Inha HTG workshop: Modern issues in Hadronic Physics

Light quark distribution functions in a heavy baryon in the large N_c limit

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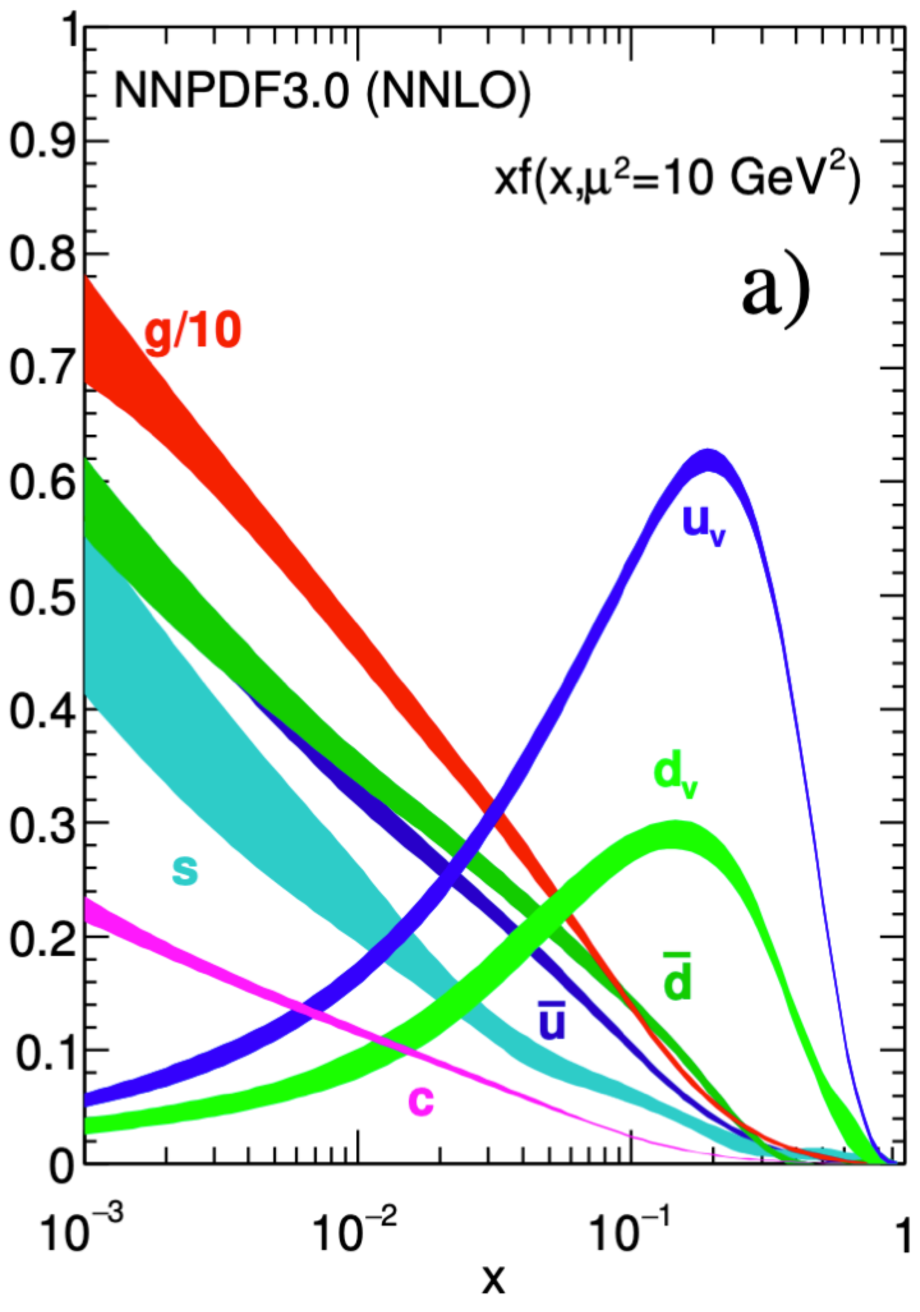
Introduction

