$$

$$\times\Bigg(\left(|\mathbb{F}_a|^2 - |\mathbb{F}'_a|^2\right)\Delta_{aa} + \left(|\mathbb{F}_b|^2 - |\mathbb{F}'_b|^2\right)\Delta_{bb} + \left(|\mathbb{F}_c|^2 - |\mathbb{F}'_c|^2\right)\Delta_{cc}$$

$$+\left(|\mathbf{F}_+|^2 - |\mathbf{F}'_+|^2\right)\Delta_{pp} + \left(|\mathbf{F}_-|^2 - |\mathbf{F}'_-|^2\right)\Delta_{mm} + \left(\mathrm{Re}\left(\mathbb{F}_a\mathbb{F}_b^*\right) - \mathrm{Re}\left(\mathbb{F}'_a\mathbb{F}'^*_b\right)\right)\Delta_{ab}$$

$$+\left(\mathrm{Re}\left(\mathbb{F}_a\mathbf{F}_+^*\right) - \mathrm{Re}\left(\mathbb{F}'_a\mathbf{F}'^*_+\right)\right)\Delta_{ap} + \left(\mathrm{Re}\left(\mathbb{F}_b\mathbf{F}_+^*\right) - \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}'^*_+\right)\right)\Delta_{bp} + \left(\mathrm{Re}\left(\mathbb{F}_a\mathbf{F}_-^*\right) - \mathrm{Re}\left(\mathbb{F}'_a\mathbf{F}'^*_m\right)\right)\Delta_{am}$$

$$+\left(\mathrm{Re}\left(\mathbb{F}_b\mathbf{F}_-^*\right) - \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}'^*_-\right)\right)\Delta_{bm} + \left(\mathrm{Re}\left(\mathbb{F}_c\mathbf{F}_-^*\right) - \mathrm{Re}\left(\mathbb{F}'_c\mathbf{F}'^*_-\right)\right)\Delta_{cm} + \left(\mathrm{Re}\left(\mathbf{F}_+\mathbf{F}_-^*\right) - \mathrm{Re}\left(\mathbf{F}'_+\mathbf{F}'^*_-\right)\right)\Delta_{pm}$$

$$+\cos\theta\Bigg(\left(|\mathbb{F}_a|^2 + |\mathbb{F}'_a|^2\right)\Sigma_{aa} + \left(|\mathbb{F}_b|^2 + |\mathbb{F}'_b|^2\right)\Sigma_{bb} + \left(|\mathbf{F}_+|^2 + |\mathbf{F}'_+|^2\right)\Sigma_{pp} + \left(|\mathbf{F}_-|^2 + |\mathbf{F}'_-|^2\right)\Sigma_{mm}$$

$$+\left(\mathrm{Re}\left(\mathbb{F}_a\mathbb{F}_b^*\right) + \mathrm{Re}\left(\mathbb{F}'_a\mathbb{F}'^*_b\right)\right)\Sigma_{ab} + \left(\mathrm{Re}\left(\mathbb{F}_a\mathbf{F}_-^*\right) + \mathrm{Re}\left(\mathbb{F}'_a\mathbf{F}'^*_m\right)\right)\Sigma_{am}$$

$$+\left(\mathrm{Re}\left(\mathbb{F}_b\mathbf{F}_-^*\right) + \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}'^*_-\right)\right)\Sigma_{bm} + \left(\mathrm{Re}\left(\mathbf{F}_+\mathbf{F}_-^*\right) + \mathrm{Re}\left(\mathbf{F}'_+\mathbf{F}'^*_-\right)\right)\Sigma_{pm}\Bigg)\Bigg), \qquad \neq\propto \boldsymbol{m_\nu^2} \quad (47)$$

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (2)

$$\frac{d^3\Gamma_{\leftrightarrow}^{D/M}}{dE_\mu^2\, d\sin\theta} = \frac{2\sqrt{E_\mu^2 - m_\mu^2}}{(4\pi)^6\, m_B E_\mu}\left(\left(\frac{m_B}{2} - E_\mu\right)^2 - m_\nu^2\right)\langle|\mathcal{M}_{\leftrightarrow}^{D/M}|^2\rangle,$$

Consider a simple case for numerical purpose only:

(1) neglect muon & neutrino mass ➔ we consider only non-resonant contributions

(2) consider only dominant form factor, ($\mathbb{H}^{\alpha\beta} = \mathbb{F}_a\, g^{\alpha\beta} + \mathbb{F}_b\, p_B^\alpha p_B^\beta + i\,\mathbb{F}_c\, \epsilon^{\alpha\beta\mu\nu} q_{+\mu} q_{-\nu},$)

(3) assume the form factor to be a constant

$$\frac{d^3\Gamma_{\leftrightarrow}^{D}}{dE_\mu^2\, d\sin\theta} = \frac{G_F^4\, |\mathbb{F}_a|^2 (m_B - 2E_\mu)^4 K_\mu}{512\pi^6 m_B E_\mu}(E_\mu - K_\mu\cos\theta)^2,$$

$$\frac{d^3\Gamma_{\leftrightarrow}^{M}}{dE_\mu^2\, d\sin\theta} = \frac{G_F^4\, |\mathbb{F}_a|^2 (m_B - 2E_\mu)^4 K_\mu}{512\pi^6 m_B E_\mu}(E_\mu^2 + K_\mu^2\cos^2\theta),$$

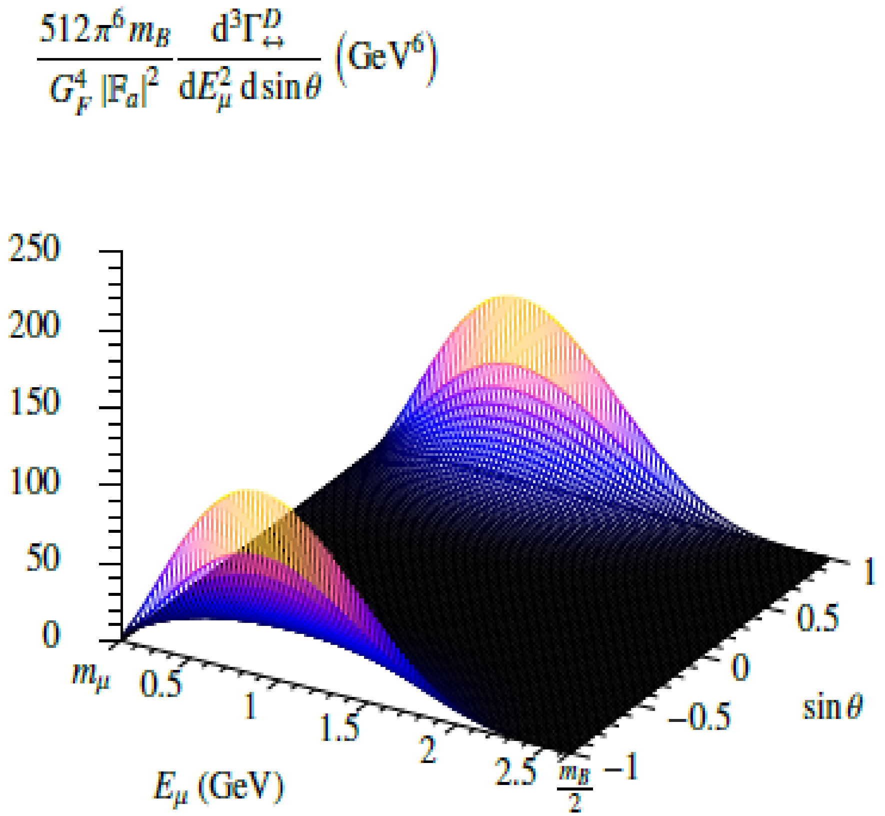
$$K_\mu = \sqrt{E_\mu^2 - m_\mu^2}$$

(4) approximate $E_\mu \approx K_\mu$

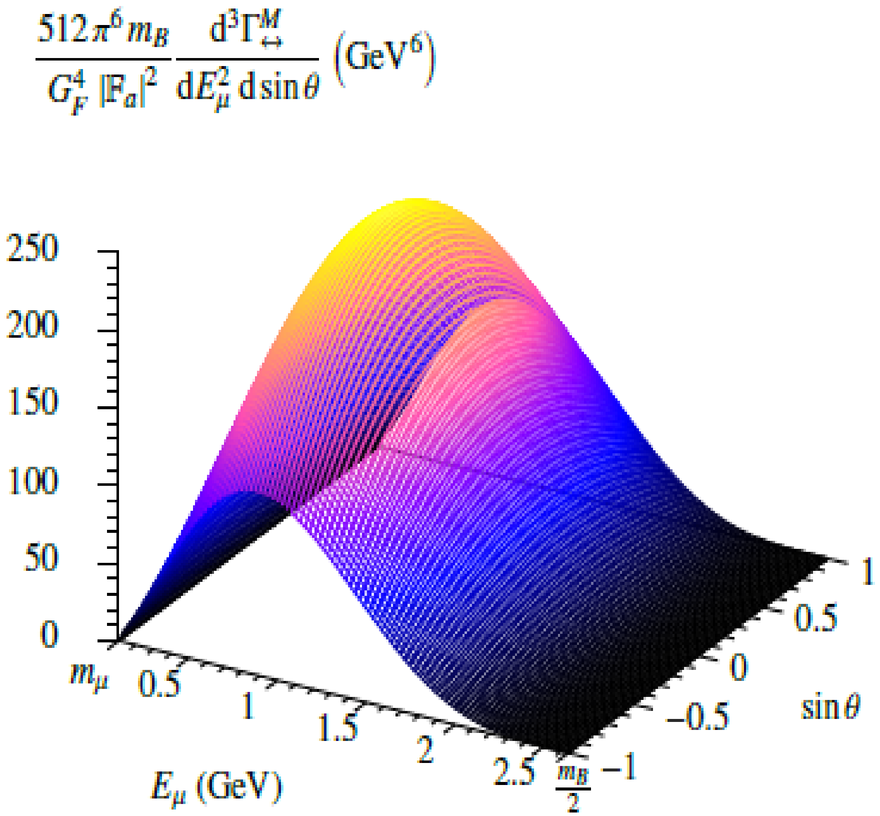
$$\frac{d^3\Gamma_{\leftrightarrow}^{D}}{dE_\mu^2\, d\sin\theta} = \frac{G_F^4\, |\mathbb{F}_a|^2 (m_B - 2E_\mu)^4 E_\mu^2}{512\pi^6 m_B}(1 - \cos\theta)^2,$$

$$\frac{d^3\Gamma_{\leftrightarrow}^{M}}{dE_\mu^2\, d\sin\theta} = \frac{G_F^4\, |\mathbb{F}_a|^2 (m_B - 2E_\mu)^4 E_\mu^2}{512\pi^6 m_B}(1 + \cos^2\theta),$$

