Energy-momentum tensor form factors with the flavor SU(3) symmetry breaking

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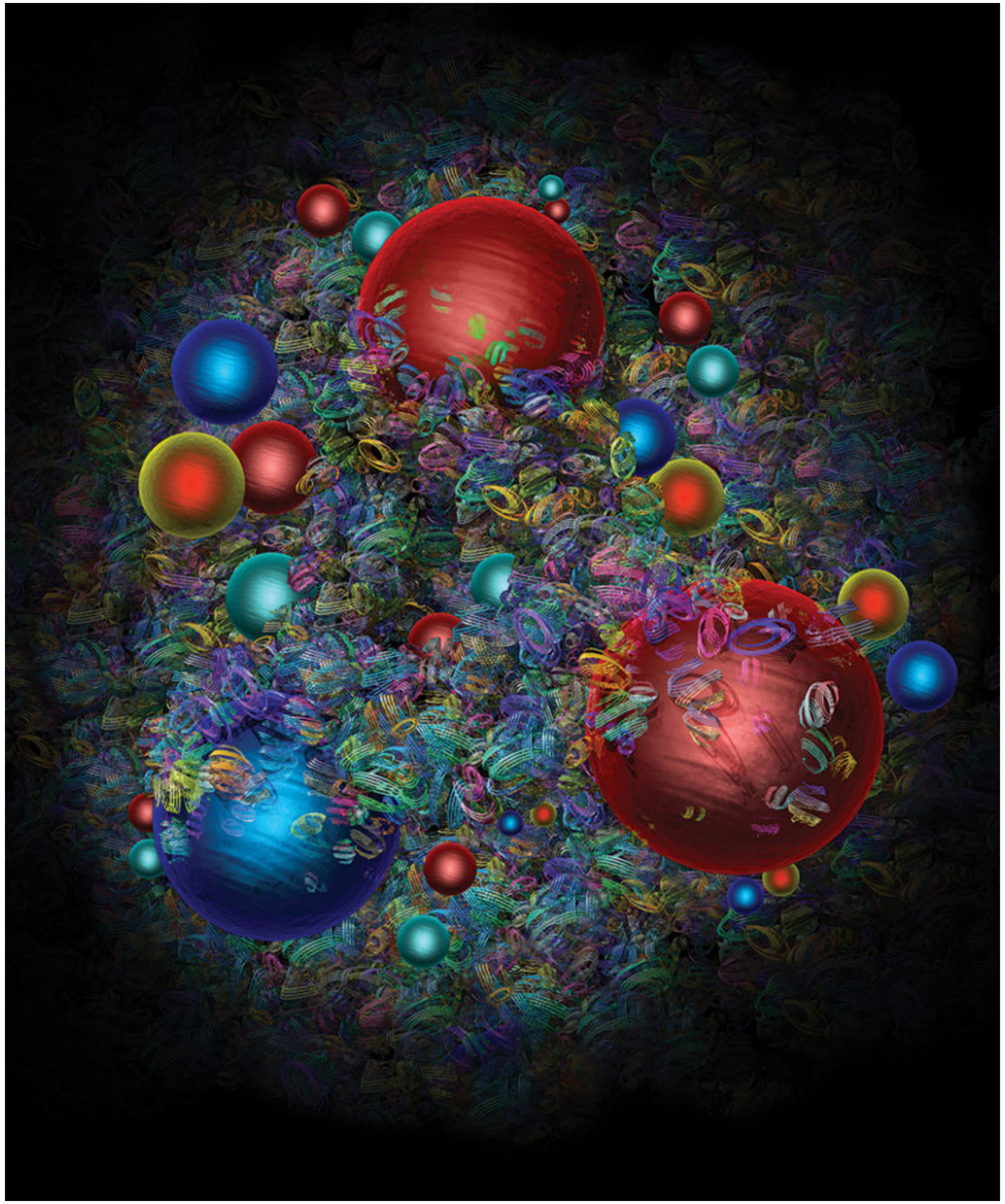
Contents

Introduction

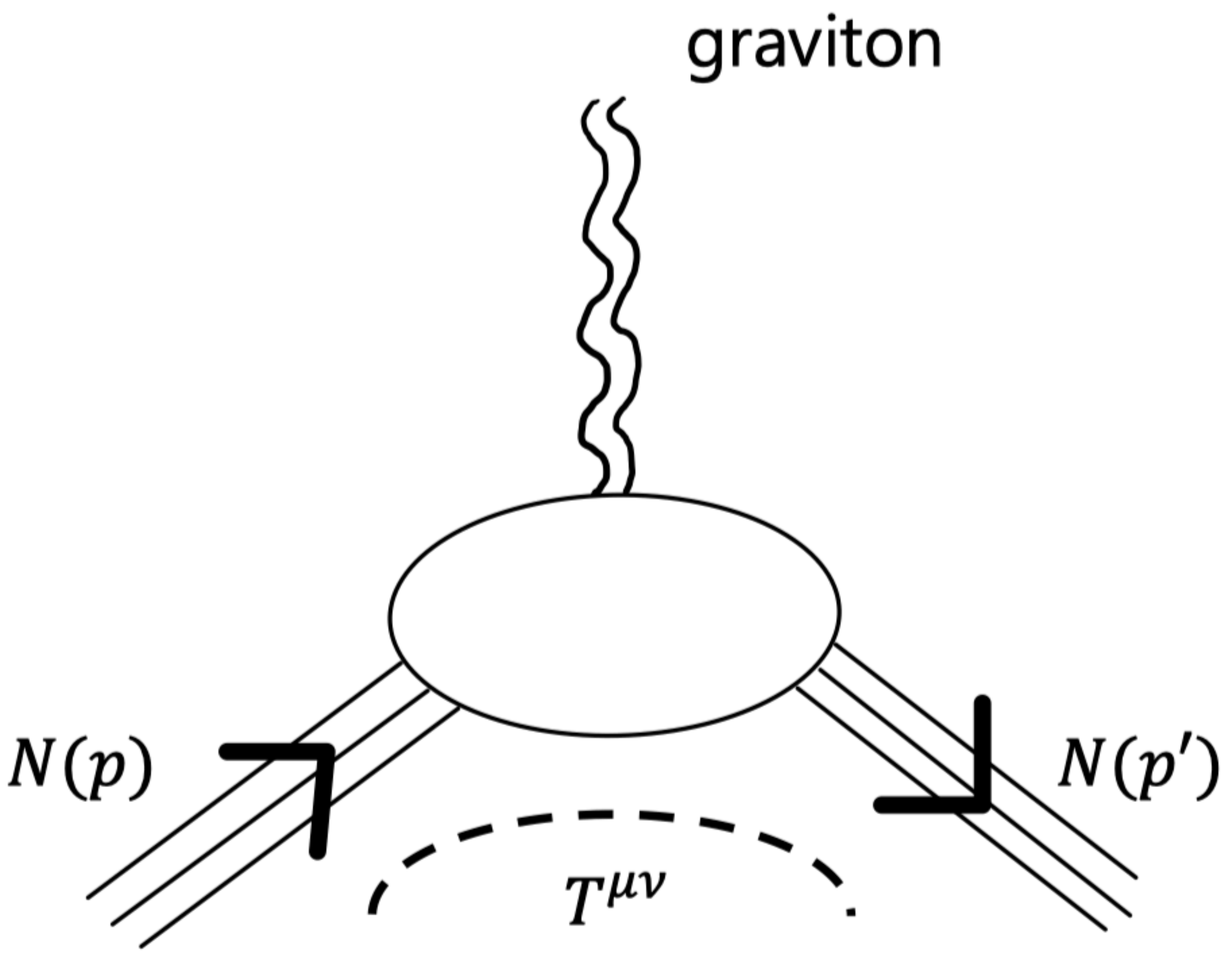
Energy-momentum tensor form factors

1. Why $m_N \sim 1\ \mathrm{GeV}$?
2. Why $S = \frac{1}{2}$?
3. Internal structure $d_1 = ?$

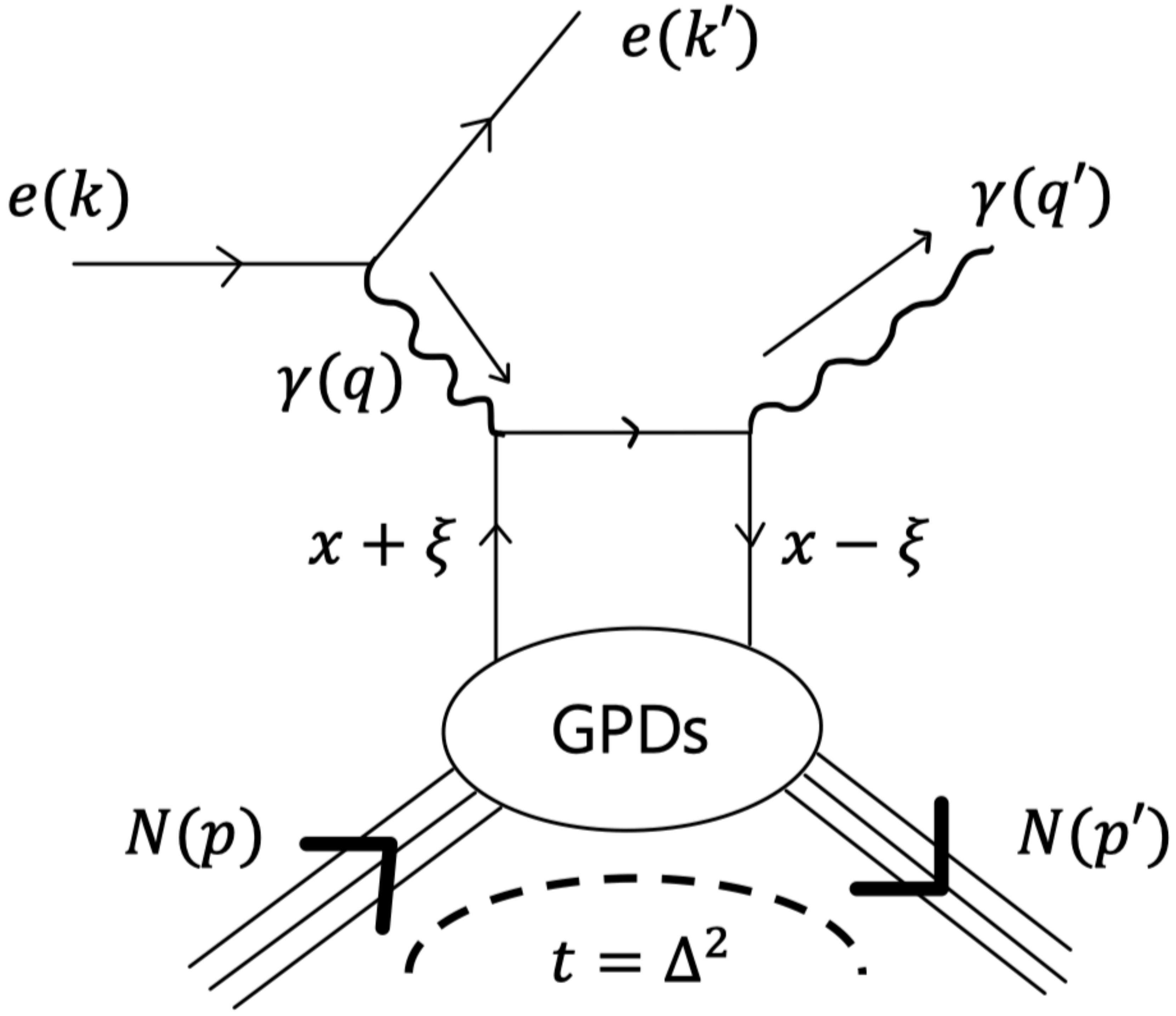


EMTFF measurement

The only direct probe - Graviton



Indirect probe
Deeply virtual Compton scattering (DVCS)