Parton distribution functions (PDFs)

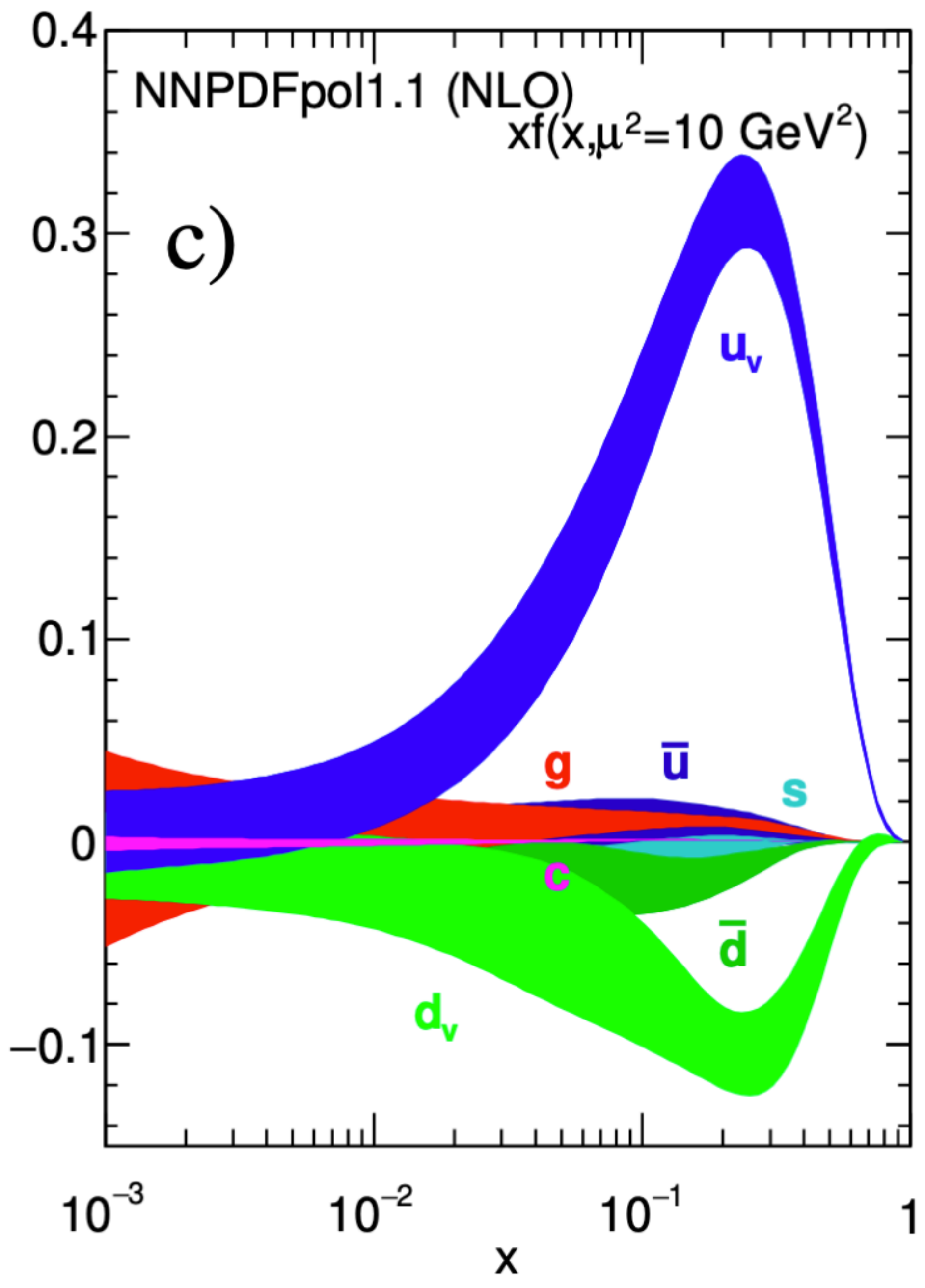
How partons (quarks and gluons) are distributed inside a hadron

Probability density (properly defined on the light-cone)