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (3)

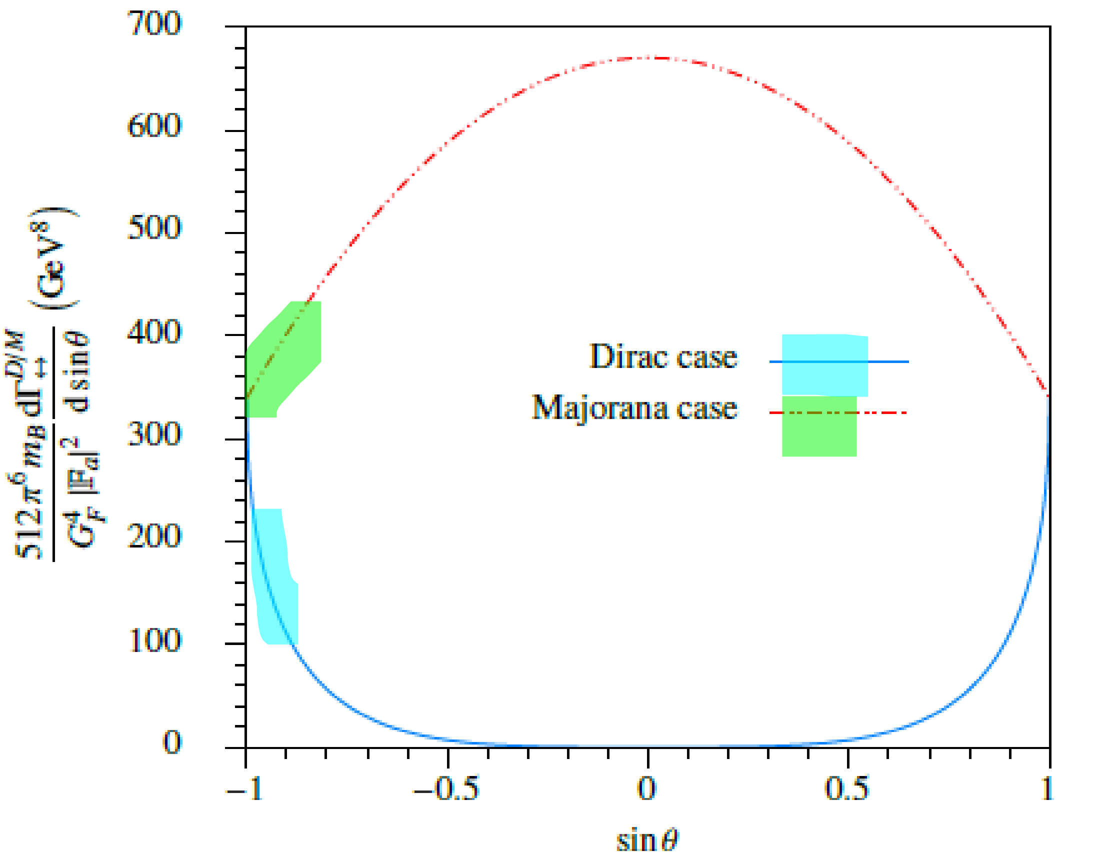


(a) Three dimensional view of the differential decay rate for Dirac case with an appropriate normalization as mentioned.

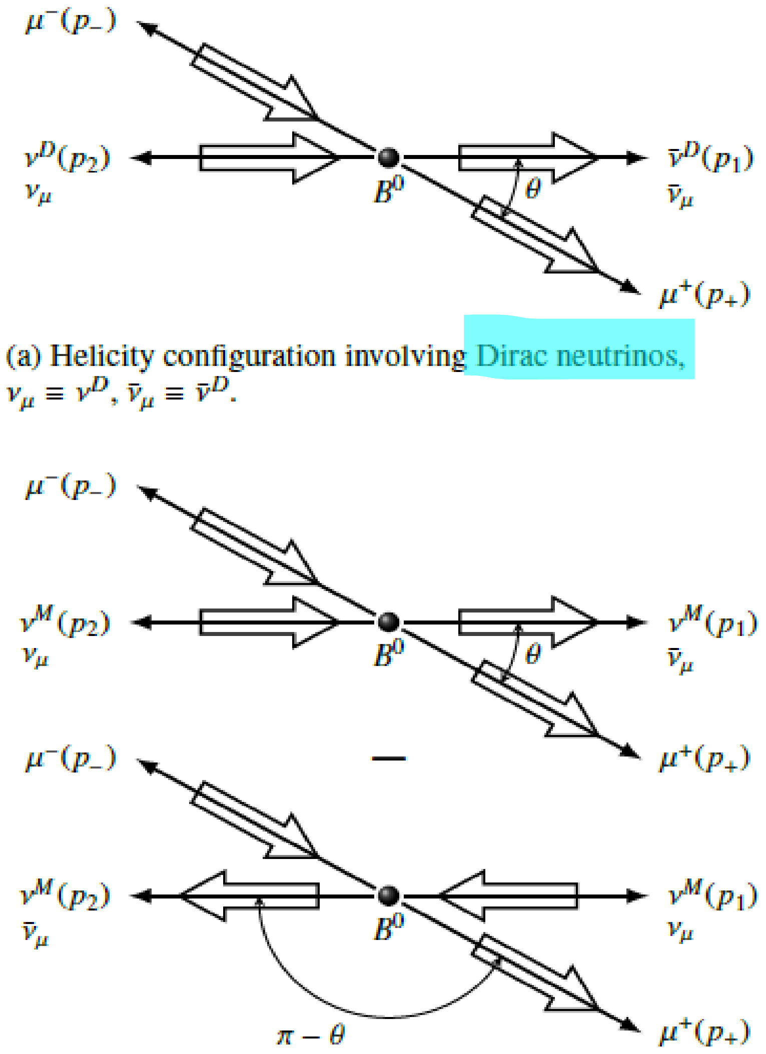


(b) Three dimensional view of the differential decay rate for Majorana case with an appropriate normalization as mentioned.

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (4)



(d) Comparison of $\sin\theta$ distribution alone between Dirac and Majorana cases. Compare with Fig. 2.



(a) Helicity configuration involving Dirac neutrinos, $\nu_\mu \equiv \nu^D$, $\bar{\nu}_\mu \equiv \bar{\nu}^D$.

(b) Helicity configuration involving Majorana neutrinos, $\nu_\mu = \bar{\nu}_\mu \equiv \nu^M$.

** Presently nu nu-bar totally missing, the angle θ is completely unknown, therefore, need to integrate out. ➔ BR(M) >> BR(D)

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (5)

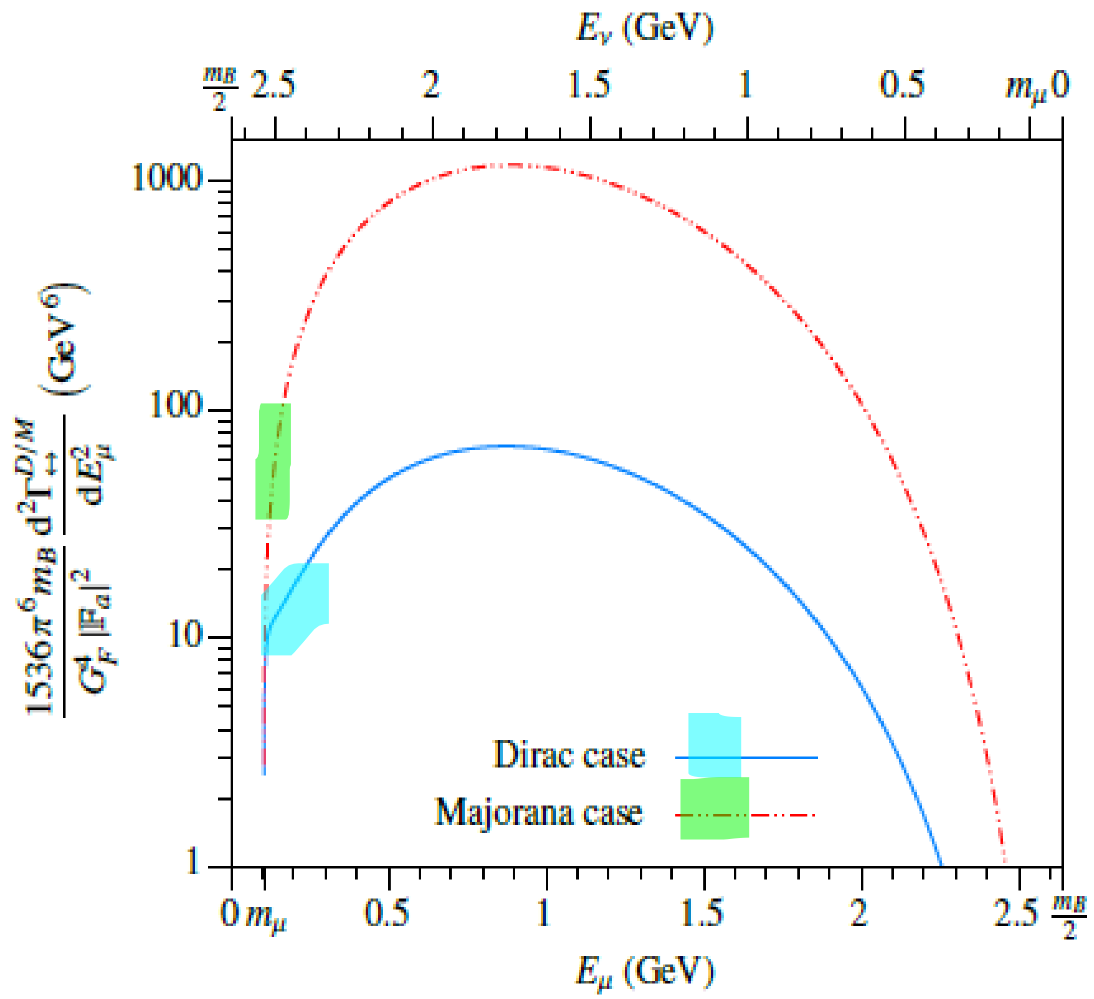
Integrating over currently unobservable angle θ, we get

$$\frac{d^2\Gamma^D_{\leftrightarrow}}{dE_\mu^2} = \frac{G_F^4\,|\mathbb{F}_a|^2}{1536\,\pi^6\,m_B\,E_\mu}\left(m_B - 2\,E_\mu\right)^4 K_\mu \times \left(10\,E_\mu^2 - 3\,\pi\,E_\mu\,K_\mu - 4\,m_\mu^2\right),$$

$$\frac{d^2\Gamma^M_{\leftrightarrow}}{dE_\mu^2} = \frac{G_F^4\,|\mathbb{F}_a|^2}{1536\,\pi^6\,m_B\,E_\mu}\left(m_B - 2\,E_\mu\right)^4 K_\mu \left(10\,E_\mu^2 - 4\,m_\mu^2\right),$$