Modern concept of form factors

Generalized Parton Distribution functions (GPDs)

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P'|\bar{\psi}\left(-\frac{\lambda n}{2}\right)\gamma^\mu \psi\left(\frac{\lambda n}{2}\right)|P\rangle$$

$$= H(x,\xi,t)\bar{u}(P')\gamma^\mu u(P) + E(x,\xi,t)\bar{u}(P')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M_B}u(P) + \cdots$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P'|\bar{\psi}\left(-\frac{\lambda n}{2}\right)\gamma^\mu \gamma_5 \psi\left(\frac{\lambda n}{2}\right)|P\rangle$$

$$= \tilde{H}(x,\xi,t)\bar{u}(P')\gamma^\mu \gamma_5 u(P) + \tilde{E}(x,\xi,t)\bar{u}(P')\frac{\gamma_5 \Delta^\mu}{2M_B}u(P) + \cdots$$

Mellin's moments for twist-2

$$\int_{-1}^{1} dx\, x^N H(x,\xi,t) = h_0(t) + h_2(t)\xi^2 + \cdots + h_{N+1}(t)\xi^{N+1}$$

$$\int_{-1}^{1} dx\, x^N E(x,\xi,t) = e_0(t) + e_2(t)\xi^2 + \cdots + e_{N+1}(t)\xi^{N+1}$$

Modern concept of form factors

Mellin's first moment

$$\int_{-1}^{1} dx H(x, \xi, t) = F_1(t)$$

$$\int_{-1}^{1} dx E(x, \xi, t) = F_2(t)$$

→ **Electromagnetic form factors (Dirac and Pauli form factor)**

$$\int_{-1}^{1} dx \tilde{H}(x, \xi, t) = G_A(t)$$

$$\int_{-1}^{1} dx \tilde{E}(x, \xi, t) = G_P(t)$$

→ **Axial-vector and pseudo-scalar form factors**

Modern concept of form factors

Mellin's second moment

$$\int_{-1}^{1} dx\ x[H(x,\xi,t) + E(x,\xi,t)] = 2J(t)$$

$$\int_{-1}^{1} dx\ xH(x,\xi,t) = M_2(t) + \frac{4}{5}d_1(t)\xi^2$$

→ **EMT form factors**

→ **EMT form factors can be measured indirectly by using the GPDs.**

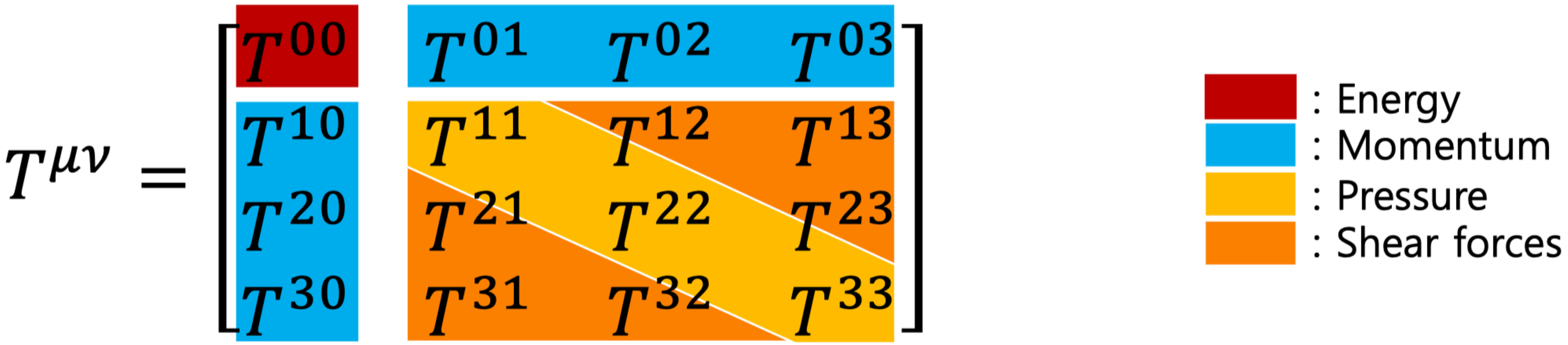
Energy-momentum tensor
and its form factors

Energy-momentum tensor current

- Belinfante-improved EMT (gauge invariant and symmetric)

$$T_Q^{\mu\nu} = \frac{i}{4}\left[\bar{\psi}\gamma^{\{\mu,}\overrightarrow{D}^{\nu\}}\psi - \bar{\psi}\gamma^{\{\mu,}\overleftarrow{D}^{\nu\}}\psi\right]$$

$$T_G^{\mu\nu} = \frac{1}{4}g^{\mu\nu}G^a_{\alpha\beta}G_a^{\alpha\beta} - G^{\mu\alpha,a}G^{\nu}_{a\alpha}$$



EMT form factors

$a = q, g$

- Baryon matrix elements

$$\langle P'|T^{\mu\nu}(0)|P\rangle = \bar{u}(P')\left[M_2(t)\frac{\bar{P}^\mu\bar{P}^\nu}{M_B} + J(t)\frac{i\bar{P}^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M_B} + d_1(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{5M_B} + M_B\bar{C}(t)g^{\mu\nu}\right]u(P)$$

Baryon mass

$$M_2(0) = \sum_a M_2^a(0) = 1$$

Baryon spin

$$J(0) = \sum_a J^a(0) = \frac{1}{2}$$

Baryon's mechanical properties
No constraint

Non-conservation term

$$\partial_\mu T^{\mu\nu} = 0 \to \sum_a \bar{C}^a(t) = 0$$

Stability conditions

Pressure and shear forces

- Static stress tensor

$$T^{ij}(\boldsymbol{r}) = \delta^{ij}\rho_p(r) + \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)\rho_s(r)$$

Pressure distribution

Shear forces distribution

$$\rho_s(r) = -\frac{1}{5M_B} r \frac{d}{dr}\frac{1}{r}\frac{d}{dr}\tilde{d}_1(r)$$

$$\rho_p(r) = \frac{2}{15M_B}\frac{1}{r^2}\frac{d}{dr} r^2 \frac{d}{dr}\tilde{d}_1(r)$$

$$\tilde{d}_1(r) = \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} d_1(t)$$

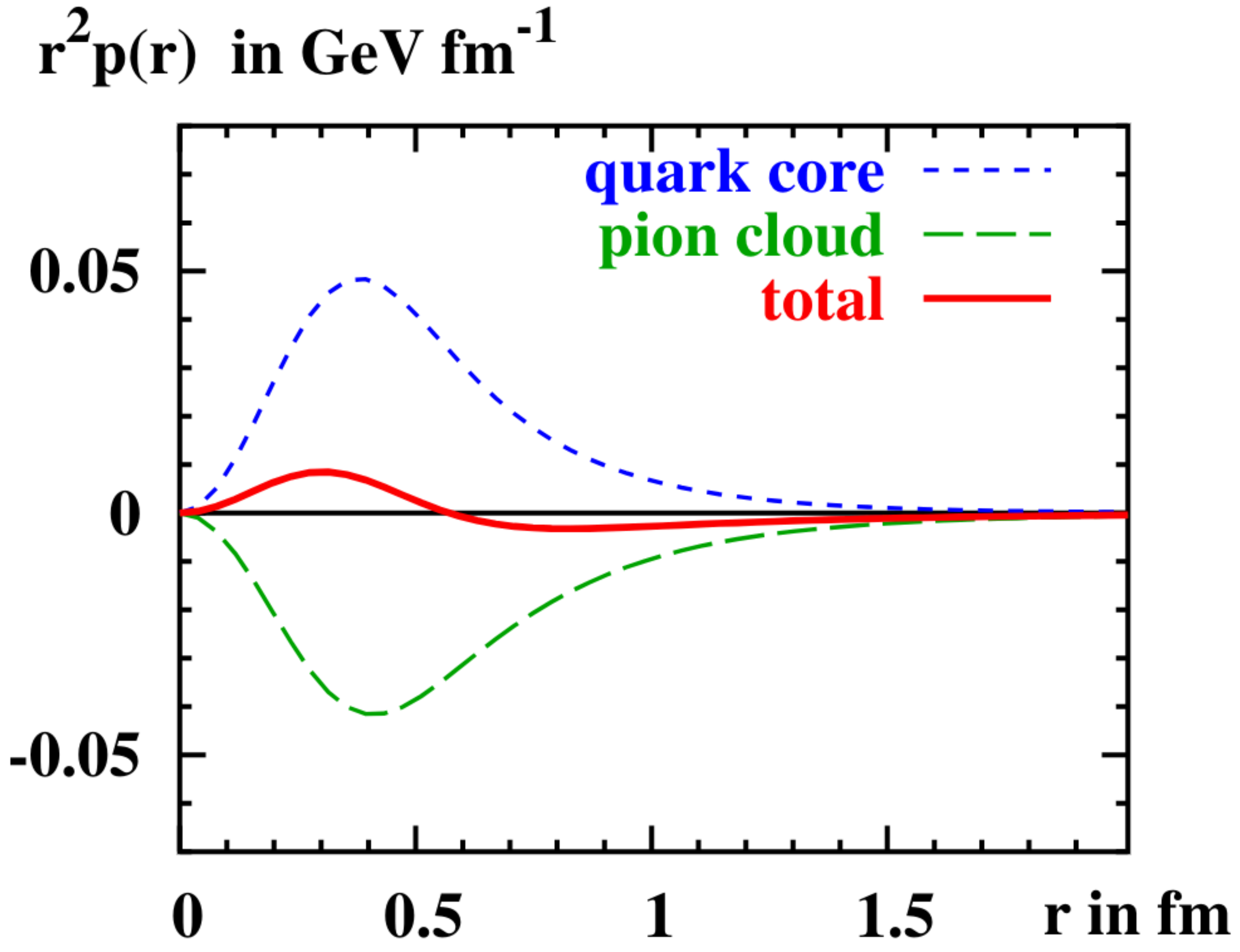
Global stability condition

- von Laue stability condition

$$\int_0^\infty dr\, r^2 \rho_p(r) = 0$$

At least, one node!

K. Goeke *et al.*, PRD 75 (2007)



In the inner region, $\rho_p > 0$ corresponds to the repulsion

In the outer region, $\rho_p < 0$ corresponds to the attraction

They are balanced to be existing the baryon.

However, even though an object satisfies the von Laue condition,
it can be unstable, which means **it is a necessary but not sufficient**.

