Inha HTG workshop :
Modern issues in Hadronic Physics

Inha University
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Analysis of Virtual Meson Production in Light-Front Dynamics

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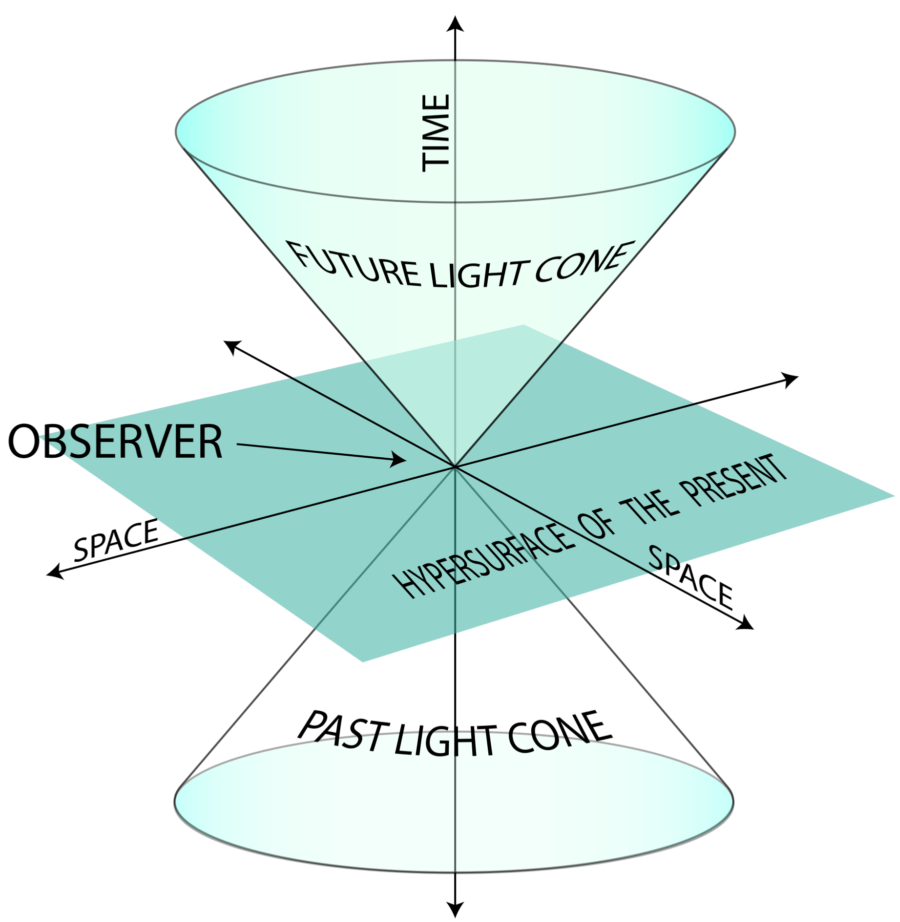
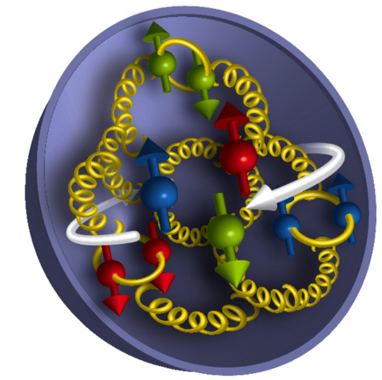


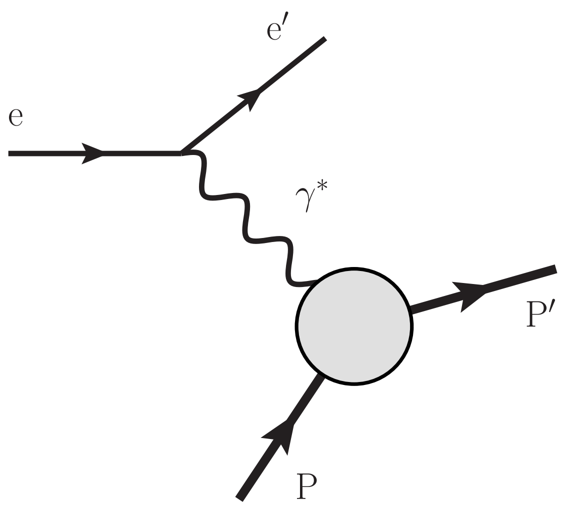
I. Virtual Meson Production (VMP)
II. Low and high energy approaches
III. Comparision in JLab kinematics
IV. Conclusion

Virtual Meson Production (VMP)

Hadron (meson and baryon) is a composite system made of quarks, anti-quarks, and gluons (parton), held together by non-perturbative QCD interaction.

complicated structure



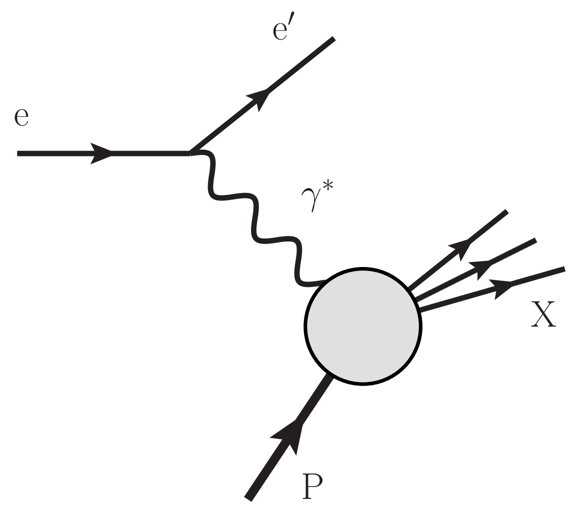


Elastic scattering

density

$$\langle P' | \bar{\psi}(0)\ \hat{\mathcal{O}}\ \psi(0) | P \rangle$$

Electro**m**agnetic
Form
Factors
(EM FFs)

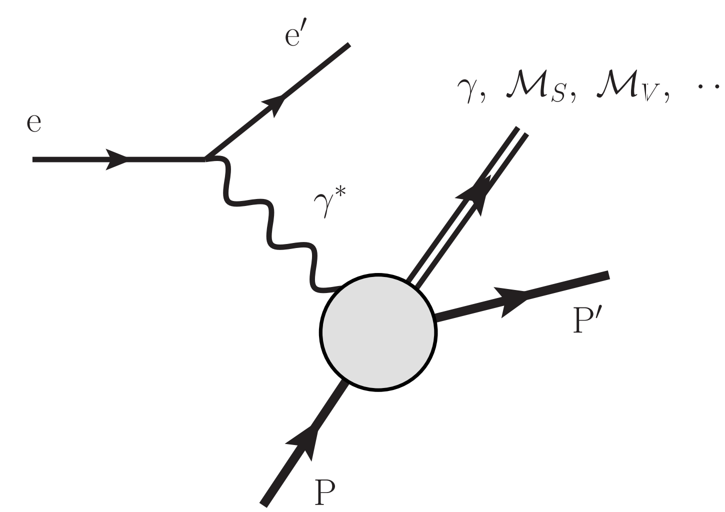


Inclusive / Inelastic

momentum

$$\langle P | \bar{\psi}(z)\ \hat{\mathcal{O}}\ \psi(0) | P \rangle$$

Parton
Distribution
Functions
(PDFs)



Exclusive / Inelastic

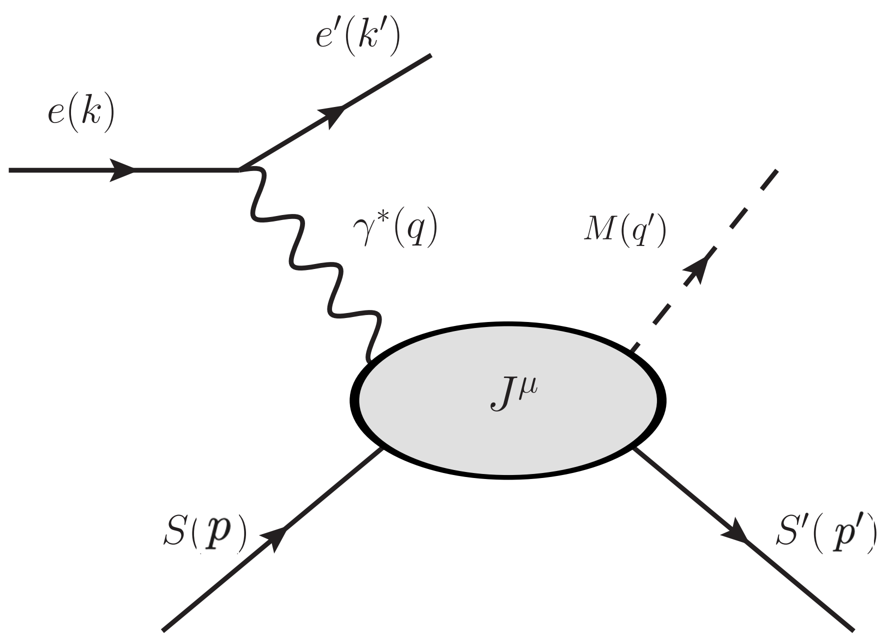
angular momentum

$$\langle P' | \bar{\psi}(z)\ \hat{\mathcal{O}}\ \psi(0) | P \rangle$$

Generalized
Parton
Distributions
(GPDs)

Low energy approach for VMP (GVMP)

Low energy approach ~ generalized structure functions (Compton Form Factors (CFFs))



Through **symmetries**, construct hadronic currents.