** $BR(D-M) = \int (D_{\leftrightarrow} - M_{\leftrightarrow}) \neq m_\nu^2$

due to very restricted phase space of B2B muons



(c) Comparison of muon energy distributions between Dirac and Majorana cases in the back-to-back scenario.

Discussions on $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (1)

(1) Branching ratio of B2B muons
(only w/ non-resonant contributions)

$$\mathcal{B}^D_{\leftrightarrow} = \Gamma^D_{\leftrightarrow}/\Gamma_B \approx 1.1 \times 10^{-12}\,\text{GeV}^{-2} \times |\mathbb{F}_a|^2,$$

$$\mathcal{B}^M_{\leftrightarrow} = \Gamma^M_{\leftrightarrow}/\Gamma_B \approx 1.8 \times 10^{-11}\,\text{GeV}^{-2} \times |\mathbb{F}_a|^2,$$

(2) Adding $B^0\,(\bar{B}^0) \to \mu^-\mu^+\nu_\mu\bar{\nu}_\mu$ $B^0\,(\bar{B}^0) \to e^+e^-\nu_e\bar{\nu}_e$, increasing BR four-fold

(3) Futuristic detectors, e.g. FASER, MATHUSLA, SHiP, GAZELLA, could enable to probe the angular distribution, $D \propto (1-\cos\theta)^2, M \propto (1+\cos\theta^2)$

(4) Bg processes, B2B muons + "missing momentum" small due to additional vertices, phase space suppression

1. $B^0 \to \tau^+\nu_\tau\mu^-\bar{\nu}_\mu \to \mu^-\mu^+\nu_\mu\bar{\nu}_\mu\nu_\tau\bar{\nu}_\tau$, and
2. $B^0 \to \tau^+\tau^- \to \mu^-\mu^+\nu_\mu\bar{\nu}_\mu\nu_\tau\bar{\nu}_\tau$.

(5) Many similar processes, $H \to \mu^+\mu^-\nu_\mu\bar{\nu_\mu}, D \to \mu^+\mu^-\nu_\mu\bar{\nu_\mu}, J/\psi \to \mu^+\mu^-\nu_\mu\bar{\nu_\mu}$
$\psi(2S) \to \pi^+\pi^-\nu_\tau\bar{\nu_\tau}, K^0 \to \mu^+\mu^-\nu_\mu\bar{\nu_\mu}, ..$

Discussions on $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, -- B2B muons (2)

Further Discussions:

(1) Discussion on Form Factor — Phys. Rev. D 83, 037503 (2011)

(2) On Collinear case : $(p_{1=}p_2), m_{\nu\nu}^2 = 4m_\nu^2 \approx 0$ [missing mass^2=0, measurable]

(3) On pDMCT

(4) On neutrino mass generation mechanism, scale dependence, mixing matrix (PMNS), any symmetry on phase space, …

(5) On non-zero nu mass, Helicity-Chirality, Weyl-Dirac-Majorana, …

Comments on New physics effects to pDMCT

Choose Process: $X(p_X) \to Y(p_Y)\, \nu(p_1)\, \overline{\nu}(p_2)$

(a) X, Y = single/multi-particle states, Y can also be null,

(b) 4-momenta p_X, p_Y are well measured .

Decay Amplitudes: Showing p_1, p_2 dependencies alone for brevity of expression,

(a) DIRAC case: $\mathscr{M}^D = \mathscr{M}(p_1, p_2)$,

(b) MAJORANA case: $\mathscr{M}^M = \frac{1}{\sqrt{2}}\Big(\underbrace{\mathscr{M}(p_1,p_2)}_{\text{Direct amplitude}} - \underbrace{\mathscr{M}(p_2,p_1)}_{\text{Exchange amplitude}}\Big)$.

$$|\mathscr{M}^D|^2 - |\mathscr{M}^M|^2 = \frac{1}{2}\Big(\underbrace{|\mathscr{M}(p_1,p_2)|^2}_{\text{Direct term}} - \underbrace{|\mathscr{M}(p_2,p_1)|^2}_{\text{Exchange term}}\Big) + \underbrace{\mathrm{Re}\left(\mathscr{M}(p_1,p_2)^* \mathscr{M}(p_2,p_1)\right)}_{\text{Interference term}}.$$

In general, $\underbrace{|\mathscr{M}(p_1,p_2)|^2}_{\text{Direct term}} \neq \underbrace{|\mathscr{M}(p_2,p_1)|^2}_{\text{Exchange term}}$. (e.g. SM z-> nu nu-bar)

Special cases $\underbrace{|\mathscr{M}(p_1,p_2)|^2}_{\text{Direct term}} = \underbrace{|\mathscr{M}(p_2,p_1)|^2}_{\text{Exchange term}}$,

$$\underbrace{|\mathscr{M}(p_1,p_2)|^2}_{\text{Direct term}} = \underbrace{|\mathscr{M}(p_2,p_1)|^2}_{\text{Exchange term}} \Longrightarrow \begin{cases} p_1 = p_2 \equiv p, & (\text{special scenario } \boxed{A}) \\ \mathscr{M}(p_1,p_2) = +\mathscr{M}(p_2,p_1), & (\text{special scenario } \boxed{B}) \\ \mathscr{M}(p_1,p_2) = -\mathscr{M}(p_2,p_1). & (\text{special scenario } \boxed{C}) \end{cases}$$

A $\Longrightarrow$ $\boxed{\left|\mathscr{M}^D_{\text{collinear}}\right|^2 - \left|\mathscr{M}^M_{\text{collinear}}\right|^2 = |\mathscr{M}(p,p)|^2 \neq 0.}$

B $\Longrightarrow$ $\boxed{\left|\mathscr{M}^D_{\text{symmetric}}\right|^2 - \left|\mathscr{M}^M_{\text{symmetric}}\right|^2 = |\mathscr{M}(p_1,p_2)|^2 \neq 0.}$

$$\mathscr{M}(p_1,p_2) \propto \begin{cases} [\overline{u}(p_1)\gamma^\alpha v(p_2)], & (\text{neutral vector current}) \\ [\overline{u}(p_1)\sigma^{\alpha\beta} v(p_2)], & (\text{neutral tensor current}) \end{cases}$$

C $\Longrightarrow$ $\boxed{\left|\mathscr{M}^D_{\text{anti-symm}}\right|^2 - \left|\mathscr{M}^M_{\text{anti-symm}}\right|^2 = -|\mathscr{M}(p_1,p_2)|^2 \neq 0.}$

$$\mathscr{M}(p_1,p_2) \propto \begin{cases} [\overline{u}(p_1)\, v(p_2)], & (\text{neutral scalar current}) \\ [\overline{u}(p_1)\gamma^5 v(p_2)], & (\text{neutral pseudo-scalar current}) \\ [\overline{u}(p_1)\gamma^\alpha\gamma^5 v(p_2)], & (\text{neutral axial-vector current}) \end{cases}$$

$Z \to \nu\,\overline{\nu}$

$$|\mathscr{M}^D|^2 - |\mathscr{M}^M|^2 = \frac{g_Z^2}{3}\left(\left(C_V^2 - C_A^2\right)\left(m_Z^2 + 2m_\nu^2\right) + 6C_A^2 m_\nu^2\right).$$

$C_V = C_A = \frac{1}{2}$ $\qquad |\mathscr{M}^D|^2 - |\mathscr{M}^M|^2 = \frac{g_Z^2}{2} m_\nu^2,$

SUMMARY

Conclusion

Acknowledgements

Conclusion

(1) We consider the B decay, $B^0 \to \mu^- \mu^+ \nu_\mu \bar{\nu}_\mu$, implementing the Fermi-Dirac statistics to find the difference between Dirac and Majorana neutrino, and to test the practical Dirac-Majorana Confusion Theorem.

(2) If we consider the special kinematic configuration of back-to-back muons in the B rest frame, there exists striking difference between D and M cases, which do not depend on neutrino mass, hence, overcoming pDMCT.

