



Transverse single-spin asymmetries of very forward neutral pion

Inha HTG Workshop: Modern issues in Hadronic Physics

Hee-Jin Kim

Inha University

arXiv:2206.02184

In collaboration with Samson Clymton and Prof. Hyun-Chul Kim

Outline

- *Introduction*
- *Inclusive $pp^{\uparrow} \rightarrow \pi^0 X$ scattering*
- *Invariant cross section in the triple-Regge limit*
- *Transverse single-spin asymmetry*
- *Results*
- *Summary*

Introduction

Introduction

- The transverse single-spin asymmetry(TSSA) is one of the crucial observables in high-energy physics for understanding particle production mechanisms and hadron spin structure.
- Large p_T (> 2 GeV/c) TSSA has been investigated by QCD-based approaches: (i.e., Transverse Momentum Distributions(TMDs), Collinear twist-3 factorization, ...)

TSSA :

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

Transversely polarized proton beam

[1] Y. Fukao *et al.*, Phys. Lett. B650 (2007) 325

[2] K. Tanida *et al.*(PHENIX Collaboration), J.Phys.Conf.Ser.295(2011)

Introduction

- The transverse single-spin asymmetry(TSSA) is one of the crucial observables in high-energy physics for understanding particle production mechanisms and hadron spin structure.
- Large p_T (> 2 GeV/c) TSSA has been investigated by QCD-based approaches: (i.e., Transverse Momentum Distributions(TMDs), Collinear twist-3 factorization, ...)
- Sizable transverse single-spin asymmetries in the *very forward* direction have been continuously reported for the few decades [1,2].
- Since the produced particles have very large values of pseudorapidity and low p_T , the TSSA displays the *non-perturbative diffractive* nature.

TSSA :

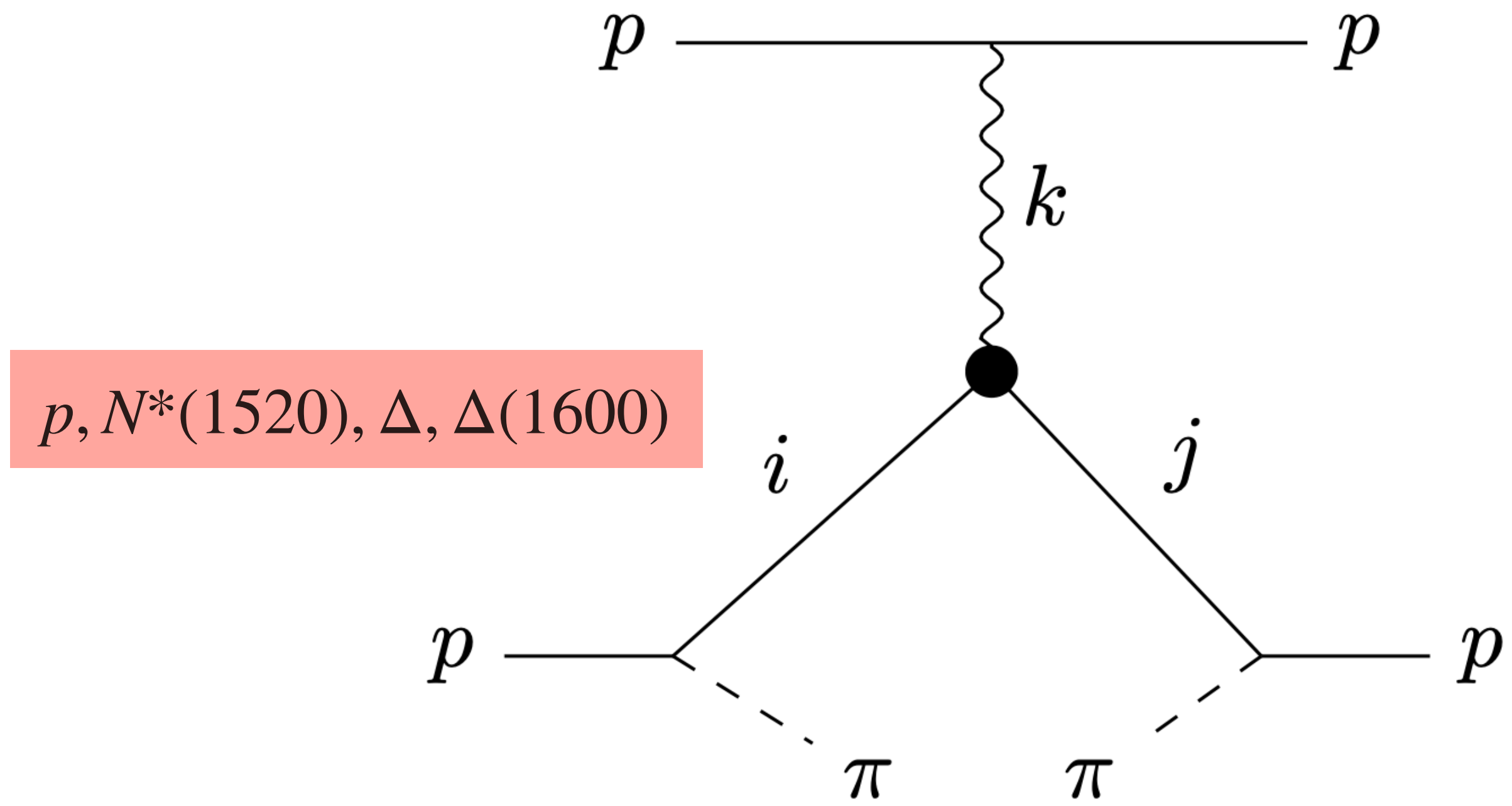
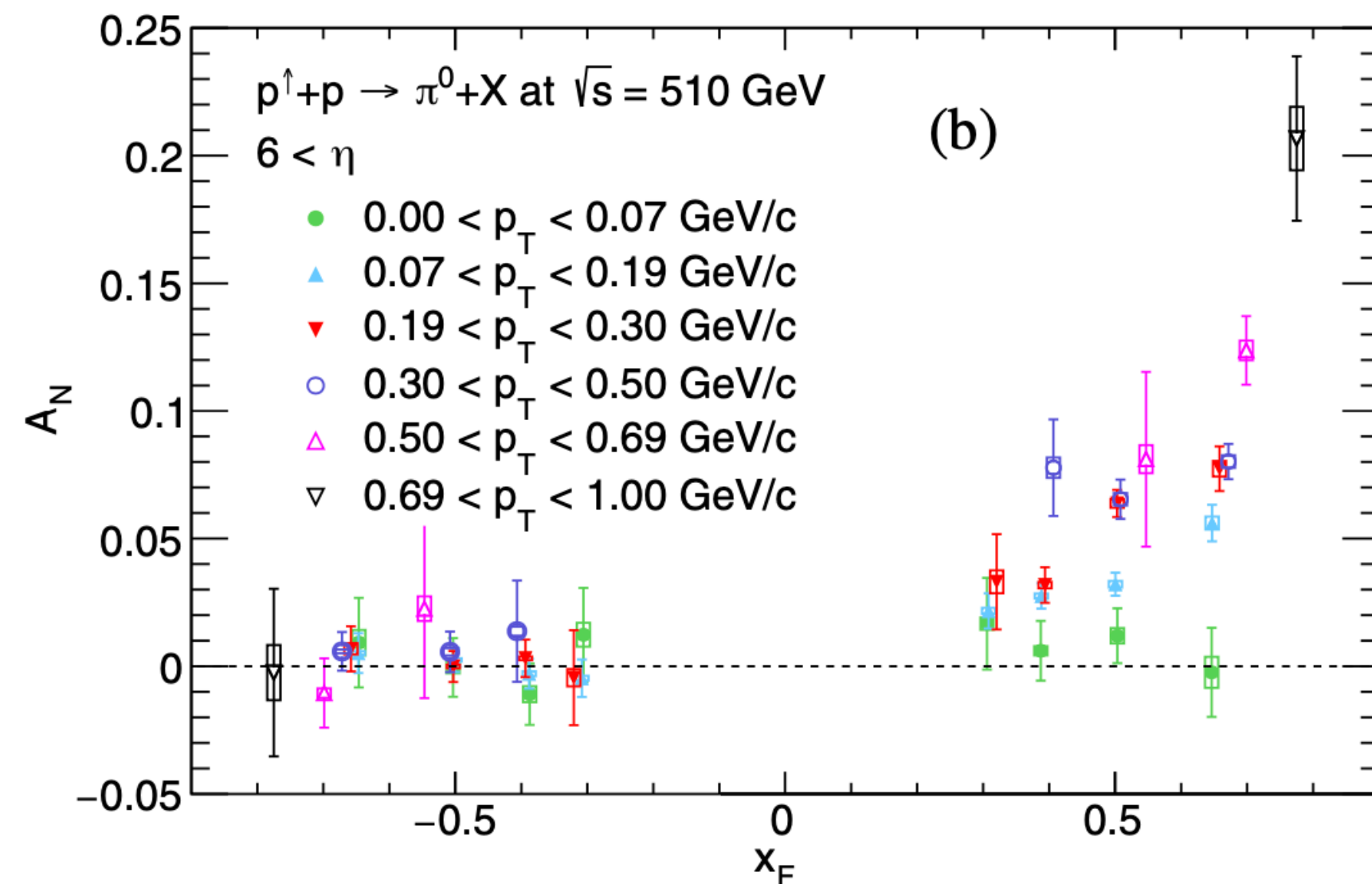
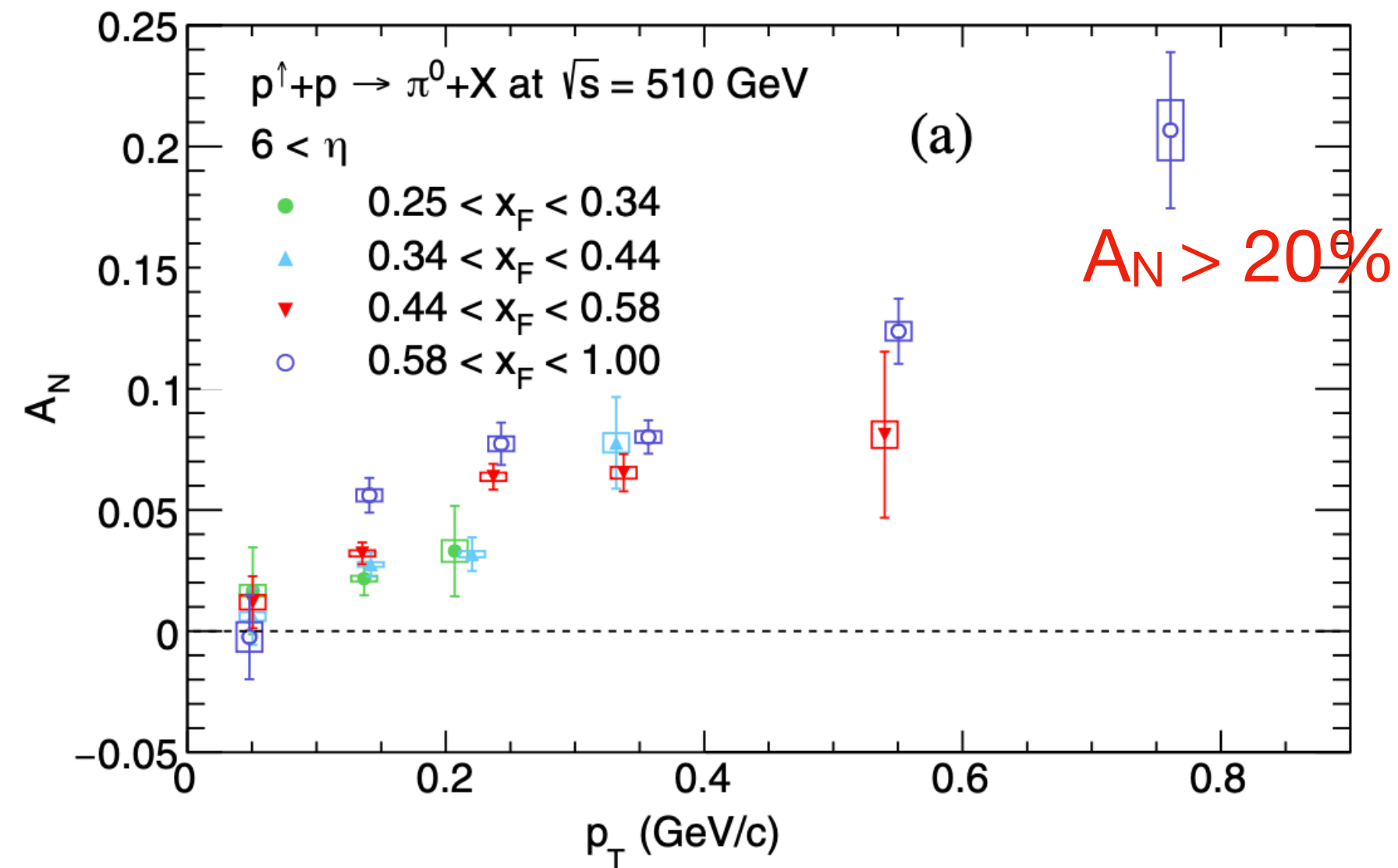
$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