DNA method provides most general hadronic currents,

for the pseudoscalar target ($J^{PC} = 0^{-+}$) : $\epsilon^{\mu\nu\alpha\beta}$

for the **scalar target** (0^{++}) : $d^{\mu\nu\alpha\beta} = g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta}$

$$J_S^\mu = (\mathscr{F}_1(Q^2, x, t)\ q_\alpha + \mathscr{F}_2(Q^2, x, t)\ \mathscr{P}_\alpha)\ \boxed{d^{\mu\nu\alpha\beta}} q_\beta \Delta_\nu$$

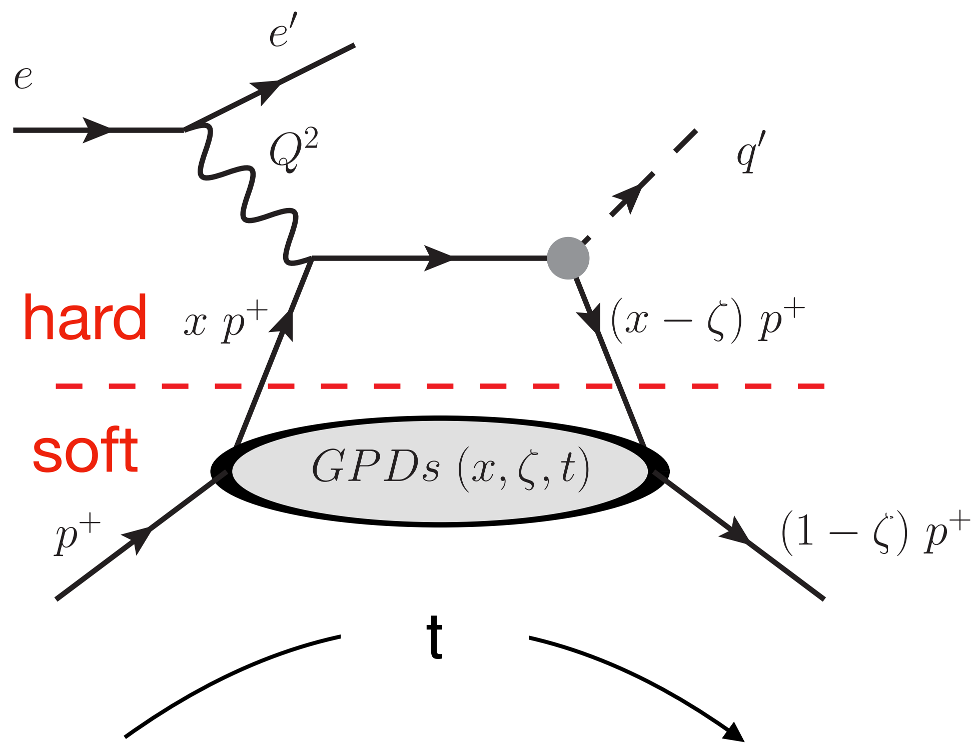
$$= \left\{q^2\ \Delta^\mu - (\Delta \cdot q)\ q^\mu\right\} \boxed{\mathscr{F}_1(Q^2, x, t)} + \left\{(\mathscr{P} \cdot q)\ \Delta^\mu - (\Delta \cdot q)\ \mathscr{P}^\mu\right\} \boxed{\mathscr{F}_2(Q^2, x, t)}$$

where $\quad \Delta = p - p' = q' - q, \quad \mathscr{P} = p + p'$

Ji, Chueng-Ryong and Choi, Ho-Meoyng and Lundeen, Andrew and Bakker, Bernard L. G. Phys. Rev. D **99**, 11 116008 (2019)

High energy approach for VMP (GPD)

High energy approach → twist expansion ~ GPD



Skewed (off-forward) parton distribution :

hard part (OPE) $\otimes$ soft part (GPD)

In the forward limit ($\zeta \to 0$), parton distribution :

hard part (OPE) $\otimes$ soft part (PDF)

$\int dx$, Electromagnetic form factor :

F(t) = $\int dx$ GPD

p^+ : Light-front (LF) longitudinal momentum of incoming target,

Q^2 : Virtuality → factorizable in $Q^2 >> |t|,\ M_t^2,\ M_s^2,\ m_{Q_1}^2,\ m_{Q_2}^2,\ \cdots,$

x : Longitudinal momentum fraction,

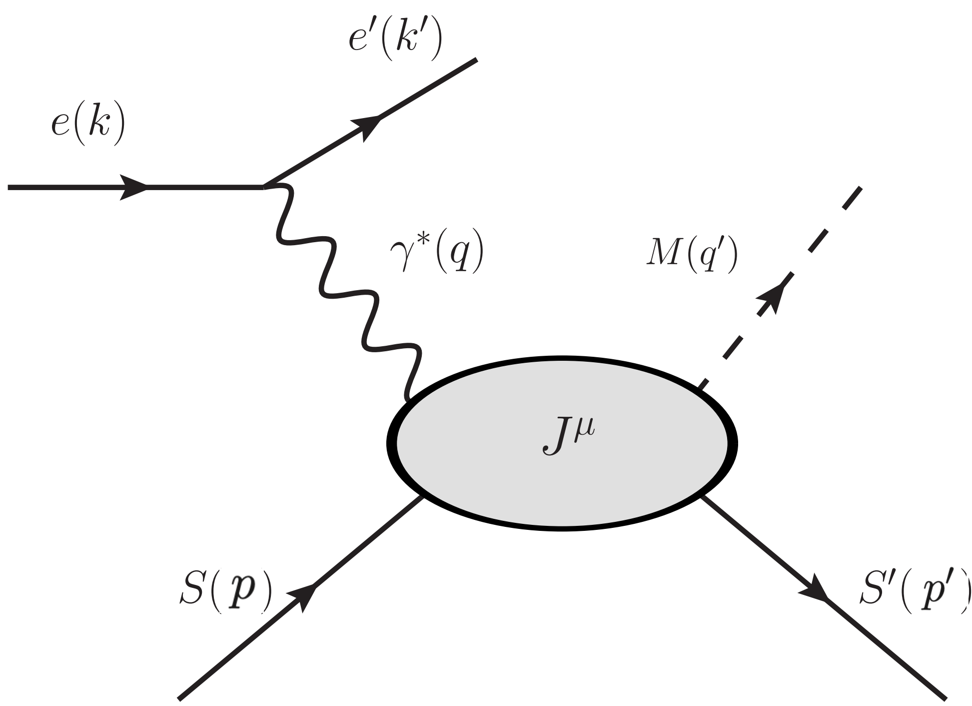
ζ : Skewness, $(p^+ - p'^+)/p^+$, asymmetry of LF longitudinal momentum of target,

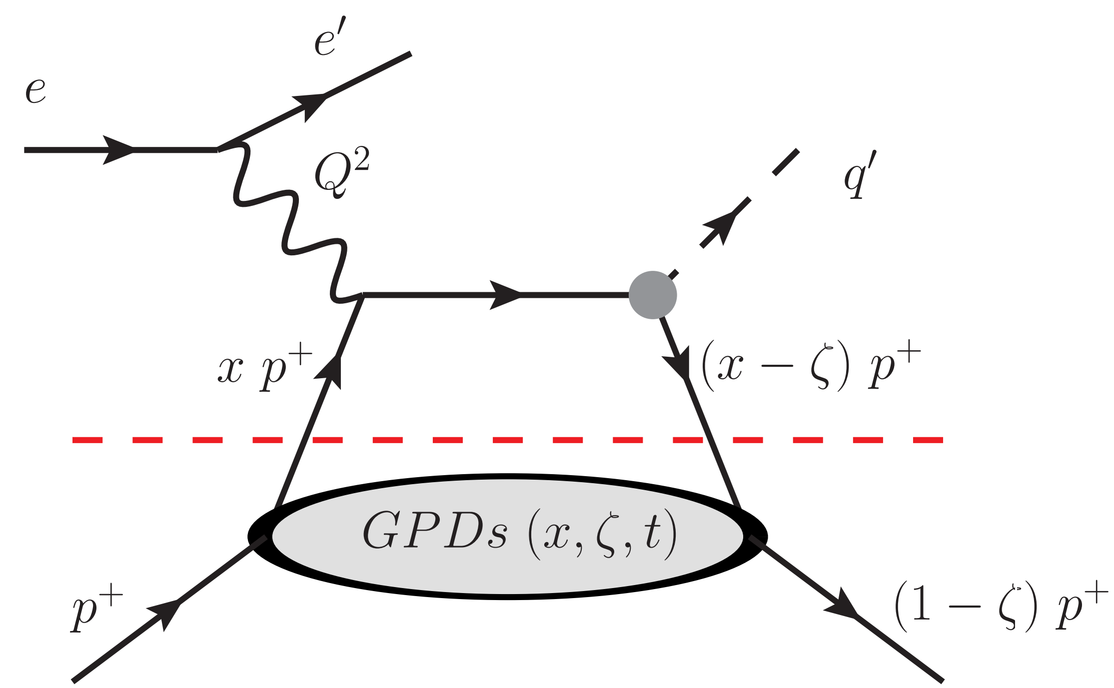
t : Square momentum transfer, $\Delta^2 = (q'-q)^2 = (p-p')^2$,