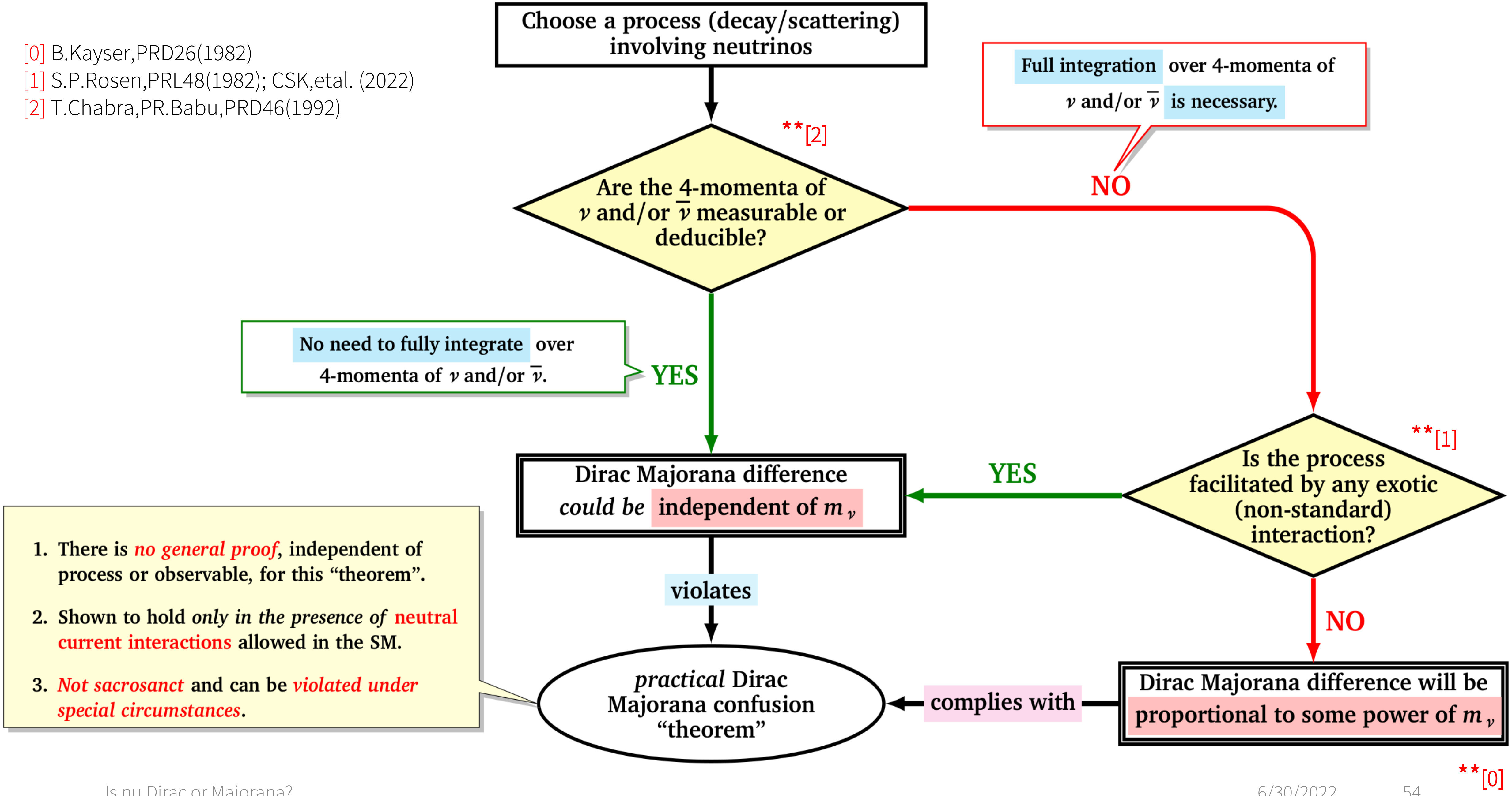
(3) We give full details of analysis, including resonant and non-resonant contributions, tiny neutrino mass dependence, helicity consideration, etc, also confirming pDMC if we integrate out full nu nu-bar phase space.

(4) Finally, we give similar decay examples, such as Higgs, K, D, J/psi, etc.

Conclusion – Final Comment

** The neutrino-less double beta decay (NDBD) has a limitation that it is dependent on the unknown tiny mass of the neutrino. If it is too small there is no possibility of establishing the nature of the neutrino through NDBD. Our proposals are the only viable alternatives to NDBD as far as probing Majorana nature of sub-eV active neutrinos is concerned.

[0] B.Kayser,PRD26(1982)
[1] S.P.Rosen,PRL48(1982); CSK,etal. (2022)
[2] T.Chabra,PR.Babu,PRD46(1992)



Conclusion – Another Final Comment

** 1. Practical Dirac Majorana confusion theorem: It is believed to suggest that all difference between Dirac and Majorana neutrinos must be proportional to some power of neutrino mass (m_ν).

Truth: (a) There is no model-independent, process-independent and observable-independent proof of this so-called "theorem". All processes where it was shown to hold involved full integration over the 4-momenta of missing neutrinos.

(b) Our manuscript is a testament to the fact that this "theorem" can be overcome, if energy and/or momentum of neutrino can be inferred or measured. The interesting question is to find out a way to realize this, which we do by measuring muon energy in the back-to-back muons configuration in the B rest frame, $E_\nu = m_B/2 - E_\mu$.

Conclusion – Another Final Comment

** 2. Massless neutrino limit: It is believed that there should be smooth transition between Majorana and Dirac neutrinos under $m_\nu \to 0$ limit.

Truth: (a) Dirac neutrino and antineutrino are fully distinguishable, while Majorana neutrino and antineutrino are quantum mechanically indistinguishable. There is no smooth limit that takes indistinguishable particles and makes them distinguishable. There is no intermediate state between distinguishable and indistinguishable particles.
(b) The reduction of neutrino degrees of freedom from 4 to 2 for $m_\nu \to 0$ is a discrete jump, and not a continuous change. So the massless neutrino is an entirely different species than the massive one even with extremely tiny mass. Therefore, the presumed smooth transitional difference between Majorana and Dirac neutrinos at $m_\nu \to 0$ is only a misperception.
(c) Majorana neutrino and antineutrino pair have to obey Fermi-Dirac statistics while Dirac neutrino and antineutrino pair do not. We emphasize that statistics of particles does not depend on a parameter like mass, but its spin.

MEET OUR COLLABORATORS

Choong Sun Kim

Professor

M V N Murthy

Professor

Dibyakrupa Sahoo

Postdoctor

THANK YOU

Choong Sun Kim

cskim@yonsei.ac.kr

http://kimcs.Yonsei.ac.kr/~cskim/

Back-up (Details)

Helicity & Chirality

Helicity configuration of back-to-back muons

Detailed study of $B^0(p_B) \rightarrow \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$

Back-to-back muons in B-rest frame

Helicity & Chirality

Dirac equation

A free fermion of mass m is described by a fermionic field $\psi(x)$ which satisfies the Dirac equation,

$$(i\partial\!\!\!/ - m)\,\psi(x) = 0, \qquad (1)$$

where $\partial\!\!\!/ \equiv \gamma^\mu \partial_\mu$ with the Dirac γ matrices having two useful representations:

Dirac representation:

$$\gamma_D^0 = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix}, \quad \gamma_D^i = \begin{pmatrix} \mathbf{0} & \sigma^i \\ -\sigma^i & \mathbf{0} \end{pmatrix}, \qquad (2)$$

and

$$\gamma_D^5 \equiv i\gamma_D^0\gamma_D^1\gamma_D^2\gamma_D^3 = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}, \qquad (3)$$

Weyl or Chiral representation:

$$\gamma_C^0 = \begin{pmatrix} \mathbf{0} & -\mathbf{1} \\ -\mathbf{1} & \mathbf{0} \end{pmatrix}, \quad \gamma_C^i = \begin{pmatrix} \mathbf{0} & \sigma^i \\ -\sigma^i & \mathbf{0} \end{pmatrix}, \qquad (4)$$

and

$$\gamma_C^5 \equiv i\gamma_C^0\gamma_C^1\gamma_C^2\gamma_C^3 = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix}, \qquad (5)$$

where $i = 1,2,3$, $\mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\mathbf{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, and the Pauli σ matrices are given by $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, and $\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Note: $\boxed{\gamma_D^i = \gamma_C^i}$, $\boxed{\gamma_D^0 = \gamma_C^5}$ and $\boxed{\gamma_D^5 = -\gamma_C^0}$.

Helicity & Chirality

Helicity / Spin projection operator

For a spin 1/2 fermion, the spin could have projection along the direction of 3-momentum (helicity ≡ $h = +1$) or opposite to it ($h = -1$). The helicity operator is given by

$$\widehat{h} \equiv \frac{\vec{S} \cdot \vec{P}}{s\,|\vec{P}|}, \tag{6}$$

where $\vec{S}$ is the spin operator and $\vec{P}$ is the 3-momentum operator and $s = 1/2$ for the spin 1/2 fermion. Thus, the field $\psi(x)$ can be split into a positive helicity part $\psi^{(+)}(x)$ and a negative helicity part $\psi^{(-)}(x)$ which are eigenfunctions of the helicity operator, i.e.

$$\widehat{h}\,\psi^{(h)}(x) = h\,\psi^{(h)}(x), \tag{7}$$

for $h = \pm 1$, and

$$\psi(x) = \psi^{(+)}(x) + \psi^{(-)}(x). \tag{8}$$

Chirality projection operator

The matrix γ^5 is the *chirality matrix*. If $\psi_R(x)$ and $\psi_L(x)$ are the right and left chiral fields, then they satisfy the following eigenvalue equations,

$$\gamma^5\,\psi_R(x) = +\psi_R(x), \tag{9}$$

$$\gamma^5\,\psi_L(x) = -\psi_L(x), \tag{10}$$

and

$$\psi(x) = \psi_R(x) + \psi_L(x). \tag{11}$$

In other words,

$$\psi_R(x) = \frac{1+\gamma^5}{2}\psi(x) \equiv P_R\,\psi(x), \tag{12}$$

$$\psi_L(x) = \frac{1-\gamma^5}{2}\psi(x) \equiv P_L\,\psi(x), \tag{13}$$

Helicity & Chirality

where, in the *chiral representation*, we have

$$P_R = \frac{1+\gamma^5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{14}$$

$$P_L = \frac{1-\gamma^5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{15}$$

Writing the general 4-component Dirac spinor ψ in terms of two 2-component (Weyl) spinors χ_R and χ_L as

$$\psi = \begin{pmatrix} \chi_R \\ \chi_L \end{pmatrix}, \tag{16}$$

we get (*in the chiral representation*)

$$\psi_R = P_R\psi = \begin{pmatrix} \chi_R \\ 0 \end{pmatrix}, \quad \psi_L = P_L\psi = \begin{pmatrix} 0 \\ \chi_L \end{pmatrix}. \tag{17}$$