Proton, global analyses, plots from PDG 2021



R. D. Ball et al. (NNPDF), JHEP 04, 040 (2015)



E. R. Nocera et al. (NNPDF), Nucl. Phys. B887, 276 (2014)

Parton distribution functions (PDFs)

Universality

PDFs do not distinguish different types of reactions

eg. Deep inelastic scattering (ep), Drell-Yan process (pp)

Fitting model PDFs using various reactions (Global analysis)

Justification of factorization is essential but mostly assumed

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution (1970')

Perturbative evolution of PDFs

$$\frac{dq_i(x,\mu^2)}{\partial\mu^2} = P_{qq} \otimes q_i + P_{qg} \otimes g$$

Splitting functions P_{ij}: probability of perturbative emission of i from j

Theoretical understanding of PDFs

PDFs are non-perturbative!

- Direct computation (x-dependence) from QCD is not possible

Effective models (at low renormalization scale)

- provide initial conditions of the QCD evolution
- To understand the detailed mechanism in terms of the effective degrees of freedom
- Positivity, sum-rules, predictions…

Chiral quark-soliton model

[D. Diakonov, V. Y. Petrov, P. V. Pobylitsa, M. Polyakov, and C. Weiss, Nuclear Physics B 480, 341 (1996)]

- **quark and antiquark distribution at low renormalization scale, μ~600 MeV**
- **Positivity, sum-rules**
- Predictions: polarized antiquark flavor asymmetry

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Lattice QCD

- **Large Momentum Effective Theory (LaMET): quasi-PDFs** [Ji, Phys. Rev. Lett. 110, 262002 (2013)]

$$q(x,\mu,P^z) = \int \frac{dz}{4\pi} e^{-ixP^z z} \langle P|\bar{\psi}(0)\gamma^z \exp\left[-ig\int_0^z dz' A^z(z')\right]\psi(z)|P\rangle + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{(P^z)^2}, \frac{M_N^2}{(P^z)^2}\right)$$

$x \in (-\infty, +\infty)$

μ: renormalization scale

P_z: nucleon momentum

Theoretical understanding of PDFs

PDFs are non-perturbative!

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Effective models (at low renormalization scale)

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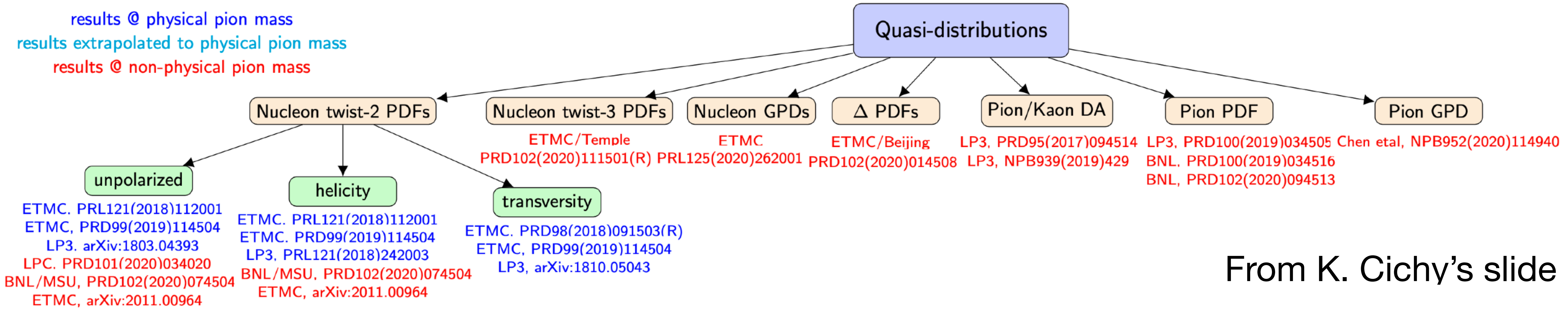


From K. Cichy's slide

Heavy baryon in the chiral quark-soliton model

Heavy quark symmetry →

Nc-1 chiral quark-soliton in the large Nc

+ Heavy quark in the heavy quark limit

Recent studies on

→ baryon mass spectrum [J.Y-. Kim H.-Ch. Kim, G.-S. Yang, PRD 2018]

→ EM ffs: good agreements with lattice calculations, Axial & Tensor [J.Y-. Kim H.-Ch. Kim, PRD 2018/EPJC 2019,2020]

→ EMT form factors [J.Y-. Kim, H.-Ch. Kim, M. Polyakov, HDS, PRD 2021]

Axial transitions, Jung-Min's talk

Light quark distribution functions in a heavy baryon

Momentum distribution of the light quarks in a heavy baryon vs. nucleon?