M. Mai *et. al.*, PRD 86 076001 (2012)
M. Mai *et. al.*, PRD 86 096002 (2012)

Local stability condition

- Strong force fields

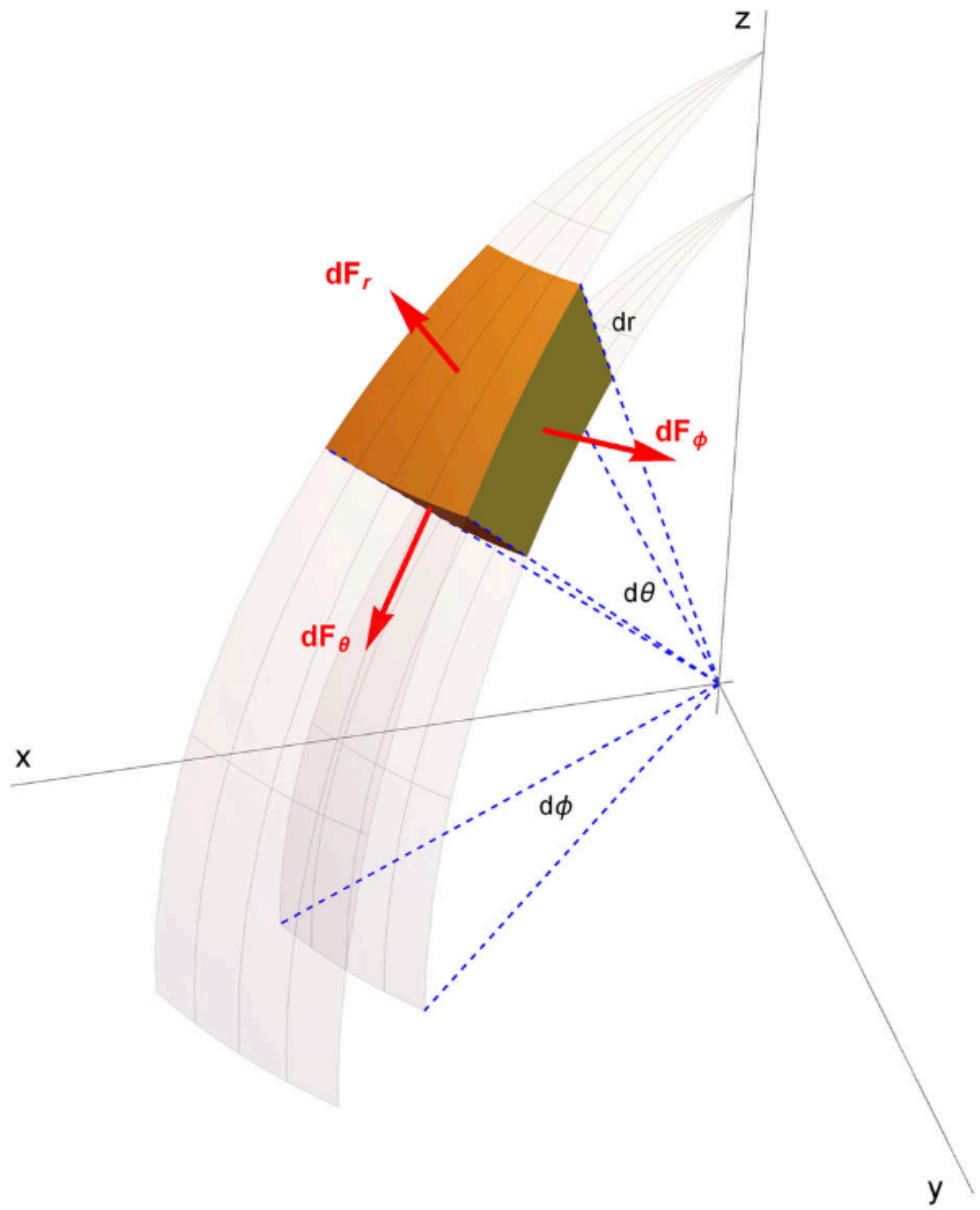
$$dF^i_{(r,\theta,\phi)} = T^{ij} dS_{(r,\theta,\phi)} \boldsymbol{e}^j_{(r,\theta,\phi)}$$

$$\rho^r_p(r) = \frac{dF_r}{dS_r} = \frac{2}{3}\rho_s(r) + \rho_p(r)$$

$$\rho^\theta_p(r) = \frac{dF_\theta}{dS_\theta} = -\frac{1}{3}\rho_s(r) + \rho_p(r)$$

$$\rho^\phi_p(r) = \frac{dF_\phi}{dS_\phi} = -\frac{1}{3}\rho_s(r) + \rho_p(r)$$

J. Y. Kim *et al.*, PRD 103 (2021)



- Outward direction

$$\rho^r_p(r) > 0$$

Chiral quark-soliton model

Chiral quark-soliton model

- Effective chiral Lagrangian (massive Goldstone bosons)

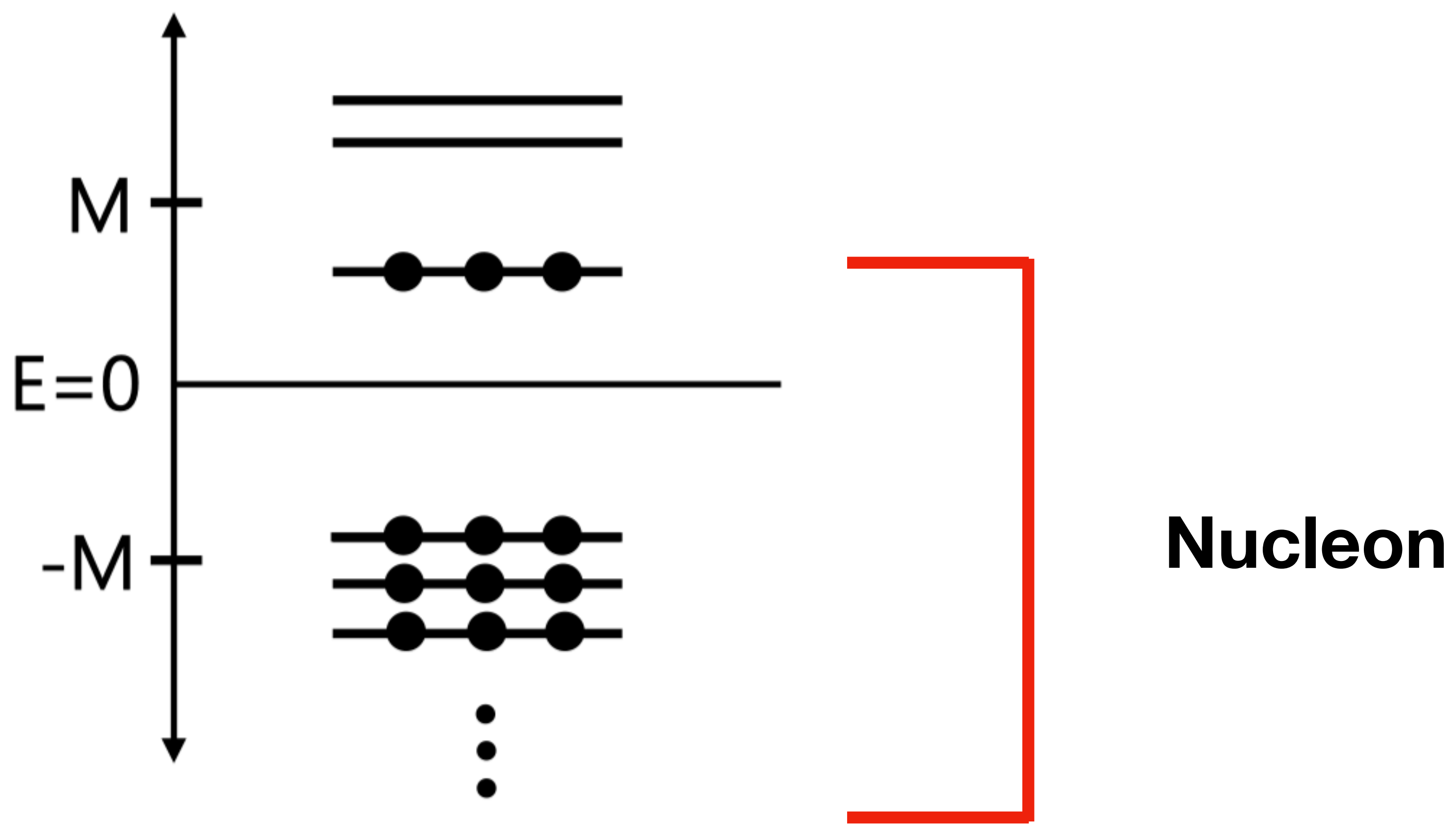
$$\mathcal{L} = \bar{\psi}[i\partial\!\!\!/ - \hat{m} - \mathrm{M}U_5]\psi$$

$$U_5 = e^{i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}} = U\frac{1+\gamma_5}{2} + U^\dagger\frac{1-\gamma_5}{2}$$

$$U = e^{i\boldsymbol{\tau}\cdot\boldsymbol{\pi}}$$

- Mean field approximation

$$\frac{\delta M_N[U]}{\delta U} = 0$$



Form factors and densities

- Form factors and its densities

$$M_2(t) - \frac{t}{5M_B^2} d_1(t) = \frac{1}{M_B} \int d^3r j_0(r\sqrt{-t}) \rho_E(r)$$