Thus the operators P_R and P_L are called the *chirality projection operators*. The chiral spinors ψ_R and ψ_L satisfy the field equations,

$$i\partial\!\!\!/\,\psi_R = m\psi_L, \tag{18}$$

$$i\partial\!\!\!/\,\psi_L = m\psi_R. \tag{19}$$

This shows that space-time evolution of the chiral spinors ψ_R and ψ_L are related to one another by the mass m. If we consider the case of massless fermions, i.e. $m = 0$, then we obtain the *Weyl equations*:

$$i\partial\!\!\!/\,\psi_R = 0, \tag{20}$$

$$i\partial\!\!\!/\,\psi_L = 0. \tag{21}$$

Helicity & Chirality

Dirac spinors

For both helicity projections, we can have positive and negative frequency solutions of the Dirac equation. Thus,

$$\psi^{(h)}(x) = \int \frac{d^3p}{(2\pi)^3 2E}\Big[a^{(h)}(p)\, u^{(h)}(p)\, e^{-ip\cdot x} + b^{(h)\dagger}(p)\, v^{(h)}(p)\, e^{ip\cdot x}\Big], \qquad (22)$$

where the coefficients $a^{(h)}(p)$ and $b^{(h)}(p)$ are given by,

$$a^{(h)}(p) = \int d^3x\, u^{(h)\dagger}(p)\, \psi(x)\, e^{ip\cdot x}, \qquad (23)$$

$$b^{(h)}(p) = \int d^3x\, \psi^{\dagger}(x)\, u^{(h)}(p)\, e^{ip\cdot x}, \qquad (24)$$

and they satisfy the condition that

$$\int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[\left|a^{(h)}(p)\right|^2 + \left|b^{(h)}(p)\right|^2\right] = 1. \qquad (25)$$

The Dirac equations satisfied by the four 4-component Dirac spinors $u^{(h)}(p)$ and $v^{(h)}(p)$ are

$$\left(\not{p} - m\right) u^{(h)}(p) = 0, \qquad (26)$$

$$\left(\not{p} + m\right) v^{(h)}(p) = 0, \qquad (27)$$

where $\not{p} \equiv \gamma^{\mu} p_{\mu}$. For the Dirac spinor associated with either positive or negative frequency solution, we can further distinguish the left and right chiral spinors, i.e.

$$u^{(h)}(p) = u_R^{(h)}(p) + u_L^{(h)}(p), \qquad (28)$$

$$v^{(h)}(p) = v_R^{(h)}(p) + v_L^{(h)}(p). \qquad (29)$$

Let us introduce the 2-component helicity eigenstate spinors $\chi^{(h)}(\vec{p})$ which satisfy the eigenvalue equation

$$\frac{\vec{p}\cdot\vec{\sigma}}{|\vec{p}|}\chi^{(h)}(\vec{p}) = h\,\chi^{(h)}(\vec{p}). \qquad (30)$$

The explicit form of Dirac spinors can be written using these 2-component spinors. The explicit form of Dirac spinors also depends on the representation of the Dirac γ matrices.

Helicity & Chirality

(1) **In the Dirac representation** we have,

$$u_D^{(h)}(p) = \begin{pmatrix} \sqrt{E+m}\,\chi^{(h)}(\vec{p}) \\ h\sqrt{E-m}\,\chi^{(h)}(\vec{p}) \end{pmatrix}$$

$$= \sqrt{E+m}\begin{pmatrix} \chi^{(h)}(\vec{p}) \\ h\dfrac{|\vec{p}|}{E+m}\chi^{(h)}(\vec{p}) \end{pmatrix}, \qquad (31)$$

$$v_D^{(h)}(p) = \begin{pmatrix} -\sqrt{E-m}\,\chi^{(-h)}(\vec{p}) \\ h\sqrt{E+m}\,\chi^{(-h)}(\vec{p}) \end{pmatrix}$$

$$= \sqrt{E+m}\begin{pmatrix} -\dfrac{|\vec{p}|}{E+m}\chi^{(-h)}(\vec{p}) \\ h\chi^{(-h)}(\vec{p}) \end{pmatrix}. \qquad (32)$$

For *non-relativistic case* we have $|\vec{p}| \ll m$ and $E \simeq m$, such that

$$u_D^{(h)}(p) = \sqrt{2m}\begin{pmatrix} \chi^{(h)}(\vec{p}) \\ h\dfrac{|\vec{p}|}{2m}\chi^{(h)}(\vec{p}) \end{pmatrix}, \qquad (33)$$

$$v_D^{(h)}(p) = \sqrt{2m}\begin{pmatrix} -\dfrac{|\vec{p}|}{2m}\chi^{(-h)}(\vec{p}) \\ h\chi^{(-h)}(\vec{p}) \end{pmatrix}. \qquad (34)$$

Since, $\dfrac{|\vec{p}|}{2m} \ll 1$, in the non-relativistic case, the two upper components of $u^{(h)}(p)$ are called the *larger components* and the two lower components are called the *smaller components*. The opposite is true for $v^{(h)}(p)$. *This makes Dirac representation a useful choice while studying non-relativistic fermions.*

Helicity & Chirality

(2) In the Weyl or Chiral representation we have

$$u_C^{(h)}(p) = \begin{pmatrix} -\sqrt{E + h\,|\vec{p}|}\,\chi^{(h)}(\vec{p}) \\ \sqrt{E - h\,|\vec{p}|}\,\chi^{(h)}(\vec{p}) \end{pmatrix}, \tag{35}$$

$$v_C^{(h)}(p) = -h \begin{pmatrix} \sqrt{E - h\,|\vec{p}|}\,\chi^{(-h)}(\vec{p}) \\ \sqrt{E + h\,|\vec{p}|}\,\chi^{(-h)}(\vec{p}) \end{pmatrix}. \tag{36}$$

Thus,

$$u_C^{(+)}(p) = \begin{pmatrix} -\sqrt{E + |\vec{p}|}\,\chi^{(+)}(\vec{p}) \\ \sqrt{E - |\vec{p}|}\,\chi^{(+)}(\vec{p}) \end{pmatrix}$$

$$= \sqrt{E + |\vec{p}|} \begin{pmatrix} -\chi^{(+)}(\vec{p}) \\ \dfrac{m}{E + |\vec{p}|}\,\chi^{(+)}(\vec{p}) \end{pmatrix}, \tag{37}$$

$$u_C^{(-)}(p) = \begin{pmatrix} -\sqrt{E - |\vec{p}|}\,\chi^{(-)}(\vec{p}) \\ \sqrt{E + |\vec{p}|}\,\chi^{(-)}(\vec{p}) \end{pmatrix}$$

$$= \sqrt{E + |\vec{p}|} \begin{pmatrix} -\dfrac{m}{E + |\vec{p}|}\,\chi^{(-)}(\vec{p}) \\ \chi^{(-)}(\vec{p}) \end{pmatrix}, \tag{38}$$

$$v_C^{(+)}(p) = -\begin{pmatrix} \sqrt{E - |\vec{p}|}\,\chi^{(-)}(\vec{p}) \\ \sqrt{E + |\vec{p}|}\,\chi^{(-)}(\vec{p}) \end{pmatrix}$$

$$= -\sqrt{E + |\vec{p}|} \begin{pmatrix} \dfrac{m}{E + |\vec{p}|}\,\chi^{(-)}(\vec{p}) \\ \chi^{(-)}(\vec{p}) \end{pmatrix}, \tag{39}$$

$$v_C^{(-)}(p) = \begin{pmatrix} \sqrt{E + |\vec{p}|}\,\chi^{(+)}(\vec{p}) \\ \sqrt{E - |\vec{p}|}\,\chi^{(+)}(\vec{p}) \end{pmatrix}$$

$$= \sqrt{E + |\vec{p}|} \begin{pmatrix} \chi^{(+)}(\vec{p}) \\ \dfrac{m}{E + |\vec{p}|}\,\chi^{(+)}(\vec{p}) \end{pmatrix}. \tag{40}$$

Helicity & Chirality

For *ultra-relativistic case* we have $m \ll E$ and $\vec{p} \simeq E$, such that

$$u_C^{(+)}(p) = \sqrt{2E}\begin{pmatrix} -\chi^{(+)}(\vec{p}) \\ \frac{m}{2E}\,\chi^{(+)}(\vec{p}) \end{pmatrix}, \quad (41)$$

$$u_C^{(-)}(p) = \sqrt{2E}\begin{pmatrix} -\frac{m}{2E}\,\chi^{(-)}(\vec{p}) \\ \chi^{(-)}(\vec{p}) \end{pmatrix}, \quad (42)$$

$$v_C^{(+)}(p) = -\sqrt{2E}\begin{pmatrix} \frac{m}{2E}\,\chi^{(-)}(\vec{p}) \\ \chi^{(-)}(\vec{p}) \end{pmatrix}, \quad (43)$$

$$v_C^{(-)}(p) = \sqrt{2E}\begin{pmatrix} \chi^{(+)}(\vec{p}) \\ \frac{m}{2E}\,\chi^{(+)}(\vec{p}) \end{pmatrix}. \quad (44)$$

Since, in the chiral representation, the upper two components of the 4-component Dirac spinor form the Right Weyl spinor and the lower two components form the Left Weyl spinor, let us introduce the following notation,

$$u_C^{(h)}(p) = \sqrt{2E}\begin{pmatrix} u_{C,R}^{(h)}(p) \\ u_{C,L}^{(h)}(p) \end{pmatrix}, \quad v_C^{(h)}(p) = \sqrt{2E}\begin{pmatrix} v_{C,R}^{(h)}(p) \\ v_{C,L}^{(h)}(p) \end{pmatrix}. \quad (45)$$

Using this notation and using the fact that for ultra-relativistic case $\frac{m}{2E} \ll 1$, it is easy to show that the *larger components* are

$$u_{C,R}^{(+)}(p) = -\chi^{(+)}(\vec{p}), \quad (46a)$$

$$u_{C,L}^{(-)}(p) = +\chi^{(-)}(\vec{p}), \quad (46b)$$

$$v_{C,L}^{(+)}(p) = -\chi^{(-)}(\vec{p}), \quad (46c)$$

$$v_{C,R}^{(-)}(p) = +\chi^{(+)}(\vec{p}), \quad (46d)$$

Helicity & Chirality

and the *smaller components* are

$$u^{(+)}_{C,L}(p) = +\frac{m}{2E}\,\chi^{(+)}(\vec{p}), \qquad (47a)$$

$$u^{(-)}_{C,R}(p) = -\frac{m}{2E}\,\chi^{(-)}(\vec{p}), \qquad (47b)$$

$$v^{(+)}_{C,R}(p) = -\frac{m}{2E}\,\chi^{(-)}(\vec{p}), \qquad (47c)$$

$$v^{(-)}_{C,L}(p) = +\frac{m}{2E}\,\chi^{(+)}(\vec{p}). \qquad (47d)$$

In simple terms, these equations state that for a fermionic **particle** in ultra-relativistic case:

(i) positive helicity state is mostly right-handed, and

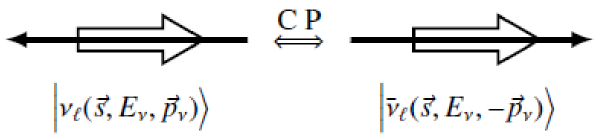
(ii) negative helicity state is mostly left-handed.