PDF studies for heavy flavor in heavy baryon/meson (heavyquark-diquark) [Guo, Thomas, Williams, PRD64 (2001)] [J. Lan et al. PRD102 (2020)]

DIS not possible / Related to the fragmentation functions by crossing of the DIS and e^+e^- (Drell-Levi-Yan) [Drell, Levy, Yan, PR 1969, PRD 1970]

A model for heavy baryon

$M_Q/\Lambda_{QCD} \to \infty$

Heavy quark symmetry → Light quark degrees of freedom does not distinguish heavy flavor/spin

Structure of a heavy baryon is governed by the light quarks

Heavy quark → free, static color source

Light quarks → chiral quark-soliton model

$N_c \to \infty$

Interaction is suppressed by ~ $\frac{1}{M_Q}, \frac{1}{N_c}$

Derive model PDFs and study their properties

Final numerical step: finite M_Q, N_c

Outline

Light quarks: chiral quark-soliton model

Light quark and antiquark isoscalar unpolarized and
isovector longitudinally polarized quark distributions

- Derivation of quark distribution functions in the χQSM
- Numerical results
- Sum rules
- Inequalities

Polarized antiquark asymmetry in Proton and heavy baryon

Nucleon and heavy baryon in the Chiral quark-soliton model

Effective partition function from the instanton vacuum

[D. Diakonov, V. Petrov, and P. Pobylitsa, Nucl. Phys. B 306, 809 (1988)]

$$Z = \int \mathcal{D}\pi^a d\psi^\dagger d\psi \ \exp \int d^4x \psi^\dagger(x)(i\partial\!\!\!/ + iMU^{\gamma_5})\psi(x)$$

$$U^{\gamma_5}(x) = U(x)\frac{1+\gamma_5}{2} + U^\dagger(x)\frac{1-\gamma_5}{2} \qquad U(x) = \exp\left[\frac{i}{F_\pi}\pi^a(x)\tau^a\right]$$

From QCD to the low energy effective theory via the **instantons**

Instanton parameters: average size $\bar{\rho} \sim 1/3 \text{ fm}$ & distance $\bar{R} \sim 1 \text{ fm}$ (no more parameters, Λ_{QCD})

Intrinsic renormalisation scale $\Lambda \sim 1/\bar{\rho} \approx 600 \text{ MeV}$

Spontaneous chiral symmetry breaking & dynamically generated quark mass $M = 350 \text{ MeV}$