Transversely polarized proton beam

[1] Y. Fukao *et al.*, Phys. Lett. B650 (2007) 325

[2] K. Tanida *et al.*(PHENIX Collaboration), J.Phys.Conf.Ser.295(2011)

- Recently the TSSA of very forward neutral pion was measured in RHICf experiment [3].



- The interferences between baryon trajectories yield the transverse single-spin asymmetry.
- Since the RHICf energy is sufficiently large, $d\sigma$ can be approximated to the triple-Regge diagram.

Inclusive $pp^{\uparrow} \rightarrow \pi^0 X$ scattering

Kinematics

Inclusive $pp^\uparrow \rightarrow \pi^0 X$ reaction:

- Kinematics for single diffractive(SD) process**

$$p_1 = (E_1, 0, 0, p_z), \quad p_2 = (E_2, 0, 0, -p_z), \quad p_3 = (E_3, \mathbf{p}_T, p'_z)$$

Missing mass : $M_X^2 \equiv (p_1 + p_2 - p_3)^2$

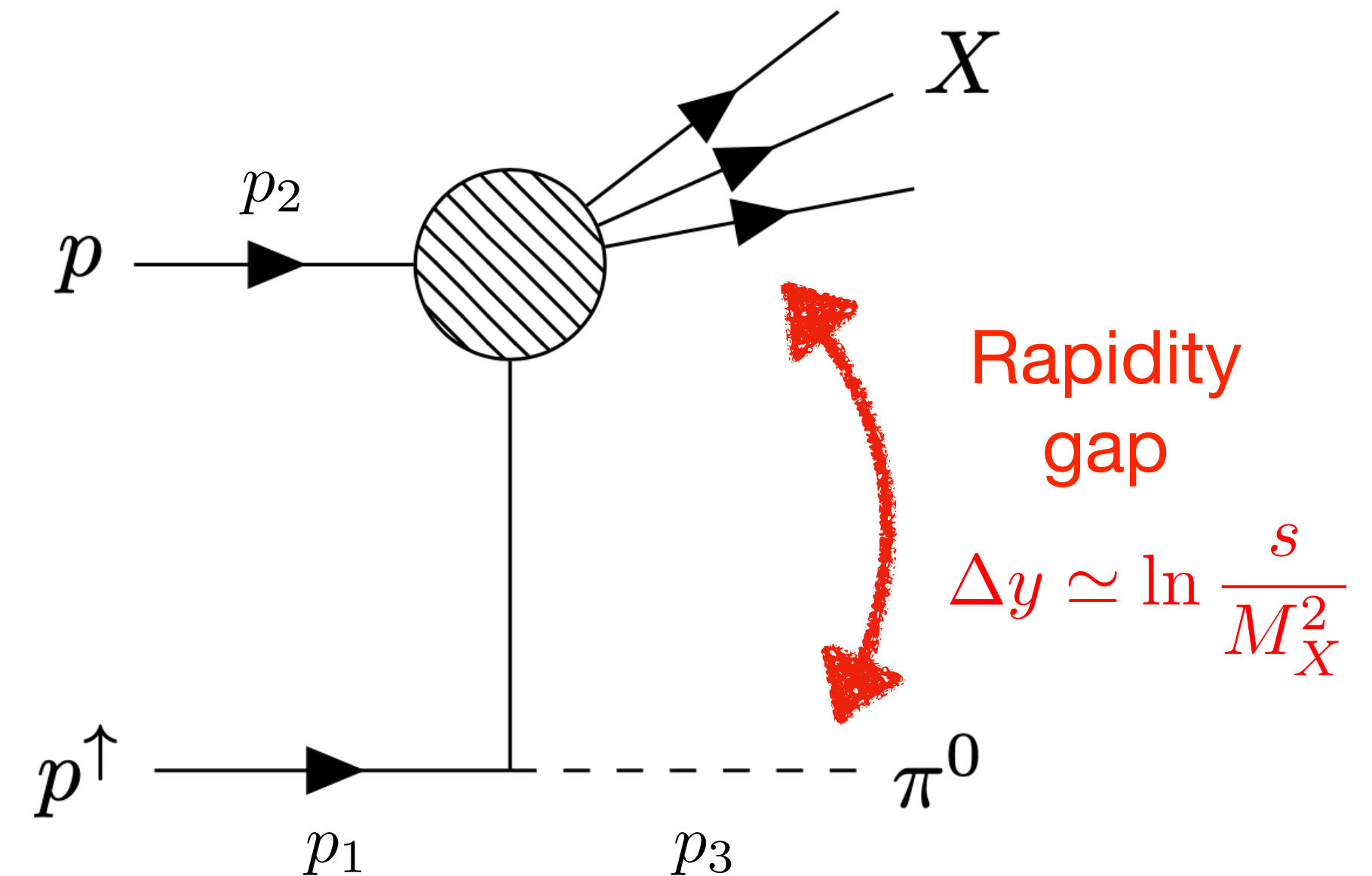
In high-energy hadron scattering the transverse momentum of the produced particle is quite small on average.

When $s, M_X^2 \gg m_i^2$ and $|p_T| < 1 \text{ GeV}/c$, one can define

Feynman variable : $x_F = \frac{p'_z}{p_z} \simeq 1 - \frac{M_X^2}{s}$

momentum transfer : $t = (p_1 - p_3)^2 \simeq (1 - x_F)m_N^2 - \frac{\mathbf{p}_T^2}{x_F}$

The SD processes can be described in terms of s , x_F , and p_T^2 .