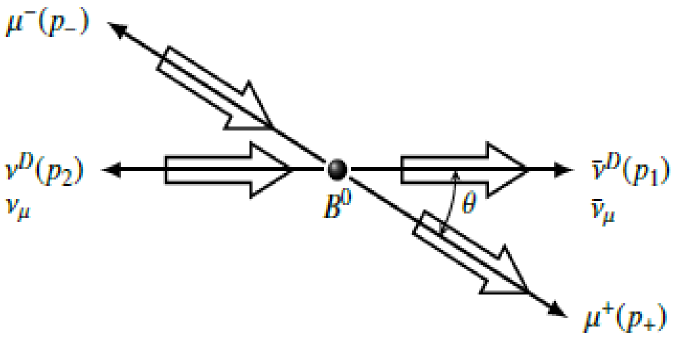
Similarly, for a fermionic **anti-particle** in ultra-relativistic case:

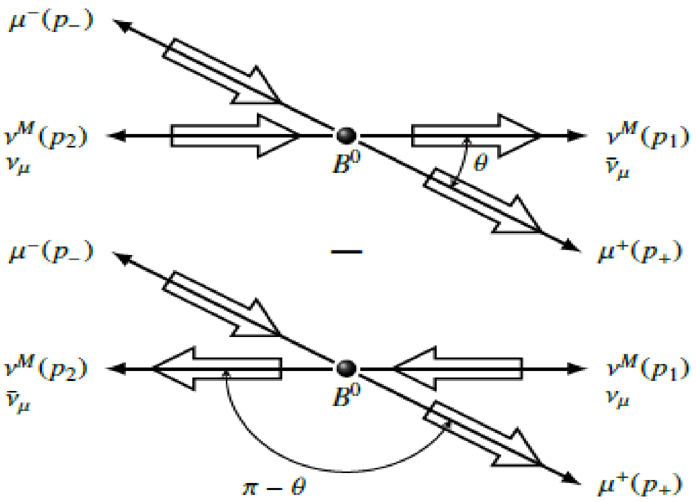
(i) positive helicity state is mostly left-handed, and

(ii) negative helicity state is mostly right-handed.

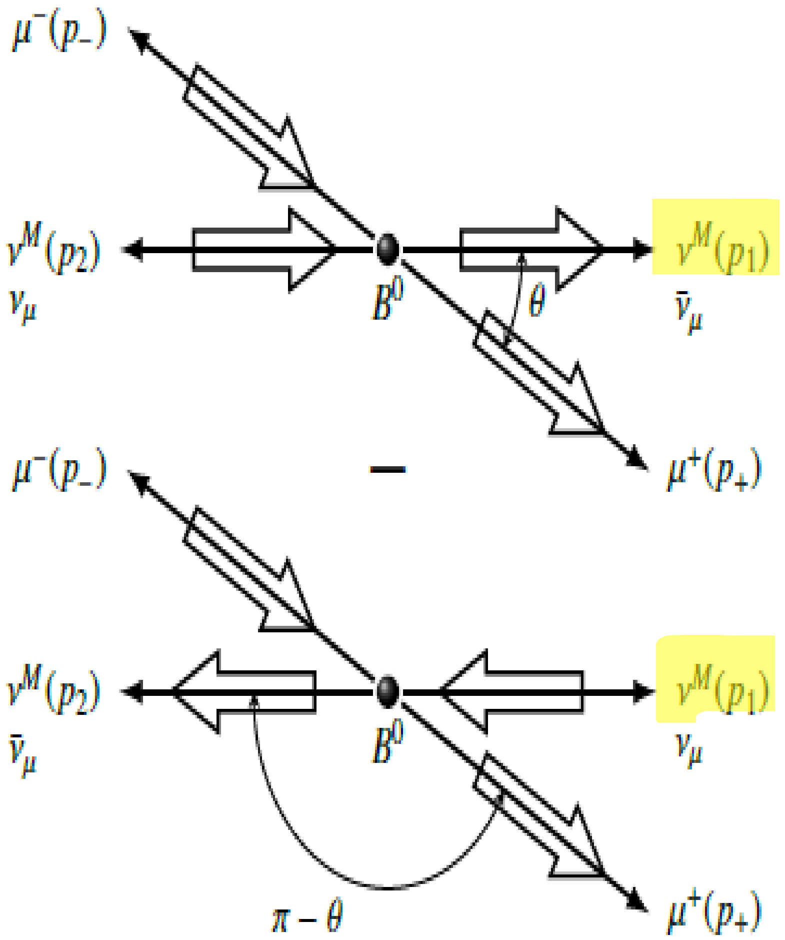
$$\mathrm{C\,P}\,|\nu_\ell(\vec{s}, E_\nu, \vec{p}_\nu)\rangle = \eta_P\,|\bar{\nu}_\ell(\vec{s}, E_\nu, -\vec{p}_\nu)\rangle\,,$$





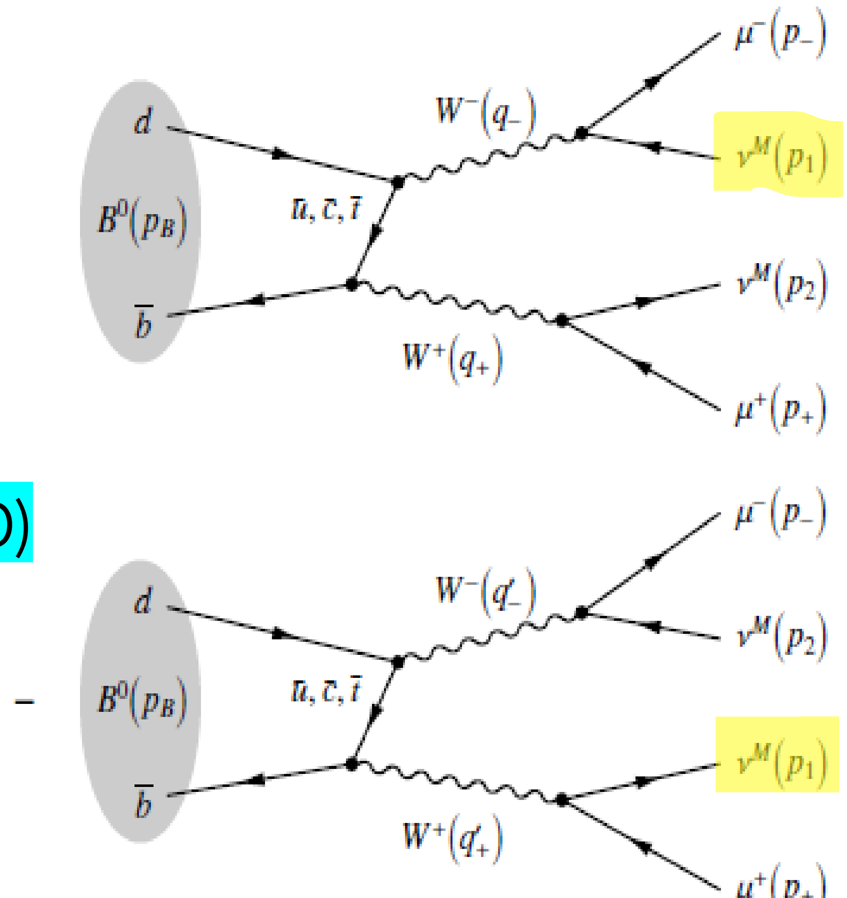


Helicity Configuration of back-to-back muons



Helicity-flip (X)

Momenta exchange (O)



Detailed study of $B^0(p_B) \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2)$,

