# Light-Front Dynamics in Hadron Physics

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Inha HTG workshop: Modern issues in Hadronic Physics

July 7, 2022

# Outline

- QCD Local and Global Symmetries
- Color Confinement and Chiral Symmetry
- Dirac's Proposition for Relativistic Dynamics
- Distinguished Features of Light-Front Dynamics
- Large $N_C$ QCD in 1+1 dim. ('tHooft Model)
- Application to Hadron Phenomenology

# Quantum Chromodynamics (QCD)

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{i,j=1}^{3} \overline{q_i}\,(i\,\not{D} - m_q)_{ij}\, q_j - \frac{1}{4}\sum_{\alpha=1}^{8} G^{\alpha}_{\mu\nu} G^{\alpha,\mu\nu}$$

**Local $SU(3)_c$ Gauge Theory**

**interrelated with**

**Global Symmetry**

**Isospin symmetry**

**Chiral symmetry**

$$\mathrm{SU(2)_R \times SU(2)_L}$$

**Spontaneous symmetry breakdown**

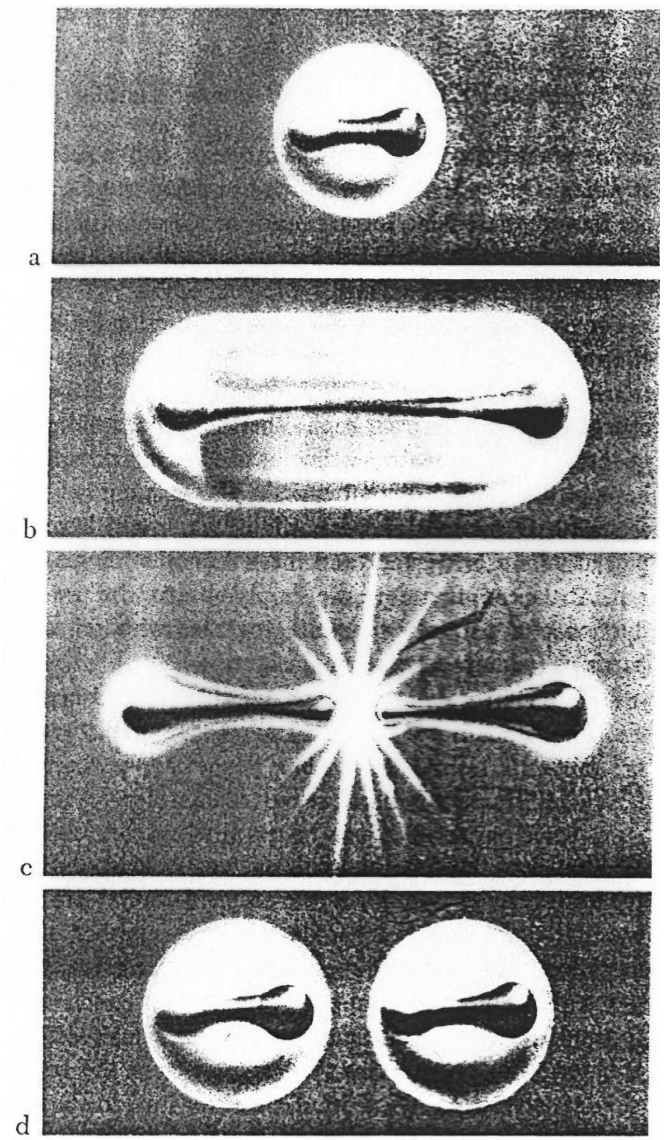
**Goldstone Bosons**

$$F_\pi^2 M_\pi^2 = -(m_u + m_d)\,\langle 0|\,\bar{u}u\,|0\rangle$$

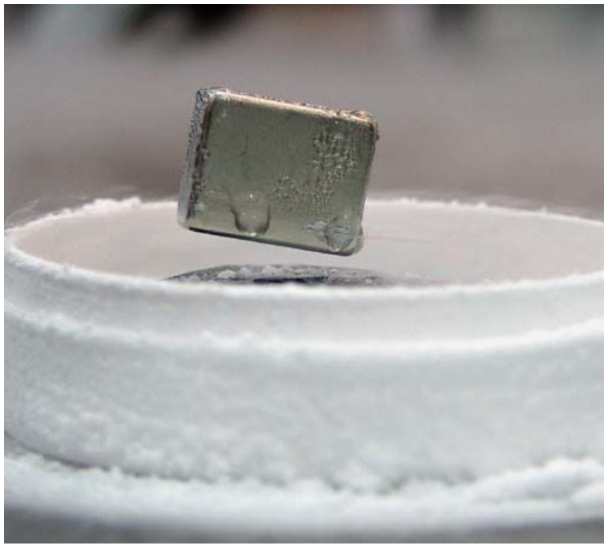
**Effective field theory**

Confinement and
Vacuum Condensation are
intimately linked to
each other.

Vacuum Condensation
indicates that
the vacuum symmetry is
Broken due to
the condensation.

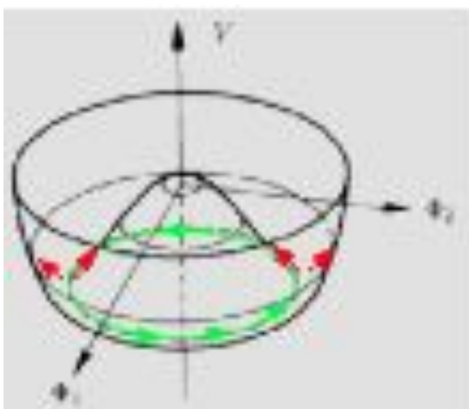


# Meissner Effect of Superconductor

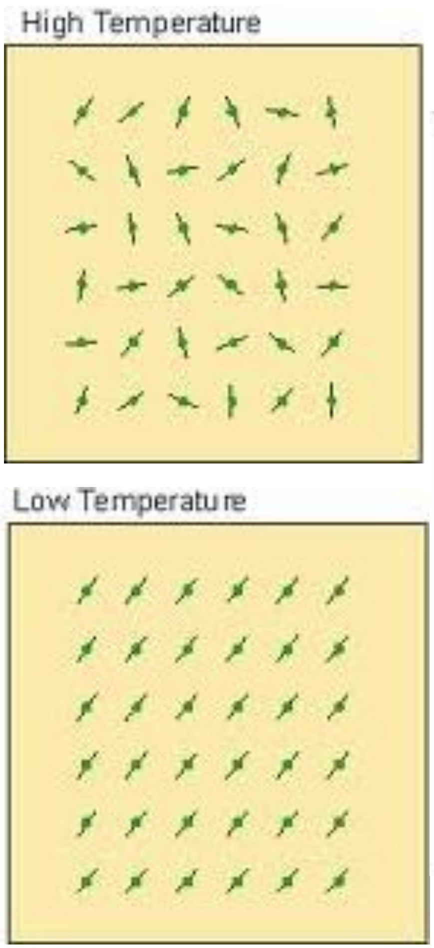


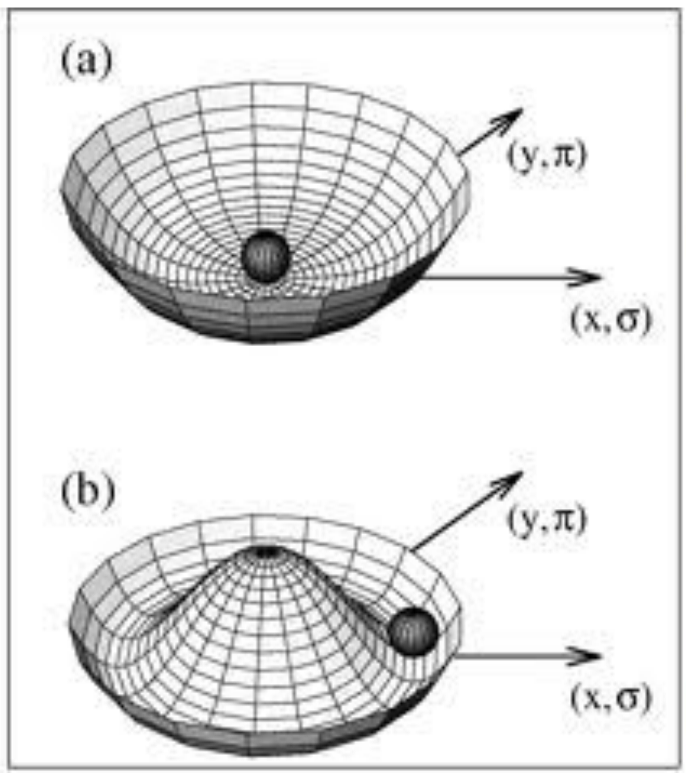
$$m\dot{\vec{x}} = \vec{p} - e\vec{A}$$

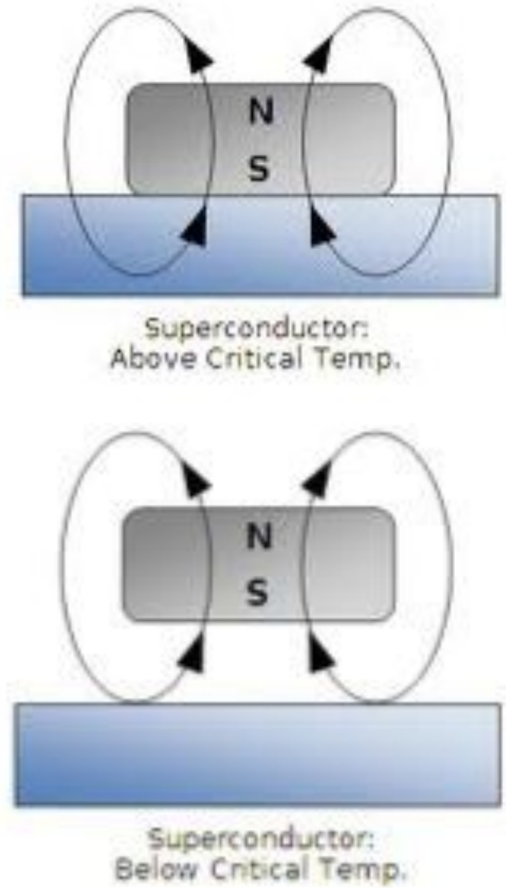
$$D_\mu \phi = \partial_\mu \phi - eA_\mu \phi$$



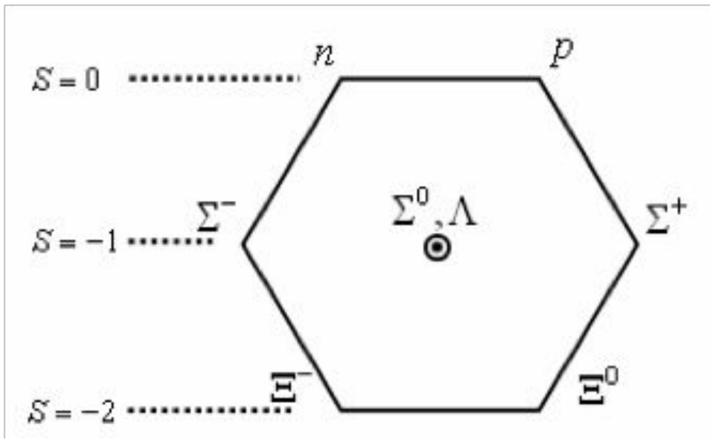

$$\phi = \begin{pmatrix} 0 \\ V + \eta \end{pmatrix} \Rightarrow m_\eta = \sqrt{\lambda V}\ ,\ m_W = gV\ , \cdots$$

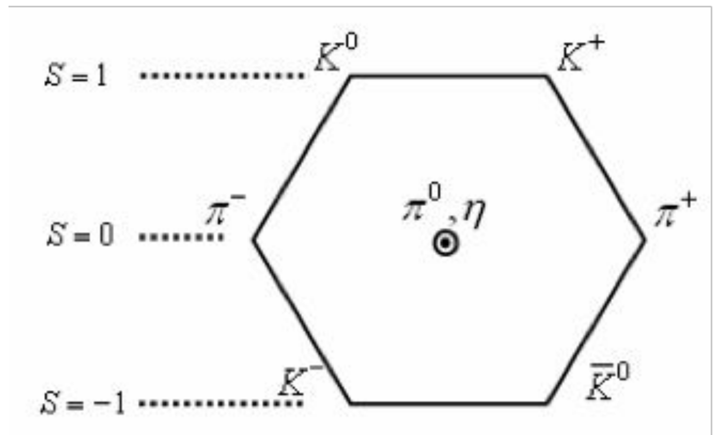

High Temperature
Low Temperature


(a)
(y,π)
(x,σ)
(b)
(y,π)
(x,σ)


N
S
Superconductor:
Above Critical Temp.
N
S
Superconductor:
Below Critical Temp.