"kick" transverse momentum depending on scattering angle.

$$t = -\frac{\zeta^2 M_t^2 + \Delta_\perp^2}{1-\zeta}$$

Low vs High energy approaches



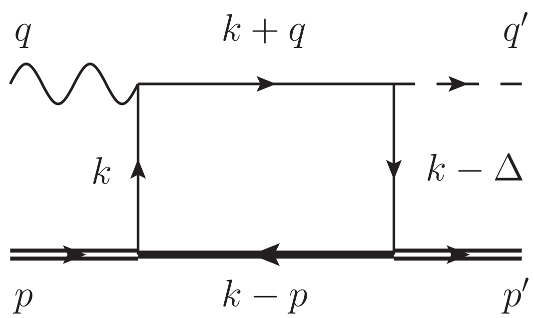


Spin-0 target	GVMP	GPDs (Twist-2)
Factorizable	X	O
Diagrams	S-, U- box, and Cat's ears	Parts of S-, U- box
Ward Identity	Satisfied	Not satisfied
Kinematics	All	Limited
# of CFFs	2	1
Exp.	Compton Form Factors, Beam Spin Asymmetry, …	

Box, Cat's ears, and Effective tree diagrams

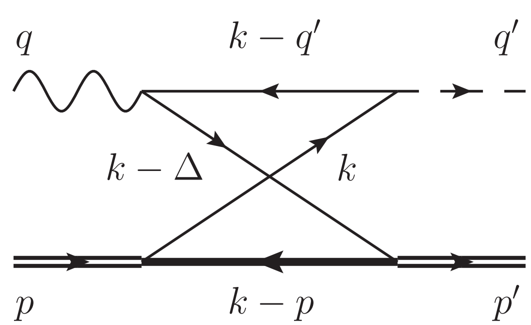
Partons inside hadron can be fermions (quarks and anti-quarks) or spin-1 boson (gluons).
In this work, we introduce **scalar currents** $J(0) = \phi^\dagger(0)\phi(0)$.

Box-S



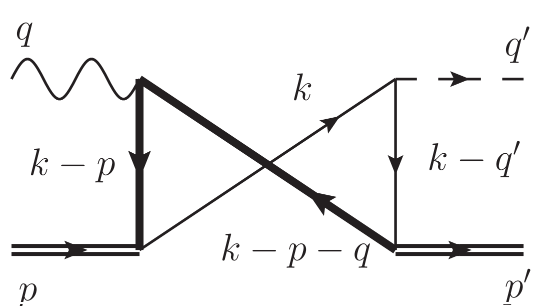
$$\mathcal{M}_s^\mu \sim i\int d^2k \frac{2k^\mu + q^\mu}{(k^2-m^2)((k+q)^2-m^2)((k-\Delta)^2-m^2)((k-p)^2-M^2)}$$

Box-U



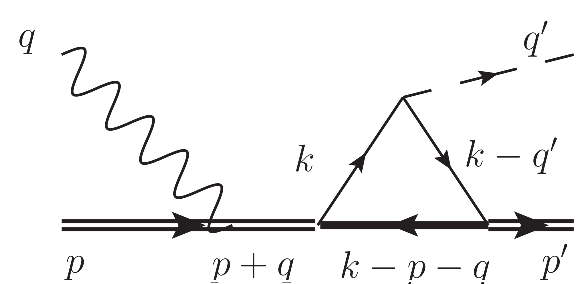
$$\mathcal{M}_u^\mu \sim i\int d^2k \frac{2k^\mu - \Delta^\mu - q'^\mu}{(k^2-m^2)((k-q')^2-m^2)((k-\Delta)^2-m^2)((k-p)^2-M^2)}$$

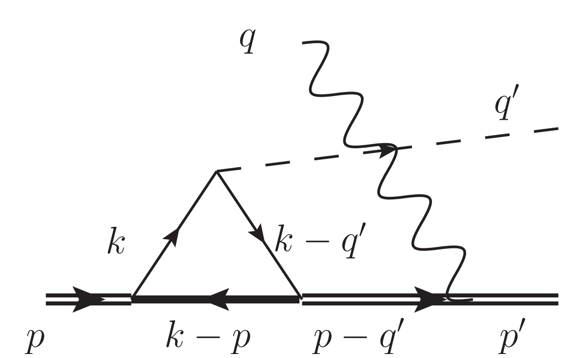
Cat's ears



$$\mathcal{M}_c^\mu \sim i\int d^2k \frac{2k^\mu - 2p^\mu - q^\mu}{(k^2-m^2)((k-q')^2-m^2)((k-p-q)^2-M^2)((k-p)^2-M^2)}$$

For the charged target, +





Operator Product Expansion (OPE)

- **Twist** expansion : **Bilocal** current = sum of **local** currents.

$$\hat{T}\{J^\mu(z)J^\nu(0)\} = \sum_n C_n(z)\ \hat{\mathcal{O}}_n(0) \qquad \text{for large } Q^2$$