$$J(t) = 3 \int d^3r \frac{j_1(r\sqrt{-t})}{r\sqrt{-t}} \rho_J(r)$$

$$d_1(t) = \frac{5M_B}{t} \int d^3r j_2(r\sqrt{-t}) \rho_s(r)$$

- Corresponding densities

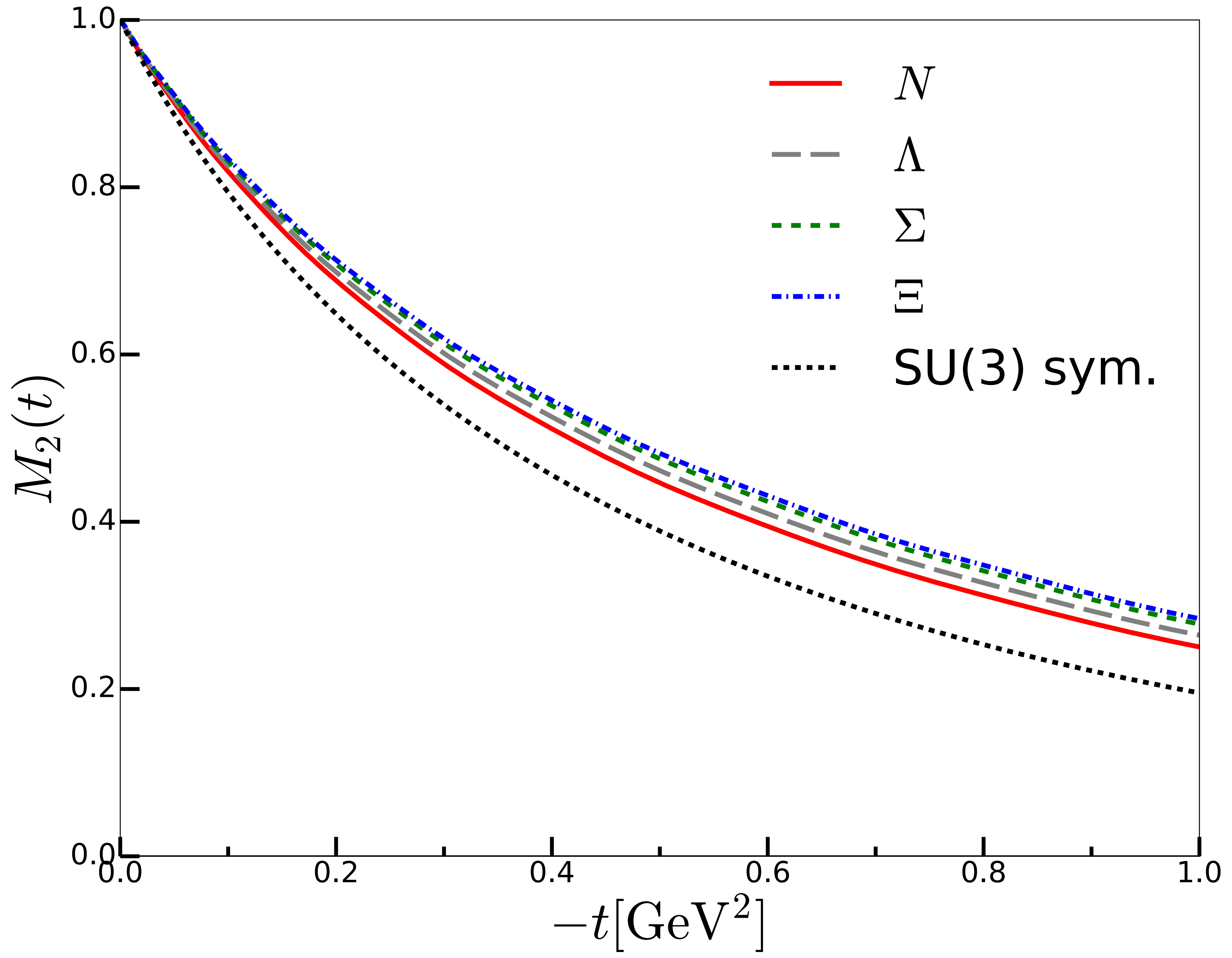
$$\rho_E(r) = \frac{T^{00}(\boldsymbol{r})}{2M_B}$$

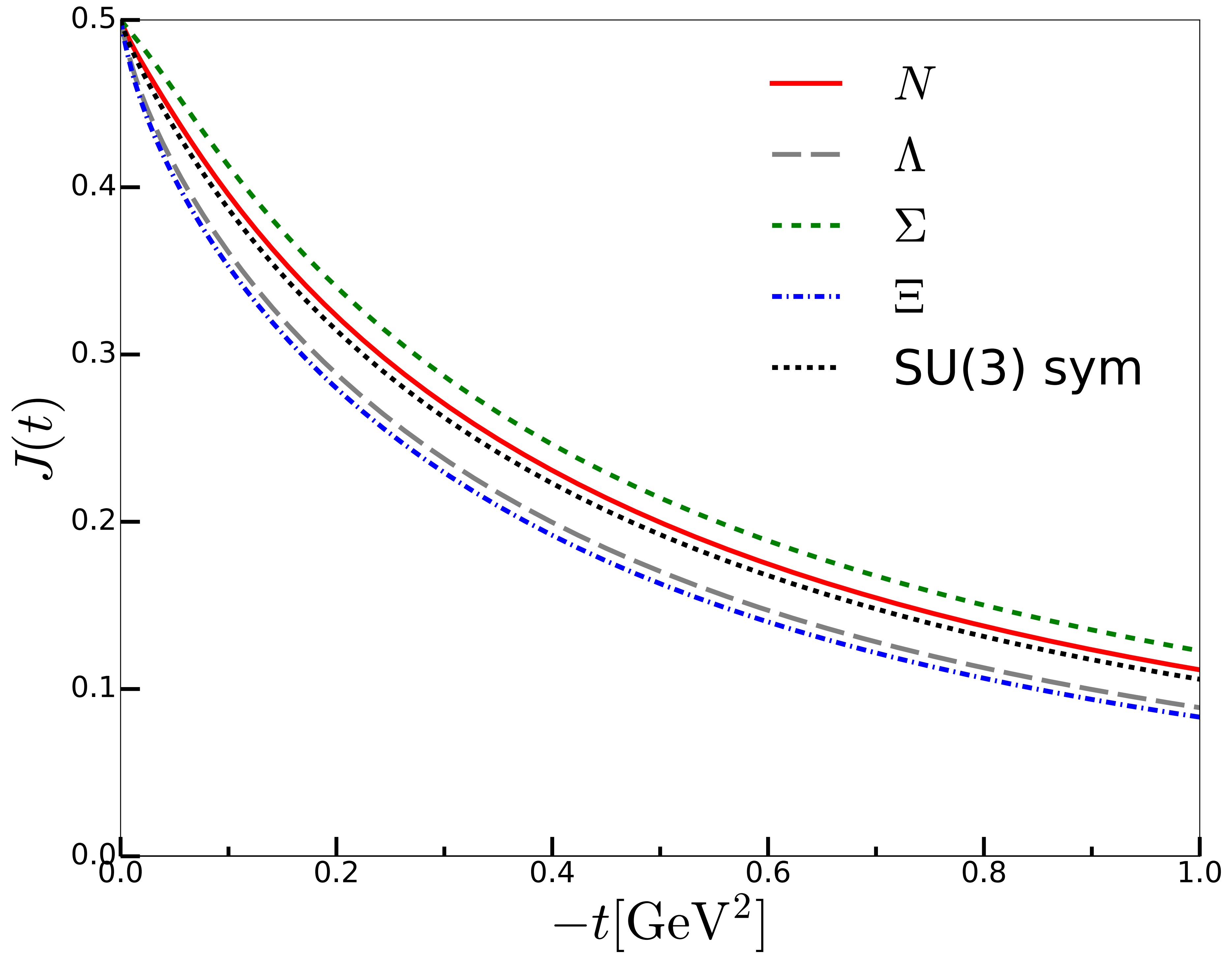
$$\rho_J(r) = -\frac{1}{6M_B} \varepsilon^{kl3} \hat{r}^l T^{0k}(\boldsymbol{r})$$