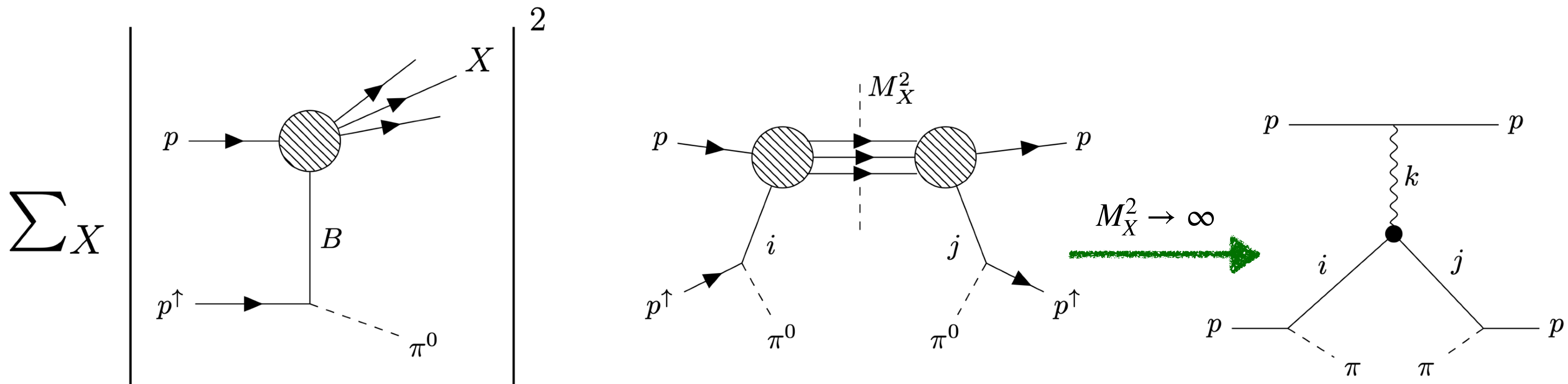


Invariant cross section in the triple-Regge limit

Invariant differential cross-section

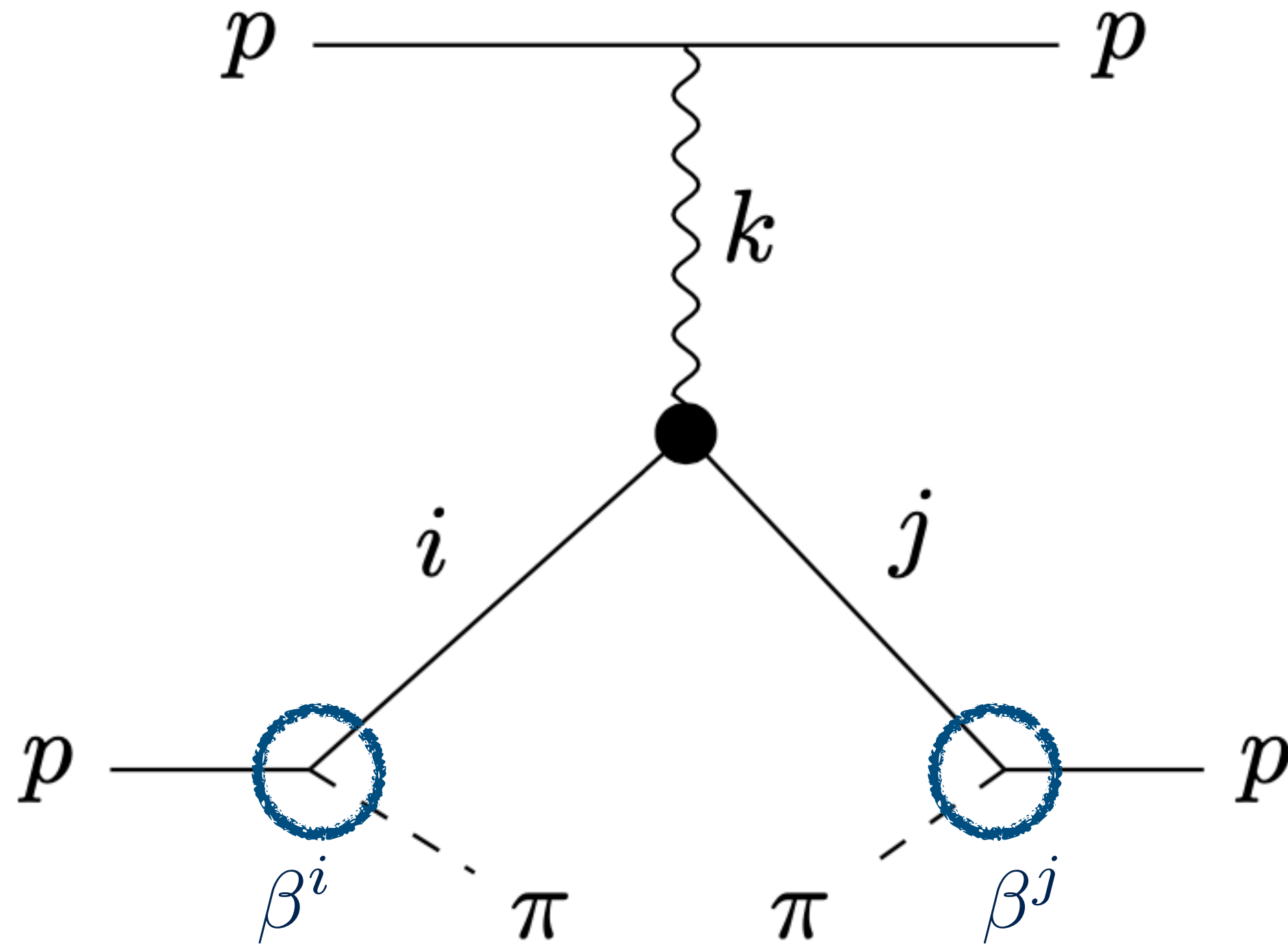
- The Lorentz-invariant differential cross section of π^0 :

$$d\sigma^h \equiv E \frac{d^3\sigma^h}{d^3\mathbf{p}} = \frac{1}{s} \sum |A_{p \rightarrow \pi^0}^{\text{tot}}(s, p_T; h)|^2$$



$$d\sigma^h = \frac{1}{s} \sum_{i,j} \sum_{\lambda,\mu} \beta_{h\lambda}^i \beta_{h\mu}^{j*} \mathcal{P}_i \mathcal{P}_j^* \sum_k G_k^{ij}(t) \gamma_k^{pp}(0) \left(\frac{M_X^2}{s_0} \right)^{\alpha_k(0)}$$

Since the Regge approach does not provide the vertex structure, we need to employ the effective Lagrangians:



$$\mathcal{L}_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi} \bar{\psi} \gamma_\mu \gamma_5 \boldsymbol{\tau} \cdot \psi \partial^\mu \pi,$$

$$\mathcal{L}_{\pi NN^*} = -i \frac{f_{\pi NN^*}}{m_\pi} \bar{\psi}_{N^*}^\mu (g_{\mu\nu} + a \gamma_\mu \gamma_\nu) \gamma_5 \mathbf{T} \cdot \psi \partial^\nu \pi$$

$$\mathcal{L}_{\pi N\Delta} = -\frac{f_{\pi N\Delta}}{m_\pi} \bar{\psi}_\Delta^\mu (g_{\mu\nu} + a \gamma_\mu \gamma_\nu) \mathbf{T} \cdot \psi \partial^\nu \pi,$$

$$\mathcal{L}_{\pi N\Delta^*} = -\frac{f_{\pi N\Delta^*}}{m_\pi} \bar{\psi}_{\Delta^*}^\mu (g_{\mu\nu} + a \gamma_\mu \gamma_\nu) \mathbf{T} \cdot \psi \partial^\nu \pi,$$

Residue functions with Born approximation

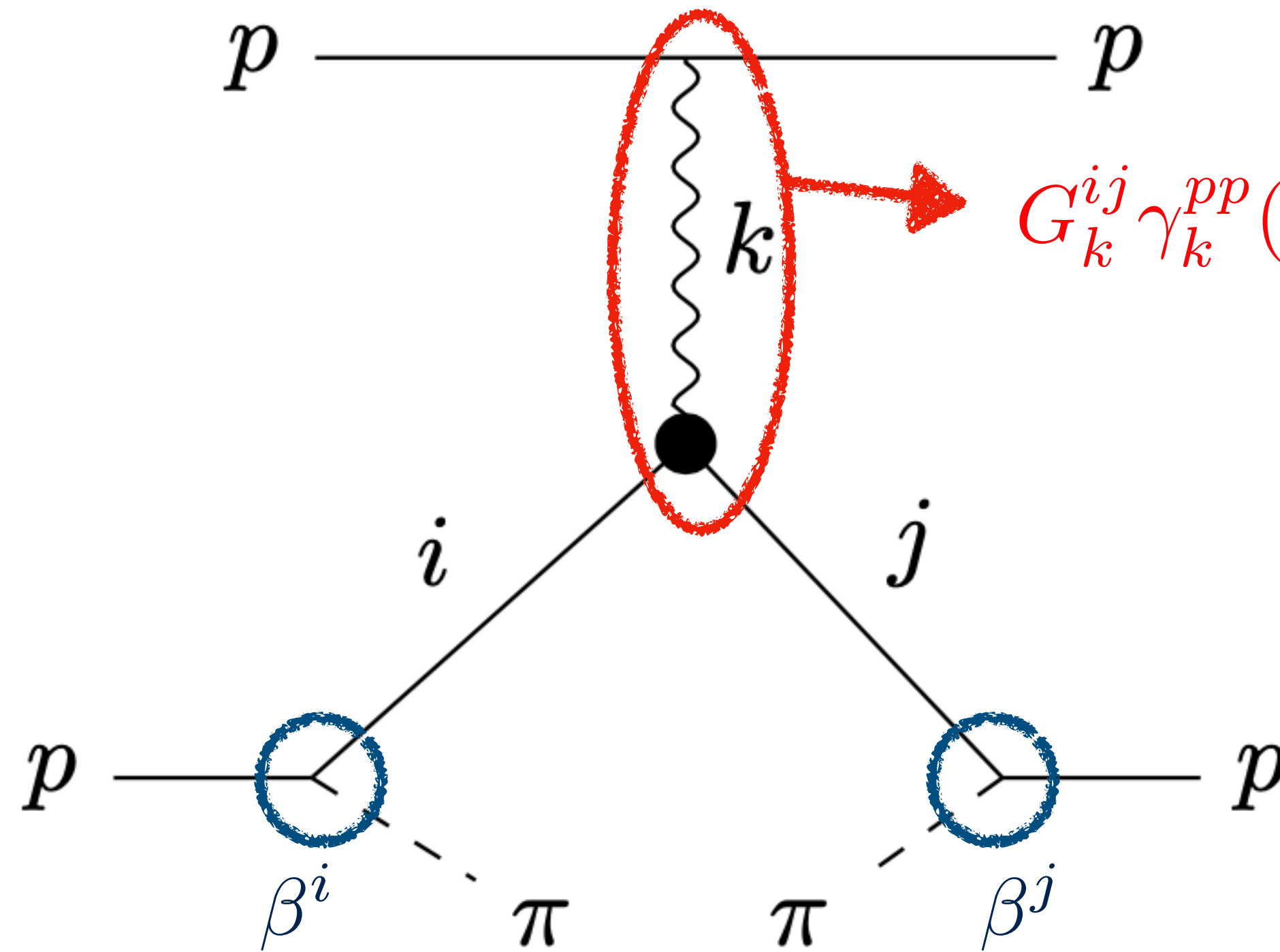
$$\beta_{ss'}^N(p_T) = \bar{u}_N(s', q) \not{k} \gamma_5 u_p(s, p),$$

$$\beta_{ss'}^{N^*}(p_T) = i \bar{u}_{N^*}^\mu(s', q) (k_\mu + a \gamma_\mu \not{k}) \gamma_5 u_p(s, p),$$

$$\beta_{ss'}^\Delta(p_T) = \bar{u}_\Delta^\mu(s', q) (k_\mu + a \gamma_\mu \not{k}) u_p(s, p),$$

$$\beta_{ss'}^{\Delta^*}(p_T) = \bar{u}_{\Delta^*}^\mu(s', q) (k_\mu + a \gamma_\mu \not{k}) u_p(s, p),$$

$$d\sigma^h = \frac{1}{s} \sum_{i,j} \sum_{\lambda,\mu} \beta_{h\lambda}^i \beta_{h\mu}^{j*} \mathcal{P}_i \mathcal{P}_j^* \sum_k G_k^{ij}(t) \gamma_k^{pp}(0) \left(\frac{M_X^2}{s_0} \right)^{\alpha_k(0)}$$



Triple-Regge coupling and ppk vertex function

$$\gamma_k^{pp}(t_{pp} = 0) = \sum_{\lambda} \beta_{\lambda\lambda}^k(0)$$

No momentum transfer between the unpolarized proton by the generalized optical theorem.