Fully field theoretic: successfully describes a wide class of baryon properties

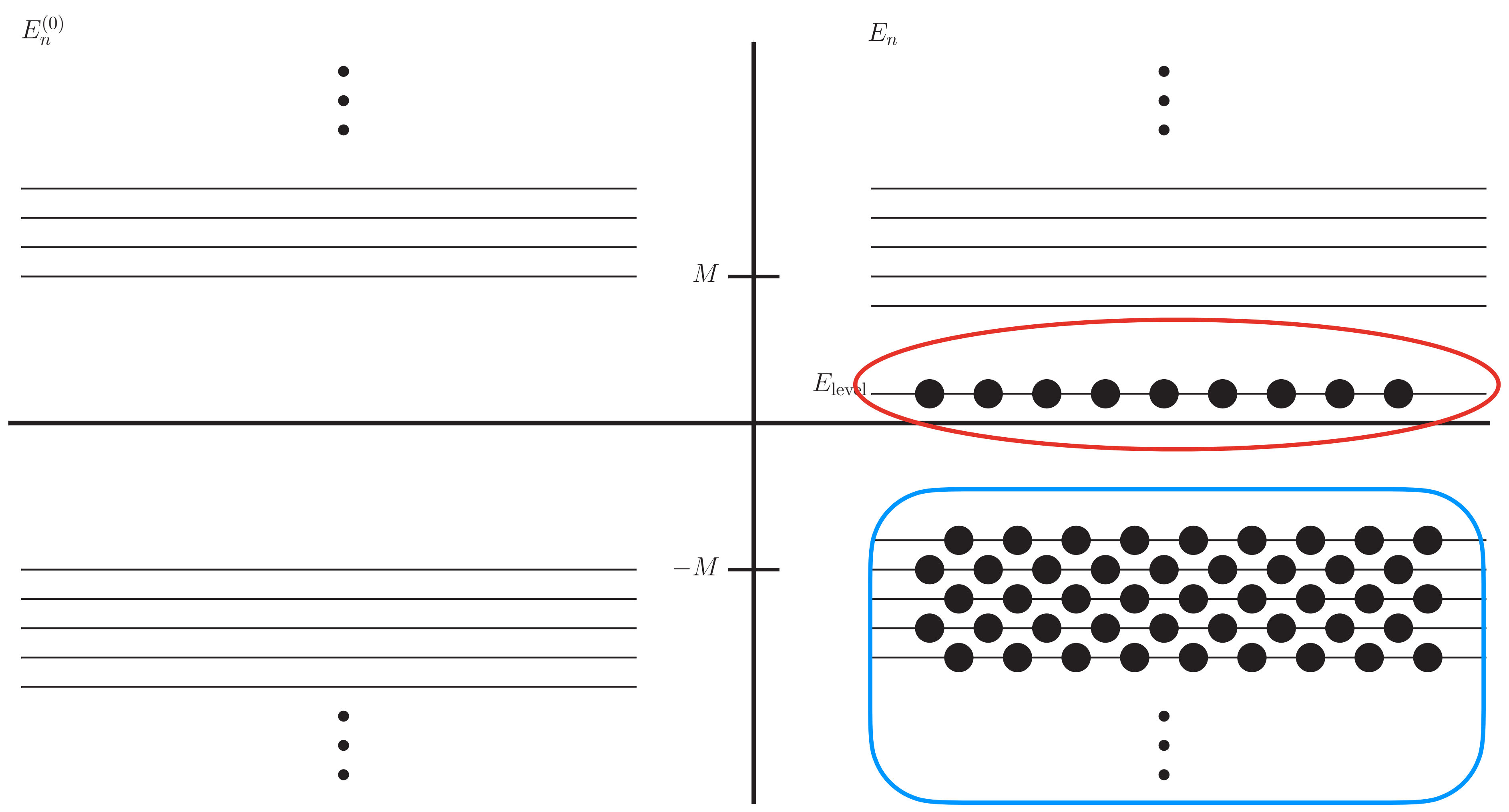
Baryon: chiral soliton in the large Nc, quarks are bound by a self-consistent mean-field

Interplays the quark-model and (topological) soliton picture of the baryons

[E. Witten, Nucl. Phys. B 160, 57 (1979)]

Nucleon as a chiral soliton in the large N_c limit

Quarks are bound by a common pion mean-field, self-consistently generated by their interactions