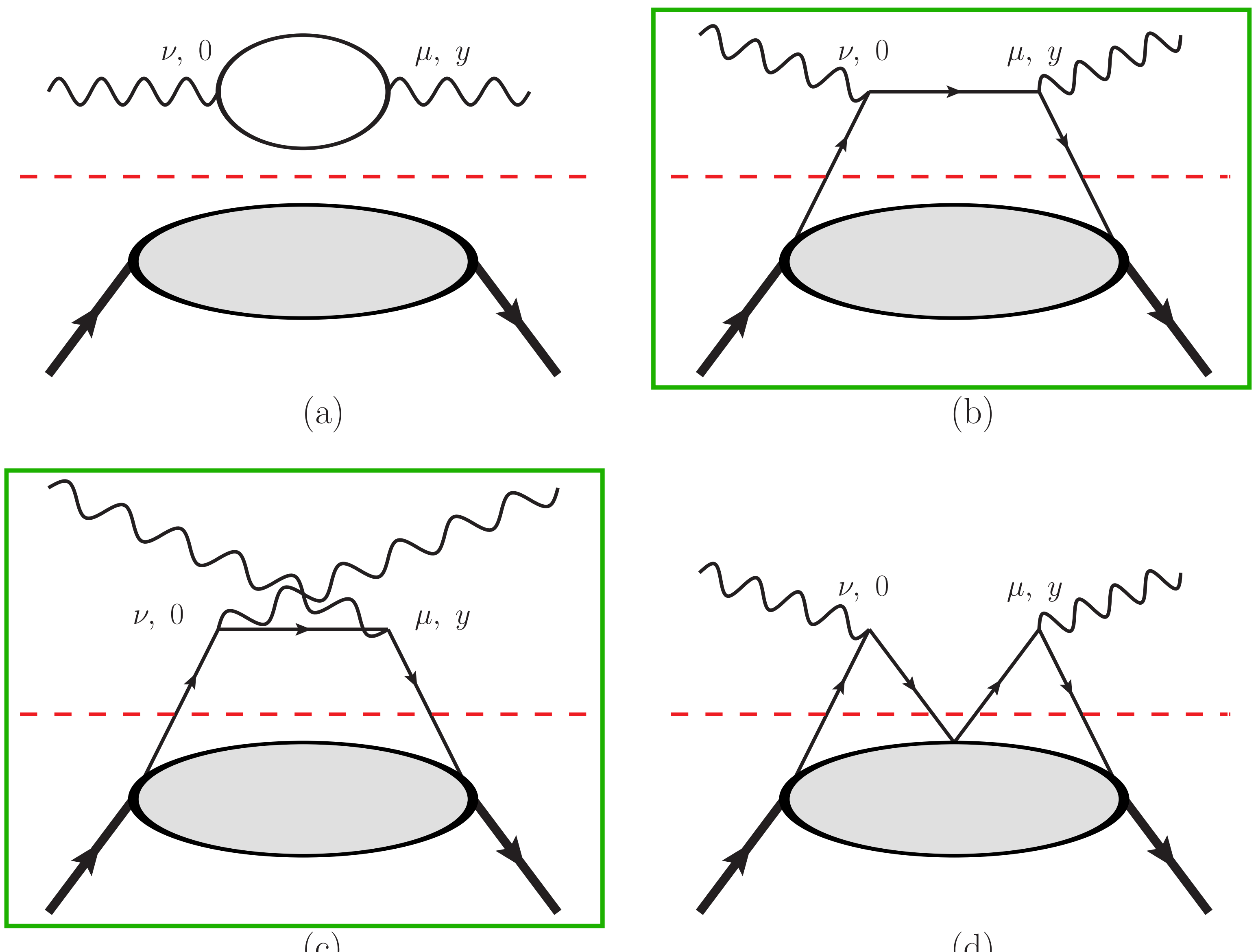
$$\hat{T}\{J^\mu(z)J^\nu(0)\} =$$

$$-\mathrm{Tr}[\gamma^\mu S_F(z)\gamma^\nu S_F(-z)] \quad \text{(a)}$$

$$+ : \bar{\psi}_q(z)\gamma^\mu S_F(z)\gamma^\nu \psi_q(0) : \quad \text{(b)}$$

$$+ : \bar{\psi}_q(0)\gamma^\nu S_F(-z)\gamma^\mu \psi_q(z) : \quad \text{(c)}$$

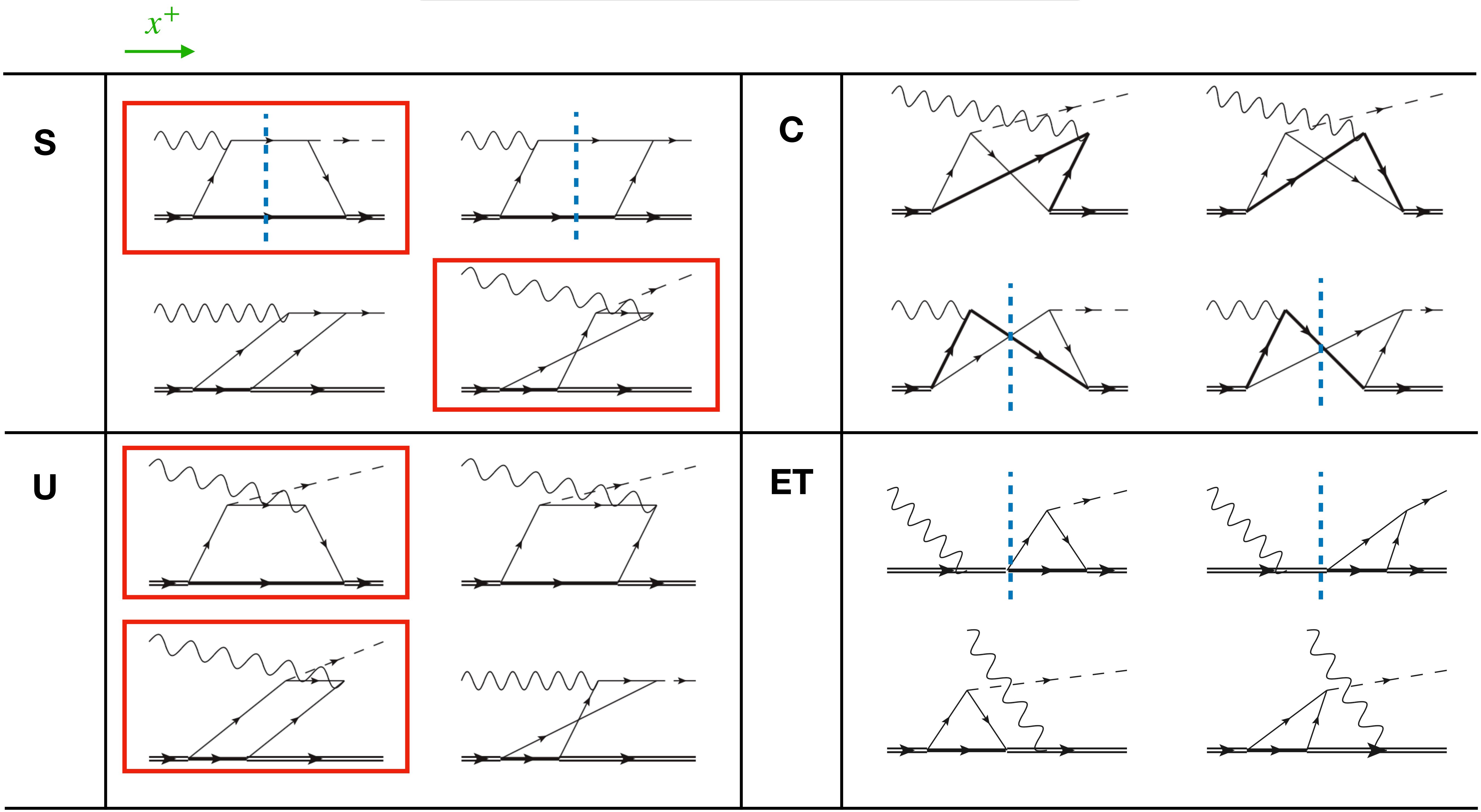
$$+ : \bar{\psi}_q(z)\gamma^\mu \psi_q(z)\bar{\psi}_q(0)\gamma^\nu \psi_q(0) : \quad \text{(d)}$$



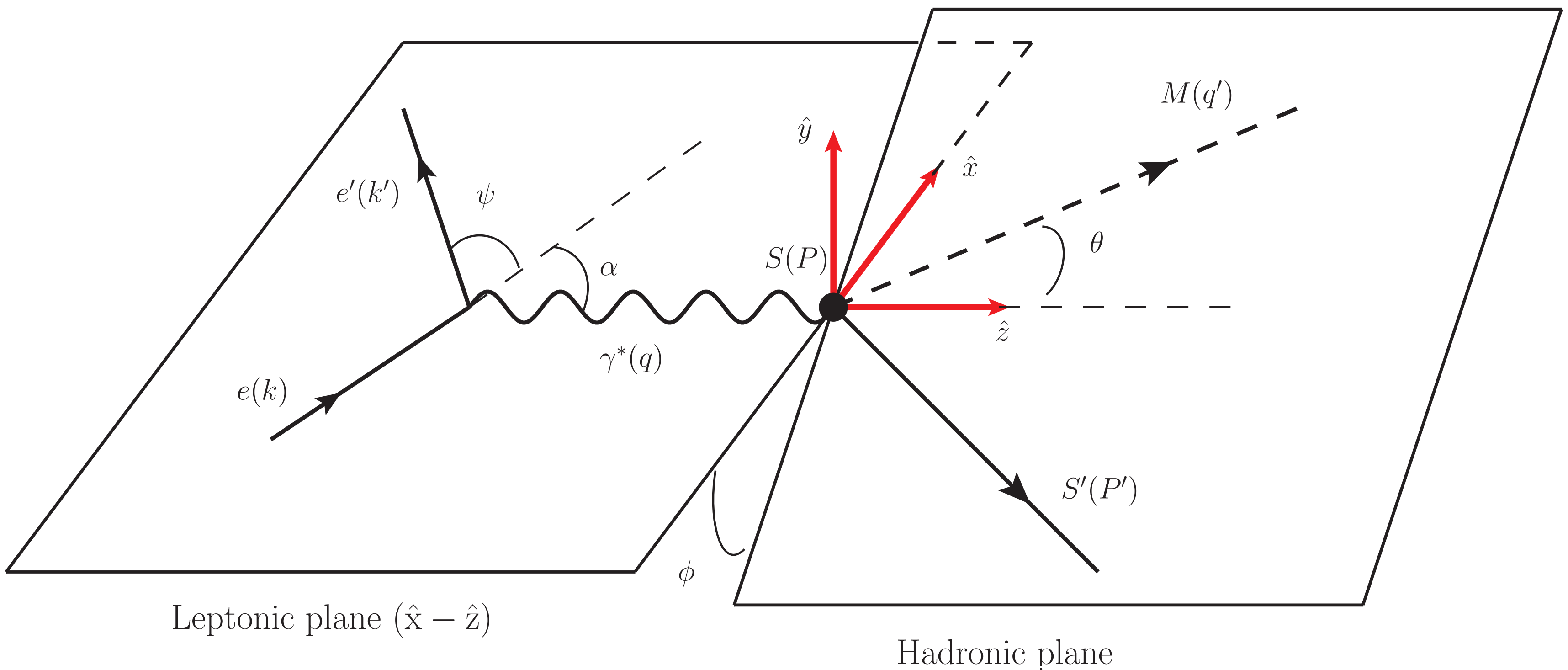
: Momentum of the target is not changed.