# Chiral symmetry breaking in QCD vacuum



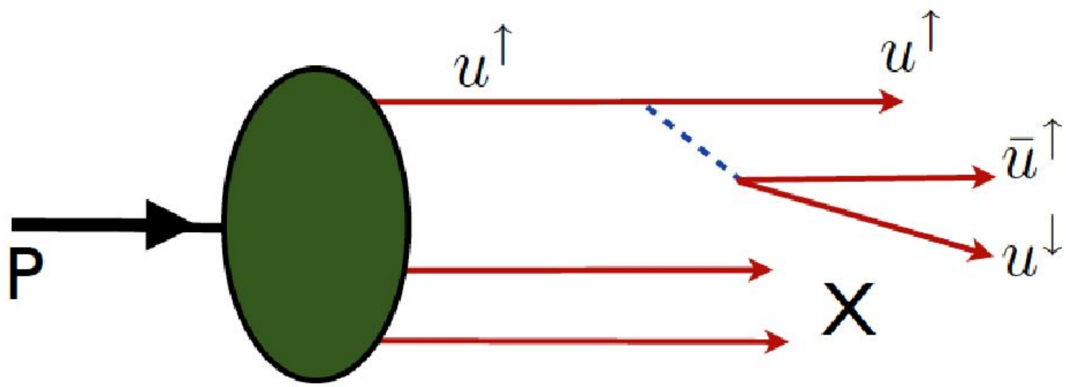




No parity doubled baryons

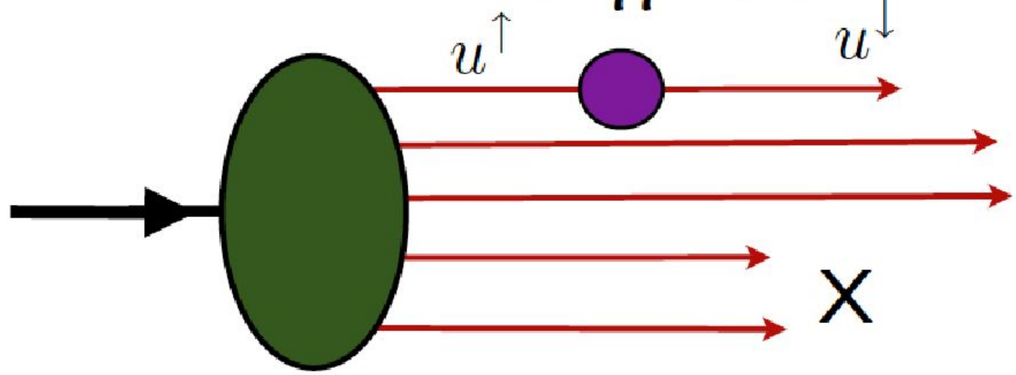
Pseudo Goldstone meson

Low energy PCAC phenomenology and Chiral effective theory

# Chiral Symmetry in QCD

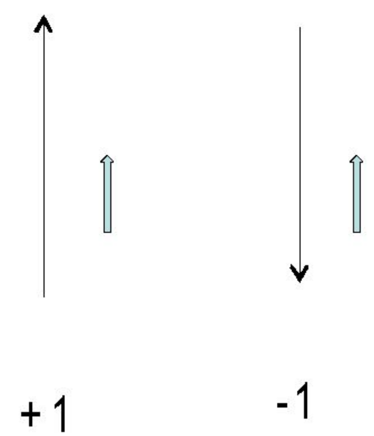



*In effect:*



# Consistency with Relativistic Dynamics

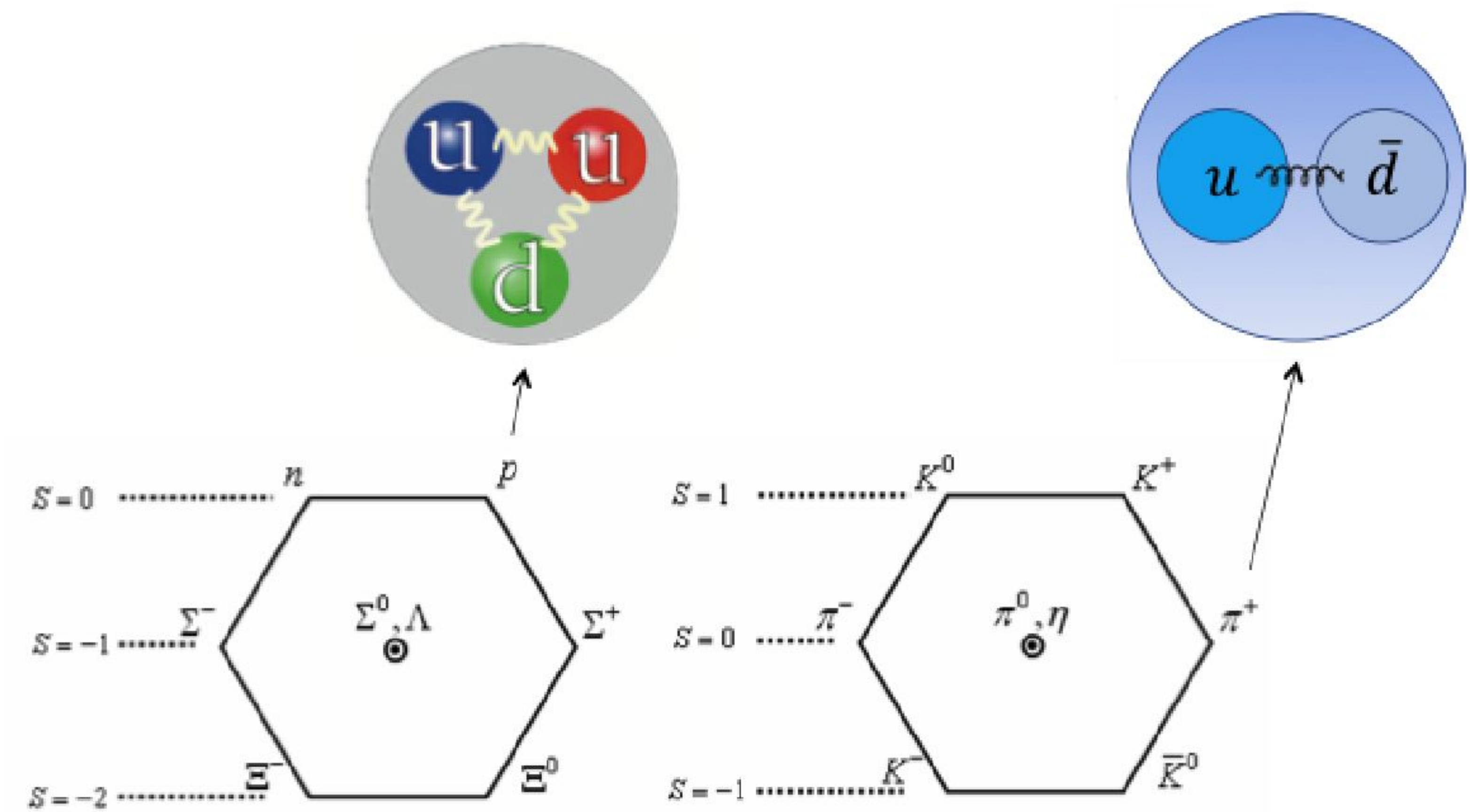
**Helicity is invariant for m=0**




Helicity = Chirality for m=0

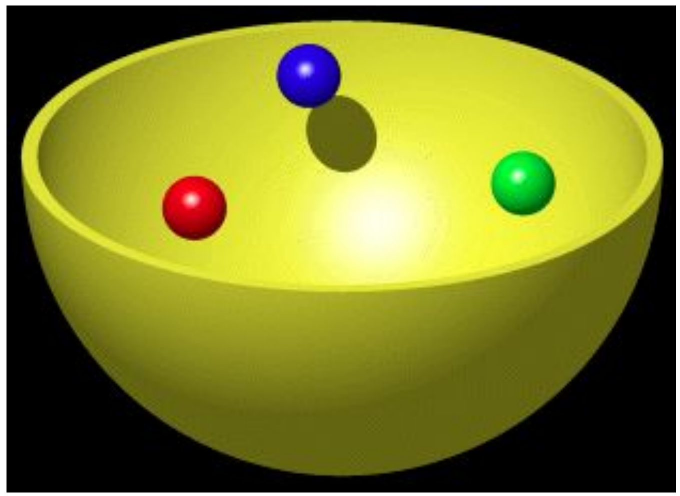
Chiral Symmetry Breaking due to dynamical mass generation

# How do we understand the Quark Model in Quantum Chromodynamics?



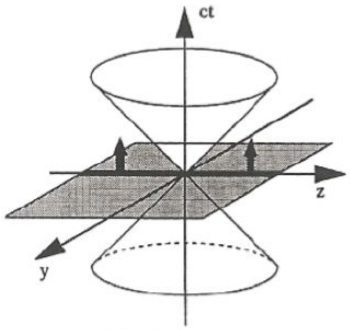
$$M_p = 938.272046 \pm 0.000021\, MeV$$

$$M_n = 939.565379 \pm 0.000021\, MeV$$



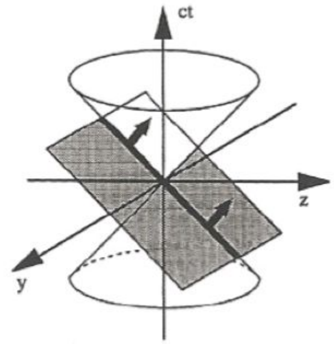
$$m_u = 2.3^{+0.7}_{-0.5}\ MeV \quad ; \quad m_d = 4.8^{+0.7}_{-0.3}\ MeV$$

# Dirac's Proposition for Relativistic Dynamics



The instant form


1949

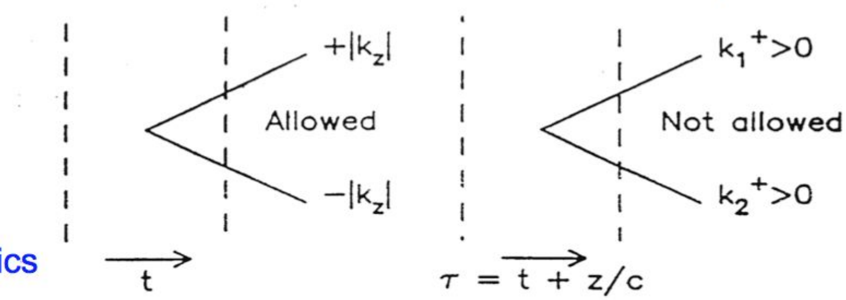


The front form

| *Equal t* | | *Equal* $\tau$ |
|---|---|---|
| $p^0$ | $\leftrightarrow$ | $p^- = p^0 - p^3$ |
| $(p^1, p^2)$ | $\leftrightarrow$ | $\vec{p}_\perp$ |
| $p^3$ | $\leftrightarrow$ | $p^+ = p^0 + p^3$ |

## Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2} \qquad\qquad p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$

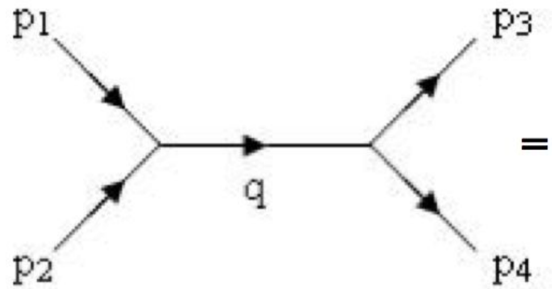


(a) (b)

**IFD**
Instant Form Dynamics

**LFD**
Light-Front Dynamics

$$“e^+e^- \to \mu^+\mu^-”$$




$$= \frac{1}{q^2 - m^2} = \frac{1}{s - m^2} \qquad q^2 = (p_1 + p_2)^2 \neq m^2$$

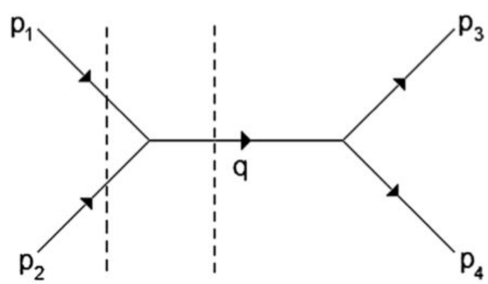
Four-momentum conservation but off-mass-shell

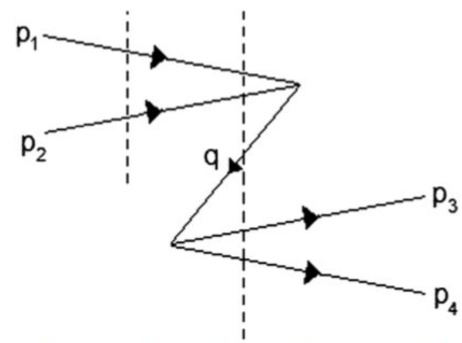
Feynman Diagram: Invariant under all 10 Poincaré generators

t→ (time evolution; time ordered process in QFT; Energy is not conserved within $\Delta t$)

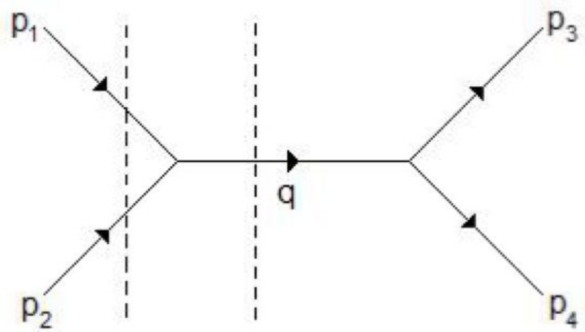
$$(\Delta E)(\Delta t) \sim \hbar$$

Three-momentum conservation but on-mass-shell

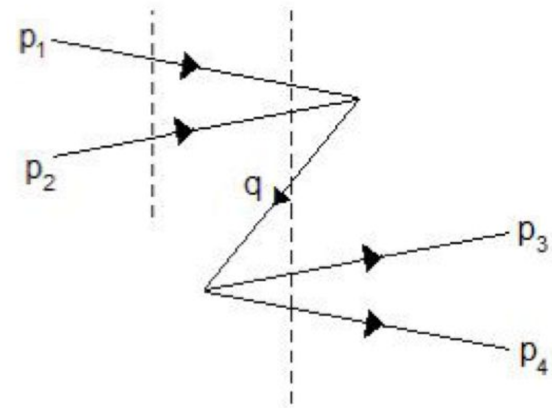



Individual Time-Ordered Diagrams: Invariant only under translation and rotation (6 kinematic generators)




(a)




(b)

$$\Sigma^a_{\mathrm{IFD}} = \frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 - q^0}\right)$$

$$\Sigma^b_{\mathrm{IFD}} = -\frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 + q^0}\right)$$

$$\Sigma^a_{IFD} + \Sigma^b_{IFD} = \frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 - q^0} - \frac{1}{p_1^0 + p_2^0 + q^0}\right)$$