$$\rho_s(r) = \frac{3}{4M_B} Y_2^{ik}(\Omega_r) T^{ik}(\boldsymbol{r})$$

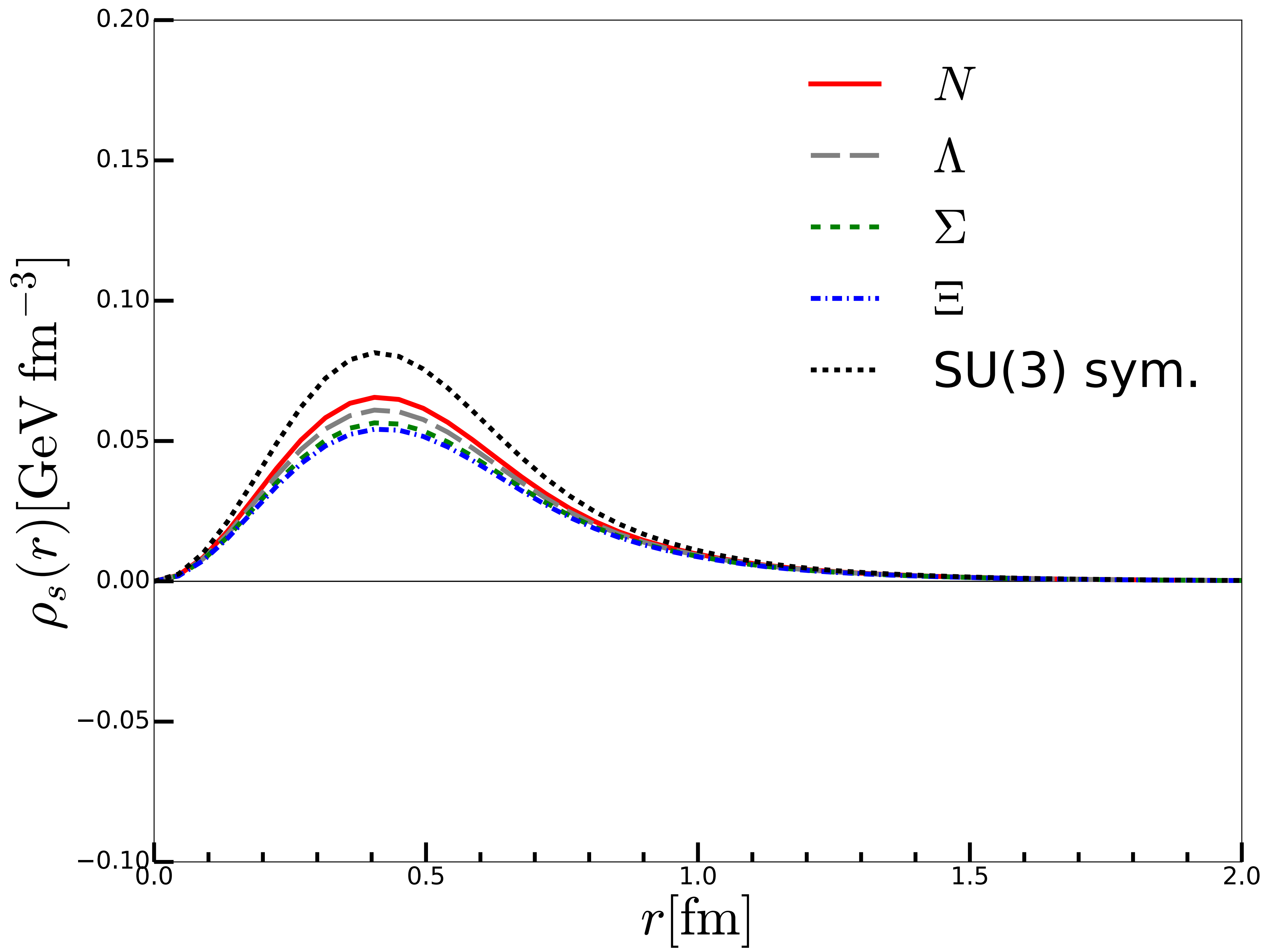
Results

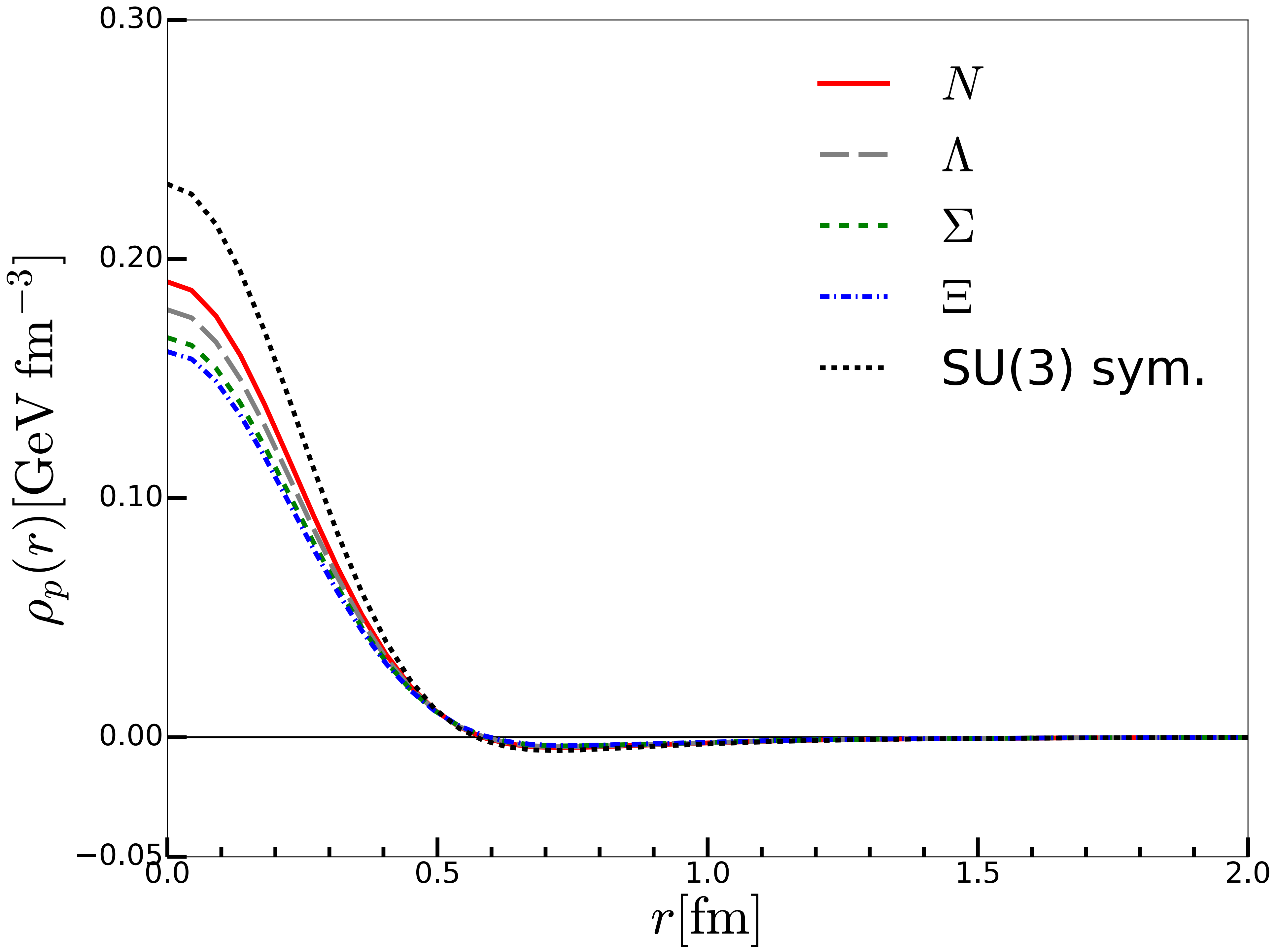
Energy & angular momentum form factors





Shear forces & Pressure distribution

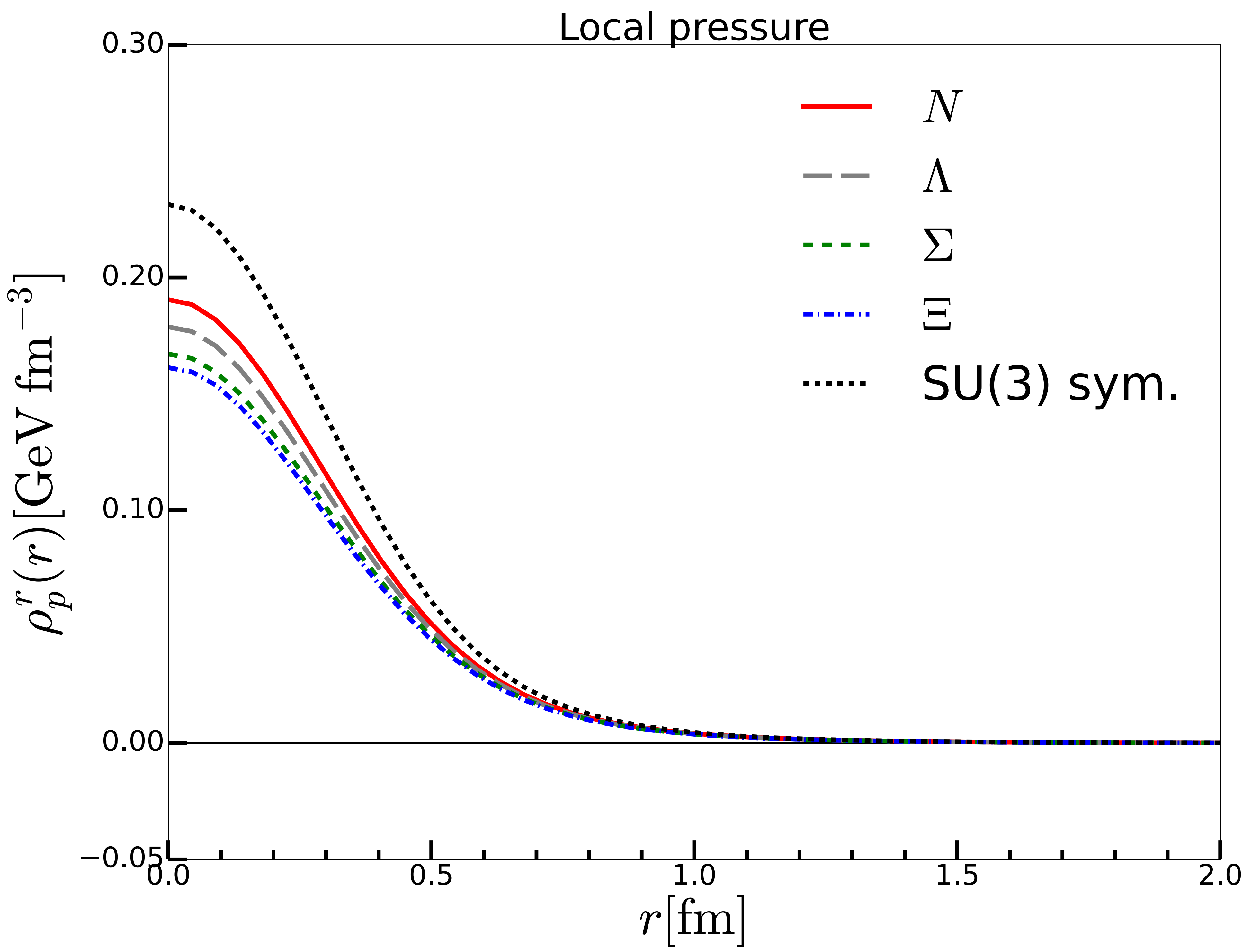


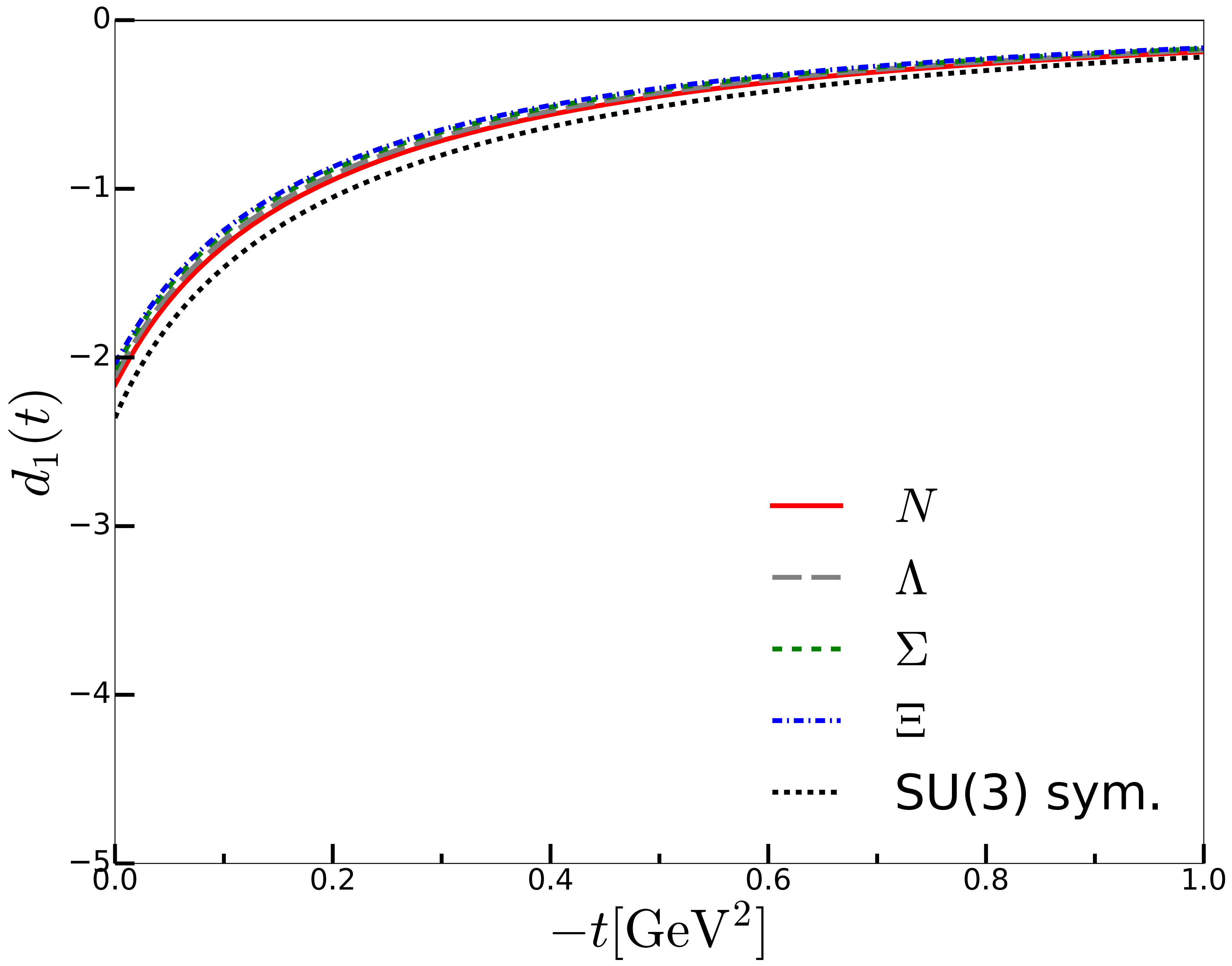


$$\partial_i T^{ij} = 0 \longrightarrow \frac{2}{3}\frac{\partial \rho_s(r)}{\partial r} + \frac{2\rho_s(r)}{r} + \frac{\partial \rho_p(r)}{\partial r} = 0$$

$$\int_0^\infty dr\ r^2 \rho_p(r) = 0$$

Local stability condition & d_1 form factor





$$\rho_p^r(r) > 0$$

Numerical table

	$\rho_E(0)$ GeV/fm^3	$\langle r_E^2 \rangle$ fm^2	$\langle r_J^2 \rangle$ fm^2	$\langle r^2 \rangle_{\mathrm{mech}}$ fm^2	$\rho_p(0)$ GeV/fm^3	r_0 fm^2	$r_{p,\mathrm{min}}$ fm^2	$r_{s,\mathrm{max}}$ fm^2	d_1
N	2.47	0.49	1.26	0.67	0.19	0.583	0.728	0.415	−2.16
Λ	2.70	0.44	2.53	0.68	0.18	0.588	0.733	0.417	−2.11
Σ	2.92	0.40	0.62	0.69	0.17	0.593	0.737	0.419	−2.07
Ξ	3.03	0.37	2.84	0.70	0.16	0.595	0.740	0.420	−2.04
SU(3) sym.	1.69	0.67	1.57	0.63	0.23	0.571	0.714	0.411	−2.36