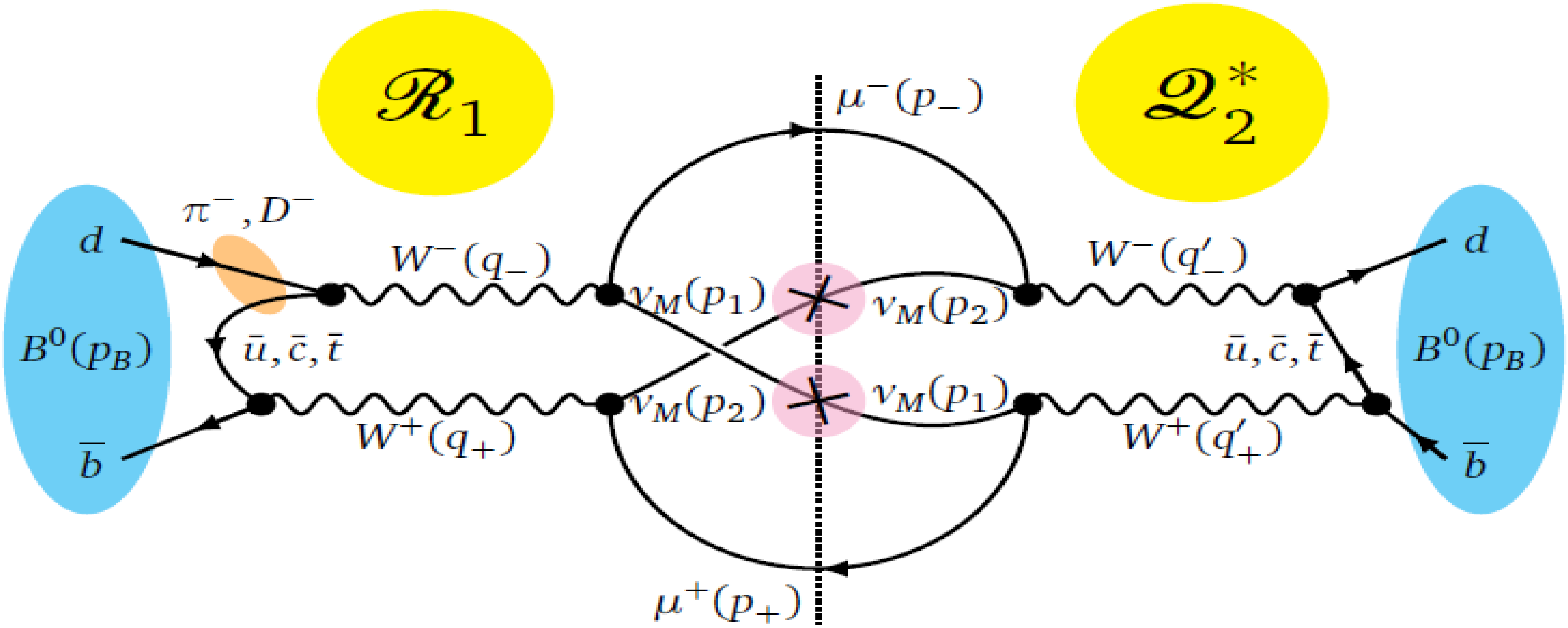
$$\frac{d^5\Gamma^{D/M}}{dm_{\mu\mu}^2\, dm_{\nu\nu}^2\, d\cos\theta_m\, d\cos\theta_n\, d\phi} = \frac{Y\,Y_m\,Y_n \left\langle \left|\mathscr{M}^{D/M}\right|^2 \right\rangle}{(4\pi)^6\, m_B^2\, m_{\mu\mu}\, m_{\nu\nu}},$$

$$\begin{aligned}\left\langle \left|\mathscr{M}^D\right|^2 \right\rangle = G_F^4 \Big(& |\mathbb{F}_a|^2 S_{aa}^D + |\mathbb{F}_b|^2 S_{bb}^D + |\mathbb{F}_c^2| S_{cc}^D + |\mathbf{F}_+|^2 S_{pp}^D + |\mathbf{F}_-|^2 S_{mm}^D + \mathrm{Re}\left(\mathbb{F}_a\mathbb{F}_b^*\right) R_{ab}^D + \mathrm{Re}\left(\mathbb{F}_a\mathbb{F}_c^*\right) R_{ac}^D + \mathrm{Re}\left(\mathbb{F}_a\mathbf{F}_+^*\right) R_{ap}^D \\
& + \mathrm{Re}\left(\mathbb{F}_a\mathbf{F}_-^*\right) R_{am}^D + \mathrm{Im}\left(\mathbb{F}_a\mathbb{F}_b^*\right) I_{ab}^D + \mathrm{Im}\left(\mathbb{F}_a\mathbb{F}_c^*\right) I_{ac}^D + \mathrm{Im}\left(\mathbb{F}_a\mathbf{F}_+^*\right) I_{ap}^D + \mathrm{Im}\left(\mathbb{F}_a\mathbf{F}_-^*\right) I_{am}^D + \mathrm{Re}\left(\mathbb{F}_b\mathbb{F}_c^*\right) R_{bc}^D \\
& + \mathrm{Re}\left(\mathbb{F}_b\mathbf{F}_+^*\right) R_{bp}^D + \mathrm{Re}\left(\mathbb{F}_b\mathbf{F}_-^*\right) R_{bm}^D + \mathrm{Re}\left(\mathbb{F}_c\mathbf{F}_+^*\right) R_{cp}^D + \mathrm{Im}\left(\mathbb{F}_b\mathbb{F}_c^*\right) I_{bc}^D + \mathrm{Im}\left(\mathbb{F}_b\mathbf{F}_+^*\right) I_{bp}^D + \mathrm{Im}\left(\mathbb{F}_b\mathbf{F}_-^*\right) I_{bm}^D \\
& + \mathrm{Im}\left(\mathbb{F}_c\mathbf{F}_+^*\right) I_{cp}^D + \mathrm{Re}\left(\mathbb{F}_c\mathbf{F}_-^*\right) R_{cm}^D + \mathrm{Re}\left(\mathbf{F}_+\mathbf{F}_-^*\right) R_{pm}^D + \mathrm{Im}\left(\mathbb{F}_c\mathbf{F}_-^*\right) I_{cm}^D + \mathrm{Im}\left(\mathbf{F}_+\mathbf{F}_-^*\right) I_{pm}^D \Big),\end{aligned}$$

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2),$

$$
\begin{aligned}
\left\langle \left|\mathscr{M}^M\right|^2 \right\rangle = \frac{G_F^4}{2}\Big(& |\mathbb{F}_a|^2 S^M_{aa} + |\mathbb{F}_b|^2 S^M_{bb} + |\mathbb{F}_c^2| S^M_{cc} + |\mathbf{F}_+|^2 S^M_{pp} + |\mathbf{F}_-|^2 S^M_{mm} + |\mathbb{F}'_a|^2 S^M_{a'a'} + |\mathbb{F}'_b|^2 S^M_{b'b'} + |\mathbb{F}'_c|^2 S^M_{c'c'} + |\mathbf{F}'_+|^2 S^M_{p'p'} \\
& + |\mathbf{F}'_-|^2 S^M_{m'm'} + \mathrm{Re}\left(\mathbb{F}_a\mathbb{F}_b^*\right) R^M_{ab} + \mathrm{Re}\left(\mathbb{F}_a\mathbb{F}_c^*\right) R^M_{ac} + \mathrm{Re}\left(\mathbb{F}_a\mathbf{F}_+^*\right) R^M_{ap} + \mathrm{Re}\left(\mathbb{F}_a\mathbf{F}_-^*\right) R^M_{am} + \mathrm{Re}\left(\mathbb{F}_b\mathbb{F}_c^*\right) R^M_{bc} \\
& + \mathrm{Re}\left(\mathbb{F}_b\mathbf{F}_+^*\right) R^M_{bp} + \mathrm{Re}\left(\mathbb{F}_b\mathbf{F}_-^*\right) R^M_{bm} + \mathrm{Re}\left(\mathbb{F}_c\mathbf{F}_+^*\right) R^M_{cp} + \mathrm{Re}\left(\mathbb{F}_c\mathbf{F}_-^*\right) R^M_{cm} + \mathrm{Re}\left(\mathbb{F}'_a\mathbb{F}'^*_b\right) R^M_{a'b'} + \mathrm{Re}\left(\mathbb{F}'_a\mathbb{F}'^*_c\right) R^M_{a'c'} \\
& + \mathrm{Re}\left(\mathbb{F}'_a\mathbf{F}'^*_+\right) R^M_{a'p'} + \mathrm{Re}\left(\mathbb{F}'_a\mathbf{F}'^*_-\right) R^M_{a'm'} + \mathrm{Re}\left(\mathbb{F}'_b\mathbb{F}'^*_c\right) R^M_{b'c'} + \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}'^*_+\right) R^M_{b'p'} + \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}'^*_-\right) R^M_{b'm'} + \mathrm{Re}\left(\mathbb{F}'_c\mathbf{F}'^*_+\right) R^M_{c'p'} \\
& + \mathrm{Re}\left(\mathbb{F}'_c\mathbf{F}'^*_-\right) R^M_{c'm'} + \mathrm{Re}\left(\mathbf{F}_+\mathbf{F}_-^*\right) R^M_{pm} + \mathrm{Re}\left(\mathbf{F}'_+\mathbf{F}'^*_-\right) R^M_{p'm'} + \mathrm{Im}\left(\mathbb{F}_a\mathbb{F}_b^*\right) I^M_{ab} + \mathrm{Im}\left(\mathbb{F}_a\mathbb{F}_c^*\right) I^M_{ac} + \mathrm{Im}\left(\mathbb{F}_a\mathbf{F}_+^*\right) I^M_{ap} \\
& + \mathrm{Im}\left(\mathbb{F}_a\mathbf{F}_-^*\right) I^M_{am} + \mathrm{Im}\left(\mathbb{F}_b\mathbb{F}_c^*\right) I^M_{bc} + \mathrm{Im}\left(\mathbb{F}_b\mathbf{F}_+^*\right) I^M_{bp} + \mathrm{Im}\left(\mathbb{F}_b\mathbf{F}_-^*\right) I^M_{bm} + \mathrm{Im}\left(\mathbb{F}_c\mathbf{F}_+^*\right) I^M_{cp} + \mathrm{Im}\left(\mathbb{F}_c\mathbf{F}_-^*\right) I^M_{cm} \\
& + \mathrm{Im}\left(\mathbb{F}'_a\mathbb{F}'^*_b\right) I^M_{a'b'} + \mathrm{Im}\left(\mathbb{F}'_a\mathbb{F}'^*_c\right) I^M_{a'c'} + \mathrm{Im}\left(\mathbb{F}'_a\mathbf{F}'^*_+\right) I^M_{a'p'} + \mathrm{Im}\left(\mathbb{F}'_a\mathbf{F}'^*_-\right) I^M_{a'm'} + \mathrm{Im}\left(\mathbb{F}'_b\mathbb{F}'^*_c\right) I^M_{b'c'} + \mathrm{Im}\left(\mathbb{F}'_b\mathbf{F}'^*_+\right) I^M_{b'p'} \\
& + \mathrm{Im}\left(\mathbb{F}'_b\mathbf{F}'^*_-\right) I^M_{b'm'} + \mathrm{Im}\left(\mathbb{F}'_c\mathbf{F}'^*_+\right) I^M_{c'p'} + \mathrm{Im}\left(\mathbb{F}'_c\mathbf{F}'^*_-\right) I^M_{c'm'} + \mathrm{Im}\left(\mathbf{F}_+\mathbf{F}_-^*\right) I^M_{pm} + \mathrm{Im}\left(\mathbf{F}'_+\mathbf{F}'^*_-\right) I^M_{p'm'} \\
& + m_\nu^2\Big(\mathrm{Re}\left(\mathbb{F}_a\mathbb{F}'^*_a\right) R^M_{aa'} + \mathrm{Re}\left(\mathbb{F}_a\mathbb{F}'^*_b\right) R^M_{ab'} + \mathrm{Re}\left(\mathbb{F}_a\mathbb{F}'^*_c\right) R^M_{ac'} + \mathrm{Re}\left(\mathbb{F}_a\mathbf{F}'^*_+\right) R^M_{ap'} + \mathrm{Re}\left(\mathbb{F}_a\mathbf{F}'^*_-\right) R^M_{am'} + \mathrm{Re}\left(\mathbb{F}_b\mathbb{F}'^*_a\right) R^M_{ba'} \\
& \quad + \mathrm{Re}\left(\mathbb{F}_b\mathbb{F}'^*_b\right) R^M_{bb'} + \mathrm{Re}\left(\mathbb{F}_b\mathbb{F}'^*_c\right) R^M_{bc'} + \mathrm{Re}\left(\mathbb{F}_b\mathbf{F}'^*_+\right) R^M_{bp'} + \mathrm{Re}\left(\mathbb{F}_b\mathbf{F}'^*_-\right) R^M_{bm'} + \mathrm{Re}\left(\mathbb{F}_c\mathbb{F}'^*_a\right) R^M_{ca'} + \mathrm{Re}\left(\mathbb{F}_c\mathbb{F}'^*_b\right) R^M_{cb'} \\
& \quad + \mathrm{Re}\left(\mathbb{F}_c\mathbb{F}'^*_c\right) R^M_{cc'} + \mathrm{Re}\left(\mathbb{F}_c\mathbf{F}'^*_+\right) R^M_{cp'} + \mathrm{Re}\left(\mathbb{F}_c\mathbf{F}'^*_-\right) R^M_{cm'} + \mathrm{Re}\left(\mathbb{F}'_a\mathbf{F}_+^*\right) R^M_{a'p} + \mathrm{Re}\left(\mathbb{F}'_a\mathbf{F}_-^*\right) R^M_{a'm} + \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}_+^*\right) R^M_{b'p} \\
& \quad + \mathrm{Re}\left(\mathbb{F}'_b\mathbf{F}_-^*\right) R^M_{b'm} + \mathrm{Re}\left(\mathbb{F}'_c\mathbf{F}_+^*\right) R^M_{c'p} + \mathrm{Re}\left(\mathbb{F}'_c\mathbf{F}_-^*\right) R^M_{c'm} + \mathrm{Im}\left(\mathbb{F}_a\mathbb{F}'^*_a\right) I^M_{aa'} + \mathrm{Im}\left(\mathbb{F}_a\mathbb{F}'^*_b\right) I^M_{ab'} + \mathrm{Im}\left(\mathbb{F}_a\mathbb{F}'^*_c\right) I^M_{ac'} \\
& \quad + \mathrm{Im}\left(\mathbb{F}_a\mathbf{F}'^*_+\right) I^M_{ap'} + \mathrm{Im}\left(\mathbb{F}_a\mathbf{F}'^*_-\right) I^M_{am'} + \mathrm{Im}\left(\mathbb{F}_b\mathbb{F}'^*_a\right) I^M_{ba'} + \mathrm{Im}\left(\mathbb{F}_b\mathbb{F}'^*_b\right) I^M_{bb'} + \mathrm{Im}\left(\mathbb{F}_b\mathbb{F}'^*_c\right) I^M_{bc'} + \mathrm{Im}\left(\mathbb{F}_b\mathbf{F}'^*_+\right) I^M_{bp'} \\
& \quad + \mathrm{Im}\left(\mathbb{F}_b\mathbf{F}'^*_-\right) I^M_{bm'} + \mathrm{Im}\left(\mathbb{F}_c\mathbb{F}'^*_a\right) I^M_{ca'} + \mathrm{Im}\left(\mathbb{F}_c\mathbb{F}'^*_b\right) I^M_{cb'} + \mathrm{Im}\left(\mathbb{F}_c\mathbb{F}'^*_c\right) I^M_{cc'} + \mathrm{Im}\left(\mathbb{F}_c\mathbf{F}'^*_+\right) I^M_{cp'} + \mathrm{Im}\left(\mathbb{F}_c\mathbf{F}'^*_-\right) I^M_{cm'} \\
& \quad + \mathrm{Im}\left(\mathbb{F}'_a\mathbf{F}_+^*\right) I^M_{a'p} + \mathrm{Im}\left(\mathbb{F}'_a\mathbf{F}_-^*\right) I^M_{a'm} + \mathrm{Im}\left(\mathbb{F}'_b\mathbf{F}_+^*\right) I^M_{b'p} + \mathrm{Im}\left(\mathbb{F}'_b\mathbf{F}_-^*\right) I^M_{b'm} + \mathrm{Im}\left(\mathbb{F}'_c\mathbf{F}_+^*\right) I^M_{c'p} + \mathrm{Im}\left(\mathbb{F}'_c\mathbf{F}_-^*\right) I^M_{c'm}\Big)\Big),
\end{aligned}
$$