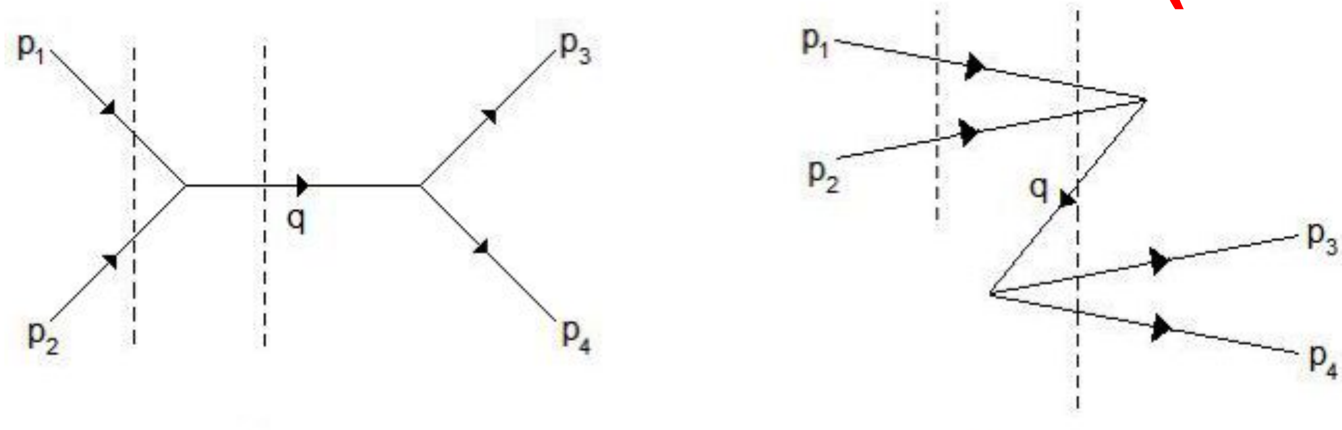
$$= \frac{1}{(p_1^0 + p_2^0)^2 - (q^0)^2}$$

$$= \frac{1}{\{(p_1^0 + p_2^0)^2 - (\vec{p}_1 + \vec{p}_2)^2\} - \{(q^0)^2 - \vec{q}^{\,2}\}}$$ : Three-momentum conservation

$$= \frac{1}{(p_1 + p_2)^2 - q^2}$$ : $q^2 = m^2$ ; on -mass-shell

$$= \frac{1}{s - m^2}$$

# Infinite Momentum Frame (IMF) Approach




(a) (b)

$$\frac{1}{E_1 + E_2 - Eq}$$

$$-\frac{1}{Eq + E_3 + E_4}$$

$$= -\frac{1}{Eq + E_1 + E_2}$$

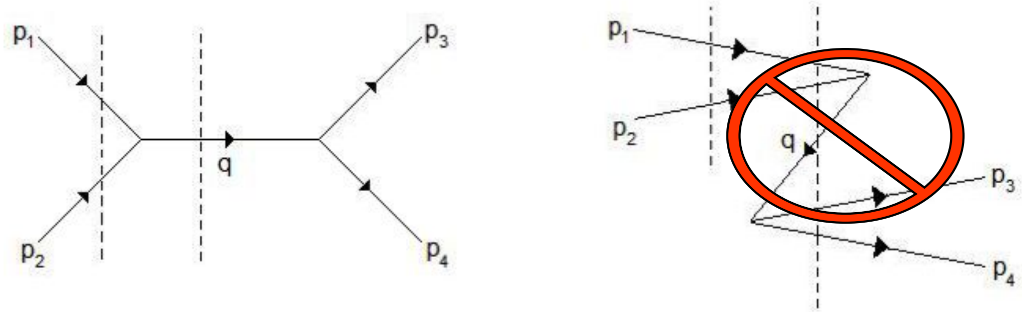
$$\rightarrow \quad 0$$

S.Weinberg, PR158,1638(1967)
"Dynamics at Infinite Momentum"

**Note that this is still in the instant form (IFD).**

# However, in LFD, (b) drops for any reference frame (not just for IMF)

τ (= t+z/c) →




(a) (b)

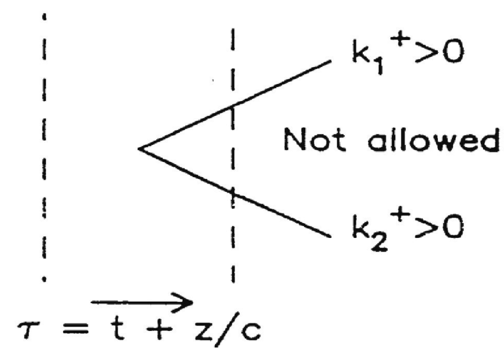
$$\Sigma^a_{LFD}+\Sigma^b_{LFD} = \frac{1}{q^+}\left(\frac{1}{p_1^-+p_2^--q^-}+0\right)$$

$$= \frac{1}{q^+\left(\frac{(p_1+p_2)^2+(\vec{p}_{1\perp}+\vec{p}_{2\perp})^2}{(p_1+p_2)^+}-\frac{m^2+\vec{q}_\perp^{\,2}}{q^+}\right)}$$

$$= \frac{1}{(p_1+p_2)^2-m^2}$$

$$= \frac{1}{s-m^2}$$

$$p^- = \frac{\vec{p}_\perp^{\,2}+m^2}{p^+}$$




Except zero-mode

$$k_1^+ = k_2^+ = 0$$

# QED Example

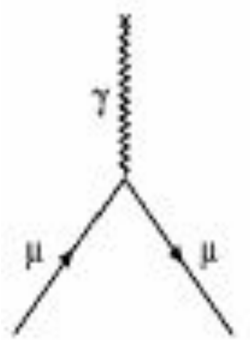
## Anomalous Magnetic Moment

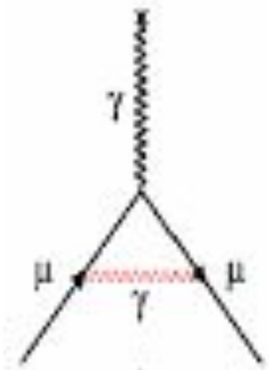
- Magnetic moment of a particle is related to its spin.

$$\vec{\mu} = g\frac{e\hbar}{2mc}\vec{S}$$

- For Dirac pointlike particle, g=2. However, the loop correction in QFT yields the non-zero g-2,

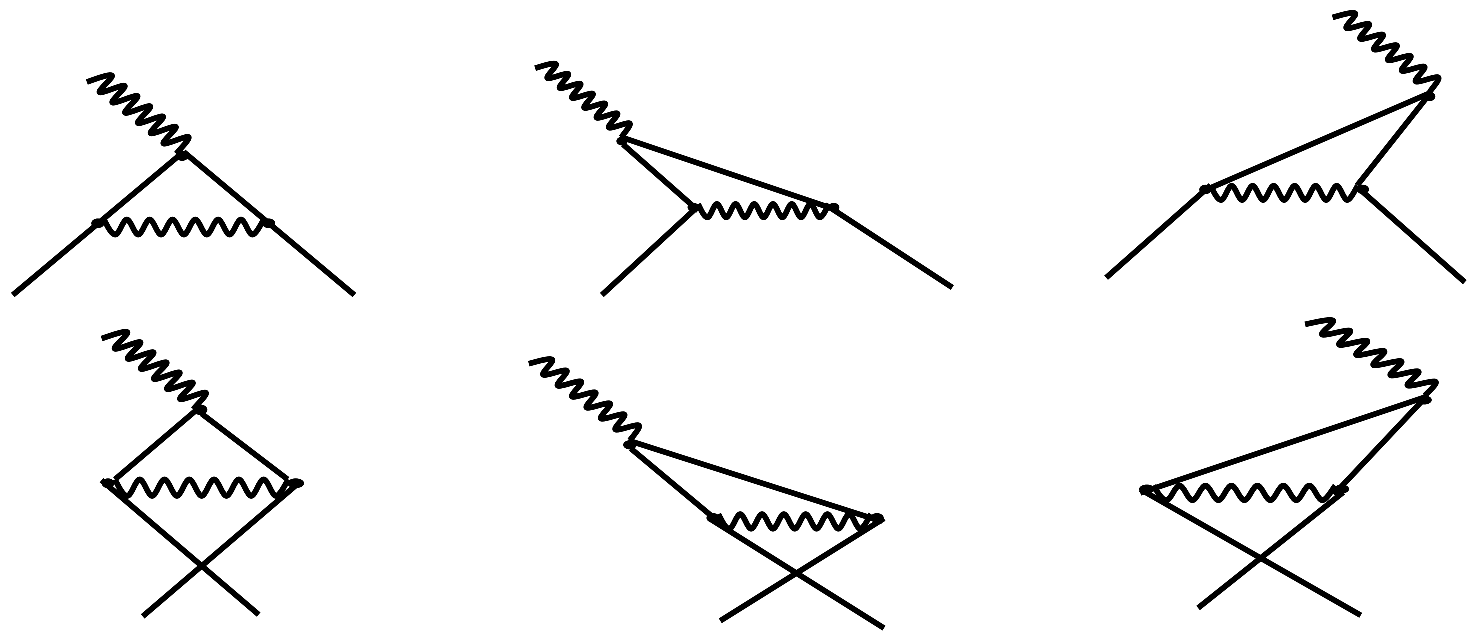
i.e. anomalous magnetic moment.