Hedgehog Ansatz

$$U = \exp[i\gamma_5 \hat{n}^a \tau^a P(r)]$$

Dirac spectra (n): Grandspin K= J + T and Parity P

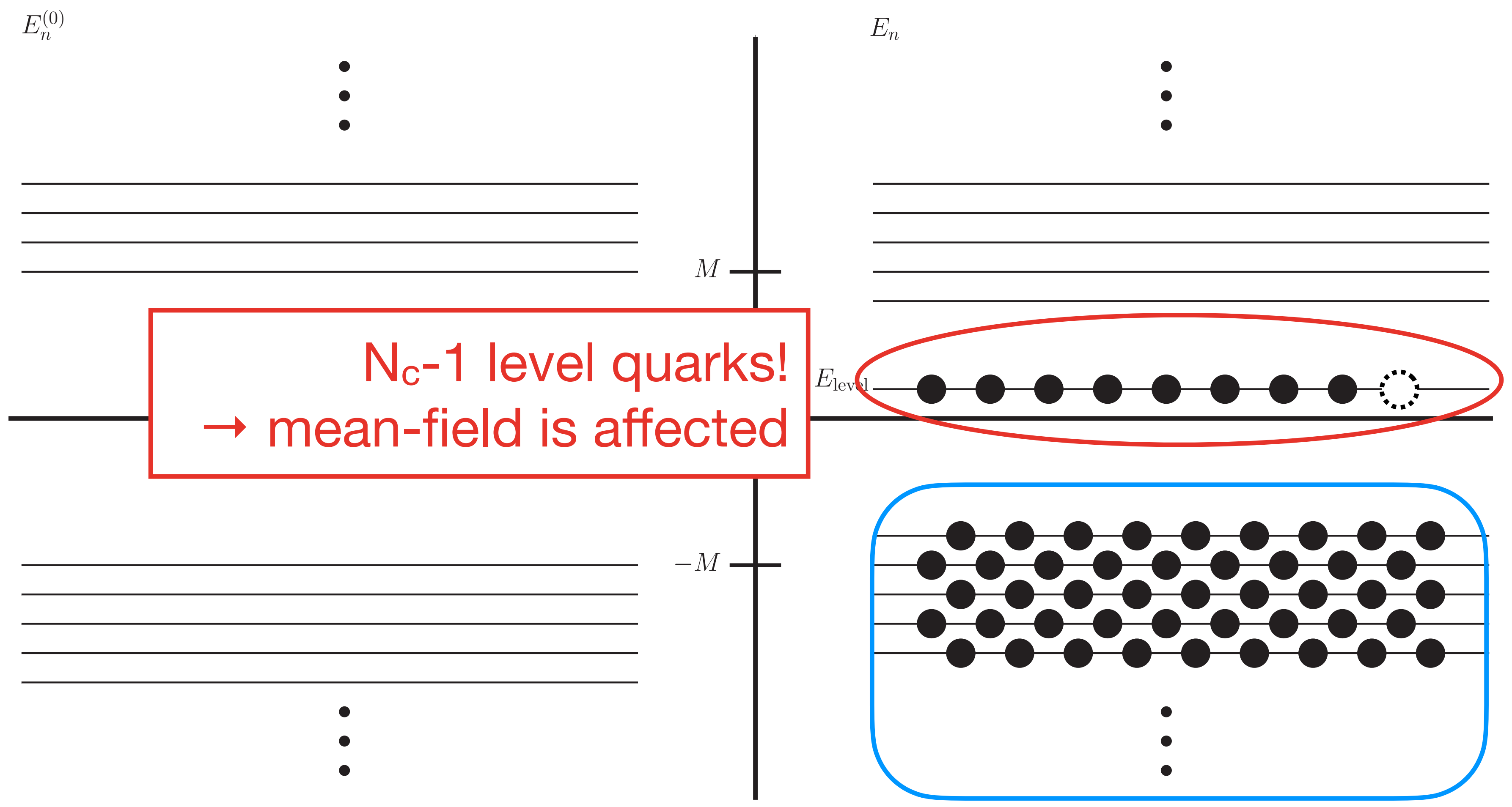
$$H\Phi_n(\vec{x}) = E_n \Phi_n(\vec{x})$$

Classical soliton energy

$$\frac{\delta}{\delta U}(N_c E_{\rm level} + E_{\rm cont.})|_{U=U_c} = 0 \quad \rightarrow \quad M_{sol} = N_c E_{\rm level}(U_c) + E_{\rm cont.}(U_c)$$

