# Study of the radial excitation of hadron



#### Ahmad Jafar Arifi

Inha HTG Workshop: Modern issues in hadron physics

## Contents



## **Radial excitation**

#### **Radial excitation**



#### • Radial excitation

- -> a.k.a. Breathing mode,
- -> The same spin-parity with the g. s.

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- –> Has "nodal" structure,
- Hadron: QCD
  - -> E.g. Roper resonance
  - -> Puzzle: mass, decay, etc.
  - –> Similar behaviors



http://be.nucl.ap.titech.ac.jp/cluster/



#### Hadrons: 1S and 2S state baryon





<> A broad resonance ( $\Gamma = 72$  MeV). Orthogonality?



#### Methodology



**Internal structure:** 

- -> Production, Cross section, polarization observables, etc
- -> **Spectrum**, Mass, mass splitting, etc
- -> **Decay**, Decay rates, branching fraction, etc

Resonance: mass, decay rate, spin, parity, isospin, etc

# **Three-body decay**Dalitz plot

Spin-parity of  $\Lambda_c(2765)$ 



#### Decay of $\Lambda_b^*(6072)$



Arifi, et. al. PRD101 (R) 111502 (2020)

## **Comparison with LHCb analysis**



• Background shape is from the kinematical reflection.

non-resonant contribution is relatively large.

# **Light-front quark model** Mass spectra and wave functions

#### Problem of Non-rel Quark model

• Strong decay of  $\Lambda_b(6072)$ 



[1] Wave function -> HO (gaussian) [2] Quark-pion interaction

$$\mathcal{L}_{\pi q q} = \frac{g_A^q}{2f_\pi} \bar{q} \gamma^\mu \gamma_5 \vec{\tau} q \cdot \partial_\mu \vec{\pi}$$

--> Nonrelativistic expansion

$$\propto g(\sigma \cdot q - \frac{\omega}{2m} \sigma \cdot (p_i + p_f))$$

$$<> \Lambda_b(6072) \rightarrow \Gamma \sim 5 \text{ MeV} \text{ (narrow)}$$
$$\Gamma_{exp} = 72 \text{ MeV}$$

- Orthogonality of w.f.?
- Relativistic effect?

#### Light-front quark model

- Constituent quark model
- Light-front dynamics

Hadrons:  $q\bar{q}$ , qqq

[1] Trial wave function
 —> Gaussian (H.O. basis).

[2] Effective potentials -> Cornel potential, etc

$$\begin{split} \phi_{1S}(x,\mathbf{k}_{\perp}) &= \frac{4\pi^{3/4}}{\beta^{3/2}} \sqrt{\frac{\partial k_z}{\partial x}} e^{-\vec{k}^2/2\beta^2}, \\ \phi_{2S}(x,\mathbf{k}_{\perp}) &= \frac{4\pi^{3/4}}{\sqrt{6}\beta^{7/2}} \left(2\vec{k}^2 - 3\beta^2\right) \sqrt{\frac{\partial k_z}{\partial x}} e^{-\vec{k}^2/2\beta^2}, \end{split}$$

[3] Variational Parameters  $\beta$ -> Fixed from mass spectra

$$M_{q\bar{q}} = \left\langle \Psi \right| \left[ H_0 + V_{q\bar{q}} \right] \left| \Psi \right\rangle$$

$$\frac{\partial \left\langle \Psi \right| \left[ H_0 + V_0 \right] \left| \Psi \right\rangle}{\partial \beta} = 0$$

#### Problem!!

-> Using pure HO basis
 -> Can't explain 2S decay constant.

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#### Problem of decay constant

<> Decay constants of Upsilon (Exp)  $-> f(\Upsilon(1S)) = 689 \text{ MeV}$  $-> f(\Upsilon(2S)) = 497 \text{ MeV}$ 

<> If we use 2S HO wave function —> If we use the same  $\beta$  parameters —> always f( $\Upsilon(2S)$ ) > f( $\Upsilon(1S)$ )

<> To solve the problem:

- –> Modify the wave function
- -> Simply use different  $\beta$  parameters

#### Wave function of 1S and 2S states

<> Minimal mixing

- -> The same  $\beta$  for 1S and 2S states
- -> keep orthogonality
- -> doesn't change 1S WF





<> Only need a small mixing  $-> \theta = 12^{\circ}$  $-> |\cos \theta|^2 = 95.7 \%$ ,  $|\sin \theta|^2 = 4.3 \%$ .