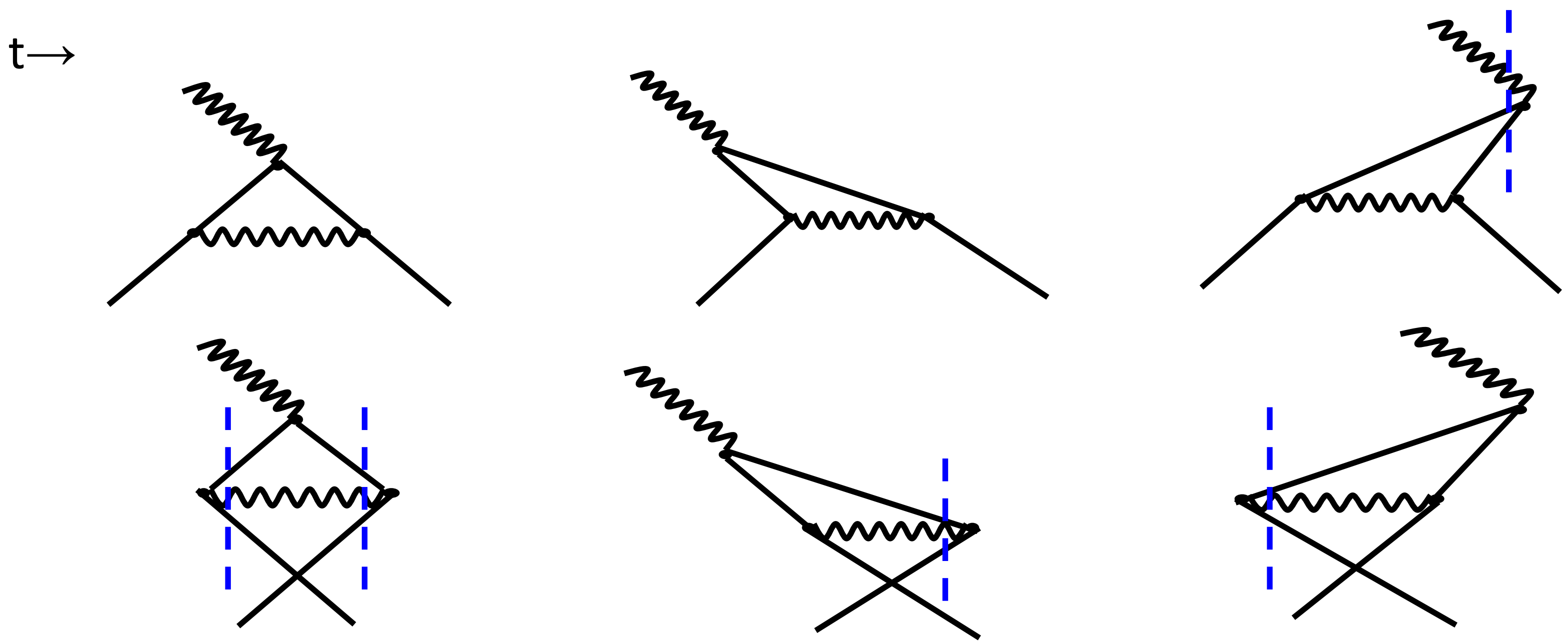



# g-2 calculation

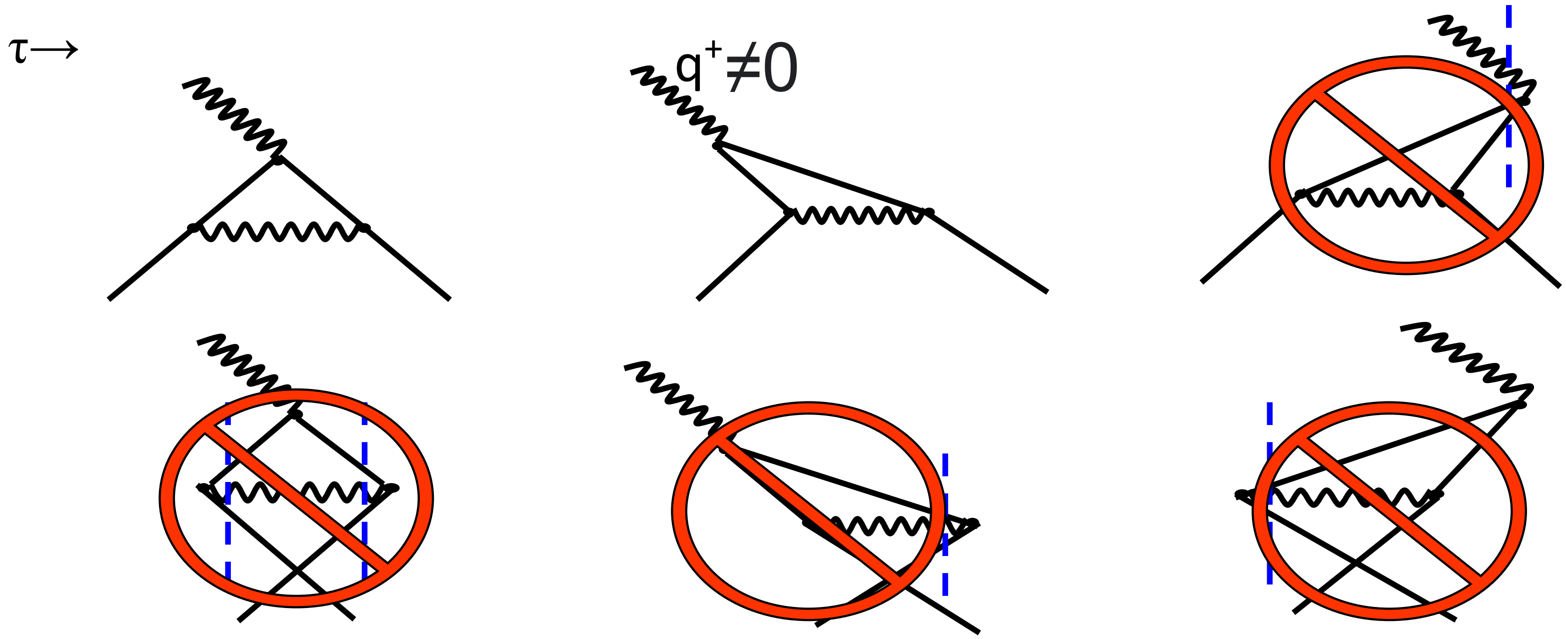
t→



# g-2 calculation

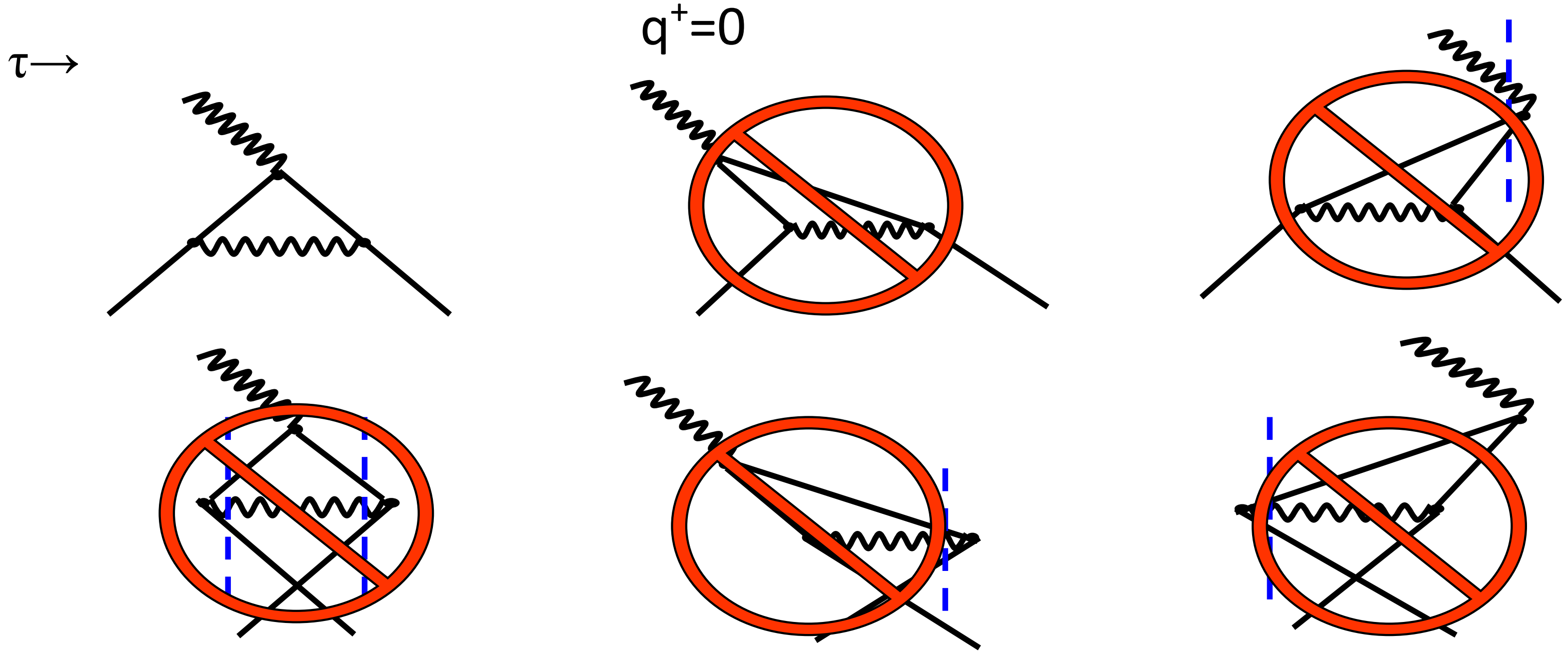


# g-2 calculation




- Vacuum fluctuations are suppressed in LFD and clean hadron phenomenology is possible.

# g-2 calculation

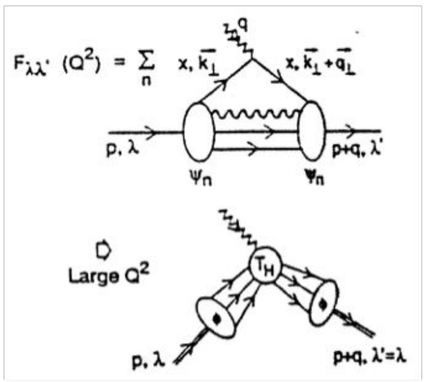



- Vacuum fluctuations are suppressed in LFD and clean hadron phenomenology is possible. Note that the dynamical difference between $q^+=0$ and $q^+\neq 0$.

# Applications to Hadron Phenomenology

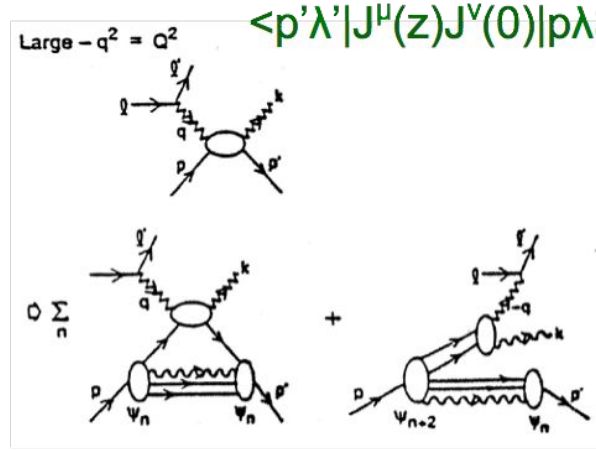
Form Factors $l\,p \to l'\,p'$

$\langle p'\,\lambda'|J^+(0)|p\,\lambda\rangle$

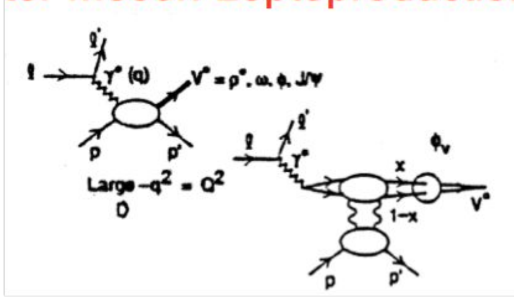



Virtual Compton $\gamma^*\,p \to \gamma'\,p'$

$\langle p'\lambda'|J^\mu(z)J^\nu(0)|p\lambda\rangle$

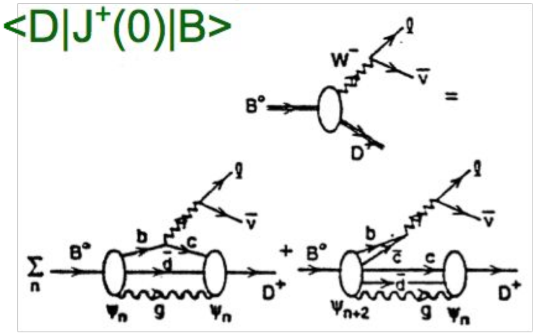



Vector Meson Leptoproduction $\gamma^* p \to V^* p'$

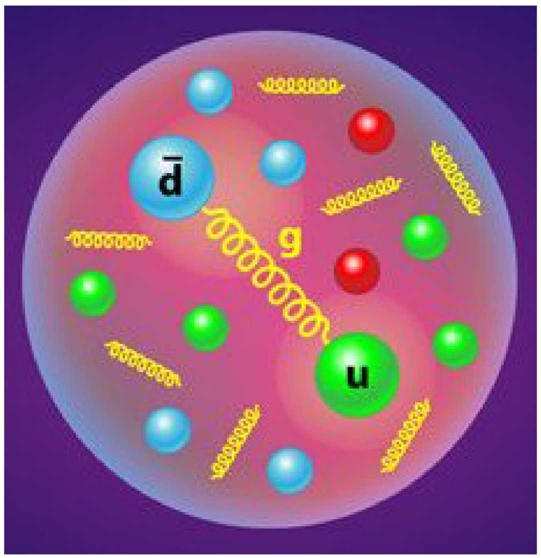



Weak Decay

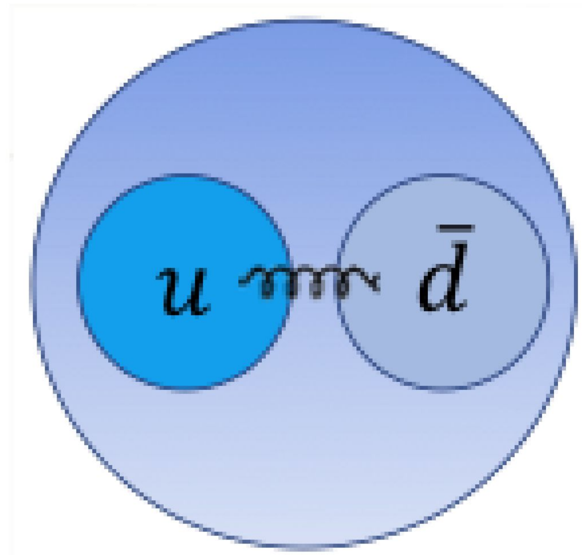
$\langle D|J^+(0)|B\rangle$



# Take advantage of LFD and Construct the Light-Front Quark Model (LFQM)




QCD




LFQM

# Bakamjian-Thomas Construction in LFD

B. Bakamjian and L. H. Thomas, Phys. Rev. 92, 1300 (1953).

Add interactions to the non-interacting representations without spoiling the Poincare Algebra satisfied by the interacting physical system.

$$M := M_0 + V$$

with

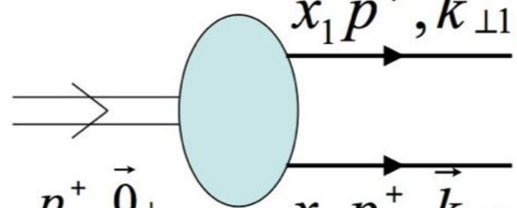
$$[\mathbf{E}_\perp, V]_- = [K^3, V]_- = [\mathbf{j}_{f0}, V]_- = [\mathbf{P}_\perp, V]_- = [P^+, V]_- = 0.$$

# Effective Constituent Quark Model for Low $Q^2$

$$|Meson\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \ldots$$
$$\approx \Psi_{Q\bar{Q}}|Q\bar{Q}\rangle,$$

*where*

$$|Q\rangle = \psi^{Q}_{q}|q\rangle + \psi^{Q}_{qg}|qg\rangle + \ldots$$
$$|\bar{Q}\rangle = \psi^{\bar{Q}}_{\bar{q}}|\bar{q}\rangle + \psi^{\bar{Q}}_{\bar{q}g}|\bar{q}g\rangle + \ldots$$




$$\Psi_{Q\bar{Q}}(x_i, \vec{k}_{\perp i}, \lambda_i) = \Phi(x_i, \vec{k}_{\perp i})\chi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Radial
(Dependent on the model potential)

H = T + V
V includes Coulomb, Confinement, Spin-Spin,Spin-Orbit interactions.

Spin-Orbit
(Interaction independent Melosh transformation)

$J^{PC} = 0^{++}(f_0, a_0, \ldots)$
$0^{-+}(\pi, K, \eta, \eta', \ldots)$
$1^{--}(\rho, K^*, \omega, \phi, \ldots)$
$\ldots$

PHYSICAL REVIEW C **92**, 055203 (2015)

# Variational analysis of mass spectra and decay constants ···

Ho-Meoyng Choi,[1] Chueng-Ryong Ji,[2] Ziyue Li,[2] and Hui-Young Ryu[1]

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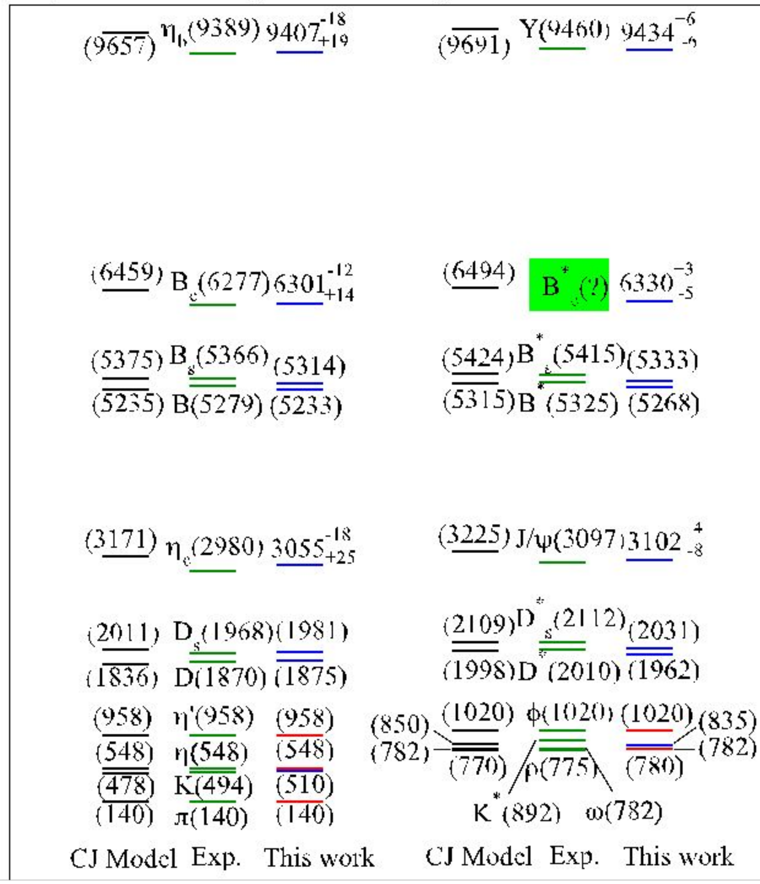
(Received 15 September 2015; published 13 November 2015)



PHYSICAL REVIEW D **102**, 036005 (2020)

# Chiral anomaly and the pion properties in the light-front quark model

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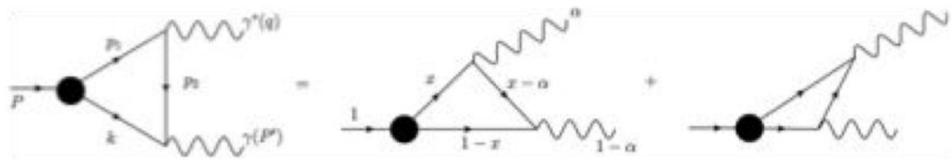


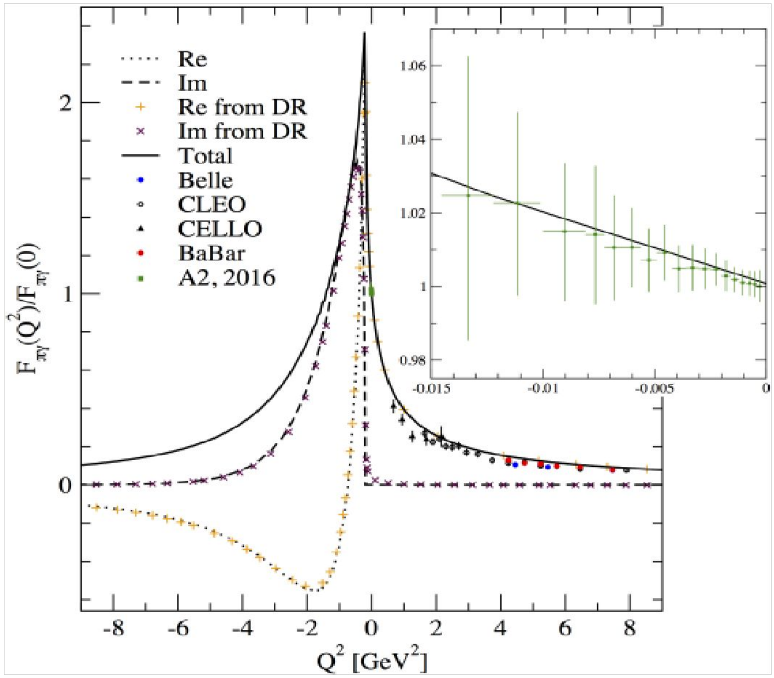
$$\Gamma_{\pi^0\to\gamma\gamma} = \frac{\pi}{4}\alpha_{\rm em}^2 M_\pi^3 |F_{\pi\gamma}(0)|^2$$

$$\Psi^\pi_{Q\bar{Q}} \equiv \Psi_\pi(x_i, \mathbf{k}_{i\perp}, \lambda_i) = \phi_R(x_i, \mathbf{k}_{i\perp})\chi(x_i, \mathbf{k}_{i\perp}, \lambda_i),$$

$$\chi_{\lambda_1\lambda_2}(x, \mathbf{k}_\perp) = \mathcal{N}\bar{u}_{\lambda_1}(k_1)\Gamma_\pi v_{\lambda_2}(k_2),$$