Residue functions with Born approximation

$$\begin{aligned} \beta_{ss'}^N(p_T) &= \bar{u}_N(s', q) \not{k} \gamma_5 u_p(s, p), \\ \beta_{ss'}^{N^*}(p_T) &= i \bar{u}_{N^*}^\mu(s', q) (k_\mu + a \gamma_\mu \not{k}) \gamma_5 u_p(s, p), \\ \beta_{ss'}^\Delta(p_T) &= \bar{u}_\Delta^\mu(s', q) (k_\mu + a \gamma_\mu \not{k}) u_p(s, p), \\ \beta_{ss'}^{\Delta^*}(p_T) &= \bar{u}_{\Delta^*}^\mu(s', q) (k_\mu + a \gamma_\mu \not{k}) u_p(s, p), \end{aligned}$$

$$d\sigma^h = \frac{1}{s} \sum_{i,j} \sum_{\lambda,\mu} \beta_{h\lambda}^i \beta_{h\mu}^j \mathcal{P}_i \mathcal{P}_j^* \sum_k G_k^{ij}(t) \gamma_k^{pp}(0) \left(\frac{M_X^2}{s_0} \right)^{\alpha_k(0)}$$

$$\alpha_p(t) = -0.35 + 0.99t$$

$$\alpha_{N^*}(t) = -0.73 + 0.95t$$

$$\alpha_{\Delta}(t) = 0.16 + 0.89t$$

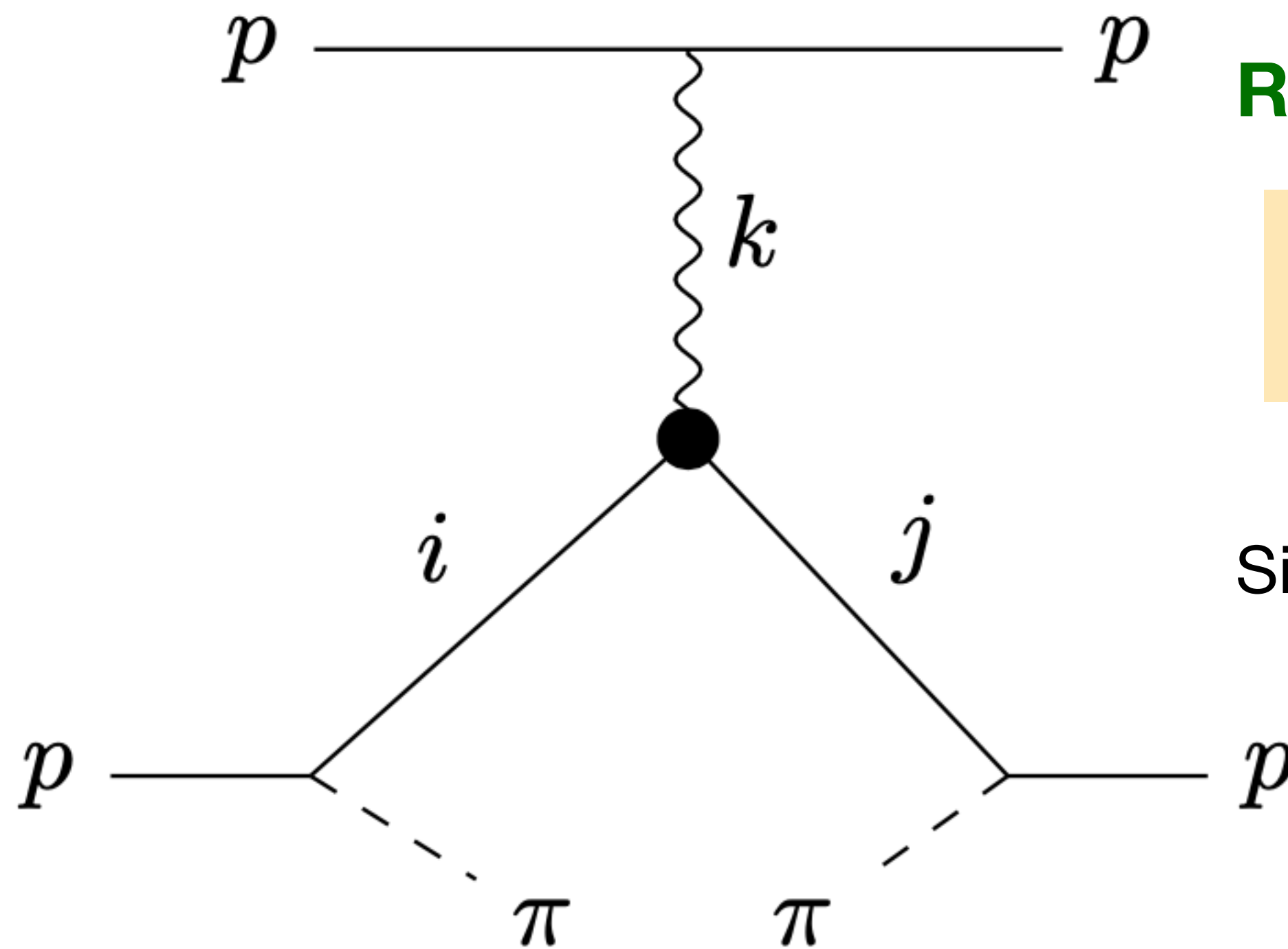
$$\alpha_{\Delta^*}(t) = -0.56 + 0.80t$$

Reggeized propagator

$$\mathcal{P}_i(t) = \alpha'_i \xi_i(t) \Gamma(J_i - \alpha_i(t)) (1 - x_F)^{-\alpha_i(t)}$$

Signature factor:
$$\xi_i(t) = \frac{1 + \tau_i \exp\{-i\pi(\alpha_i(t) - 0.5)\}}{2}$$

- Note that β should be real-valued function, so the phase of a Regge-pole contribution to A_N comes from the signature factor



Transverse single-spin asymmetry

Transverse single-spin asymmetry(TSSA)

- **TSSA**

TSSA is defined as a fraction of spin-dependent cross section and spin-averaged one:

$$A_N = \frac{d\Delta\sigma_{\perp}}{d\sigma} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

Spin-dependent differential cross section

$$d\Delta\sigma_{\perp} = \frac{1}{s} \sum_{i,j} \sum_{\lambda} \beta_{+\lambda}^i \beta_{-\lambda}^j 2\text{Im} \mathcal{P}_i \mathcal{P}_j^* \sum_k G_k^{ij}(t) \gamma_k^{pp}(0) \left(\frac{M_X^2}{s_0} \right)^{\alpha_k(0)}$$

Spin-averaged differential cross section

$$d\sigma = \frac{1}{s} \sum_{i,j} \sum_{\lambda} \beta_{+\lambda}^i \beta_{+\lambda}^j 2\text{Re} \mathcal{P}_i \mathcal{P}_j^* \sum_k G_k^{ij}(t) \gamma_k^{pp}(0) \left(\frac{M_X^2}{s_0} \right)^{\alpha_k(0)}$$

Parity invariance : $\beta_{\lambda_1 \lambda_2}^i = \eta_1 \eta_2 \eta_i (-)^{\lambda_1 - \lambda_2} \beta_{-\lambda_1, \lambda_2}$