- <> Huge impact to observables.
  - -> Mass spectra,
  - -> Decay constants,
  - -> Charge radii, etc

#### əpctp

#### Variational and potential parameters





<> In the mixed scenario:

- -> use the same quark mass.
- $\rightarrow \beta$  systematically decrease.
- -> Potential look the same.

$$V_{q\bar{q}} = a + br - \frac{4\alpha_s}{3r}$$

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#### Mass spectra and gaps





<> Competing contribution: —> Confinement int

$$\Delta M_{conf} \propto \frac{1}{\beta}$$

-> Coulomb int  $\Delta M_{colmb} \propto \beta$  <> Hyperfine int -> Small but, very important -> Mixing is needed ->  $\Delta M_P > \Delta M_V$   $\Delta M_{hyp} \propto (S_q \cdot S_{\bar{q}})(\cos 2\theta - 2\sqrt{6} \sin 2\theta)$ ->  $\theta_c \approx 6^\circ$ 

#### **Decay constant**



<> Related to the wave function at origin.

<> Current component independent calculation.

#### **Charge radius**



# Global analysis

model parameters & error analysis

#### LWFW + Hamiltonian

<> Model parameters:

- Quark mass
- Beta parameters -> via variational analysis
- Potential parameters
- <> Parameter determination: Previous work —> trial-error analysis Plan —> Global fit
- <> Fitting method (Frequentist) —> Single fit (Minuit, Minos, Hesse) —> Sampling (Monte-Carlo Bootstrap)



#### Monte-Carlo Bootstrap



<> Based on Monte-Carlo approach -> Sampling -> Not a single fit -> Very expensive

> <> Example: JPAC collab

VII sheet

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- pole determination.
- 10.000 samples.

## Summary

- Numerous discoveries of hadron resonances
  - -> Model extension to excited state is important
  - -> Study of various flavor quark content
  - -> Radial excitation -> interesting features
- Multi-body decays
  - -> Extraction of resonance parameters
- Light-front quark model
  - –> Application to the structure and properties
  - -> LFWF, Hamiltonian, Higher Fock state
  - -> Current independent issue
  - -> Extension to the multi-quark states
- Global analysis/fit
  - -> Monte-Carlo Bootstrap/ Bayesian
  - -> Robust error analysis and parameter determination



# Thank you very much

https://ajarifi.github.io

# Chiral quark model Strong decay

#### **Strong decay of** $\Lambda_b(6072)$

• Chiral quark model (Non-relativistic)



[1] Wave function -> HO (gaussian) [2] Quark-pion interaction  $\mathcal{L}_{\pi q q} = \frac{g_A^q}{2f_\pi} \bar{q} \gamma^\mu \gamma_5 \vec{\tau} q \cdot \partial_\mu \vec{\pi}$ 

--> Nonrelativistic expansion

$$\propto g(\sigma \cdot q - \frac{\omega}{2m} \sigma \cdot (p_i + p_f))$$

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$$\Gamma_{exp} = 72 \text{ MeV}$$

- Orthogonality of w.f.?
- Relativistic effect?

#### **Relativistic correction**

<> Foldy-Wouthyson-Tani trans. $H = H(1/m^0) + H(1/m) + H(1/m^2) +$							
				NR	RC		
negligible	small		large				
$\left< \Sigma_b \right  1 \left  \Lambda_b \right>$	$\propto q^2  \left< \Sigma_b \right  p_i \left  \Lambda_b \right>$	$\propto q \qquad \Big\langle \Sigma_b$	$\left  p_i^2 \left  \Lambda_b \right\rangle \propto a^2$				
	NR		RC				
State	Multiplet	Channel	$\Gamma_{\rm NR}$	$\Gamma_{\rm NR+RC}$	$\Gamma_{\rm Exp}$		
$\overline{ egin{array}{c} \Sigma_b(5810)^+ \ \Sigma_b(5830)^+ \end{array} }$	$\Sigma_b(1S,1/2(1)^+)\ \Sigma_b(1S,3/2(1)^+)$	$egin{array}{c} \Lambda_b \pi \ \Lambda_b \pi \end{array}$	11.9–12.3 20.4–21.4	0.62–5.11 1.08–8.80	$\begin{array}{c} 4.83 \pm 0.31 \\ 9.34 \pm 0.47 \end{array}$	~( — )	
$\Lambda_b(5912)^0 \ \Lambda_b(5920)^0$	$\Lambda_b(1P_\lambda, 1/2(1)^-)$ $\Lambda_b(1P_\lambda, 3/2(1)^-)$	$\Sigma_b \pi  onumber \Sigma_b^* \pi$	0.001-0.003 0.004-0.008	0.001-0.003 0.004-0.009	< 0.25 < 0.19	~(0)	
$\Lambda_b(6072)^0$	$\Lambda_b(2S_{\lambda\lambda},1/2(0)^+)$	$rac{\Sigma_b\pi}{\Sigma_b^*\pi}$	0.72–2.17 1.08–3.00	4.97–20.8 7.81–31.5			
		Sum	1.80–5.17	12.8–52.3	$72 \pm 11$	~(+)	