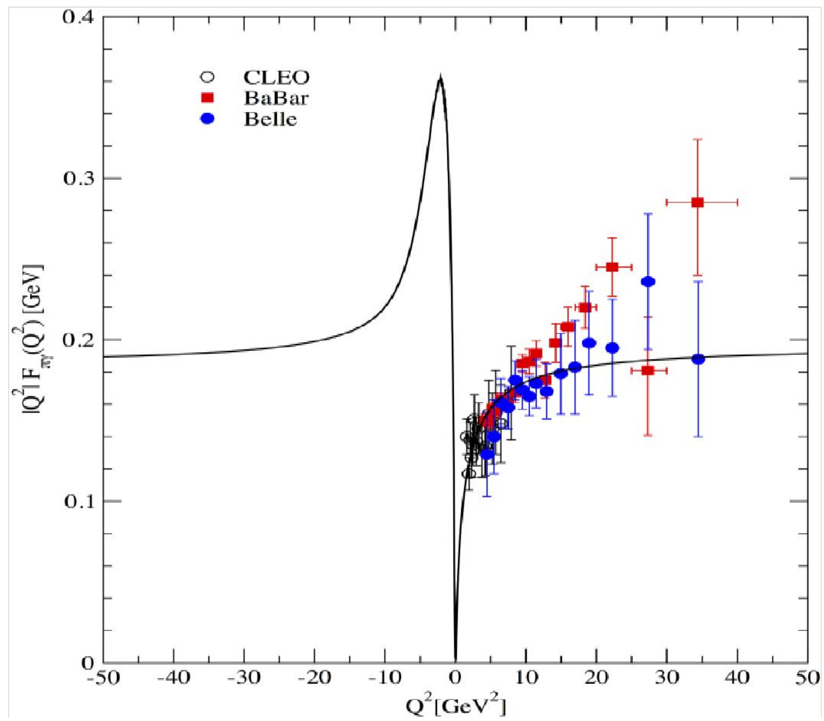
$$\Gamma_\pi = (A_\pi + B_\pi \not{P})\gamma_5$$

$$F_{\pi\gamma}(q^2) = \frac{e_u^2 - e_d^2}{\sqrt{2}}\frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 \frac{dx}{(1-x)}\int d^2\mathbf{k}_\perp \frac{\psi_\pi(x, \mathbf{k}_\perp)}{M_0^2 - q^2}$$

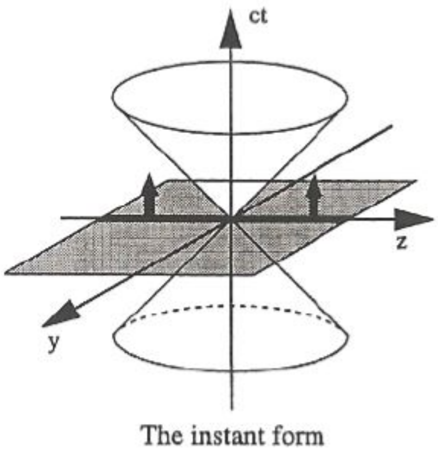



Both spacelike and timelike form factors can be computed in LFQM.

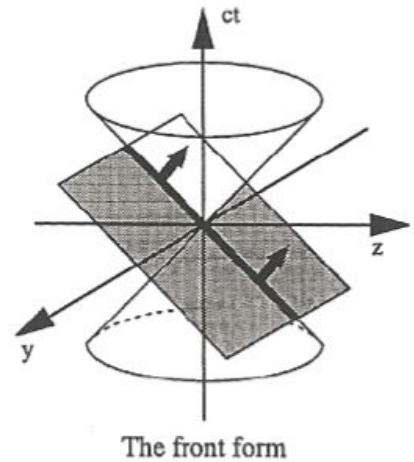
H.-M. Choi, H.-Y. Ryu, C.-R.Ji, PRD96, 056008 (2017)



# Can IFD and LFD be linked?




1949




**Yes, they can!**

1992~2021

Traditional approach
evolved from NR dynamics

Close contact with
Euclidean space

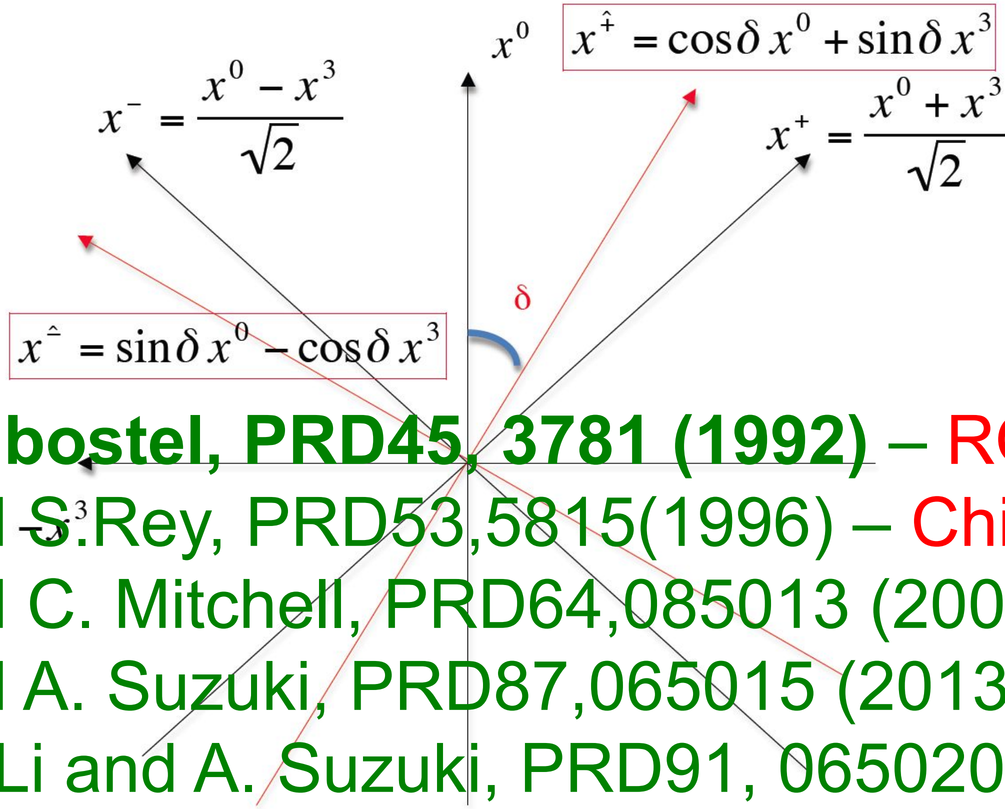
T-dept QFT, LQCD, IMF, etc.

Innovative approach
for relativistic dynamics

Strictly in Minkowski space

DIS, PDFs, DVCS, GPDs, etc.

# Interpolation between IFD and LFD




**K. Hornbostel, PRD45, 3781 (1992)** – RQFT

C.Ji and S.Rey, PRD53,5815(1996) – Chiral Anomaly

C.Ji and C. Mitchell, PRD64,085013 (2001) – Poincare Algebra

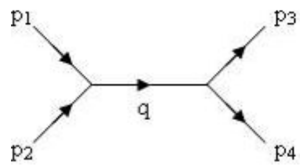
C.Ji and A. Suzuki, PRD87,065015 (2013) – Scattering Amps

C.Ji, Z. Li and A. Suzuki, PRD91, 065020 (2015) – EM Gauges

Z.Li, M. An and C.Ji, PRD92, 105014 (2015) – Spinors

C.Ji, Z.Li, B.Ma and A.Suzuki, PRD98, 036017(2018) – QED

B.Ma and C.Ji, PRD104, 036004(2021), – $QCD_{1+1}$




$$\delta = 0$$

$$p_0 = p^0$$

$$-p_3 = p^3$$

$$0 < \delta < \pi/4$$

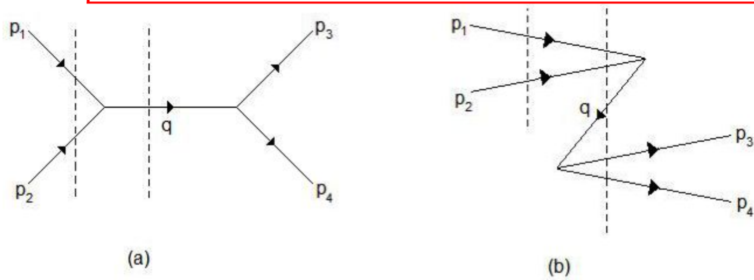
$$p_{\hat{+}} = p^0 \cos\delta - p^3 \sin\delta$$

$$p_{\hat{-}} = p^0 \sin\delta + p^3 \cos\delta$$

$$\delta = \pi/4$$

$$p_+ = p^-$$

$$p_- = p^+$$




$$\frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 - q^0} - \frac{1}{p_1^0 + p_2^0 + q^0}\right)$$

$$\frac{1}{2\omega_q}\left(\frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} - \omega_q}{\mathbb{C}}} - \frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}}}\right)$$

$$\omega_q = \sqrt{q_{\hat{-}}^2 + \mathbb{C}\left(\vec{\mathbf{q}}_{\hat{\perp}}^2 + m^2\right)}$$

$$\mathbb{C} = \cos 2\delta$$

$$\mathbb{S} = \sin 2\delta$$

$$\frac{1}{P^+}\frac{1}{\left\{P^- - \frac{(\vec{\mathbf{P}}_{\perp}^2 + m^2)}{2P^+}\right\}}$$

$$\frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}} \rightarrow \frac{2}{\mathbb{C}} - \frac{\vec{\mathbf{q}}_{\hat{\perp}}^2 + m^2}{2q_{\hat{-}}} + \mathcal{O}(\mathbb{C})$$

$$\rightarrow \infty \quad \text{as } \mathbb{C} \rightarrow 0$$

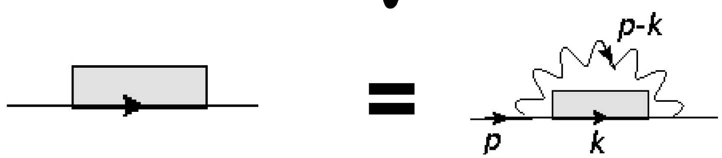
# Large $N_C$ QCD in 1+1 dim. ('tHooft Model)

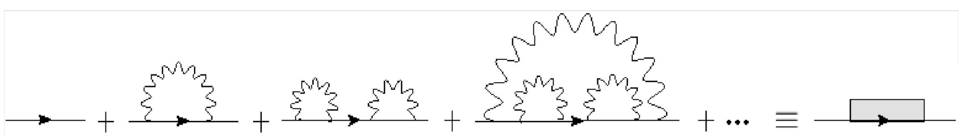
**Interpolating 't Hooft model between instant and front forms**

Bailing Ma and Chueng-Ryong Ji

PRD104, 036004(2021)

## MASS GAP EQUATION



$$\equiv i\,\Sigma(p)$$

$$\Sigma(p_{\hat{-}}) = i\frac{\lambda}{2\pi}\int \frac{dk_{\hat{-}}\,dk_{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\gamma^{\hat{+}}\frac{1}{\not{k} - m - \Sigma(k_{\hat{-}}) + i\epsilon}\gamma^{\hat{+}}$$

# Fermion Propagator

Free Propagator

$$S_f(p) = \frac{1}{\not{p} - m + i\varepsilon}$$

$$\Sigma(p) = \Sigma_s(p) + \Sigma_v(p)\not{p}$$

Interacting Propagator

$$S(p) = \frac{1}{\not{p} - m - \sum(p) + i\varepsilon}$$

$$= \frac{F(p)}{\not{p} - M(p) + i\varepsilon}$$

$$F(p) = \left(1 - \Sigma_v(p)\right)^{-1}$$ "Wave function renormalization factor"

$$M(p) = \frac{m + \Sigma_s(p)}{1 - \Sigma_v(p)}$$ "Renormalized fermion mass function"

# Energy-Momentum Dispersion Relation

**Free particle** | **Interacting particle**

$$E = \sqrt{p_z^2 + m^2}$$

$$\frac{F(p'_\perp)E(p'_\perp)}{\sqrt{C}} = \sqrt{p'^2_\perp + M(p'_\perp)^2} \equiv \tilde{E}(p'_\perp)$$

$$\theta_f = \tan^{-1}(p_z / m)$$

$$\theta(p'_\perp) = \theta_f(p'_\perp) + 2\zeta(p'_\perp)$$

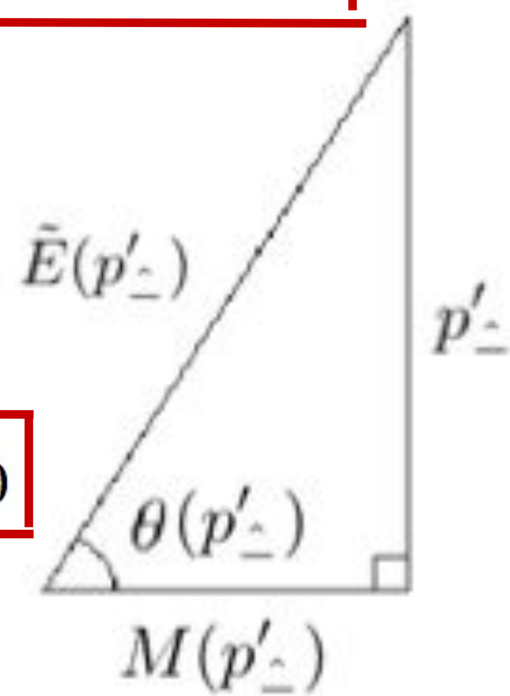
$$\beta = p_z / E$$

$$= \sin\theta_f$$

$$= \tanh\eta$$

$$\begin{pmatrix} b^i(p'_\perp) \\ d^{+i}(p'_\perp) \end{pmatrix} = \begin{pmatrix} \cos\zeta(p'_\perp) & -\sin\zeta(p'_\perp) \\ \sin\zeta(p'_\perp) & \cos\zeta(p'_\perp) \end{pmatrix} \begin{pmatrix} b^i_f(p'_\perp) \\ d^{+i}_f(p'_\perp) \end{pmatrix}$$