: Handbag dominance

LF time-ordered amplitudes



Beam Spin Asymmetry



Beam **S**pin **A**symmetry (BSA) of the scalar meson electroproduction off the scalar target :

$$\frac{d\sigma^S_{\lambda=+1} - d\sigma^S_{\lambda=-1}}{d\sigma^S_{\lambda=+1} + d\sigma^S_{\lambda=-1}} = \frac{d\sigma^S_{BSA}}{d\sigma^S_T\,(1+\epsilon\,\cos(2\phi)) + d\sigma^S_L\,\epsilon_L + d\sigma^S_{LT}\,\cos\phi\sqrt{\epsilon_L(1+\epsilon)/2}}$$

$$\sim \mathscr{F}_1\,\mathscr{F}^*_2 - \mathscr{F}_2\,\mathscr{F}^*_1$$

Leading twist GPD does not provide BSA, because it has only one GPD for the spin-0 target.

Compton Form Factors

- **CFFs from DNA method is satisfied with Ward identity : $q \cdot \mathscr{M} = 0$**
- **Two Lorentz covariant vectors are not independent in (1+1)-dimensions.**

$$\mathscr{M}^{\mu}_{tot} = \left\{ (\Delta \cdot q)\ q^{\mu} - q^{2}\ \Delta^{\mu} \right\} \mathscr{F}_{1} + \left\{ (\Delta \cdot q)\ \mathscr{P}^{\mu} - (\mathscr{P} \cdot q)\ \Delta^{\mu} \right\} \mathscr{F}_{2}$$

$$= A^{\mu}\ \mathscr{F}_{1} + B^{\mu}\ \mathscr{F}_{2}$$ in (3+1)-dimensions

$$= A^{\mu}\ (\mathscr{F}_{1} + c\mathscr{F}_{2}) = A^{\mu}\ \mathscr{F}_{A}$$ in (1+1)-dimensions

where $\mathscr{P} = p + p', \quad \Delta = p - p' = q' - q, \quad c = B^{+}/A^{+} = B^{-}/A^{-}.$

Kinematics & GPDs limit

Kinematics in (1+1)-LFD

$$k = [\, xp^+, \ k^- \,],$$

$$p = \left[\, p^+, \ \frac{M_t}{p^+} \,\right],$$

$$q = \left[\, \left(\frac{\zeta' M_s^2}{Q^2} - \zeta\right) p^+, \ \left(\frac{1}{\zeta'} - \frac{t}{\zeta Q^2}\right) \boxed{Q^2} \frac{}{p^+} \,\right],$$

$$\Delta = q' - q = p - p' = \left[\, \zeta p^+, \ \frac{t}{\zeta p^+} \,\right],$$

$$p' = \left[\, (1-\zeta)\, p^+, \ \left(M_t^2 - \frac{t}{\zeta}\right) \frac{1}{p^+} \,\right],$$

$$q' = \left[\, \frac{\zeta' M_s^2 p^+}{Q^2}, \ \frac{\boxed{Q^2}}{\zeta' p^+} \,\right],$$

where $\zeta = \dfrac{p^+ - p'^+}{p^+}$.

→

GPDs limit ($Q^2 >> |t|, M_s, \cdots$)

$$k = [\, xp^+, \ k^- \,],$$

$$p = \left[\, p^+, \ \frac{M_t}{p^+} \,\right],$$

$$q = \left[-\zeta p^+, \ \frac{Q^2}{\zeta' p^+} \,\right] \simeq \left[-\zeta p^+, \boxed{\frac{Q^2}{\zeta p^+}} \right],$$

$$\Delta = \left[\, \zeta p^+, \ \frac{t}{\zeta p^+} \,\right],$$

$$p' = \left[\, (1-\zeta)\, p^+, \ \left(M_t^2 - \frac{t}{\zeta}\right) \frac{1}{p^+} \,\right],$$

$$q' = \left[\, 0, \ \frac{Q^2}{\zeta' p^+} \,\right] \simeq \left[\, 0, \boxed{\frac{Q^2}{\zeta p^+}} \right],$$

where there is no M_s dependence, and $\zeta \simeq \zeta'$.

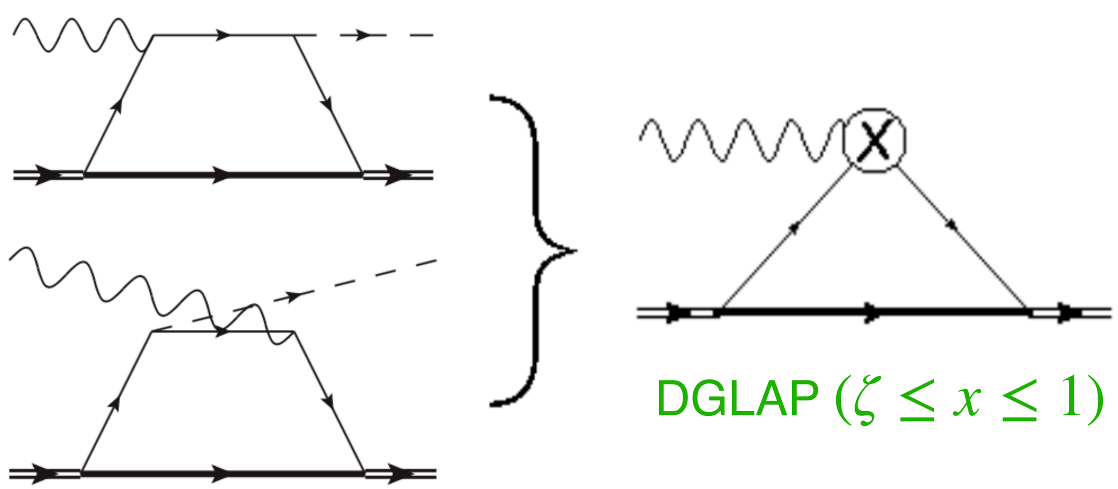
GPD does not distinguish between DVMP & DVCS.

CR Ji, BLG Bakker,
Int. J. Mod. Phys. E 22, 1330002 (2013)

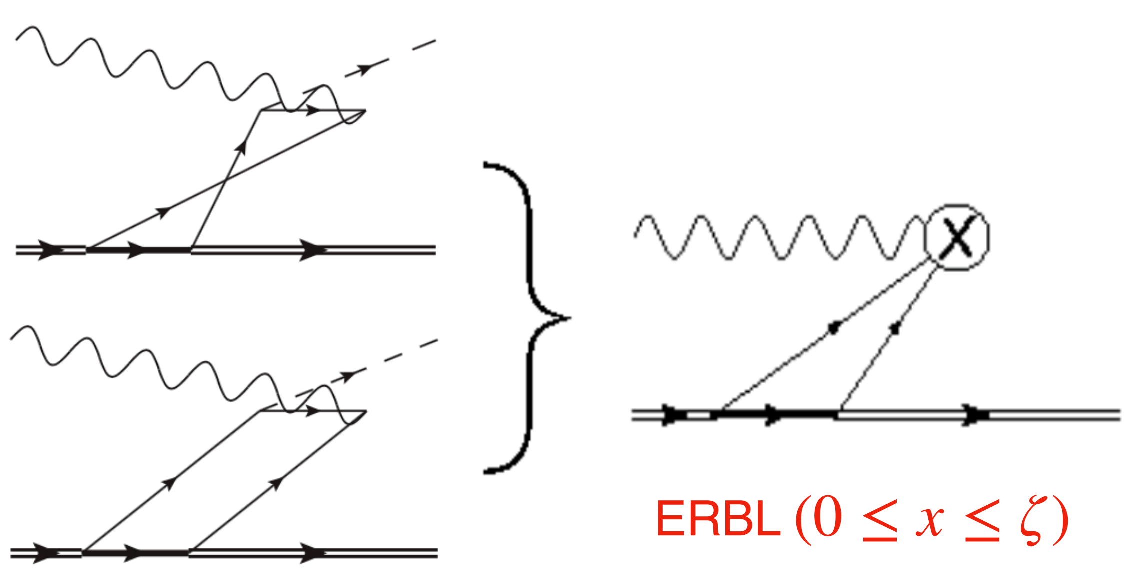
GPD Formulation

- In GPD limit ($Q^2 >> |t|, M_s^2, \cdots$), q^- and q'^- are dominant, it leads to the reduced scattering amplitudes with GPD functions.

s-ch : $\frac{1}{(k+q)^2 - m^2} \sim \frac{1}{q^-\ (k^+ + q^+)} \sim \frac{1}{q^-\ \boxed{(x-\zeta)}\ p^+}$, u-ch : $\frac{1}{(k-q)^2 - m^2} \sim -\frac{1}{q'^-\ (k^+ - q'^+)} \sim -\frac{1}{q^-\ \boxed{x}\ p^+}$