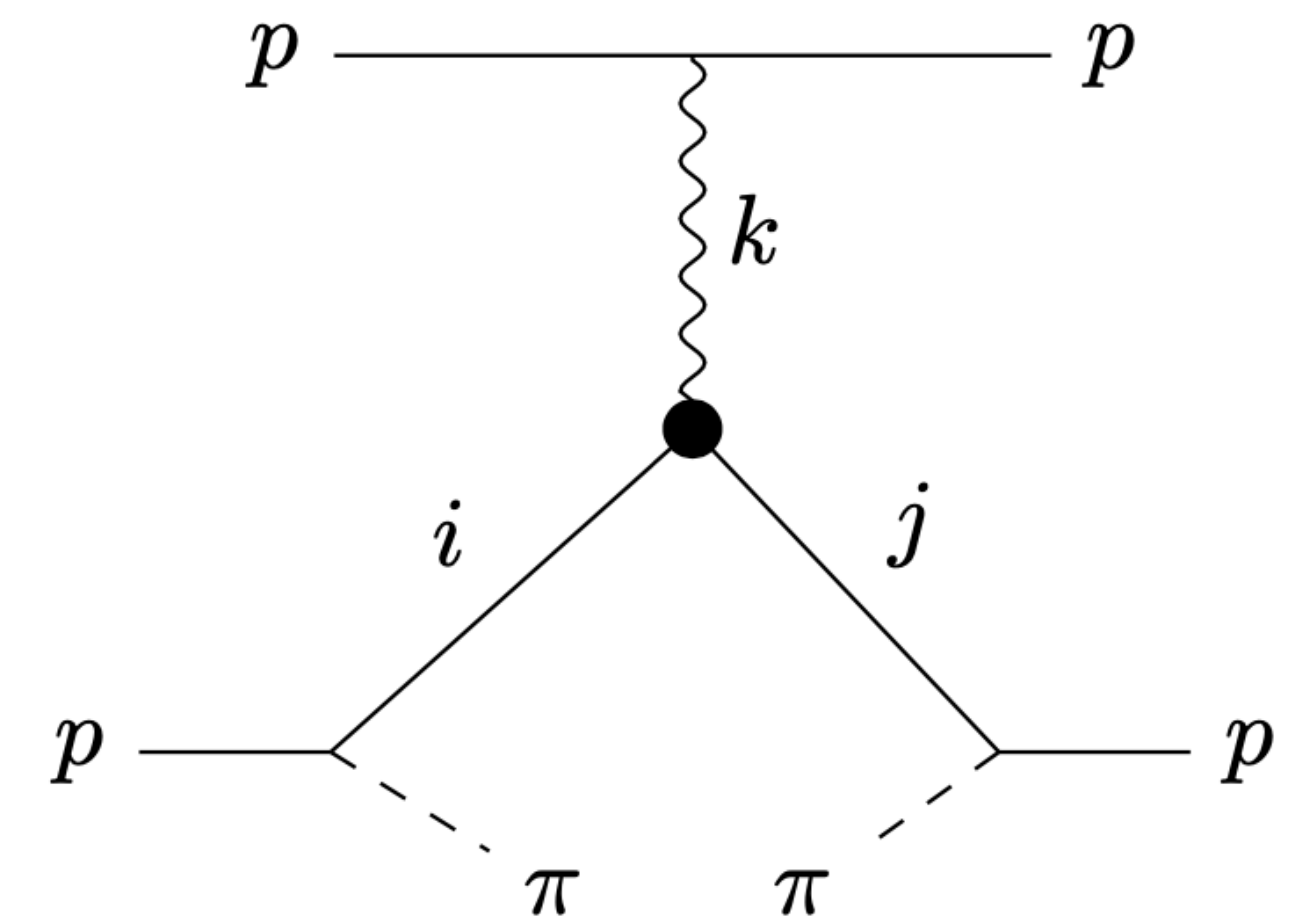
1. $d\sigma$ vanishes if k is an unnatural parity state.

unnatural parity states : ~~π, ω, a_1, \dots~~

natural parity states : **Pomeron**, ρ, a_2, \dots

$$\alpha_{\mathbb{P}}(0) \simeq 1, \quad \alpha_{\rho}(0) \approx 0.5$$

2. $d\Delta\sigma_{\perp}$ vanishes when i and j have opposite naturalities.



Spin-dependent cross section

$$d\Delta\sigma_{\perp} = d\Delta\sigma_{\perp}^N + d\Delta\sigma_{\perp}^U$$

$$= \frac{1}{s} \sum_{\lambda} \left[\beta_{+\lambda}^N \beta_{-\lambda}^{N*} \text{Im} \mathcal{P}_N \mathcal{P}_{N^*}^* G_{\mathbb{P}}^{NN^*}(t) + \beta_{+\lambda}^{\Delta} \beta_{-\lambda}^{\Delta*} \text{Im} \mathcal{P}_{\Delta} \mathcal{P}_{\Delta^*}^* G_{\mathbb{P}}^{\Delta\Delta^*}(t) \right] \gamma_{\mathbb{P}}^{pp}(0) \left(\frac{M_X^2}{s_0} \right)^{\alpha_{\mathbb{P}}(0)}$$

Spin-averaged cross section

$$d\sigma = \frac{1}{s} \sum_{\lambda} \left[\sum_i 2(\beta_{+\lambda}^i)^2 |\mathcal{P}_i|^2 G_k^{ii}(t) + \sum_{i \neq j} \beta_{+\lambda}^i \beta_{+\lambda}^j 2\text{Re} \mathcal{P}_i \mathcal{P}_j^* G_{\mathbb{P}}^{ij}(t) \right] \gamma_{\mathbb{P}}^{pp}(0) \left(\frac{M_X^2}{s_0} \right)^{\alpha_{\mathbb{P}}(0)}$$

- Parameterization of the triple-Regge couplings

Different forms of $G_{\mathbb{P}}^{ij}(t)$ are used for the diagonal and off-diagonal exchanges, respectively:

$$G_{\mathbb{P}}^{ij}(t) = \begin{cases} G_{\mathbb{P}}^{ij}(0)e^{-B_{\mathbb{P}}^{ij}|t|}, & \text{for } i = j \\ G_{\mathbb{P}}^{ij}(0)\frac{\sqrt{|t|}}{m_{\pi}}e^{-B_{\mathbb{P}}^{ij}|t|}, & \text{for } i \neq j \end{cases}$$

Redefine the parameters as

$$g_{\mathbb{P}}^{ij} \equiv G_{\mathbb{P}}^{ij}(0)/G_{\mathbb{P}}^{NN}(0), \quad b_{\mathbb{P}}^{ij} \equiv B_{\mathbb{P}}^{ij} - B_{\mathbb{P}}^{NN}$$

$$A_N = \frac{\sum_{\lambda} \left[\beta_{+\lambda}^N \beta_{-\lambda}^{N*} \text{Im } \mathcal{P}_N \mathcal{P}_{N^*}^* (\sqrt{|t|}/m_{\pi}) g_{\mathbb{P}}^{NN*} + \beta_{+\lambda}^{\Delta} \beta_{-\lambda}^{\Delta*} \text{Im } \mathcal{P}_{\Delta} \mathcal{P}_{\Delta^*}^* (\sqrt{|t|}/m_{\pi}) g_{\mathbb{P}}^{\Delta\Delta*} e^{-b_{\mathbb{P}}^{\Delta\Delta*}|t|} \right]}{\sum_{\lambda} \left[\sum_i (\beta_{+\lambda}^i)^2 |\mathcal{P}_i^2| g_{\mathbb{P}}^{ii} e^{-b_{\mathbb{P}}^{ii}|t|} + \sum_{i \neq j} \beta_{+\lambda}^i \beta_{+\lambda}^j \text{Re } \mathcal{P}_i \mathcal{P}_j^* (\sqrt{|t|}/m_{\pi}) g_{\mathbb{P}}^{ij} e^{-b_{\mathbb{P}}^{ij}|t|} \right]}$$