Summary

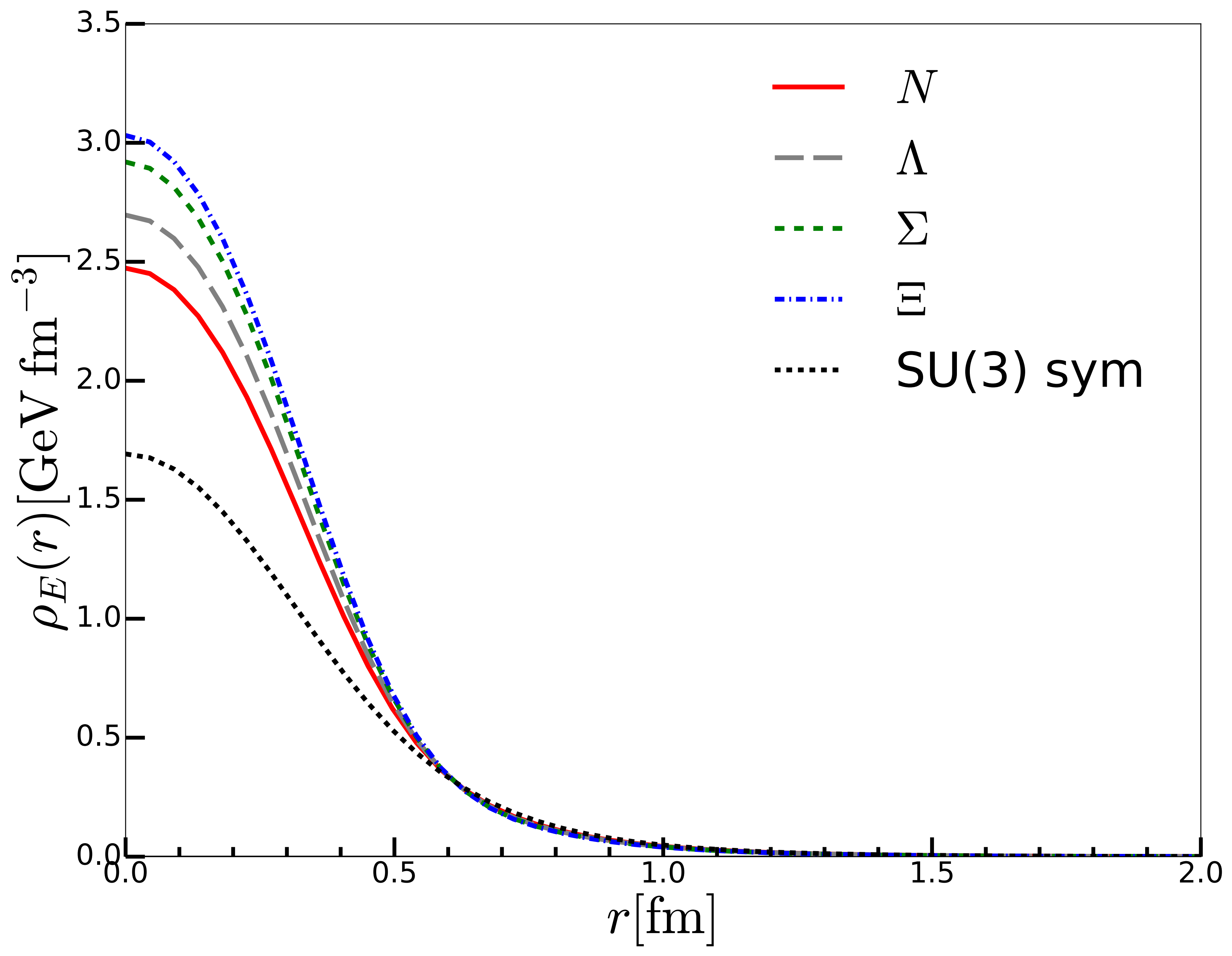
Summary

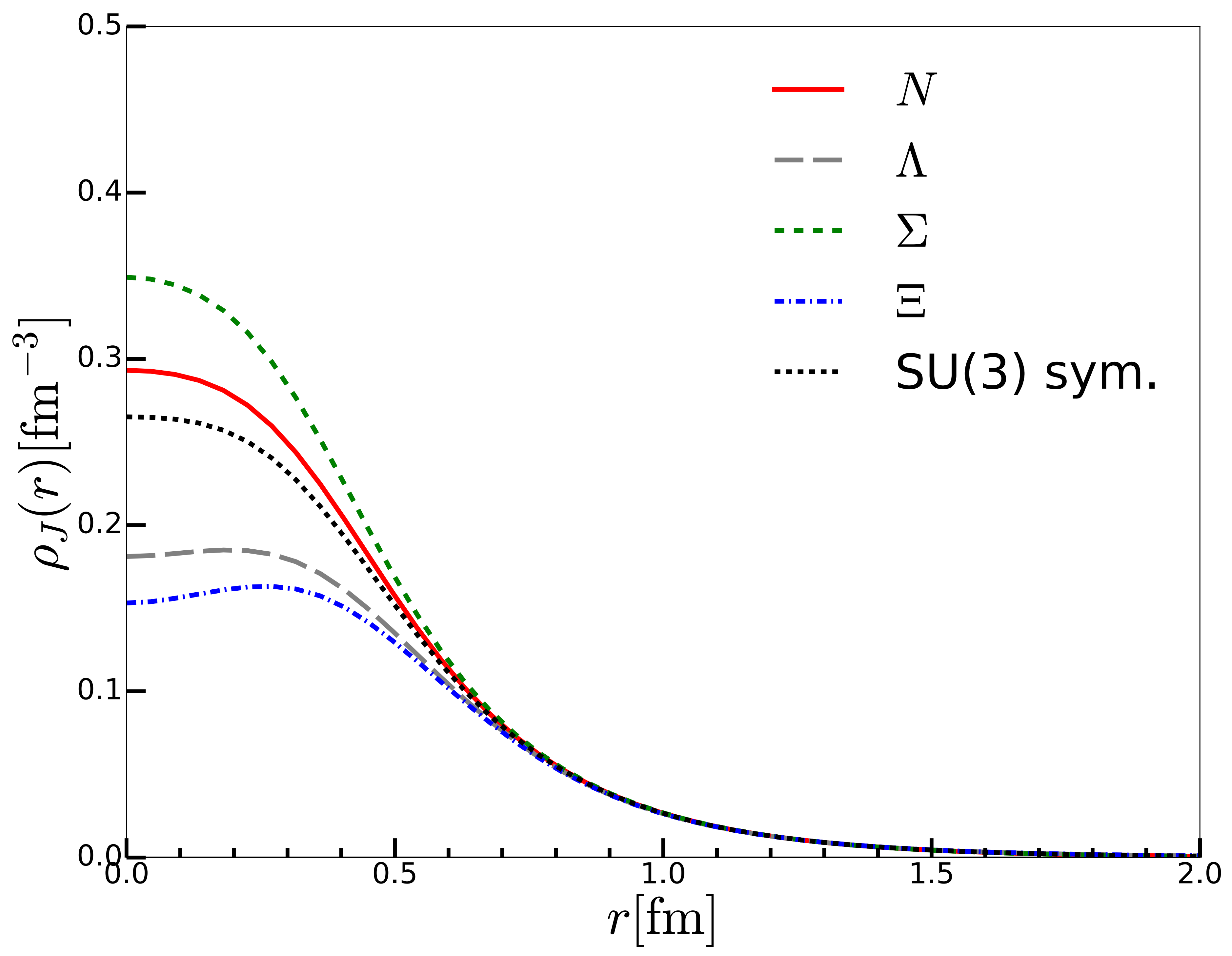
- We investigated the energy-momentum tensor form factors of baryon octet by using the chiral quark-soliton model.
- $d_1(t)$ form factor provides information on the stability conditions.
- We showed the baryon octet satisfies the stability conditions despite the flavor SU(3) symmetry breaking.

Thank you for listening

Appendix

Energy & Angular momentum distribution





Bi-local matrix elements

- Bi-local current

$$\langle B', p' | \psi^\dagger(z_1) \Gamma \psi(z_2) | B, p \rangle \qquad \Gamma = \gamma_0 \gamma^\mu$$

- Baryonic current

$$|B, p\rangle = N(p) \lim_{x_4 \to \infty} e^{ip_4 x_4} \int d^3x \; e^{i\boldsymbol{p}\cdot\boldsymbol{x}} J_B^\dagger(x) \, |0\rangle$$

$$\langle B, p| = N^*(p) \lim_{x_4 \to \infty} e^{-ip_4 y_4} \int d^3y \; e^{-i\boldsymbol{p}\cdot\boldsymbol{y}} \, \langle 0| \, J_B(y)$$

- Ioffe-type current

$$J_B(x) = \frac{1}{N_c!} \varepsilon^{\alpha_1 \ldots \alpha_{N_c}} \Gamma^{f_1 \ldots f_{N_c}} \psi_{\alpha_1 f_1}(x) \ldots \psi_{\alpha_{N_c} f_{N_c}}(x)$$

$$J_B^\dagger(x) = \frac{1}{N_c!} \varepsilon^{\beta_1 \ldots \beta_{N_c}} (\Gamma^{g_1 \ldots g_{N_c}})^* (-i\psi(x)^\dagger \gamma_4)_{\beta_1 g_1} \ldots (-i\psi(x)^\dagger \gamma_4)_{\beta_{N_c} g_{N_c}}(x)$$

Bi-local matrix elements

$$\langle B', p' | \psi^\dagger(z_1)\Gamma\psi(z_2) | B, p\rangle = N^*(p')N(p) \lim_{y_4\to\infty} \lim_{x_4\to\infty} \exp\left(-iy_4p'_4 + ix_4p_4\right)$$

$$\int d^3y d^3x \exp(-i\boldsymbol{p}' \cdot \boldsymbol{y} + i\boldsymbol{p}\cdot\boldsymbol{x}) \langle 0|T\{J_{B'}(y)\psi^\dagger(z_1)\Gamma\psi(z_2)J_B^\dagger(x)\}|0\rangle$$

$$\langle 0|\, T\{J_{B'}(y)\psi^\dagger(z_1)\Gamma\psi(z_2)J_B^\dagger(x)\}\, |0\rangle$$

$$= \frac{1}{Z_0}\int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\psi^\dagger J_{B'}(y)[-i\psi^\dagger(z_1)\gamma_4]\Gamma\psi(z_2)J_B^\dagger(x)\exp\left[\int d^4z\psi^\dagger(z)(i\partial\!\!\!/ + iMU_5 + i\hat{m})\psi(z)\right]$$