DGLAP ($\zeta \leq x \leq 1$)



ERBL ($0 \leq x \leq \zeta$)

Generalized **P**arton **D**istributions (GPDs) : angular momentum

$$\mathcal{M} \sim \int_\zeta^1 dx \left(\frac{1}{\boxed{x-\zeta}} - \frac{1}{\boxed{x}} \right) \boxed{H_{DGLAP}(x,\ t)}$$

$$+ \int_0^\zeta dx \left(\frac{1}{\boxed{x-\zeta}} - \frac{1}{\boxed{x}} \right) \boxed{H_{ERBL}(x,\ t)}$$

Parton **D**istribution **F**unctions (PDFs) : momentum

$$f(x) \sim \lim_{\zeta \to 0} H(x,\ \zeta,\ t) = H_{DGLAP}(x,\ t = 0)$$

Electro**m**agnetic **F**orm **F**actors (EM FFs) : density

$$F(t) \sim \frac{1}{2-\zeta} \left[\int_0^\zeta dx\ H_{ERBL}(x,\ t) + \int_\zeta^1 dx\ H_{DGLAP}(x,\ t) \right]$$

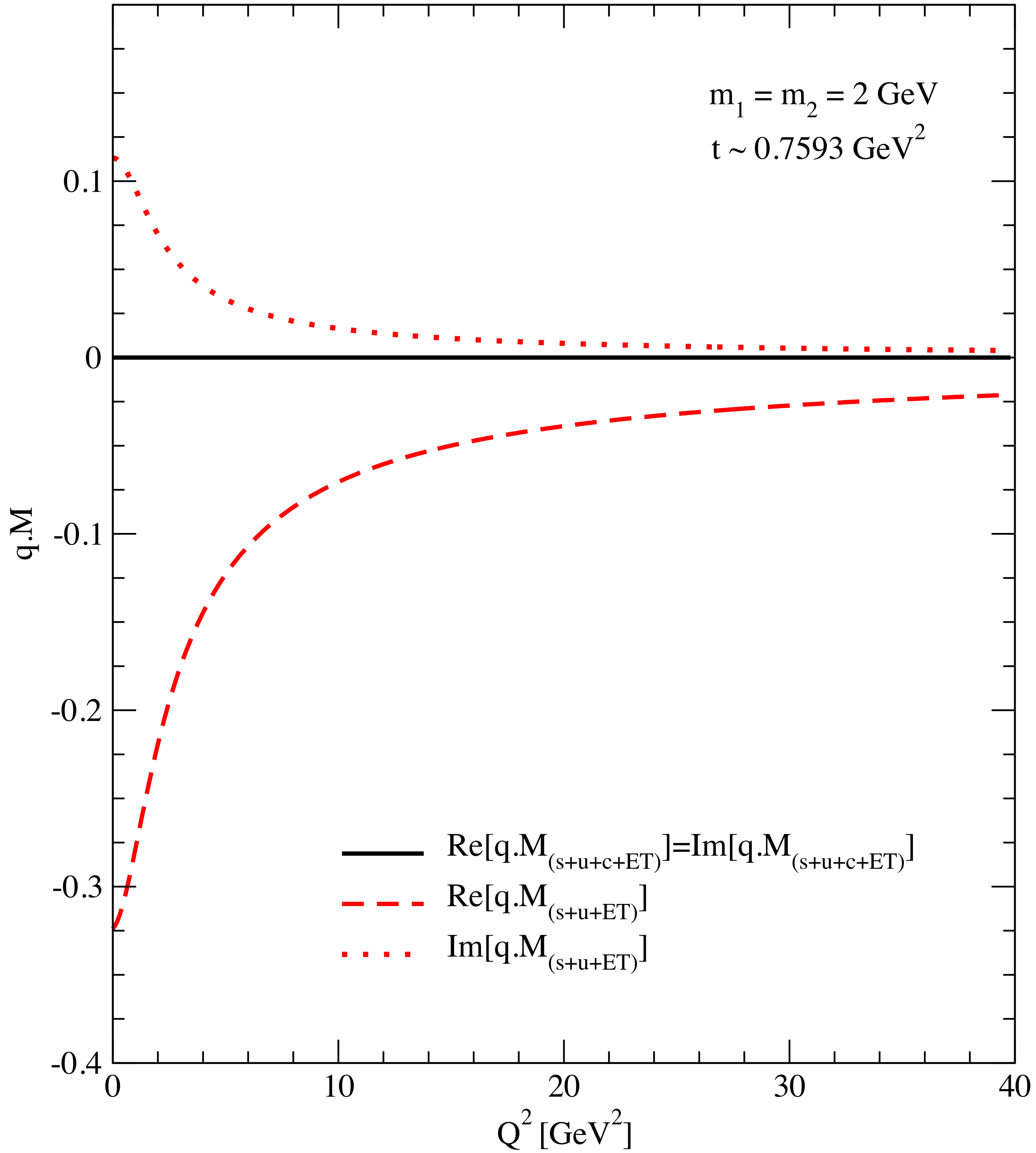
Result - Ward Identity