<> Large decay width of 2S states.

<> Relativistic effect is important. -> light-front quark model

#### Finding the missing partner: $\Xi(2S)$



- Can be studied in J-PARC experiment
- ΔM ~ 500 MeV.
- Several states in QM, 1/2+, 1/2-, 3/2-
- $\Xi(1820)$  in PDG -> 3/2-
- Exp data —> from 1980's

#### How can we find it?

- -> Decay pattern
- -> Chiral quark model



#### Exp status of $\Xi(1820)$

$$\Gamma_{\rm pdg} = 24 \pm 5 \,\,{\rm MeV}$$

LHCb, 2021	$\Gamma = 36$	$\pm 4 \text{ MeV}$
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- **BES III**, 2020  $\Gamma = 17 \pm 15 \text{ MeV}$
- **BES III**, 2015  $\Gamma = 54.4 \pm 15.7 \text{ MeV}$
- Biagi, 1987  $\Gamma = 24.6 \pm 5.3 \text{ MeV}$
- Biagi, 1981  $\Gamma = 72 \pm 20 \text{ MeV}$
- Briefel, 1976  $\Gamma = 99 \pm 57 \text{ MeV}$
- Gay, 1976  $\Gamma = 21 \pm 7 \text{ MeV}$
- Apsel, 1970  $\Gamma = 64 \pm 23 \text{ MeV}$

Inconsistencies among the data

Some experiments  $\rightarrow J^P = 3/2^-$ . Hypothesis: other nearby resonance?





#### $\Xi(1820)$ in the quark model



Signature of the 1/2+ state:

- -> Large width
- $\rightarrow$  Dominant  $\Sigma K$  channel
- -> Ratio of  $\Gamma(\Xi\pi)/\Gamma(\Xi^*\pi)$  ~ 0.5

#### Coupling constants in QM

-> Production of  $\Xi(2S)$  in J-PARC

#### E.m. Form factor



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apctp





#### Ξ(1690), ??

 $\Gamma_{\rm exp} < 30 {\rm MeV}$ 

		$\Xi\pi$	$\Xi^*\pi$	$\Lambda K$	$\Sigma K$	Sum
70, <sup>2</sup> 10,1,1,1/2 <sup>-</sup>	NR	2.0	0.002	4.1	0.6	6.7
	NR+RC	2.7	0.002	4.3	0.6	7.6
70, <sup>2</sup> 8,1,1,1/2 <sup>-</sup>	NR	2.0	0.002	16.4	9.4	27.9
	NR+RC	2.7	0.002	17.3	9.4	29.5
70,48,1,1,1/2-	NR	32.3	0.0006	16.4	2.4	51.1
	NR+RC	42.8	0.0004	17.3	2.4	62.5
56,28,1,1,3/2-	NR	0.3	1.0	0.2	~0	1.8
	NR+RC	0.2	1.1	0.2	~0	1.6
56,28,2,0,1/2+	NR	0.2	0.02	0.4	0.02	0.7
	NR+RC	2.3	0.3	1.3	0.1	3.9

 $egin{aligned} R^{\Sigma^+K^-}_{\Lambda^0ar{K}^0} &= \ 0.50 \ R^{\Xi\pi}_{\Sigmaar{K}} &< 0.09 \ R^{\Xi^*\pi}_{\Sigmaar{K}} &< 0.06 \end{aligned}$ 

• The most suitable state: 1/2-, Not possible to assign it as 1/2+

$\Xi(1620)$		$\Gamma_{exp} = 40 \pm 15 \text{ MeV}$				
			$\Xi\pi$	$\Lambda K$	Sum	
$70^{48} 1 1 1/2^{-}$	NR		24	6	30	
/0, 0,1,1,1/2 /	NR+RC		29	6	35	

• Only the 1/2- state that have a sizable width.