Bi-local matrix elements

Greek letter: Color
English letter: Flavor

$$\langle 0|\, T\{J_{B'}(y)\psi^\dagger(z_1)\Gamma\psi(z_2)J_B^\dagger(x)\}\,|0\rangle = \frac{1}{Z_0}\int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\psi^\dagger \frac{\varepsilon^{\{\alpha\}}\varepsilon^{\{\beta\}*}}{(N_c!)^2}\Gamma^{\{f\}}\Gamma^{\{g\}*}$$
$$\times\, T\{\psi_{\alpha_1 f_1}(y)\cdots\psi_{\alpha_{N_c} f_{N_c}}(y)[-i\psi^\dagger(z_1)\gamma_4]_{\gamma\eta m}\Gamma^{mn}_{\gamma\eta}\psi_{\eta n}(z_2)[-i\psi(x)\gamma_4]_{\beta_1 g_1}\cdots[-i\psi(x)\gamma_4]_{\beta_{N_c} g_{N_c}}\}$$
$$\times \exp\left[\int d^4z\psi^\dagger(z)(i\partial\!\!\!/ + iMU_5 + i\hat{m})\psi(z)\right]$$

- Wick's theorem

First contraction

$$T\{\psi_{\alpha_k f_k}(y)[-i\psi^\dagger(z_1)\gamma_4]_{\gamma m}\Gamma^{mn}_{\gamma\eta}\psi^\dagger_{\eta n}(z_2)[-i\psi^\dagger(x)\gamma_4]_{\beta_{k'} g_{k'}}\}$$
$$= -T\{\psi_{\alpha_k f_k}(y){\color{red}\psi^\dagger_{\eta n}(z_2)}\Gamma^{mn}_{\gamma\eta}{\color{red}[-i\psi^\dagger(z_1)\gamma_4]_{\gamma m}}[-i\psi^\dagger(x)\gamma_4]_{\beta_{k'} g_{k'}}\}$$
$$= -\psi_{\alpha_k f_k}(y)\psi^\dagger_{\eta n}(z_2)\Gamma^{mn}_{\gamma\eta}[-i\psi^\dagger(z_1)\gamma_4]_{\gamma m}[-i\psi^\dagger(x)\gamma_4]_{\beta_{k'} g_{k'}}$$
$$= -(-1)^2 G_{f_k g_k}(y-x)\delta_{\alpha_k\beta_{k'}}\Gamma^{mn}_{\gamma\eta}G_{nm}(z_2-z_1)\delta_{\eta\gamma}$$

The other contraction

$$T\{\psi_{\alpha_k f_k}(y)[-i\psi^\dagger(z_1)\gamma_4]_{\gamma m}\Gamma^{mn}_{\gamma\eta}\psi^\dagger_{\eta n}(z_2)[-i\psi^\dagger(x)\gamma_4]_{\beta_{k'} g_{k'}}\}$$
$$= \psi_{\alpha_k f_k}(y)[-i\psi^\dagger(z_1)\gamma_4]_{\gamma m}\Gamma^{mn}_{\gamma\eta}\psi^\dagger_{\eta n}(z_2)[-i\psi^\dagger(x)\gamma_4]_{\beta_{k'} g_{k'}}$$
$$= (-1)^2 G_{f_k m}(y-z_1)\delta_{\alpha_k\gamma}\Gamma^{mn}_{\gamma\eta}G_{n g_{k'}}(z_2-x)\delta_{\eta\beta_{k'}}$$

Bi-local matrix elements

$$\Gamma^{mn}_{\gamma\eta} = \Gamma^{mn} N_c \delta_{\gamma\eta}$$

$$\langle 0| T\{J_{B'}(y)\psi^\dagger(z_1)\Gamma\psi(z_2)J_B^\dagger(x)\}|0\rangle = \frac{1}{Z_0}\int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\psi^\dagger N_c \frac{\varepsilon^{\{\alpha\}}\varepsilon^{\{\beta\}*}}{(N_c!)^2}\Gamma^{\{f\}}\Gamma^{\{g\}*}$$

$$\times\Bigg[\sum_{\theta\in S_{N_c}}(-1)^{\pi(\theta)}\Bigg[\Big\{G_{f_1g_1}(y-x)\delta_{\alpha_1\beta_{\theta_1}}\cdots G_{f_{N_c-1}g_{\theta_{N_c-1}}}(y-x)\delta_{\alpha_{N_c-1}\beta_{\theta_{N_c-1}}}\Big\}$$

$$\times\Big\{-G_{f_kg_{\theta_{N_c}}}(y-x)\mathrm{tr}_{\gamma f}\big[\Gamma G(z_2-z_1)\big]\delta_{\alpha_k\beta_{\theta_{N_c}}}+G_{f_km}(y-z_1)\Gamma^{mn}G_{ng_{\theta_{N_c}}}(z_2-x)\delta_{\alpha_k\beta_{\theta_{N_c}}}\Big\}\Bigg]\Bigg]$$

$$\times\exp\left[\int d^4z\psi^\dagger(z)(i\partial\!\!\!/ + iMU_5 + i\hat{m})\psi(z)\right]$$

- Spectral representation

$$G(y,x) = A(y_4)\langle \boldsymbol{y}-\boldsymbol{Z}|\frac{i\gamma_4}{D_E(U_c)}|\boldsymbol{x}-\boldsymbol{Z}\rangle A^\dagger(x_4)$$

$$= \left[\theta(y_4-x_4)\sum_{E_n>0} - \theta(x_4-y_4)\sum_{E_n<0}\right]e^{-E_n(y_4-x_4)}A(y_4)\psi_n(\boldsymbol{y})\psi_n^\dagger(\boldsymbol{x})A(x_4)$$

Bi-local matrix elements

$$\langle 0 | T\{J_{B'}(y)\psi^\dagger(z_1)\Gamma\psi(z_2)J_B^\dagger(x)\} |0\rangle = \frac{1}{Z_0}\int \mathcal{D}U \exp\left[N_c \mathrm{Trln}(i\partial\!\!\!/ + iMU_5 + i\hat{m})\right]$$

$$\times N_c \prod_{i=2}^{N_c} G_{f_i g_i}(y-x)\left[G_{f_1 m}(y-z_1)\Gamma^{mn}G_{ng_1}(z_2-x) - G_{f_1 g_1}(y-x)\mathrm{tr}_{\gamma f}\left[\Gamma G(z_2-z_1)\right]\right]$$

$$= N_c\mathcal{K}_{val} + N_c\mathcal{K}_{sea}$$

$$\mathcal{K}_{val} = \frac{1}{Z_0}\int \mathcal{D}U e^{-S_{eff}} \prod_{i=2}^{N_c} G_{f_i g_i}(y-x)\left[G(y-z_1)\Gamma G(z_2-x)\right]_{f_1 g_1}$$

$$\mathcal{K}_{sea} = \frac{1}{Z_0}\int \mathcal{D}U e^{-S_{eff}} \prod_{i=1}^{N_c} G_{f_i g_i}(y-x)\left[-\mathrm{tr}_{\gamma f}[\Gamma G(z_2-z_1)\right]$$

Bi-local matrix elements

- Valence part

$$\mathcal{K}_{val} = \frac{1}{Z_0}\int \mathcal{D}U e^{-S_{eff}} \prod_{i=2}^{N_c} G_{f_i g_i}(y-x)\Big[G^{(\Omega^0\delta m^0)}(y-z_1)\Gamma G^{(\Omega^0\delta m^0)}(z_2-x)$$
$$+ G^{(\Omega^0\delta m^0)}(y-z_1)\Gamma G^{(\Omega^1\delta m^0)}(z_2-x) + G^{(\Omega^1\delta m^0)}(y-z_1)\Gamma G^{(\Omega^0\delta m^0)}(z_2-x)$$
$$+ G^{(\Omega^0\delta m^0)}(y-z_1)\Gamma G^{(\Omega^0\delta m^1)}(z_2-x) + G^{(\Omega^0\delta m^1)}(y-z_1)\Gamma G^{(\Omega^0\delta m^0)}(z_2-x)\Big]_{f_1 g_1}$$

- Sea part

$$\mathcal{K}_{sea} = \frac{1}{Z_0}\int \mathcal{D}U e^{-S_{eff}} \prod_{i=1}^{N_c} G_{f_i g_i}(y-x)\Big[-\mathrm{tr}_{\gamma f}\big[\Gamma G^{(\Omega^0\delta m^0)}(z_2-z_1)$$
$$- G^{(\Omega^1\delta m^0)}(z_2-z_1) - G^{(\Omega^0\delta m^1)}(z_2-z_1)\big]\Big]$$

Bi-local matrix elements

Saddle-point approximation

$$\mathcal{D}U \to d\boldsymbol{Z}\mathcal{D}A$$

$$\langle B', p' |\, \bar{\psi}(z_1)\gamma^{\mu}\psi(z_2)\,| B, p\rangle = \int d\boldsymbol{Z} \int \mathcal{D}A \; e^{-S_{eff}}$$

$$\times\, N^*(p') \lim_{y_4\to\infty} e^{-iy_4 p'_4} \int \; d^3y e^{-i\boldsymbol{p}\,'\cdot\boldsymbol{y}} \Gamma^{\{f\}} \prod_{i=1}^{N_c} [A(y_4)\psi(\boldsymbol{y}-\boldsymbol{Z})]_{f_i}$$

$$\times\, \mathcal{F}(\boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{Z})$$

$$N(p) \lim_{x_4\to\infty} e^{+ix_4 p_4} \int \; d^3x e^{+i\boldsymbol{p}\cdot\boldsymbol{x}} \Gamma^{\{g\}*} \prod_{i=1}^{N_c} [\psi^\dagger(\boldsymbol{x}-\boldsymbol{Z})A^\dagger(x_4)]_{g_i}$$

Bi-local matrix elements

Transformation,

$$\boldsymbol{y} - \boldsymbol{Z} \to \boldsymbol{y},\ \boldsymbol{x} - \boldsymbol{Z} \to \boldsymbol{x}$$

$$\langle B', p' |\, \bar{\psi}(z_1)\gamma^\mu \psi(z_2)\, | B, p \rangle$$

General form factors

$$= \int d\boldsymbol{Z} e^{-i\boldsymbol{q}\cdot\boldsymbol{Z}} \int \mathcal{D}A\ \Psi^\dagger_{B'}(A)\mathcal{F}(\boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{Z})\Psi_B(A) e^{-S_{eff}}$$