Nucleon quantum numbers: quantization around the rotational zero-modes

Heavy baryon: Nc-1 quark-soliton & free heavy quark



$$\frac{\delta}{\delta U}\left[(N_c-1)E_{\rm level}+E_{\rm cont.}\right]\Big|_{U=U_c}=0$$

$$M_{sol}=(N_c-1)E_{\rm level}(U_c)+E_{\rm cont.}(U_c)$$

$$M_h=M_Q+M_{sol}$$

Heavy quark mass $M_Q=(1.3,\,4.2)$ GeV as parameters to demonstrate Σc and Σb

M=420 MeV: strong quark-pion coupling is needed because of Nc-1 (vs. 350 MeV in instanton picture)

Recent studies for the heavy baryons

→ground-state mass spectrum

→EM ffs: good agreements with lattice calculations, Axial & Tensor

→Energy-momentum tensor form factors

:**Nc-1 level quarks produce a self-consistent mean-field**

~ key ingredient for the stability

[J.Y-. Kim, H.-Ch. Kim, M. Polyakov, HDS, PRD 2021]

Light quark distributions in a heavy baryon

Light quark and antiquark distribution functions in Σ_c and Σ_b

Unpolarized quark distributions

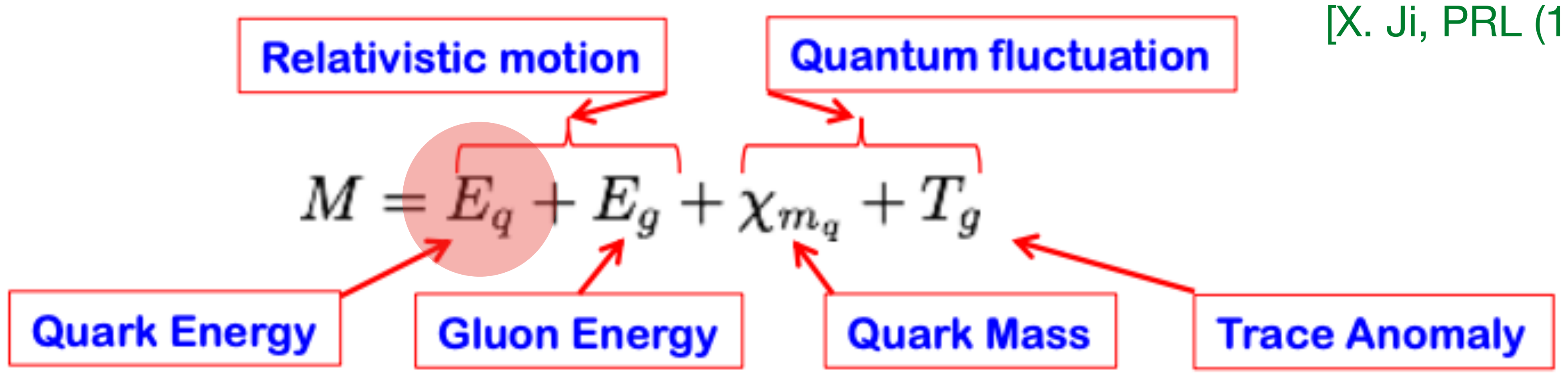
Probability to find a quark with momentum fraction x ~ dx

Baryon number and momentum sum rules

→ Momentum sum-rule: Mass form factor (EMT)

→ Hadron mass decomposition

[X. Ji, PRL (1995)]



$$\int \frac{dz^-}{4\pi} \exp[iz^-P^+x] \langle P|\bar{\psi}(0)\gamma^+\psi(z)|P\rangle = u(x) + d(x)$$

$$\int \frac{dz^-}{4\pi} \exp[iz^-P^+x] \langle P|\bar{\psi}(0)\gamma^+\tau^3\psi(z)|P\rangle = u(x) - d(x)$$

Large Nc scaling of the unpolarized and longitudinally polarized quark distributions

$$\left.\begin{matrix} u(x) + d(x) \\ \Delta u(x) - \Delta d(x) \end{matrix}\right\} \sim N_c^2\, \rho(N_c x)$$

VS

$$\left.\begin{matrix} u(x) - d(x) \\ \Delta u(x) + \Delta d(x) \end{matrix}\right\} \sim N_c\, \rho(N_c x)$$