#### **Relativistic corrections**

• Foldy-Woutysen-Tani (FWT) transformation

 $H=eta m+\mathcal{O}+\mathcal{E},$  Remove large-small component (odd operator).

- Unitary transformation
  - $\Psi' = e^{iS} \Psi,$  $S = -rac{ieta \mathcal{O}}{2m}.$

- Dirac equation

$$\begin{split} H\Psi &= i \frac{\partial \Psi}{\partial t}, \\ H' &= e^{iS} \left( H - i \frac{\partial}{\partial t} \right) e^{-iS}. \end{split}$$

- Expanding the Hamiltonian

$$H' = H + i[S, H] - \frac{1}{2}[S, [S, H]] - \frac{i}{6}[S, [S, [S, H]]] - \dot{S} - \frac{i}{2}[S, \dot{S}] + \frac{1}{6}[S, [S, \dot{S}]] + \dots$$

- We obtain

$$H' = \beta m + \mathcal{E}' + \mathcal{O}', \qquad \mathcal{E}' = \mathcal{E} + \frac{1}{2m}\beta\mathcal{O}^2 - \frac{1}{8m^2}[\mathcal{O}, [\mathcal{O}, \mathcal{E}]] - \frac{i}{8m^2}[\mathcal{O}, \dot{\mathcal{O}}],$$
$$\mathcal{O}' = \frac{\beta}{2m}[\mathcal{O}, \mathcal{E}] + i\frac{\beta\dot{\mathcal{O}}}{2m} - \frac{\mathcal{O}^3}{3m^2},$$
$$39 \qquad \qquad \mathbf{\partial}\mathbf{OCC}$$

#### **Pion interaction**

• FWT transformation gives a correction order by order  $H = H(1/m^{0}) + H(1/m) + H(1/m^{2}) + \dots$ NR RC • Hamiltonian Pseudovector type  $H = \beta m + \vec{\alpha} \cdot \vec{p} + g \ \partial_t \pi \gamma_5 + g \ \vec{\alpha} \cdot \vec{\nabla} \pi \gamma_5,$  $= \vec{\alpha} \cdot \vec{n} + a \partial_{\mu} \pi \gamma_{\tau}$ 

$${\cal L}_{\pi q q} = - rac{g^q_A}{2 f_\pi} ar q \gamma_\mu \gamma_5 ec au q \cdot \partial^\mu ec \pi$$

$$H = eta m + \mathcal{O} + \mathcal{E}, \qquad egin{array}{cc} \mathcal{O} = ec{lpha} \cdot ec{p} + g \; \partial_t \pi \gamma_5 \ \mathcal{E} = g \; ec{lpha} \cdot ec{
abla} \pi \gamma_5. \end{array}$$

• Leading term up to 1/m

$$H_{NR} = g \left[ \boldsymbol{\sigma} \cdot \boldsymbol{q} - \frac{\omega_{\pi}}{2m} \boldsymbol{\sigma} \cdot \left( \boldsymbol{p}_i + \boldsymbol{p}_f \right) \right]$$

the same as obtained by non-rel reduction.

• The correction up to  $1/m^2$ 

$$\begin{split} H_{RC} &= \frac{g}{8m^2} \bigg[ m_\pi^2 \boldsymbol{\sigma} \cdot \boldsymbol{q} - 2\boldsymbol{\sigma} \cdot (\boldsymbol{p}_i + \boldsymbol{p}_f) \times (\boldsymbol{q} \times \boldsymbol{p}_i) \bigg] \text{ E. M.} \text{ $-->$ spin-orbit coupling} \\ & \text{ important term} \end{split}$$

#### **Ground state:** $\Sigma_c \to \Lambda_c \pi$

Ground state	NR	NR + RC	Exp.
$\Sigma_c(2455): 1/2^+$	4.27 - 4.34	0.35 - 1.95	1.89 MeV
$\Sigma_c(2520): 3/2^+$	29.8 - 31.4	2.70 - 14.1	14.78 MeV
	2 x	reduced	

- Suppression of  $g_A^q$  coupling constant.
- Dominant term is  $(\sigma \cdot q)$  term.
- The overlap of the wave functions is unity in wave-length limit.

$$\begin{array}{c|c|c|c|c|c|} & \left< \Lambda_c \right| 1 & \left| \Sigma_c \right> \propto 1 & \left< \Lambda_c \right| p_i & \left| \Sigma_c \right> \propto q & \left< \Lambda_c \right| p_i^2 & \left| \Sigma_c \right> \propto a^2 \\ \hline & \text{large} & \text{small} & & \text{large} \end{array}$$