- **Ward identity does not hold without cat's ears diagrams.**

$$q \cdot \mathscr{M} = 0 \quad :$$

$$q^+ \, \mathrm{Re}[\mathscr{M}^-] \; = -\, q^- \, \mathrm{Re}[\mathscr{M}^+]$$

$$q^+ \, \mathrm{Im}[\mathscr{M}^-] \; = -\, q^- \, \mathrm{Im}[\mathscr{M}^+]$$

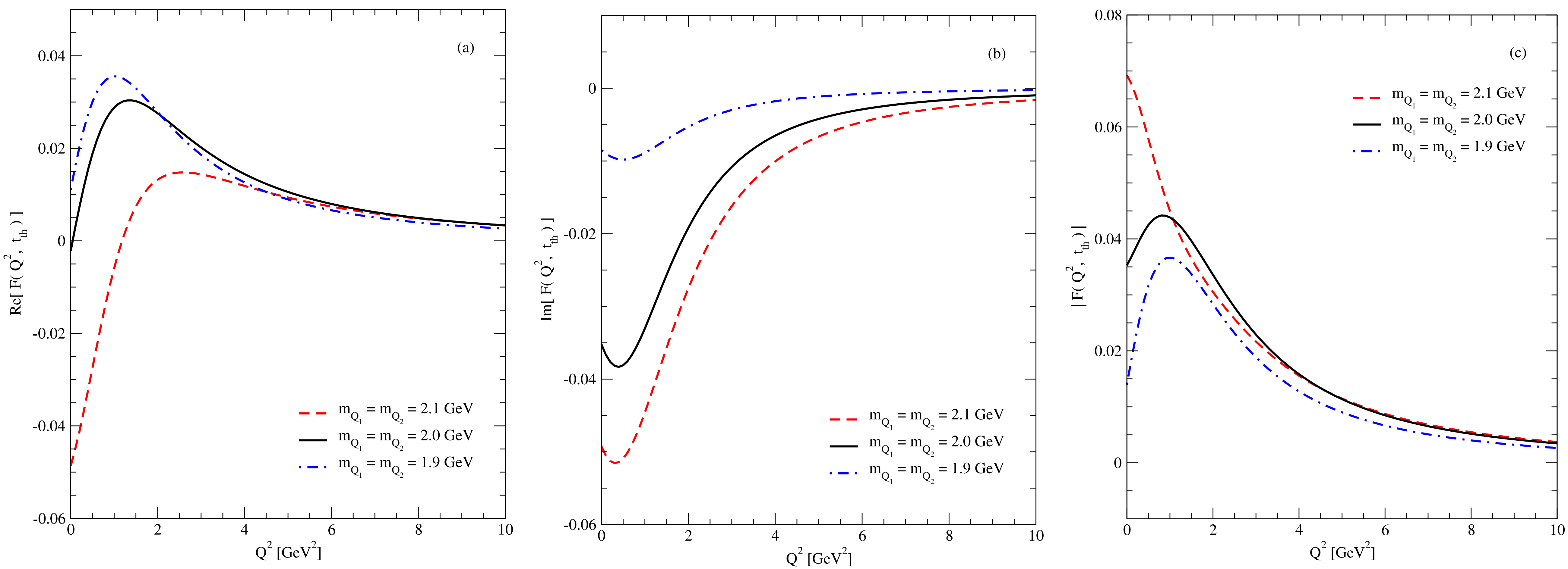


Result - Compton Form Factors

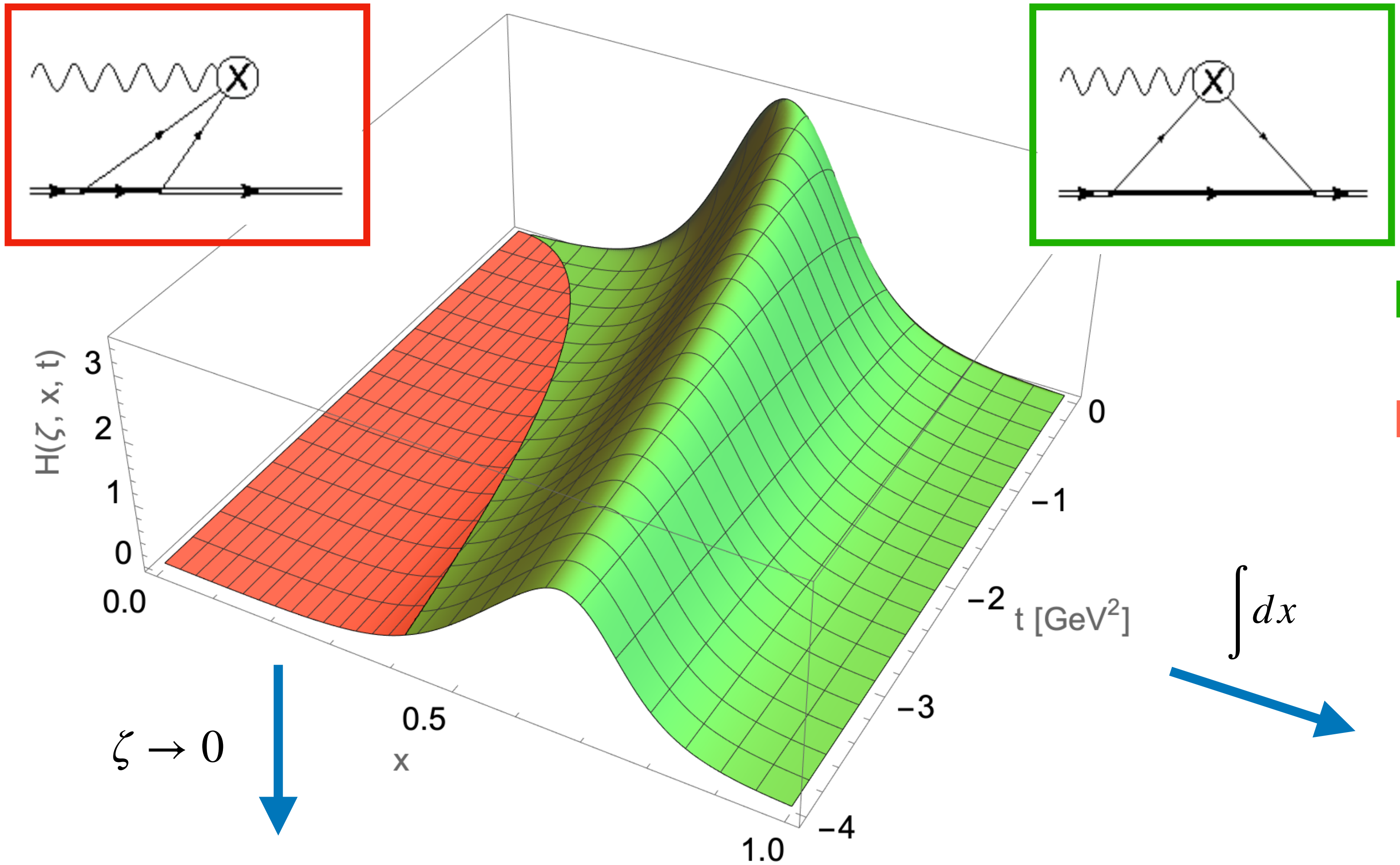
- Parameter set :

$$m_{Q_1} = 2,\ \ m_{Q_2} = 2,\ \ M_t\ ({}^4He) = 3.7,\ \ M_s\ (f_0) = 0.98\ \text{GeV},\ \ t = -\,0.7593\ \text{GeV}^2$$

- The imaginary part related to the beam spin asymmetry (BSA) is relatively not small.



Result - GPDs, PDFs, and FFs



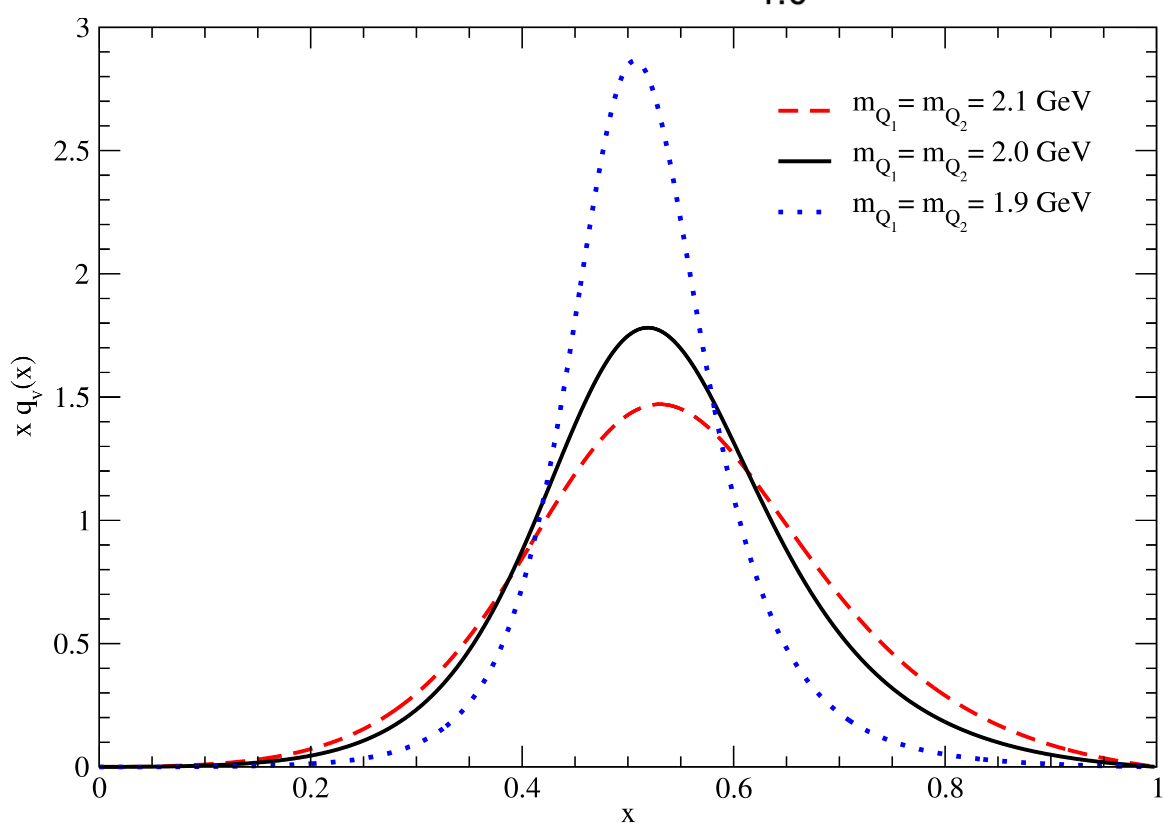
$$\zeta = \frac{1}{2\ M_t^2}\left(t + \sqrt{t^2 - 4\ t\ M_t^2}\right)$$