- Diagrammatic representation of the A_N

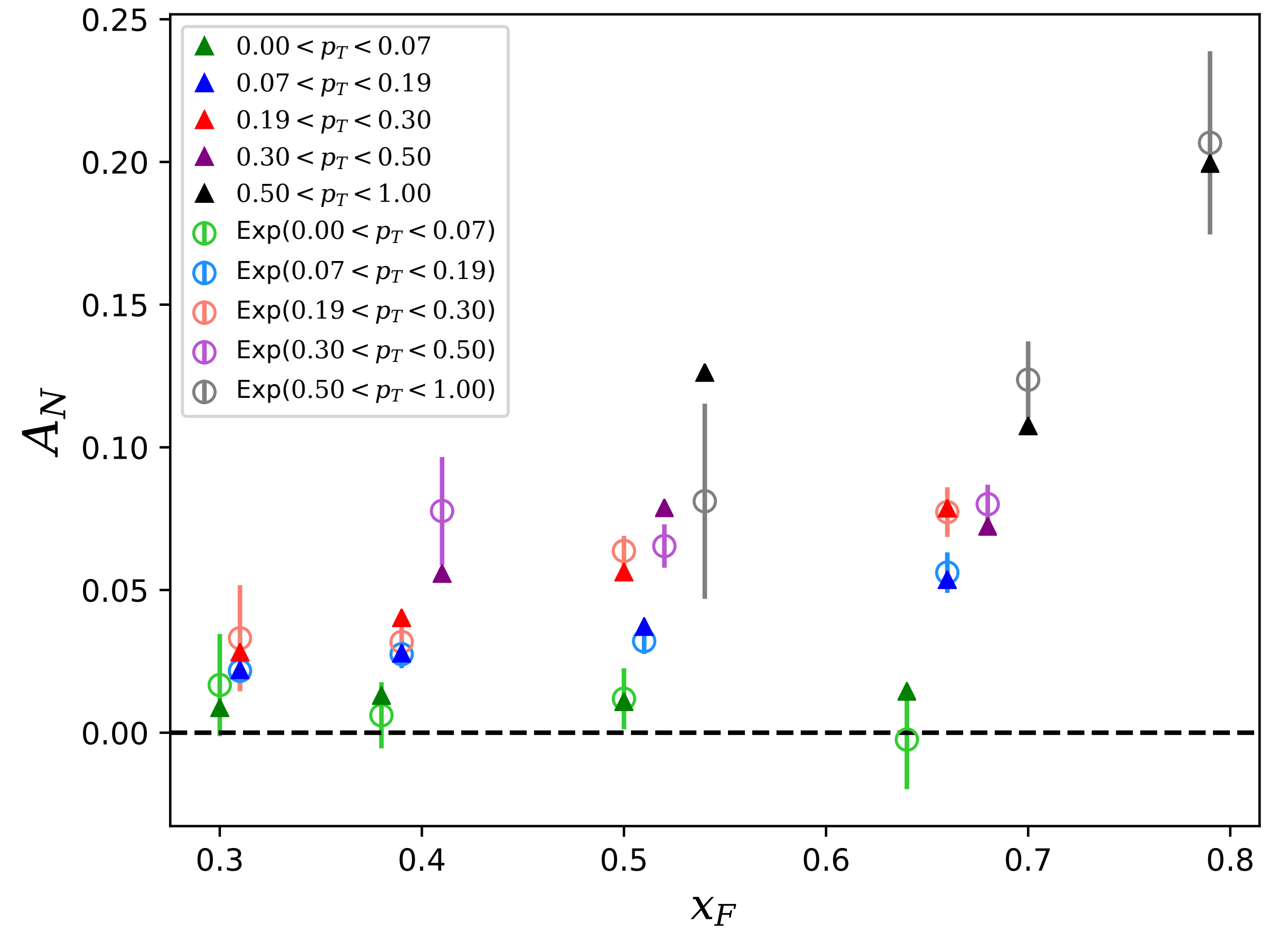
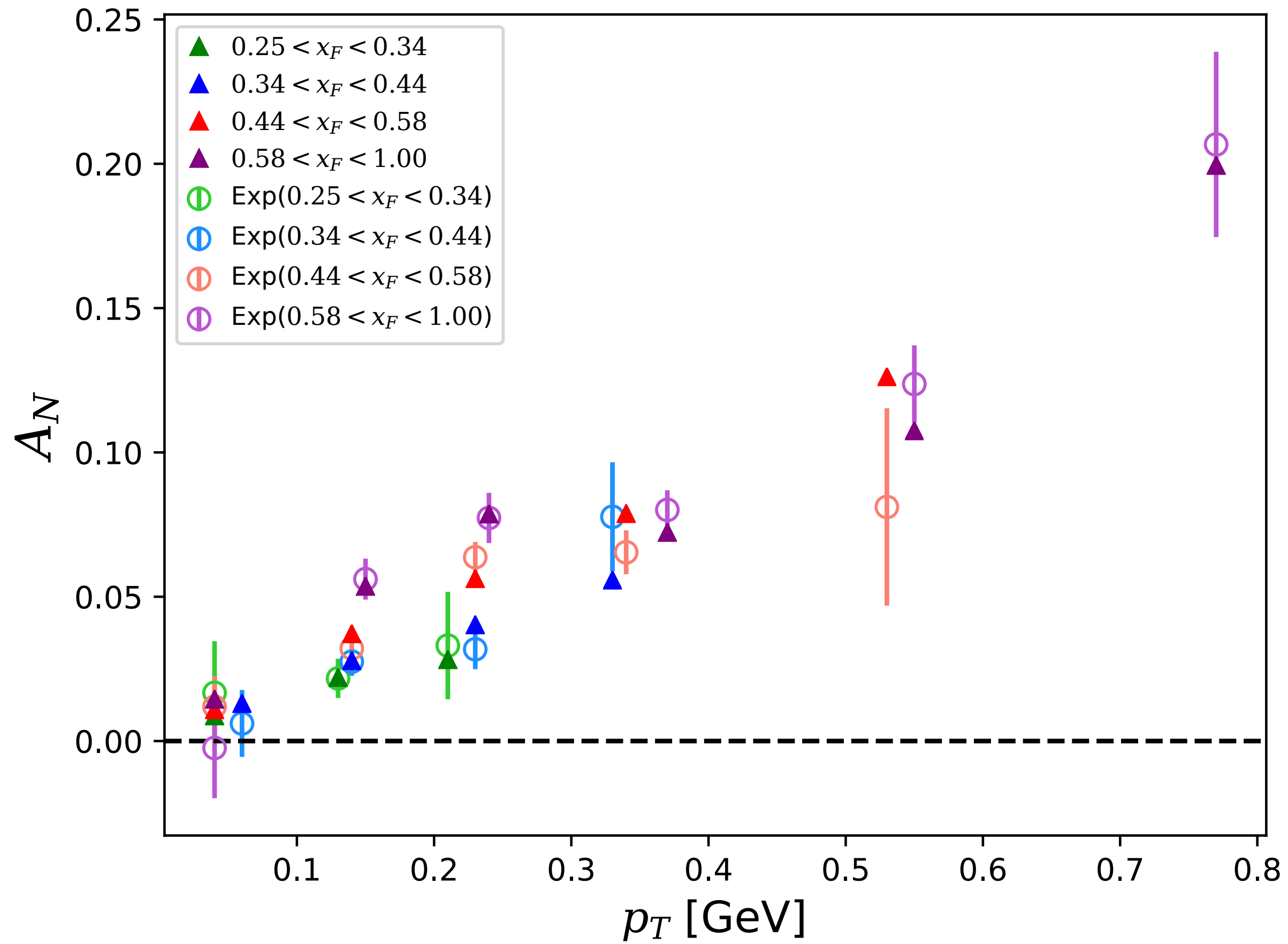
$$A_N = \frac{\sum_{\lambda} \left[\beta_{+\lambda}^N \beta_{-\lambda}^{N^*} \text{Im } \mathcal{P}_N \mathcal{P}_{N^*}^* (\sqrt{|t|}/m_{\pi}) g_{\mathbb{P}}^{NN^*} + \beta_{+\lambda}^{\Delta} \beta_{-\lambda}^{\Delta^*} \text{Im } \mathcal{P}_{\Delta} \mathcal{P}_{\Delta^*}^* (\sqrt{|t|}/m_{\pi}) g_{\mathbb{P}}^{\Delta\Delta^*} e^{-b_{\mathbb{P}}^{\Delta\Delta^*} |t|} \right]}{\sum_{\lambda} \left[\sum_i (\beta_{+\lambda}^i)^2 |\mathcal{P}_i|^2 g_{\mathbb{P}}^{ii} e^{-b_{\mathbb{P}}^{ii} |t|} + \sum_{i \neq j} \beta_{+\lambda}^i \beta_{+\lambda}^j \text{Re } \mathcal{P}_i \mathcal{P}_j^* (\sqrt{|t|}/m_{\pi}) g_{\mathbb{P}}^{ij} e^{-b_{\mathbb{P}}^{ij} |t|} \right]}$$

$$A_N = \frac{-2\text{Im} \left(\begin{array}{c} p \text{---} p \\ | \\ \text{wavy } \mathbb{P} \\ | \\ \bullet \\ / \quad \backslash \\ p \quad N^* \\ / \quad \backslash \\ p \quad p \\ | \quad | \\ \text{dashed } \pi \quad \pi \end{array} + \begin{array}{c} p \text{---} p \\ | \\ \text{wavy } \mathbb{P} \\ | \\ \bullet \\ / \quad \backslash \\ p \quad \Delta \\ / \quad \backslash \\ p \quad p \\ | \quad | \\ \text{dashed } \pi \quad \pi \end{array} + i \leftrightarrow j \right)}{\sum_{i,j} \text{Re} \begin{array}{c} p \text{---} p \\ | \\ \text{wavy } \mathbb{P} \\ | \\ \bullet \\ / \quad \backslash \\ i \quad j \\ / \quad \backslash \\ p \quad p \\ | \quad | \\ \text{dashed } \pi \quad \pi \end{array}}$$

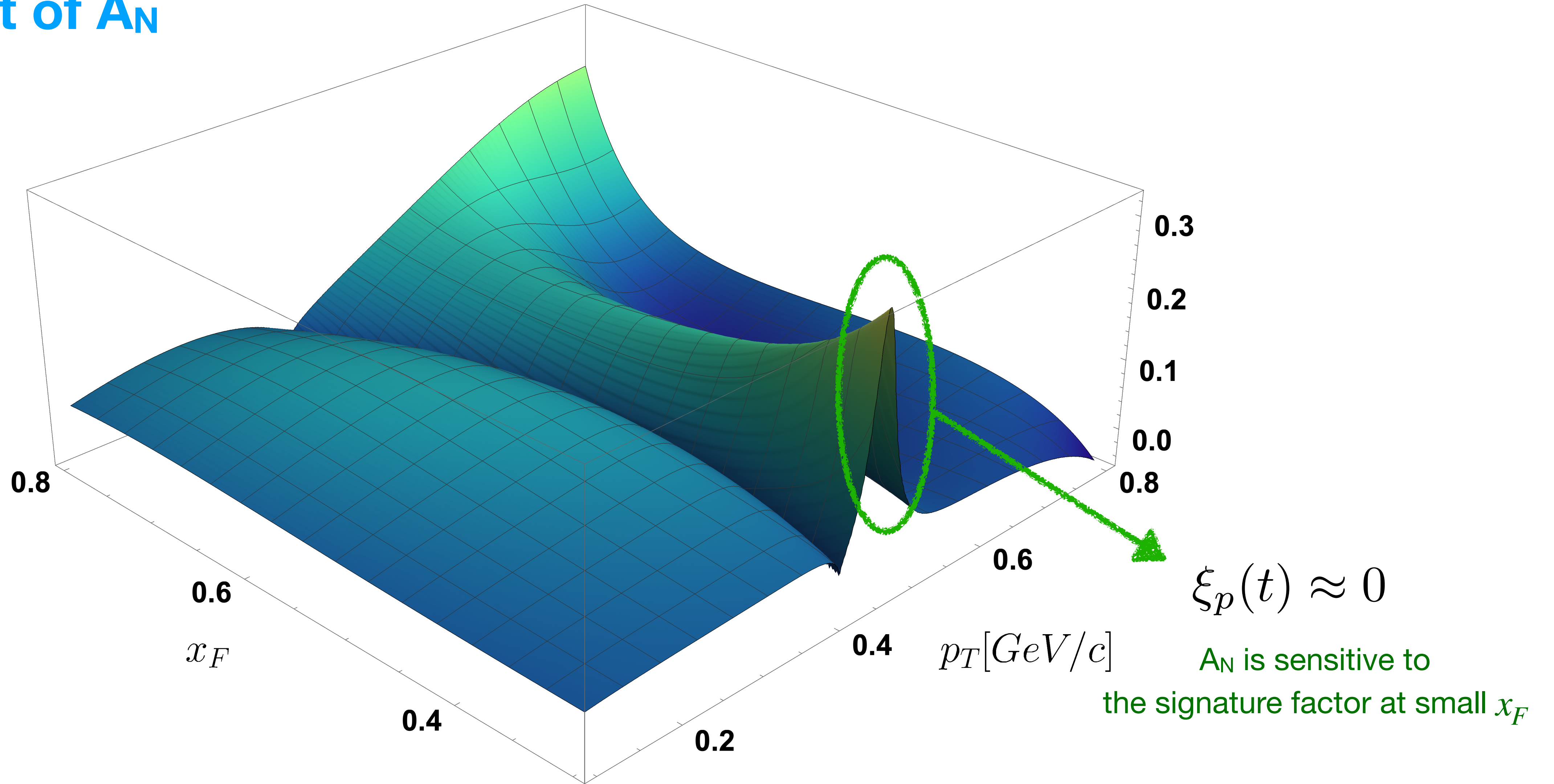
Results

Results

	g_{P}^{ij}	b_{P}^{ij} [GeV $^{-2}$]
NN^*	0.028	0.2
$\Delta\Delta^*$	-0.018	0
N^*N^*	0.10	0
$\Delta\Delta$	0.022	0
$\Delta^*\Delta^*$	0.079	0