$b^i_f\,|0>=0, d^i_f\,|0>=0$ **vs.** $b^i\,|\Omega>=0, d^i\,|\Omega>=0$




**Interpolation**

$$(E, p_z) \Rightarrow (\hat{p}^+ / \sqrt{C}, p_\perp / \sqrt{C} \equiv p'_\perp)$$

# Mass Gap Equation in Scaled Variables

$$\bar{p}'_{\underline{\wedge}} = \frac{\bar{p}_{\underline{\wedge}}}{\sqrt{\mathbb{C}}},\ \bar{E}' = \frac{\bar{E}}{\sqrt{\mathbb{C}}}, \bar{p}_{\underline{\wedge}} = \frac{p_{\underline{\wedge}}}{\sqrt{2\lambda}},\ \bar{E} = \frac{E}{\sqrt{2\lambda}}, \bar{m} = \frac{m}{\sqrt{2\lambda}}$$

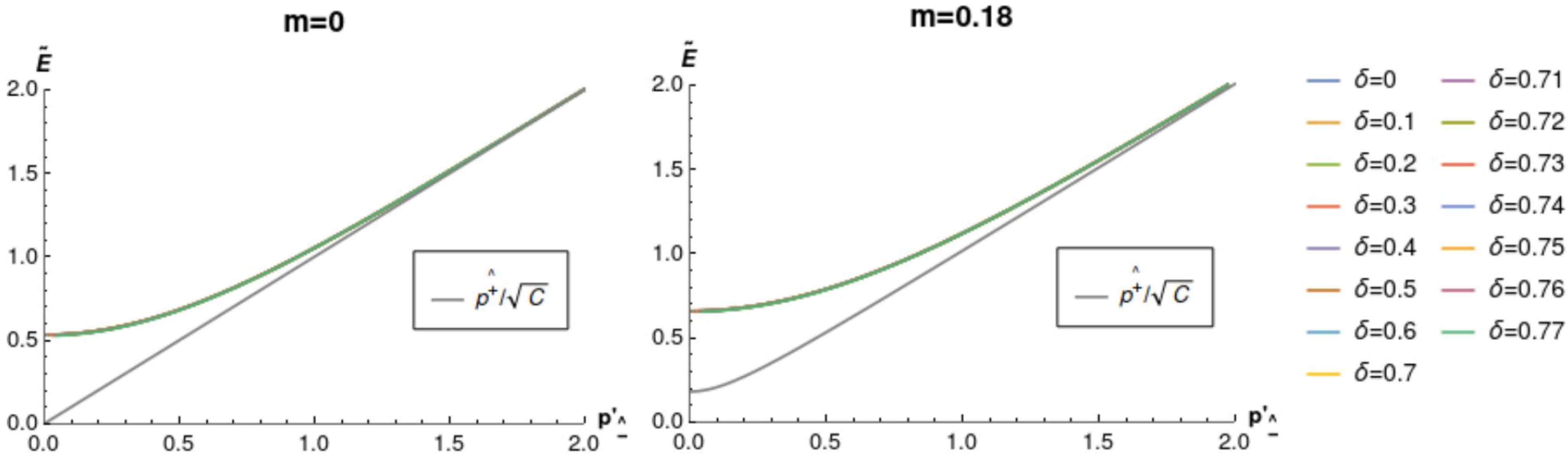
$$\bar{p}'_{\underline{\wedge}} \cos\theta(\bar{p}'_{\underline{\wedge}}) - \bar{m}\sin\theta(\bar{p}'_{\underline{\wedge}}) = \frac{1}{4}\int \frac{d\bar{k}'_{\underline{\wedge}}}{(\bar{p}'_{\underline{\wedge}} - \bar{k}'_{\underline{\wedge}})^2} \sin\left(\theta(\bar{p}'_{\underline{\wedge}}) - \theta(\bar{k}'_{\underline{\wedge}})\right)$$

$$\bar{E}'(\bar{p}'_{\underline{\wedge}}) = \bar{p}'_{\underline{\wedge}} \sin\theta(\bar{p}'_{\underline{\wedge}}) + \bar{m}\cos\theta(\bar{p}'_{\underline{\wedge}}) + \frac{1}{4}\int \frac{d\bar{k}'_{\underline{\wedge}}}{(\bar{p}'_{\underline{\wedge}} - \bar{k}'_{\underline{\wedge}})^2} \cos\left(\theta(\bar{p}'_{\underline{\wedge}}) - \theta(\bar{k}'_{\underline{\wedge}})\right)$$

$$\frac{p_{\underline{\wedge}}}{\mathbb{C}} \cos\theta(p_{\underline{\wedge}}) - \frac{m}{\sqrt{\mathbb{C}}}\sin\theta(p_{\underline{\wedge}}) = \frac{\lambda}{2}\int \frac{dk_{\underline{\wedge}}}{(p_{\underline{\wedge}} - k_{\underline{\wedge}})^2} \sin\left(\theta(p_{\underline{\wedge}}) - \theta(k_{\underline{\wedge}})\right)$$

$$E(p_{\underline{\wedge}}) = p_{\underline{\wedge}} \sin\theta(p_{\underline{\wedge}}) + \sqrt{\mathbb{C}}m\cos\theta(p_{\underline{\wedge}}) + \frac{\mathbb{C}\lambda}{2}\int \frac{dk_{\underline{\wedge}}}{(p_{\underline{\wedge}} - k_{\underline{\wedge}})^2} \cos\left(\theta(p_{\underline{\wedge}}) - \theta(k_{\underline{\wedge}})\right)$$

# Mass Gap Solutions



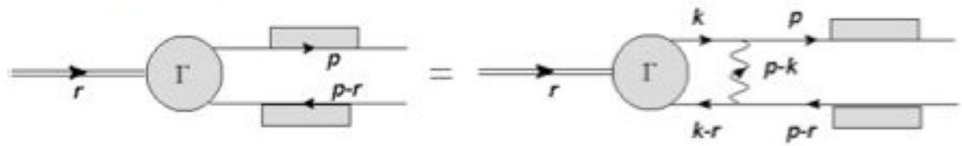

$$\tilde{E}(0) = \frac{F(0)E(0)}{\sqrt{\mathbb{C}}} = M(0)$$

| $m$ | 0 | 0.045 | 0.18 | 0.749 | 1.00 | 2.11 | 4.23 |
|---|---|---|---|---|---|---|---|
| $M(0)$ | 0.532778 | 0.563644 | 0.659112 | 1.10105 | 1.31167 | 2.30969 | 4.34358 |
| $F(0)$ | −0.495173 | −0.584175 | −0.987673 | 4.11079 | 2.17976 | 1.22134 | 1.05526 |

$$m \lesssim 0.56$$

# BOUND-STATE EQUATION

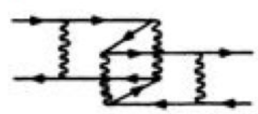
$$\Gamma(r,p) = \frac{i\lambda}{2\pi} \int \frac{dk_{\hat{-}}\, dk_{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2} S(p) \gamma^{\hat{+}} \Gamma(r,k) \gamma^{\hat{+}} S(p-r)$$





$$\left[-r_{\hat{+}} + \frac{-\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}(p_{\hat{-}} - r_{\hat{-}}) + E(p_{\hat{-}} - r_{\hat{-}})}{\mathbb{C}}\right] \hat{\phi}_+(r_{\hat{-}}, p_{\hat{-}})$$
$$= \lambda \int \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2} \left[C(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}}) \hat{\phi}_+(r_{\hat{-}}, k_{\hat{-}}) - S(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}}) \hat{\phi}_-(r_{\hat{-}}, k_{\hat{-}})\right],$$

$$\left[r_{\hat{+}} + \frac{-\mathbb{S}(p_{\hat{-}} - r_{\hat{-}}) + E(p_{\hat{-}} - r_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}}\right] \hat{\phi}_-(r_{\hat{-}}, p_{\hat{-}})$$
$$= \lambda \int \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2} \left[C(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}}) \hat{\phi}_-(r_{\hat{-}}, k_{\hat{-}}) - S(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}}) \hat{\phi}_+(r_{\hat{-}}, k_{\hat{-}})\right].$$

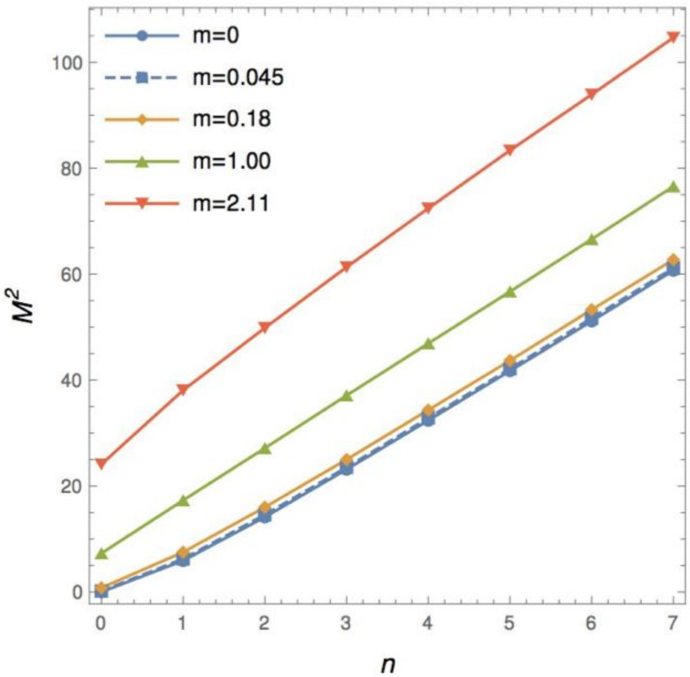


**LFD** $\left[\mathcal{M}^2 - \frac{m^2 - 2\lambda}{x} - \frac{m^2 - 2\lambda}{1-x}\right] \phi(x) = -2\lambda \int_0^1 \frac{dy}{(x-y)^2} \phi(y)$

# Meson Spectroscopy

**LFD**

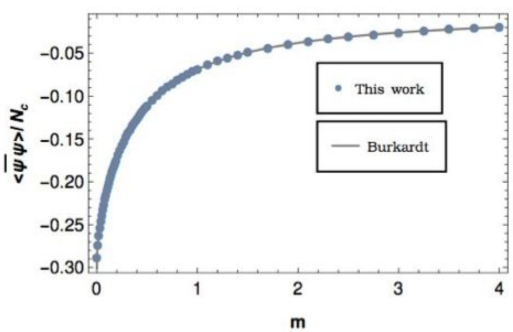
**G.'tHooft, NPB75,461 (1974)**




**IFD**

**Li,Wilets,Birse JPG:NP13,915 (1987)**

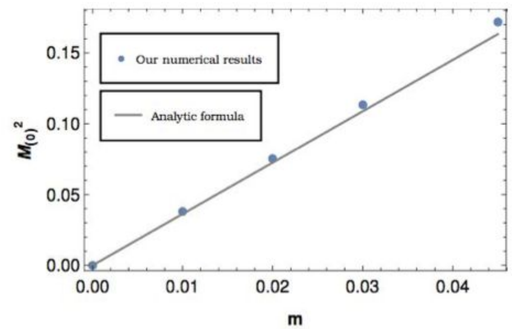
**Jia,Liang,Xiong JHEP11,151 (2017)**



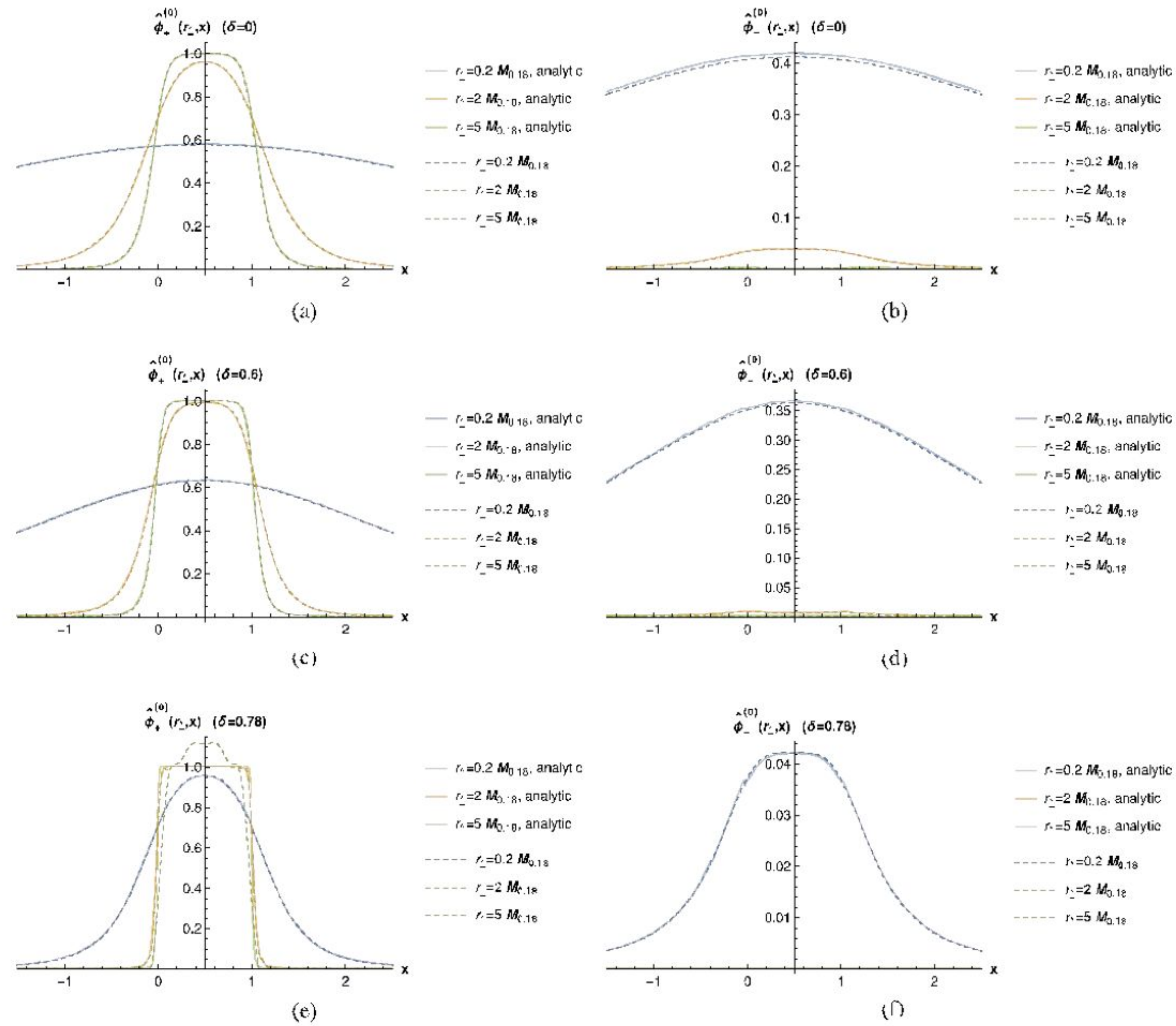

$$\mathcal{M}_\pi^2 = -\frac{4m < \bar{\psi}\psi >}{f_\pi^2} = \sqrt{\frac{8\pi^2 m^2 \lambda}{3}}$$

$$f_\pi = \sqrt{N_c/\pi}$$

$$< \bar{\psi}\psi > = -N_c/\sqrt{12}$$



# Meson Wavefunctions



# Parton Distribution Functions (PDFs)

$$q_n(x) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-}$$

$$\times \langle P_n^-, P^+ | \bar{\psi}(\xi^-)\gamma^+ \mathcal{W}[\xi^-,0]\psi(0) | P_n^-, P^+ \rangle_C,$$

$$\mathcal{W}[\xi^-,0] = \mathcal{P}\left[\exp\left(-ig_s \int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\right]$$