• The relativistic correction has opposite sign.

$$H_{NR} = g \left[ \boldsymbol{\sigma} \cdot \boldsymbol{q} - \frac{\omega_{\pi}}{2m} \boldsymbol{\sigma} \cdot \left( \boldsymbol{p}_i + \boldsymbol{p}_f \right) \right] \ H_{RC} = \frac{g}{8m^2} \left[ m_{\pi}^2 \boldsymbol{\sigma} \cdot \boldsymbol{q} - 2\boldsymbol{\sigma} \cdot \left( \boldsymbol{p}_i + \boldsymbol{p}_f \right) \times \left( \boldsymbol{q} \times \boldsymbol{p}_i \right) \right]$$

#### Negative parity state: $\Lambda_c^* \to \Sigma_c \pi$

Negative parity state	NR	NR + RC	Exp.
$\Lambda_c(2595): 1/2^-$	1.35 - 3.16	1.36 - 3.20	2.6 MeV
$\Lambda_c(2625): 3/2^-$	0.15 - 0.33	0.09 - 0.26	< 0.97 MeV

- In this case, the momentum is almost zero.
- The dominant term is  $(\sigma \cdot p_i)$  term.

$$\begin{aligned} \left\langle \Sigma_{c} \right| 1 \left| \Lambda_{c} \right\rangle \approx q & \left\langle \Sigma_{c} \right| p_{i} \left| \Lambda_{c} \right\rangle \approx a & \left\langle \Sigma_{c} \right| p_{i}^{2} \left| \Lambda_{c} \right\rangle \approx q \ a^{2} \\ \text{small} & \text{dominant} & \text{small} \end{aligned}$$

- The dominance of the S-wave decay.
- The relativistic correction is rather small.

**Roper-like state:**  $\Lambda_c^* \to \Sigma_c \pi$ 

Roper-like state	NR	NR + RC	Exp.
$Λ_c(2765): 1/2^+, λλ$	2 - 5	11 - 49	73 MeV
$\Lambda_{c}(3136): 1/2^{+}, \rho\rho$	11 - 123	314 - 1799	

• The overlap is orthogonal in the long-wavelength limit.

$$\begin{aligned} \left\langle \Sigma_{c} \right| 1 \left| \Lambda_{c} \right\rangle \approx q^{2} & \left\langle \Sigma_{c} \right| p_{i} \left| \Lambda_{c} \right\rangle \approx q & \left\langle \Sigma_{c} \right| p_{i}^{2} \left| \Lambda_{c} \right\rangle \approx a^{2} \\ & \text{negligible} & \text{small} & \text{large} \end{aligned}$$

- The  $(\sigma \cdot p_i)$  term has small contribution since it is associated to pion energy  $\omega_{\pi}$ .
- The relativistic correction is quite important.

Phys. Rev. D 103, 094003 (2021)



### Wave function of heavy baryon



Harmonic Oscillator potential

M = 1.50 GeV  
m = 0.35 GeV 
$$\Box \sim \omega_{\lambda} = 350 \text{ MeV}$$
  
k = 0.03 GeV^3

Orbital Spin  

$$Y_{c} = \left[ \left[ \psi_{l_{\lambda}}(\vec{\lambda}) \psi_{l_{\rho}}(\vec{\rho}), d \right]^{j}, s_{c} \right]^{J} \psi_{flavor} \psi_{color} \qquad J = j + s_{Q}$$

Symmetric

Anti-Symmetric

Nagahiro, et. al. PRD95 014023 (2017)

## $\Xi$ (or $\Omega$ ) baryon decay



- We use SU(3) symmetry basis for the wave function.
- We take and use the averaged mass of u/d and strange quark.
- Also, we use the same interaction Lagrangian.



$$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}=\mathbf{10}_{S}\oplus\mathbf{8}_{M}\oplus\mathbf{8}_{M}\oplus\mathbf{1}_{A}$$

• Unlike heavy baryons, now  $\lambda$  and  $\rho$  modes are mixed.





Phys. Rev. D 87, 094002 (2013)

#### Decay of $\Lambda_b^*(6072)$



## LHCb analysis on $\Lambda_b^*(6072)$



- Background shape is different with the LHCb one.
- It is from the kinematical reflection.
- Sequential decay is sufficient to describe the invariant mass distribution.

non-resonant contribution is relatively large.

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