Collective baryon wave functions

$$\Psi^\dagger_B(A) = N^*(p') \lim_{y_4 \to \infty} e^{-iy_4 p'_4} \int d^3y e^{-i\boldsymbol{p}' \cdot \boldsymbol{y}} \Gamma^{\{f\}} \prod_{i=1}^{N_c} [A(y_4)\psi(\boldsymbol{y})]_{f_i}$$

$$\Psi_B(A) = N(p) \lim_{x_4 \to \infty} e^{+ix_4 p_4} \int d^3x e^{+i\boldsymbol{p}\cdot\boldsymbol{x}} \Gamma^{\{g\}*} \prod_{i=1}^{N_c} [\psi^\dagger(\boldsymbol{x})A^\dagger(x_4)]_{g_i}$$

Bi-local matrix elements

For example…

$$
\begin{aligned}
\mathcal{F}^{(\Omega^1\delta m^0)}(\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{Z}) = -\int d\omega_4 \Bigg[& e^{-E_v(\omega_4 - z_4^1)}\Big[\theta(z_4^2-\omega_4)\sum_{E_n>0} - \theta(\omega_4 - z_4^2)\sum_{E_n<0}\Big] e^{-E_n(z_4^2-\omega_4)} \\
& \times \psi_v^\dagger(\boldsymbol{z}_1-\boldsymbol{Z})A^\dagger(z_4^1)\Gamma A(z_4^2)\psi_n(\boldsymbol{z}_2-\boldsymbol{Z})\,\langle n|\, i\Omega(\omega_4)\,|v\rangle \\
& + e^{-E_v(z_4^2-\omega_4)}\Big[\theta(\omega_4 - z_4^1)\sum_{E_n>0} - \theta(z_4^1-\omega_4)\sum_{E_n<0}\Big] e^{-E_n(\omega_4 - z_4^1)} \\
& \times \langle v|\, i\Omega(\omega_4)\,|n\rangle\, \psi_n^\dagger(\boldsymbol{z}_1-\boldsymbol{Z})A^\dagger(z_4^1)\Gamma A(z_4^2)\psi_v(\boldsymbol{z}_2-\boldsymbol{Z})\Bigg] \\
& + \int d\omega_4 \Bigg[\Big[\theta(z_4^2-\omega_4)\sum_{E_n>0} - \theta(\omega_4 - z_4^2)\sum_{E_n<0}\Big] e^{-E_{n_1}(z_4^2-\omega_4)} \\
& \times \Big[\theta(\omega_4 - z_4^1)\sum_{E_m>0} - \theta(z_4^1-\omega_4)\sum_{E_m<0}\Big] e^{-E_m(\omega_4 - z_4^1)}\,\langle n|\, i\Omega(\omega_4)\,|m\rangle \\
& \psi_{ma}^\dagger(\boldsymbol{z}_1-\boldsymbol{Z})[A^\dagger(z_4^1)\Gamma A(z_4^2)]_{ab}\psi_{nb}(\boldsymbol{z}_2-\boldsymbol{Z})\Bigg]
\end{aligned}
$$

Bi-local matrix elements

$$\mathcal{F}^{(\Omega^1\delta m^0)}(\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{Z}) = \int d\omega_4 \Big[\theta(z_4^2-\omega_4)\theta(\omega_4-z_4^1)\sum_{\substack{n=non\\m=non}} T\big\{\Omega^\alpha(\omega_4)A^\dagger_{ab}(z_4^1)A_{cd}(z_4^2)\big\}$$

$$-\,\theta(z_4^2-\omega_4)\theta(z_4^1-\omega_4)\sum_{\substack{n=non\\m=occ}} T\big\{\Omega^\alpha(\omega_4)A^\dagger_{ab}(z_4^1)A_{cd}(z_4^2)\big\}$$

$$-\,\theta(\omega_4-z_4^2)\theta(\omega_4-z_4^1)\sum_{\substack{n=occ\\m=non}} T\big\{\Omega^\alpha(\omega_4)A^\dagger_{ab}(z_4^1)A_{cd}(z_4^2)\big\}$$

$$+\,\theta(\omega_4-z_4^2)\theta(z_4^1-\omega_4)\sum_{\substack{n=occ\\m=occ}} T\big\{\Omega^\alpha(\omega_4)A^\dagger_{ab}(z_4^1)A_{cd}(z_4^2)\big\}\Big]$$

$$\times\, e^{-\omega_4(E_m-E_n)}e^{E_m z_4^1-E_n z_4^2}\langle n|\,\frac{1}{2}\lambda^\alpha\,|m\rangle\,\psi^\dagger_{ma}(\boldsymbol{z}_1-\boldsymbol{Z})\Gamma_{bc}\psi_{nd}(\boldsymbol{z}_2-\boldsymbol{Z})$$

Bi-local matrix elements

$$
\begin{aligned}
\mathcal{F}^{(\Omega^1\delta m^0)}(\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{Z}) = \int d\omega_4 \Bigg[& + \theta(z_4^2-\omega_4)\theta(\omega_4-z_4^1) \sum_{\substack{n=non\\ m=non}} A_{cd}\Omega^\alpha A^\dagger_{ab} \\
& - \theta(z_4^1-z_4^2)\theta(z_4^2-\omega_4) \sum_{\substack{n=non\\ m=occ}} A^\dagger_{ab}A_{cd}\Omega^\alpha \\
& - \theta(z_4^2-z_4^1)\theta(z_4^1-\omega_4) \sum_{\substack{n=non\\ m=occ}} A_{cd}A^\dagger_{ab}\Omega^\alpha \\
& - \theta(\omega_4-z_4^1)\theta(z_4^1-z_4^2) \sum_{\substack{n=occ\\ m=non}} \Omega^\alpha A^\dagger_{ab}A_{cd} \\
& - \theta(\omega_4-z_4^2)\theta(z_4^2-z_4^1) \sum_{\substack{n=occ\\ m=non}} \Omega^\alpha A_{cd}A^\dagger_{ab} \\
& + \theta(\omega_4-z_4^2)\theta(z_4^1-\omega_4) \sum_{\substack{n=occ\\ m=occ}} A^\dagger_{ab}\Omega^\alpha A_{cd} \Bigg] \\
& \times e^{-\omega_4(E_m-E_n)} e^{E_m z_4^1 - E_n z_4^2} \langle n| \frac{1}{2}\lambda^\alpha |m\rangle \psi^\dagger_{ma}(\boldsymbol{z}_1-\boldsymbol{Z})\Gamma_{bc}\psi_{nd}(\boldsymbol{z}_2-\boldsymbol{Z})
\end{aligned}
$$

$$
\begin{aligned}
&\langle B'(p',J')|\,\psi^\dagger(z_1)\Gamma_\mu\psi(z_2)\,|B(p,J)\rangle = 2M_B\int d^3r e^{i\boldsymbol{\Delta}\cdot\boldsymbol{r}}N_c\Bigg[\delta_{J_3'J_3}\sum_{n=occ}e^{-E_n(z_4^2-z_4^1)}\psi_n^\dagger(\boldsymbol{z}_1+\boldsymbol{r})\Gamma_\mu\psi_n(\boldsymbol{z}_2+\boldsymbol{r})\\
&+\frac{1}{2I_1}\langle J_i\rangle\Bigg[-\sum_{\substack{n=non\\m=occ}}\frac{e^{-E_m(z_4^2-z_4^1)}}{E_n-E_m}+\sum_{\substack{n=occ\\m=non}}\frac{e^{-E_n(z_4^2-z_4^1)}}{E_n-E_m}+\sum_{\substack{n=occ\\m=occ}}\frac{e^{-E_n(z_4^2-z_4^1)}}{E_n-E_m}-\sum_{\substack{n=occ\\m=occ}}\frac{e^{-E_m(z_4^2-z_4^1)}}{E_n-E_m}\Bigg]\\
&\times\langle n|\,\tau^i\,|m\rangle\,\psi_m^\dagger(\boldsymbol{z}_1+\boldsymbol{r})\Gamma_\mu\psi_n(\boldsymbol{z}_2+\boldsymbol{r})\\
&-M_8\langle D_{8i}\rangle\frac{K_1}{I_1}\Bigg[-\sum_{\substack{n=non\\m=occ}}\frac{e^{-E_m(z_4^2-z_4^1)}}{E_n-E_m}+\sum_{\substack{n=occ\\m=non}}\frac{e^{-E_n(z_4^2-z_4^1)}}{E_n-E_m}+\sum_{\substack{n=occ\\m=occ}}\frac{e^{-E_n(z_4^2-z_4^1)}}{E_n-E_m}-\sum_{\substack{n=occ\\m=occ}}\frac{e^{-E_m(z_4^2-z_4^1)}}{E_n-E_m}\Bigg]\\
&\times\langle n|\,\tau^i\,|m\rangle\,\psi_m^\dagger(\boldsymbol{z}_1+\boldsymbol{r})\Gamma_\mu\psi_n(\boldsymbol{z}_2+\boldsymbol{r})\\
&+\left[M_1\delta_{J_3'J_3}+\frac{1}{\sqrt{3}}M_8\langle D_{88}\rangle\right]\\
&\times\Bigg[-\sum_{\substack{n=non\\m=occ}}\frac{e^{-E_m(z_4^2-z_4^1)}}{E_n-E_m}+\sum_{\substack{n=occ\\m=non}}\frac{e^{-E_n(z_4^2-z_4^1)}}{E_n-E_m}+\sum_{\substack{n=occ\\m=occ}}\frac{e^{-E_n(z_4^2-z_4^1)}}{E_n-E_m}-\sum_{\substack{n=occ\\m=occ}}\frac{e^{-E_m(z_4^2-z_4^1)}}{E_n-E_m}\Bigg]\\
&\times\langle n|\,\gamma_4\,|m\rangle\,\psi_m^\dagger(\boldsymbol{z}_1+\boldsymbol{r})\Gamma_\mu\psi_n(\boldsymbol{z}_2+\boldsymbol{r})\\
&+M_8\langle D_{8i}\rangle\Bigg[-\sum_{\substack{n=non\\m=occ}}\frac{e^{-E_m(z_4^2-z_4^1)}}{E_n-E_m}+\sum_{\substack{n=occ\\m=non}}\frac{e^{-E_n(z_4^2-z_4^1)}}{E_n-E_m}+\sum_{\substack{n=occ\\m=occ}}\frac{e^{-E_n(z_4^2-z_4^1)}}{E_n-E_m}-\sum_{\substack{n=occ\\m=occ}}\frac{e^{-E_m(z_4^2-z_4^1)}}{E_n-E_m}\Bigg]\\
&\times\langle n|\,\gamma_4\tau^i\,|m\rangle\,\psi_m^\dagger(\boldsymbol{z}_1+\boldsymbol{r})\Gamma_\mu\psi_n(\boldsymbol{z}_2+\boldsymbol{r})\Bigg]
\end{aligned}
$$

D-functions

Baryon	Y	T	$D_{88}(A)$	$D_{83}(A)$
N	1	$\frac{1}{2}$	$\frac{3}{10}$	$-\frac{1}{10\sqrt{3}}$
Λ	0	0	$\frac{1}{10}$	$\frac{3}{10\sqrt{3}}$
Σ	0	0	$-\frac{1}{10}$	$-\frac{3}{10\sqrt{3}}$
Ξ	-1	$\frac{1}{2}$	$-\frac{1}{5}$	$\frac{4}{10\sqrt{3}}$