**$A^+=0$ Gauge in LFD**

## Quasi-PDFs

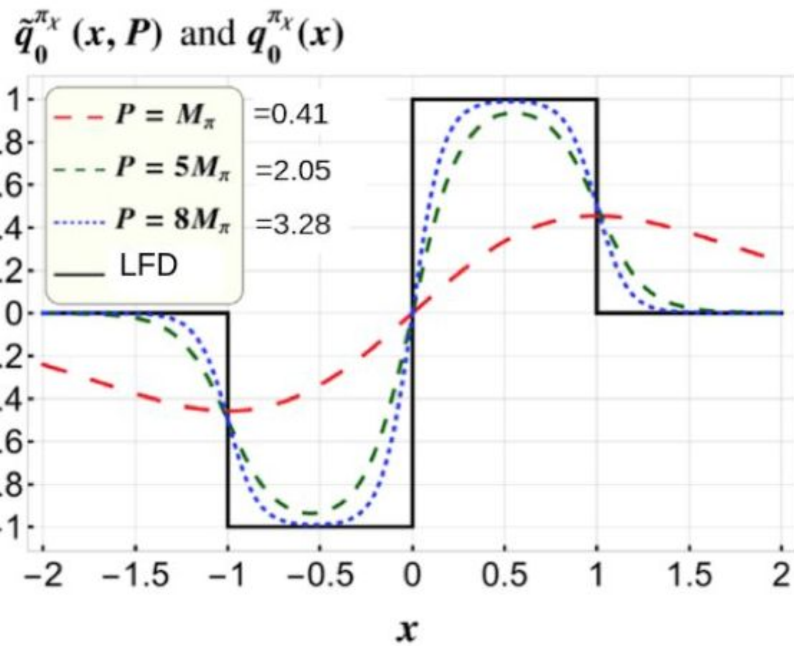
$$\tilde{q}_{(n)}(r_{\hat{-}}, x) = \int_{-\infty}^{+\infty} \frac{dx^{\hat{-}}}{4\pi} \, \mathrm{e}^{ix^{\hat{-}} r_{\hat{-}}}$$

$$\times < r^{\hat{+}}_{(n)}, r_{\hat{-}} \mid \bar{\psi}(x^{\hat{-}}) \, \gamma_{\hat{-}} \, \mathcal{W}[x^{\hat{-}}, 0] \, \psi(0) \mid r^{\hat{+}}_{(n)}, r_{\hat{-}} >_C,$$

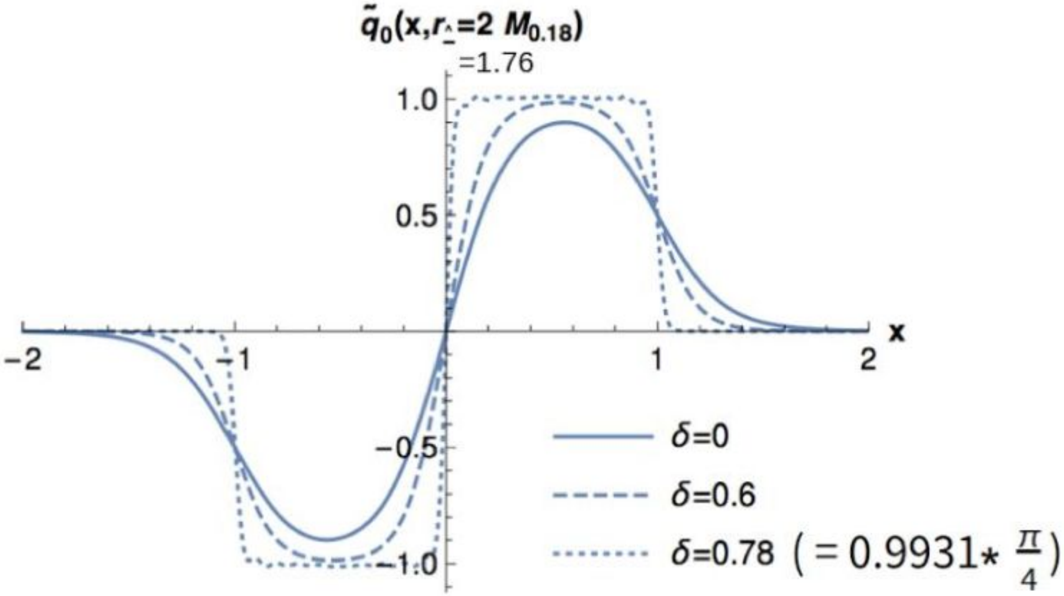
$$\mathcal{W}[x^{\hat{-}}, 0] = \mathcal{P}\left[\exp\left(-ig \int_0^{x^{\hat{-}}} dx'^{\hat{-}} A_{\hat{-}}(x'^{\hat{-}})\right)\right]$$

**Interpolating dynamics**

# Quasi-PDF




Quark quasi-PDFs and light-front PDF for the chiral pion.



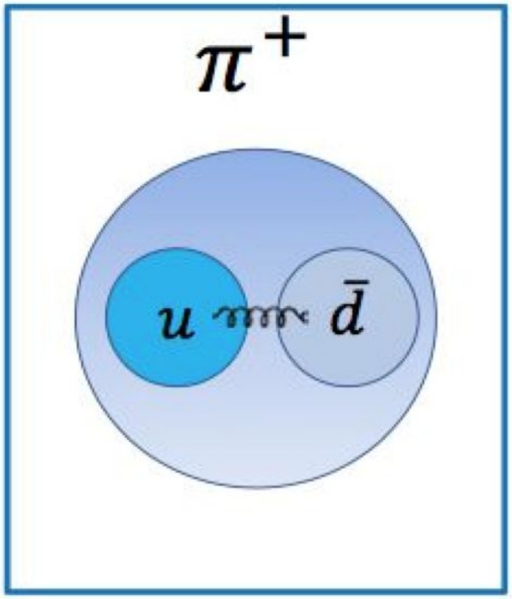

Interpolating "quasi-PDFs" for the chiral pion.

All quantities are in proper units of $\sqrt{2\lambda}$.

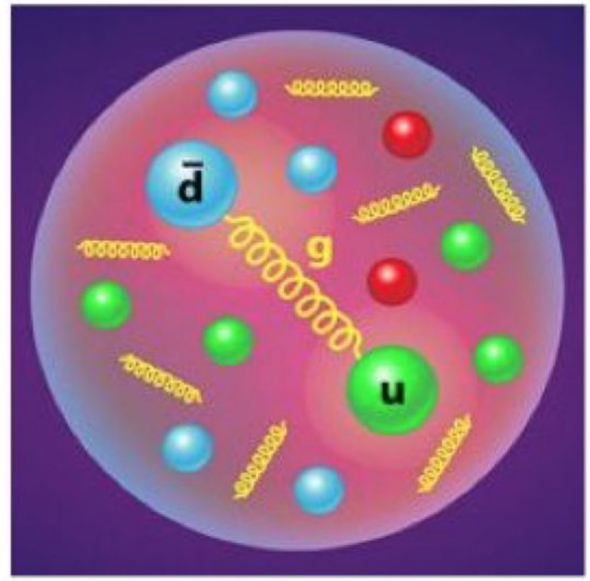

Jia, Y., Liang, S., Xiong, X., and Yu, R. (2018). Phys. Rev. D, 98:054011.

Ma, B. and Ji, C.-R. (2021). Phys. Rev. D, 104:036004.

# Pion's Dichotomy




vs.




**Constituent Quark Model**

$$M = m_1 + m_2 + A\frac{\vec{s}_1 \cdot \vec{s}_2}{m_1 m_2}$$

$$m_u = m_d = 310 MeV/c^2$$

$$A = \left(\frac{2m_u}{\hbar}\right)^2 160 MeV/c^2$$

**Quantum Chromodynamics**

**Isospin symmetry**

**Chiral symmetry**

$\mathrm{SU(2)_R \times SU(2)_L}$

**Spontaneous symmetry breakdown**

**Goldstone Bosons**

$$F_\pi^2 M_\pi^2 = -(m_u + m_d)\langle 0|\bar{u}u|0\rangle$$

**Effective field theory**

# Pion Properties

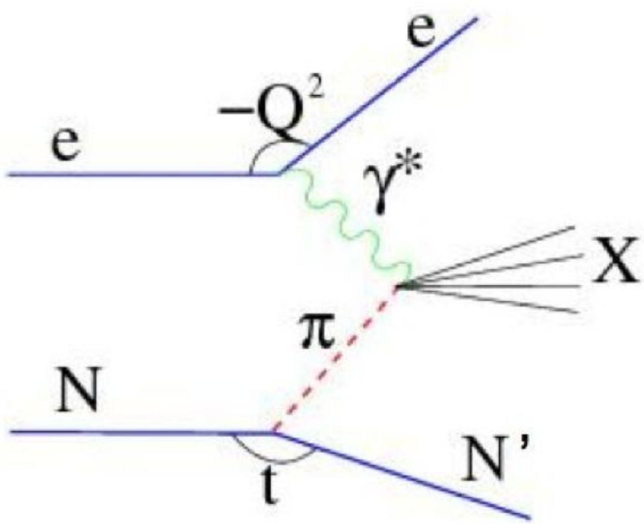
- Lightest bound state composed of quarks, antiquarks, and gluons
- Masses: $m_{\pi^\pm} = 139.57\ \mathrm{MeV}, m_{\pi^0} = 134.977\ \mathrm{MeV}$
- Lifetimes: $\tau_{\pi^\pm} = 2.603\times10^{-8}\ \mathrm{s}$ , $\tau_{\pi^0} = 8.52\times10^{-17}\ \mathrm{s}$

Charged pions decay via weak interaction

Neutral pions decay via electromagnetic interaction, *i.e.* $\pi^0 \rightarrow \gamma\gamma$

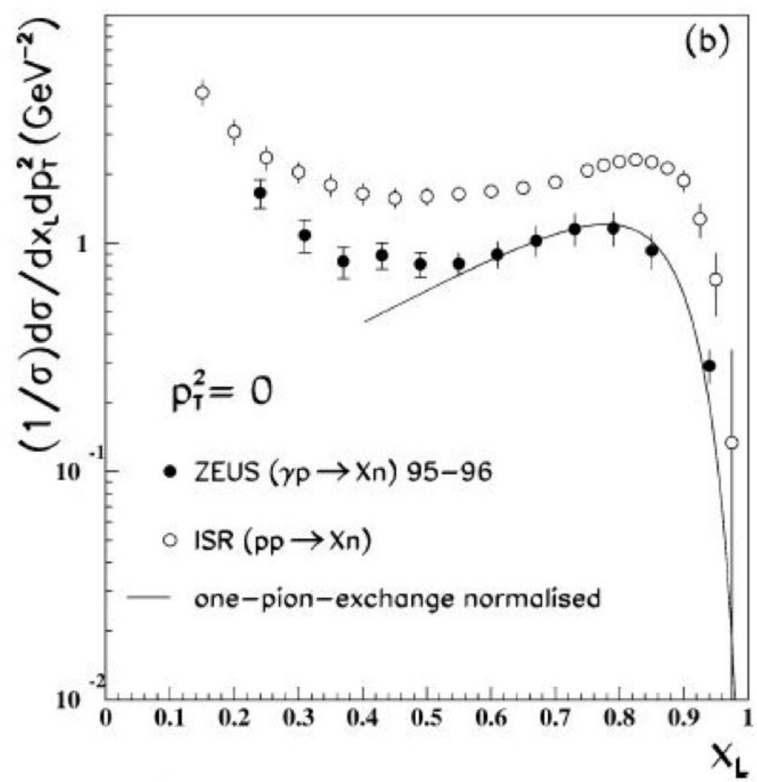
Measurement of Tagged Deep Inelastic Scattering (TDIS)

C.Keppel (Contact person)




$$e + p(or\ n) \rightarrow e' + p + X$$

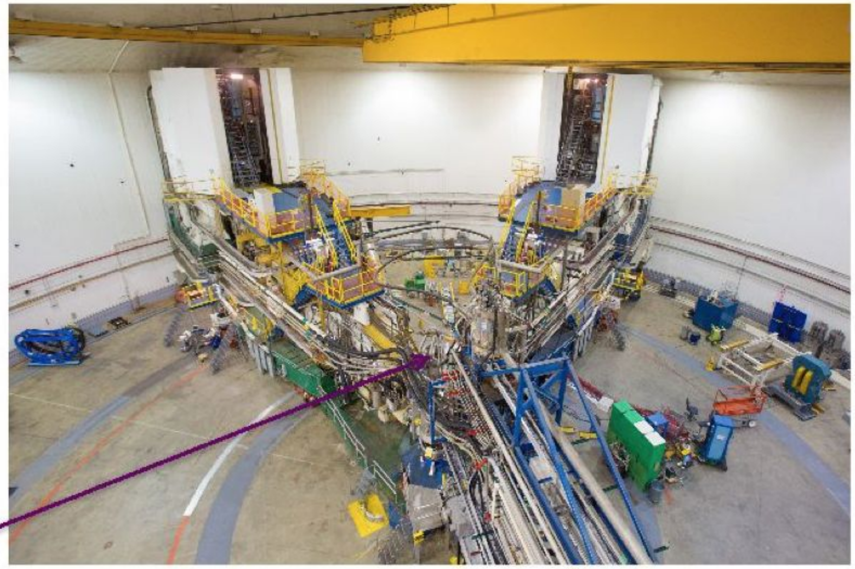
$$e + D \rightarrow e' + p + p + X$$




Leading neutron production in $e^+p$ collisions at HERA

ZEUS Collaboration, NPB 637 (2002) 3–56

# JLab Hall A TDIS Experiment



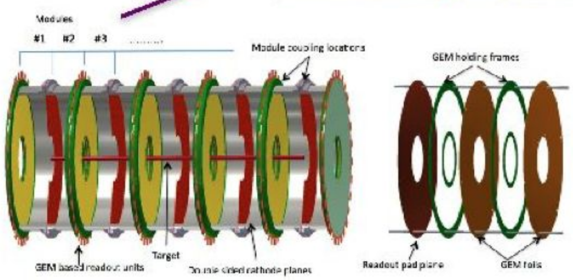
proton tag detection in GEM-based mTPC at pivot