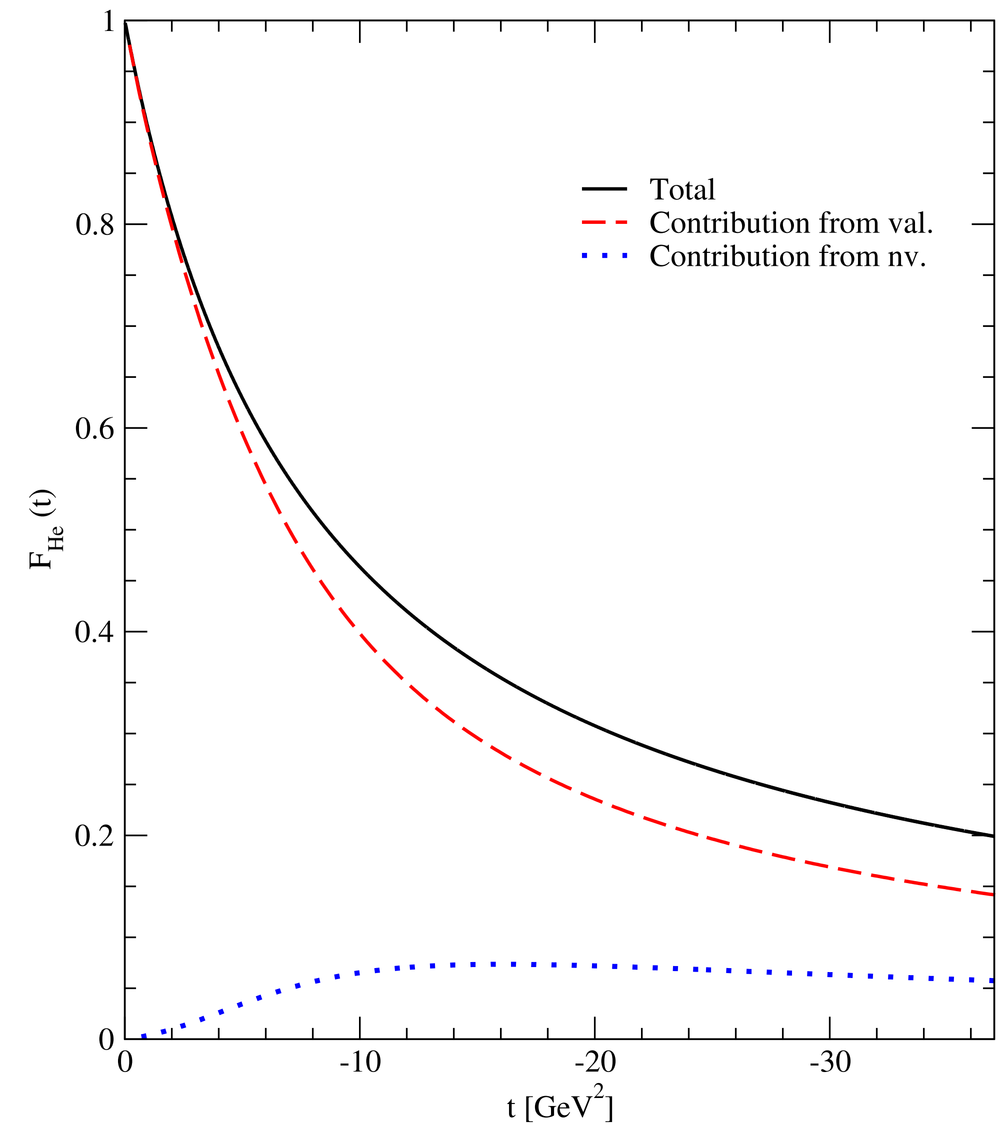
DGLAP $(\zeta \leq x \leq 1) \rightarrow$ valence

ERBL $(0 \leq x \leq \zeta) \rightarrow$ non-valence

$\zeta \rightarrow 0$

$\int dx$





Kinematic settings

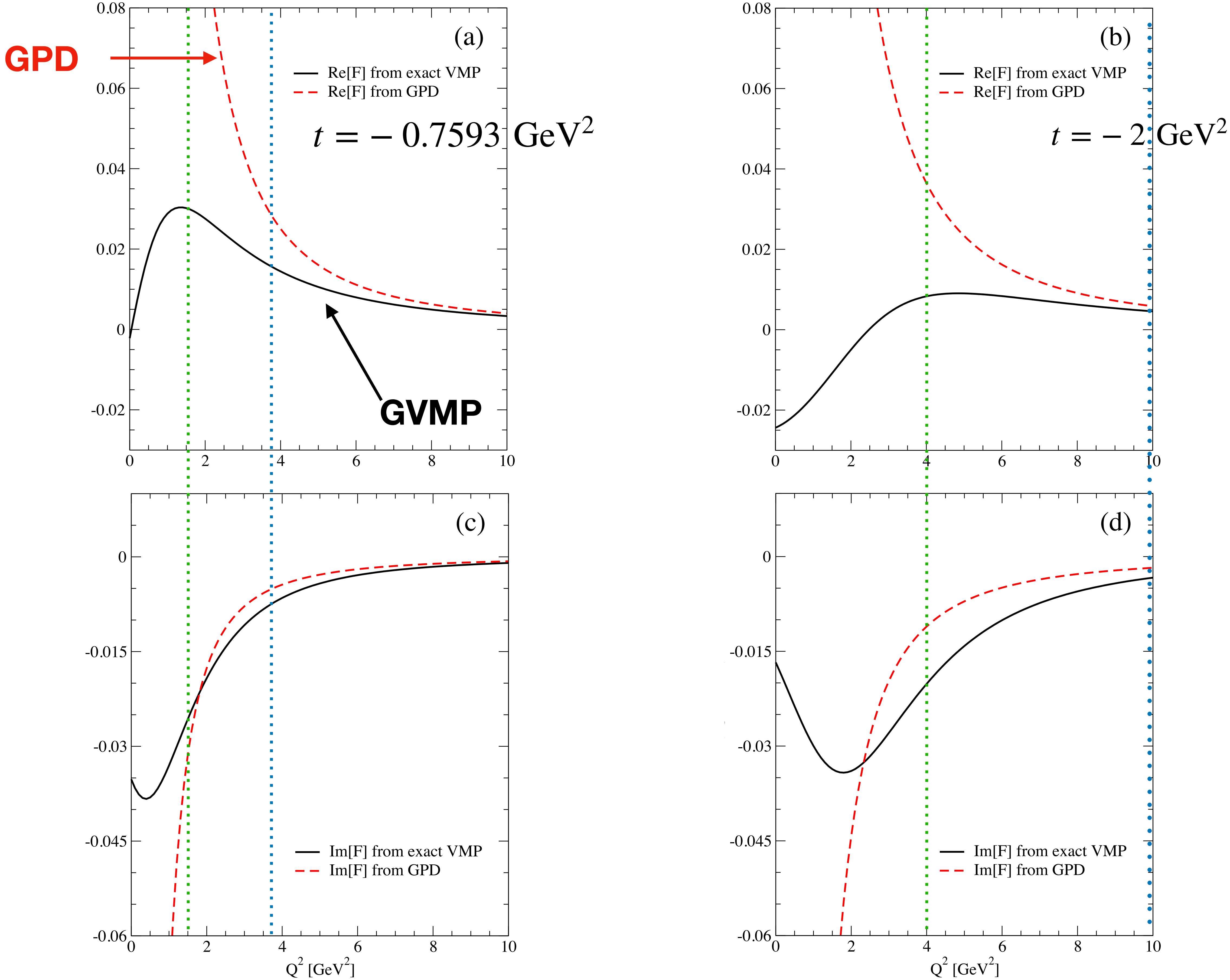
- In Jefferson laboratory experiments, for $e + {}^4He \rightarrow e' + \gamma + {}^4He$ (DVCS) :

$$1.9 < Q^2 < 9.0\ GeV^2, \quad -4.5 < t < -1.0\ GeV^2, \quad -t/Q^2 \sim 0.5$$

$Q^2(\mathrm{GeV}^2)$	x	k|(GeV)	k' (GeV)	θ_e (°)	θ_q (°)	q'(0°) (GeV)	W^2 (GeV^2)	M (GeV)	t (GeV^2)	t_min (GeV^2)	$-t/Q^2$
1.9	0.36	5.75	2.94	19.3	18.1	2.73	4.2	3.72738	−1.06554	−0.955137	0.560813
3.	0.36	6.6	2.15	26.5	11.7	4.35	6.2	3.72738	−1.32826	−1.22078	0.442755
4.	0.36	8.8	2.88	22.9	10.3	5.83	8.	3.72738	−1.54314	−1.40201	0.385786
4.55	0.36	11.	4.26	17.9	10.8	6.65	9.	3.72738	−1.68065	−1.48463	0.369374
3.1	0.5	6.6	3.2	22.5	18.5	3.11	4.1	3.72738	−1.9983	−1.83768	0.644611
4.8	0.5	8.8	3.68	22.2	14.5	4.91	5.7	3.72738	−2.64071	−2.41298	0.550148
6.3	0.5	11	4.29	21.1	12.4	6.5	7.2	3.72738	−3.09918	−2.81838	0.491934
7.2	0.5	11.	3.32	25.6	10.2	7.46	8.1	3.72738	−3.27475	−3.02728	0.454826
5.1	0.6	8.8	4.27	21.1	17.8	4.18	4.3	3.72738	−3.41689	−3.15331	0.669978
6	0.6	8.8	3.47	25.6	14.1	4.97	4.9	3.72738	−3.74772	−3.51599	0.624621
7.7	0.6	11	4.16	23.6	13.1	6.47	6	3.72738	−4.45326	−4.12602	0.578346
9.	0.6	11	3	30.2	10.2	7.62	6.9	3.72738	−4.81139	−4.53706	0.534599

Experimentally, $|t|$ increases as Q^2 increases,
it is difficult to measure experiment for enough small $-t/Q^2$.

Result - GVMP vs GPD



Conclusion

- We investigate the virtual meson production by using the ϕ^3-scalar field model in (1+1) light-front dynamics.
- The virtual meson production is theoretically accessed by the generalized hadronic current and generalized parton distribution.
- The Compton form factors from the GVMP and twist-2 GPD formulation are quite different in JLab kinematics setting.
- We expect that $-t/Q^2$ should be at least less than 0.2 for twist-2 GPD. But, this tendency should be checked in the more realistic model.