3D Plot of A_N



Summary

Summary

- We investigated the TSSA for very forward neutral pion through the Reggeon exchange processes.
- The differential cross-section for $pp^{\uparrow} \rightarrow \pi^0 X$ is approximated to the triple-Regge exchange process.
- Our results match the RHICf data of both transverse momentum and x_F distribution quite well.
- We found that diffractive reactions are a main source of the $\pi^0 A_N$ in the very forward direction.
- $pp^{\uparrow} \rightarrow \pi^{\pm} X, \eta X, \Lambda X, \Delta X \dots$

Thank you

Back up

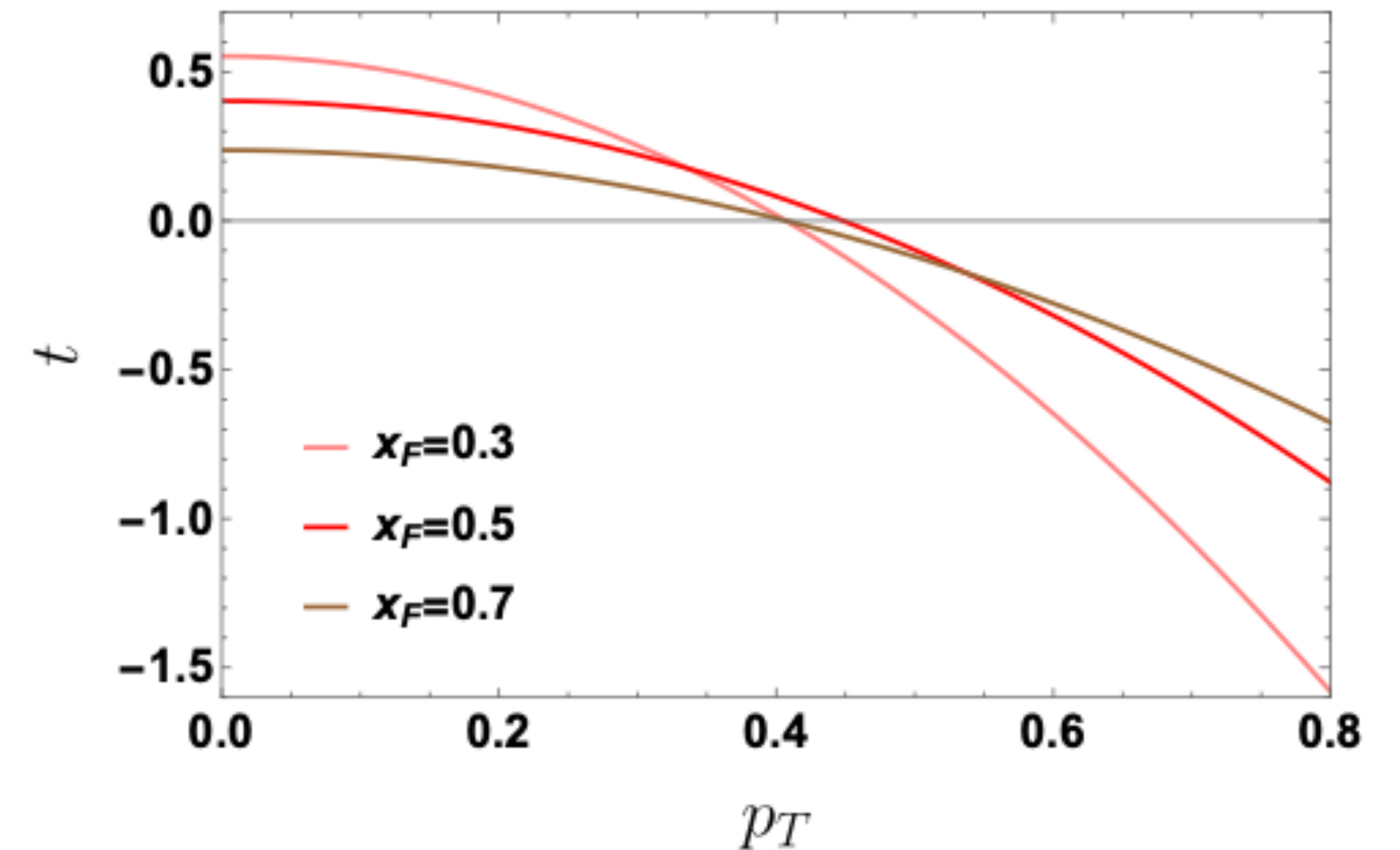
Kinematics

We are going to ignore the pion mass m_3 since it is negligible in the calculation. First, we expand the following variables up to $(m_N^2/s)^{1/2}$ order.

$$\begin{aligned}
 p_z &= \sqrt{\frac{s}{4} - m_N^2} \approx \frac{\sqrt{s}}{2} - \frac{m_N^2}{\sqrt{s}}, & p'_z &= p_z x_F \approx \frac{\sqrt{s}}{2} x_F - \frac{m_N^2}{\sqrt{s}} x_F, \\
 E_3 &= \sqrt{p_z'^2 + \mathbf{p}_T^2} \approx \frac{\sqrt{s}}{2} x_F - \frac{1}{x_F} \frac{m_N^2}{\sqrt{s}} + \frac{\mathbf{p}_T^2}{\sqrt{s}}.
 \end{aligned}
 \tag{5}$$

The squared momentum transfer is then

$$\begin{aligned}
 t &= (p_1 - p_3)^2 \\
 &= (E_1 - E_3)^2 - \mathbf{p}_T^2 - (p_z - p'_z)^2 \\
 &= \frac{s}{4}(1 - x_F)^2 + (1 - x_F) \left(m_N^2 x_F - \frac{\mathbf{p}_T^2}{x_F} \right) - \mathbf{p}_T^2 - \left[\frac{s}{4}(1 - x_F)^2 - m_N^2(1 - x_F)^2 \right] \\
 &= (1 - x_F)m_N^2 - \frac{\mathbf{p}_T^2}{x_F}.
 \end{aligned}$$



Regge poles

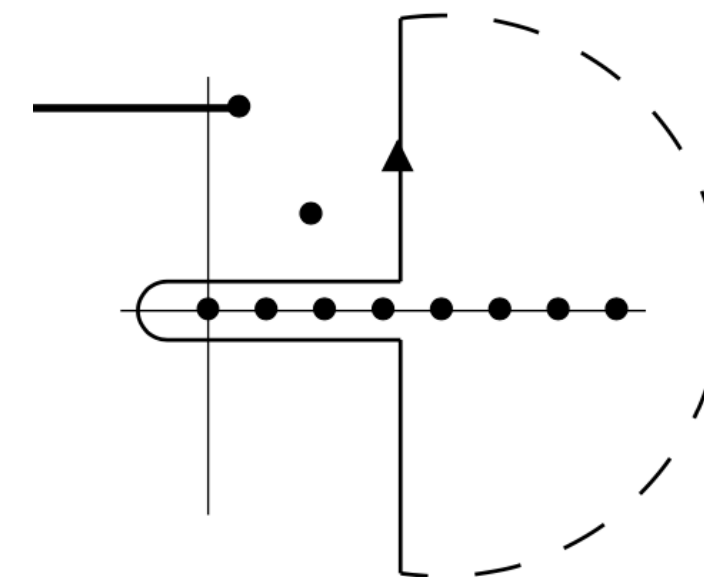
The t -channel partial-wave series for scattering amplitude: $A(s, t) = 16\pi \sum_{l=0}^{\infty} A_l(t) P_l(z_t)$

Ex) Equal-mass scattering: $z_t = 1 + \frac{2s}{t - 4m^2} \rightarrow P_l(z_t) \sim s^l$

This series is a correct representation of scattering in the physical region of the t -channel, where $t > 4m^2, s < 0$, it is irrelevant for high-energy s -channel scattering.

- Analytic continuation to the complex ℓ -plane

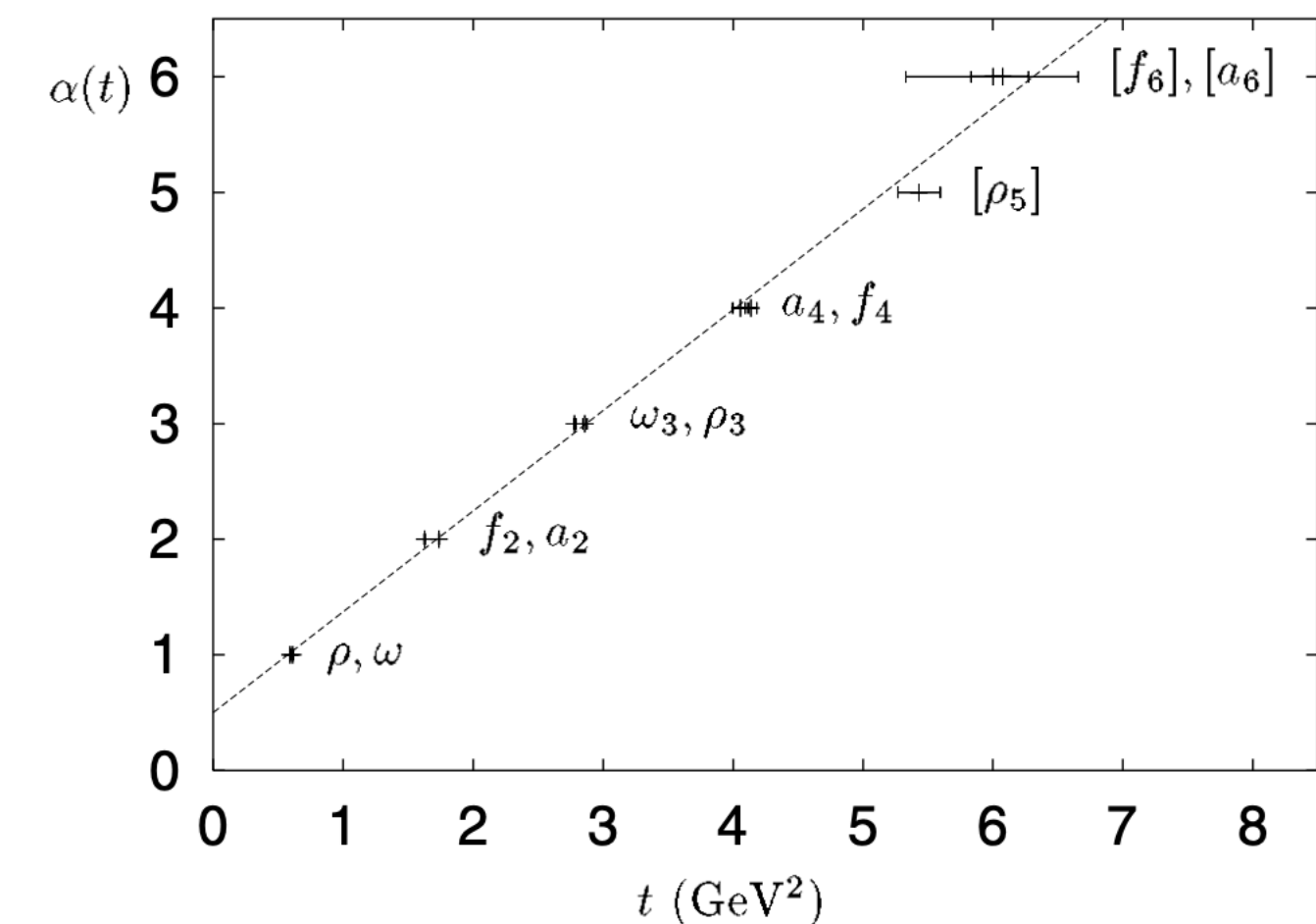
$$A(s, t) = 16\pi \sum_{l=0}^{\infty} A_l(t) P_l(z_t) \rightarrow 8\pi i \int_C dl A(l, t) \frac{P_l(-z_t) + P_l(z_t)}{\sin \pi l}$$