*Hall A with SBS:*

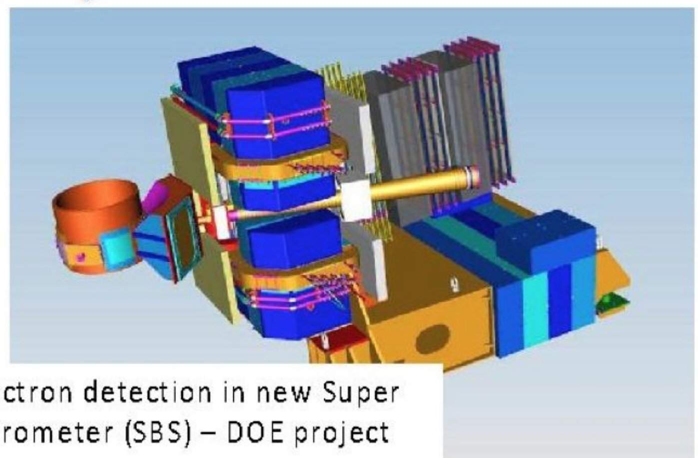
- High luminosity,
  50 ∝Amp,
  $L = 3x10^{36}/cm^2 s$
- Large acceptance
  ~70 msr

**Important for small cross sections**

e- beam

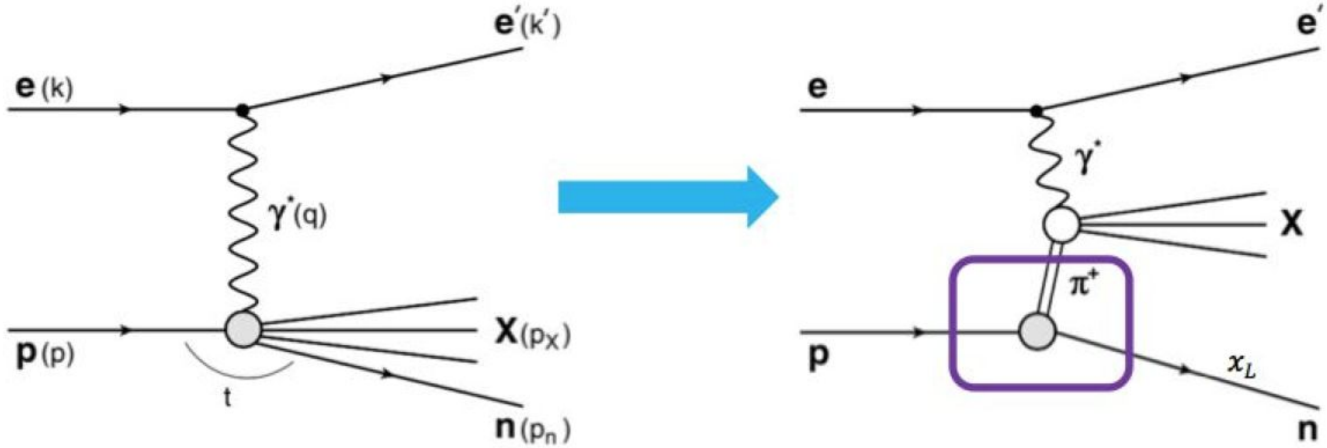



mTPC inside superconducting solenoid



Scattered electron detection in new Super Bigbite Spectrometer (SBS) – DOE project complete

# Convolution with Chiral Effective Theory




Denotes the splitting of $e.g., p \rightarrow \pi^+ n$
Notationally: $f_\pi(\bar{x}_L = 1 - x_L)$

$$(\bar{d} - \bar{u})(x) = \frac{2}{3} \int_x^1 \frac{dy}{y} \, f_\pi(y) \, \bar{q}^\pi(x/y)$$

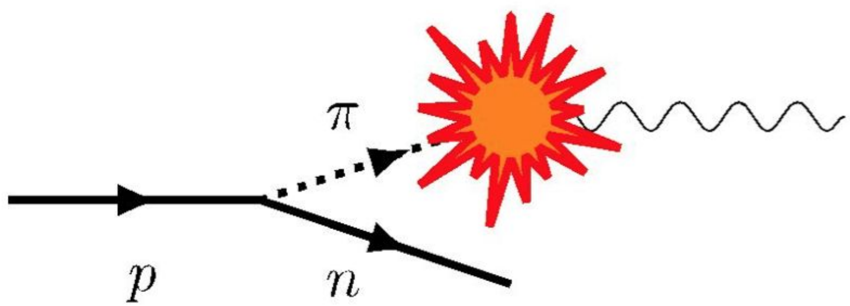
pion light-cone momentum distribution in nucleon

# First global Monte Carlo analysis of pion PDFs

P. Barry, N. Sato, W. Melnitchouk, C.Ji

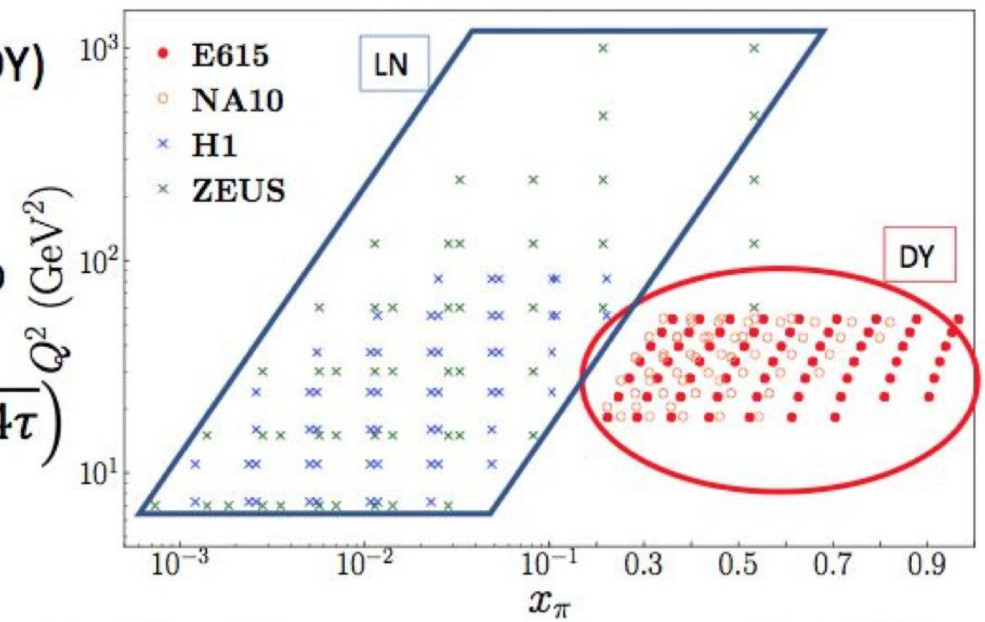
PRL121,152001(2018) Featured in Physics

- **How to probe pion structure**
  - $\pi + A \to l\bar{l} + X$ **(Drell-Yan)**
  - $\pi + A \to \gamma + X$ (prompt photons)
  - $e + p \to e' + n + X$ **(SIDIS)** $\to$ small $x_\pi$ gluon PDF



# Datasets vs. Kinematics

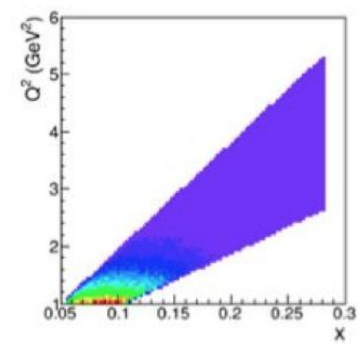
- Large $x_\pi$ -- Drell-Yan (DY)
- Small $x_\pi$ -- Leading Neutron (LN)
- Not much data overlap
- In DY:
  $$x_\pi = \frac{1}{2}\left(x_F + \sqrt{x_F^2 + 4\tau}\right)$$
- In LN:
  $$x_\pi = x_B / \bar{x}_L$$

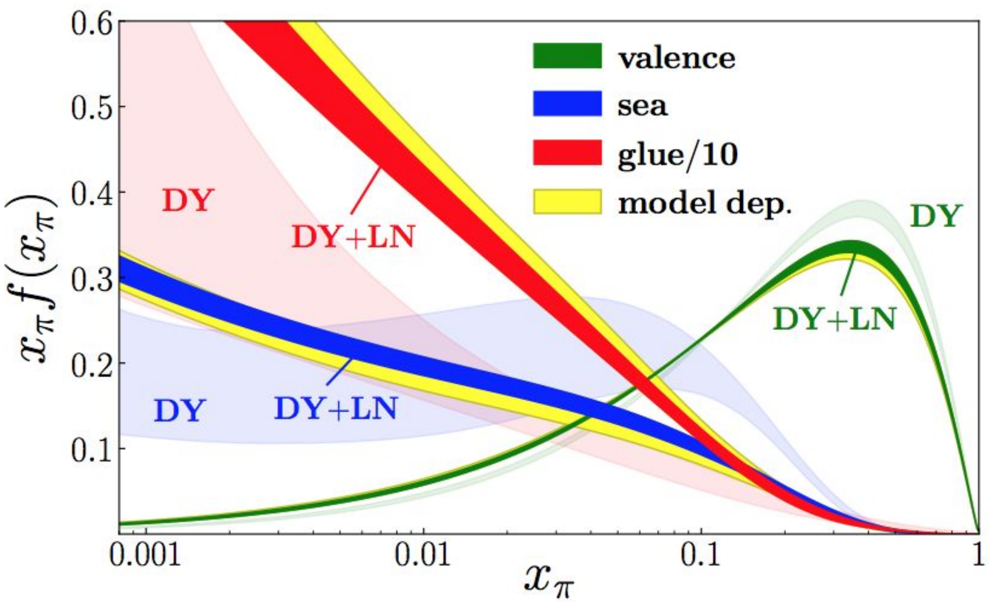



- JLab can reach much smaller $Q^2$ and larger $x$ range than in the HERA

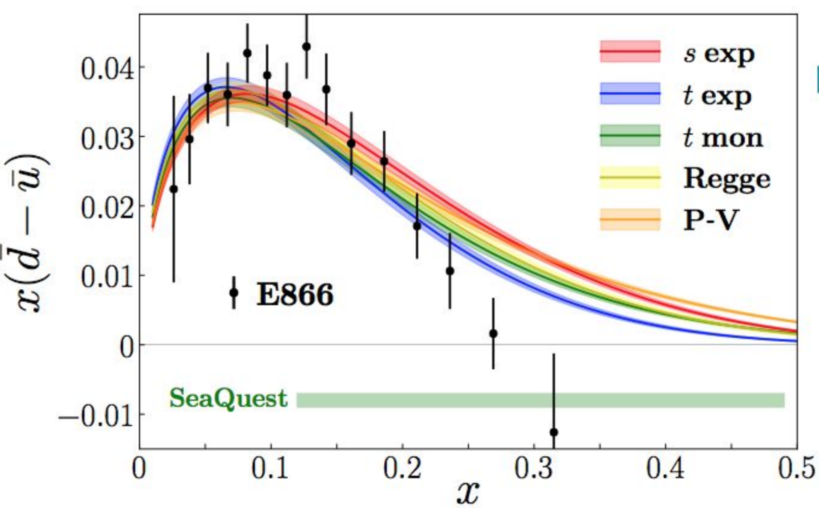
# EIC Impact on Pion PDFs

- $s = 5400$ GeV$^2$, 1.2% systematic uncertainty, integrated $\mathcal{L} = 100$fb$^{-1}$

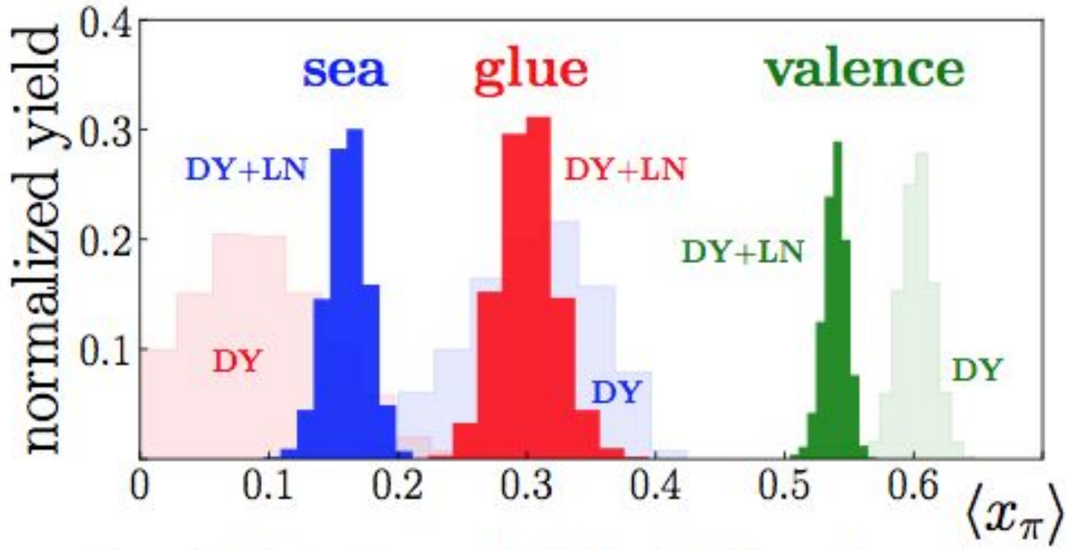





- Significant reduction of the uncertainties
- Non-overlapping uncertainties → tensions among the data
- Accuracy will be improved with future TDIS (JLab12/EIC)




- We performed an additional analysis of LN+DY+E866 → good description of E866 data except for large $x$




- Constraints from HERA significantly increase $\langle x_\pi^g \rangle$.
  **The role of the glue is more important than suggested by DY alone**
- In contrast, the strength of the sea is reduced
- Due to momentum sum rule $\langle x_\pi^{\mathrm{valence}} \rangle$ decreases

# Outlook

- LFD provides a useful tool to study highly nontrivial quatum chromodynamic phenomenology taking advantage its distinguished features such as the boost invariance and the cleaner vacuum properties.
- Corresponding the LFD results with the IFD and its IMF approach is useful to understand the complicate the confinement mechanism and the chiral symmetry aspects of QCD and the associate hadron phenomenology.
- Vigorous experimental measurements, e.g. 12 GeV upgrade of Jefferson Lab, future Electron Ion Collider projects, etc. are encouraging to provide deeper understanding on the nature of hadrons.