$\mathrm{Re}\left(\mathcal{M}(p_1, p_2)^* \mathcal{M}(p_2, p_1)\right) \propto m_\nu^2.$ **of** $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$



Since the squared diagram involves two helicity flips for the Majorana neutrinos, these contributions are directly proportional to m_ν^2.

Detailed study of $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, w/ B2B muons

Appendix B: Expressions for the various Σ_{ij} and Δ_{ij} terms

The Δ_{ij} terms appearing in Eq. (47) are given by

$$\Delta_{aa} = -16\,(m_B - 2\,E_\mu)^2\,\left((m_\mu^2 - E_\mu^2)\cos^2\theta - E_\mu^2\right), \quad \text{(B1)}$$

$$\Delta_{bb} = -4\,m_B^4\,(m_B - 2\,E_\mu)^2\,\left((m_\mu^2 - E_\mu^2)\cos^2\theta - E_\mu^2\right), \quad \text{(B2)}$$

$$\Delta_{cc} = -8\,m_\mu^2\,(m_\mu^2 - E_\mu^2)\,m_B^2\,(m_B - 2\,E_\mu)^2\,\sin^2\theta, \quad \text{(B3)}$$

$$\Delta_{pp} = -4\,m_\mu^4\,(m_B - 2\,E_\mu)^2\,\left((m_\mu^2 - E_\mu^2)\cos^2\theta - E_\mu^2\right), \quad \text{(B4)}$$

$$\Delta_{mm} = 4\,m_\mu^2\,(m_B - 2\,E_\mu)^2\,\Big((m_\mu^2 - E_\mu^2)\,(m_B^2 - m_\mu^2)\cos^2\theta + E_\mu\,(E_\mu\,m_B^2 - 2\,m_\mu^2\,m_B + E_\mu\,m_\mu^2)\Big), \quad \text{(B5)}$$

$$\Delta_{ab} = -16\,m_B^2\,(m_B - 2\,E_\mu)^2 \times \left((m_\mu^2 - E_\mu^2)\cos^2\theta - E_\mu^2\right), \quad \text{(B6)}$$

$$\Delta_{ap} = 16\,m_\mu^4\,(m_B - 2\,E_\mu)^2, \quad \text{(B7)}$$

$$\Delta_{bp} = 8\,m_\mu^4\,m_B^2\,(m_B - 2\,E_\mu)^2, \quad \text{(B8)}$$

$$\Delta_{am} = 16\,m_\mu^2\,(m_B - 2\,E_\mu)^2\,(E_\mu\,m_B - m_\mu^2), \quad \text{(B9)}$$

$$\Delta_{bm} = 8\,m_\mu^2\,m_B^2\,(m_B - 2\,E_\mu)^2\,(E_\mu\,m_B - m_\mu^2), \quad \text{(B10)}$$

$$\Delta_{cm} = 8\,m_\mu^2\,m_B^2\,(E_\mu^2 - m_\mu^2)\,(m_B - 2\,E_\mu)^2\,\sin^2\theta, \quad \text{(B11)}$$

$$\Delta_{pm} = 8\,m_\mu^4\,(m_B - 2\,E_\mu)^2 \times \left((m_\mu^2 - E_\mu^2)\cos^2\theta + E_\mu\,(m_B - E_\mu)\right), \quad \text{(B12)}$$

and the Σ_{ij} terms are given by,

$$\Sigma_{aa} = -32\,E_\mu\,\sqrt{E_\mu^2 - m_\mu^2}\,(m_B - 2\,E_\mu)^2, \quad \text{(B13)}$$

$$\Sigma_{bb} = -8\,m_B^4\,E_\mu\,\sqrt{E_\mu^2 - m_\mu^2}\,(m_B - 2\,E_\mu)^2, \quad \text{(B14)}$$

$$\Sigma_{pp} = 8\,E_\mu\,m_\mu^4\,\sqrt{E_\mu^2 - m_\mu^2}\,(m_B - 2\,E_\mu)^2, \quad \text{(B15)}$$

$$\Sigma_{mm} = -8\,m_\mu^4\,\sqrt{E_\mu^2 - m_\mu^2}\,(m_B - 2\,E_\mu)^2\,(m_B - E_\mu), \quad \text{(B16)}$$

$$\Sigma_{ab} = -32\,m_B^2\,E_\mu\,\sqrt{E_\mu^2 - m_\mu^2}\,(m_B - 2\,E_\mu)^2, \quad \text{(B17)}$$

$$\Sigma_{am} = -16\,m_\mu^2\,m_B\,\sqrt{E_\mu^2 - m_\mu^2}\,(m_B - 2\,E_\mu)^2, \quad \text{(B18)}$$

$$\Sigma_{bm} = -8\,m_\mu^2\,m_B^3\,\sqrt{E_\mu^2 - m_\mu^2}\,(m_B - 2\,E_\mu)^2, \quad \text{(B19)}$$

$$\Sigma_{pm} = 8\,m_\mu^4\,\sqrt{E_\mu^2 - m_\mu^2}\,(m_B - 2\,E_\mu)^3. \quad \text{(B20)}$$