The singularities of $A(l, t)$ are **poles** whose locations at $l = \alpha(t)$

$$A(s, t) \sim s^{\alpha(t)}$$

The functions $\alpha(t)$ are called **Regge trajectories**. The particles on the same trajectory give a **collective** energy dependence



interferences and pole contributions

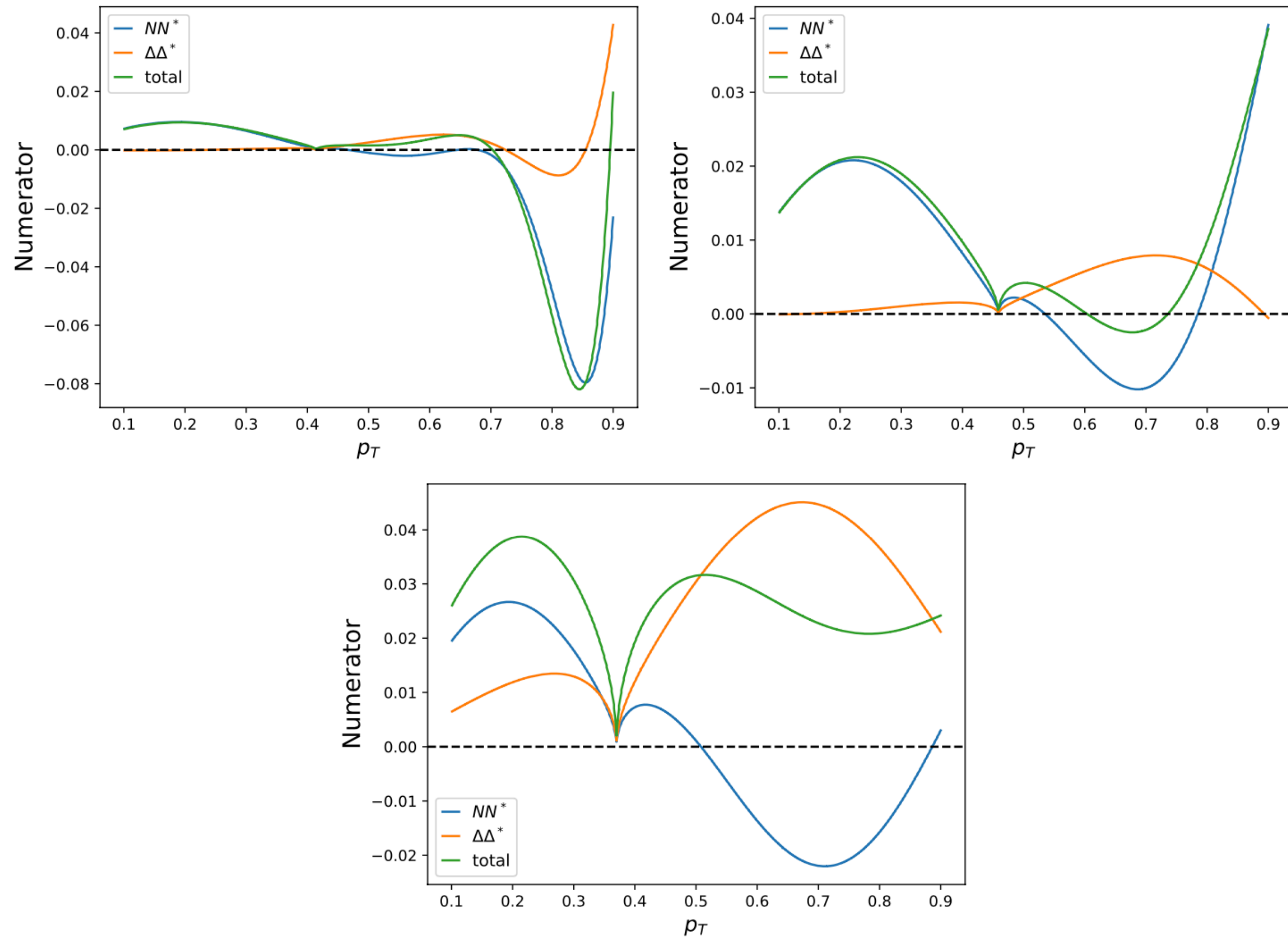


FIG. 7. Numerators when $x_F = 0.3, 0.5, 0.8$.

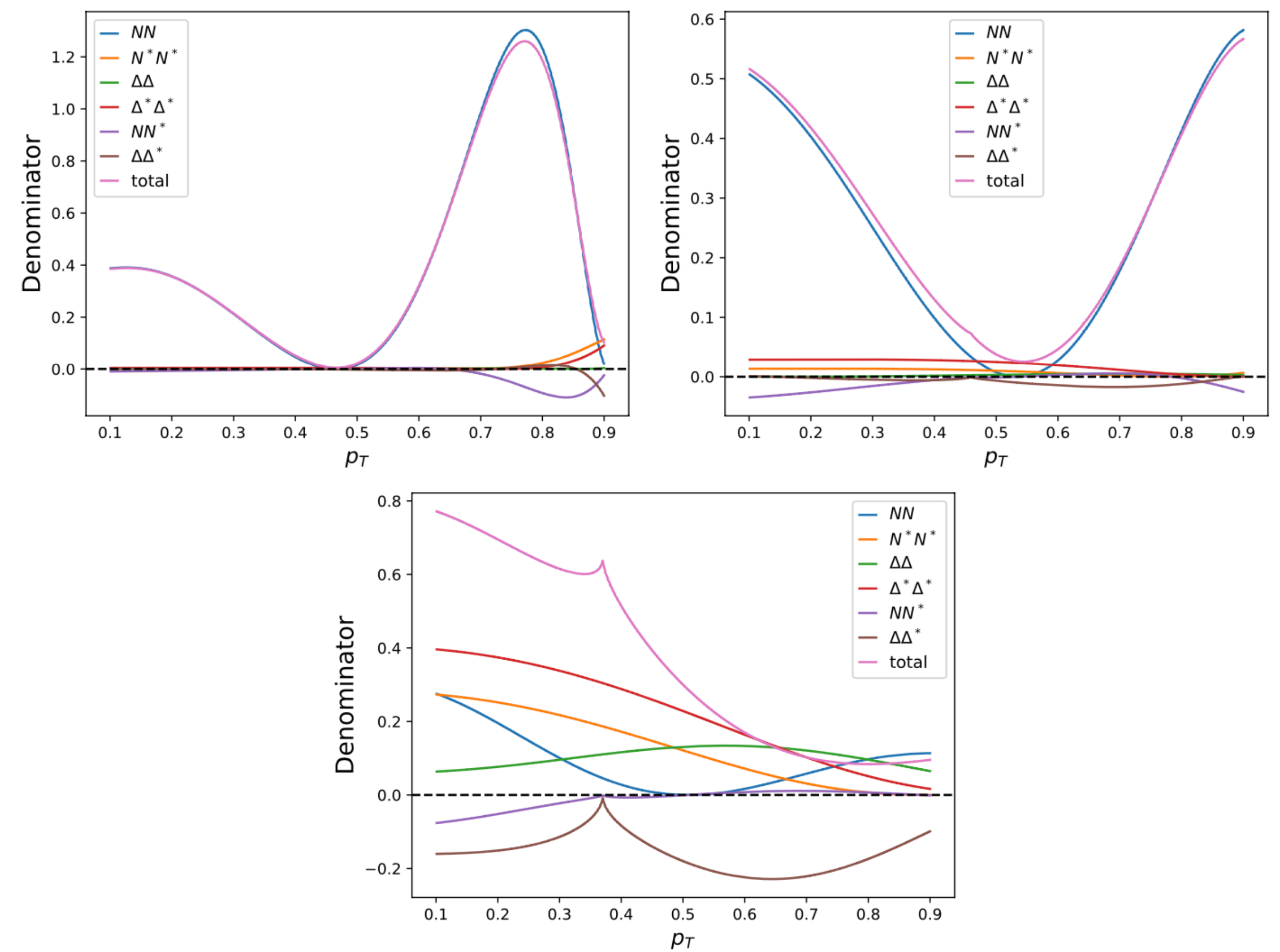


FIG. 8. Denominators when $x_F = 0.3, 0.5, 0.8$.