Light quark and antiquark distribution functions in Σ_c and Σ_b

Longitudinally polarized quark distribution

Probability to find a quark with longitudinal spin parallel to hadron spin

Spin sum-rule and axial charge

→ Hadron spin decomposition

[Jaffee, Manohar, NPB 337 (1990)]

$$1/2 = \frac{1}{2}\int_0^1 dx\, \Delta\Sigma(x,Q^2) + \int_0^1 dx\, \Delta g(x,Q^2) + \sum_q L_q + L_g$$

$$\int \frac{dz^-}{4\pi} \exp[iz^- P^+ x]\, \langle P | \bar{\psi}(0)\gamma^+\gamma^5\tau^3\psi(z) | P\rangle = \Delta u(x) - \Delta d(x)$$

$$\int \frac{dz^-}{4\pi} \exp[iz^- P^+ x]\, \langle P | \bar{\psi}(0)\gamma^+\gamma^5\psi(z) | P\rangle = \Delta u(x) + \Delta d(x)$$

Large Nc scaling of the unpolarized and longitudinally polarized quark distributions

$$\left.\begin{array}{l} u(x) + d(x) \\ \Delta u(x) - \Delta d(x) \end{array}\right\} \sim N_c^2\, \rho(N_c x)$$

vs

$$\left.\begin{array}{l} u(x) - d(x) \\ \Delta u(x) + \Delta d(x) \end{array}\right\} \sim N_c\, \rho(N_c x)$$

Light quark and antiquark distribution functions in Σ_c and Σ_b

Quark and antiquark quasi number densities $x \in (-\infty, \infty)$

$$D_f(x,v) = \frac{1}{2E_h} \int \frac{d^3k}{(2\pi)^3} \delta\left(x - \frac{k^3}{P_h}\right) \int d^3x e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \langle h_v | \bar{\psi}_f(-\boldsymbol{x}/2, t) \Gamma \psi_f(\boldsymbol{x}/2, t) | h_v \rangle$$

$$\bar{D}_f(x,v) = \frac{1}{2E_h} \int \frac{d^3k}{(2\pi)^3} \delta\left(x - \frac{k^3}{P_h}\right) \int d^3x e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \langle h_v | \mathrm{Tr}\left[\Gamma \bar{\psi}_f(-\boldsymbol{x}/2, t) \psi_f(\boldsymbol{x}/2, t)\right] | h_v \rangle$$

Quark bi-local fields in (equal-time) Euclidean separation

become exact number densities in the limit $v \to 1$, approaching the light-cone, $x \in [0,1]$

Light quark and antiquark distribution functions in Σ_c and Σ_b

Isoscalar unpolarized distributions

$$u(x)+d(x)=(N_c-1)\int\frac{d^3k}{(2\pi)^3}\Phi^\dagger_{\mathrm{level}}(\vec{k})(1+\gamma^0\gamma^3)\Phi_{\mathrm{level}}(\vec{k})\delta(k_3-xM_h+E_{\mathrm{level}})$$
$$+N_c\sum_{E_n<0}\int\frac{d^3k}{(2\pi)^3}\Phi^\dagger_n(\vec{k})(1+\gamma^0\gamma^3)\Phi_n(\vec{k})-(U\to 1),$$
$$\bar{u}(x)+\bar{d}(x)=-(u(-x)+d(-x))$$

Isovector polarized distributions

$$\Delta u(x)-\Delta d(x)=-\frac{1}{3}(2T_3)(N_c-1)\int\frac{d^3k}{(2\pi)^3}\Phi^\dagger_{\mathrm{level}}(\vec{k})(1+\gamma^0\gamma^3)\tau^3\gamma_5\Phi_{\mathrm{level}}(\vec{k})$$
$$-\frac{1}{3}(2T_3)N_c\sum_{E_n<0}\int\frac{d^3k}{(2\pi)^3}\Phi^\dagger_n(\vec{k})(1+\gamma^0\gamma^3)\tau^3\gamma_5\Phi_n(\vec{k})-(U\to 1),$$
$$\Delta\bar{u}(x)-\Delta\bar{d}(x)=\Delta u(-x)-\Delta d(-x).$$

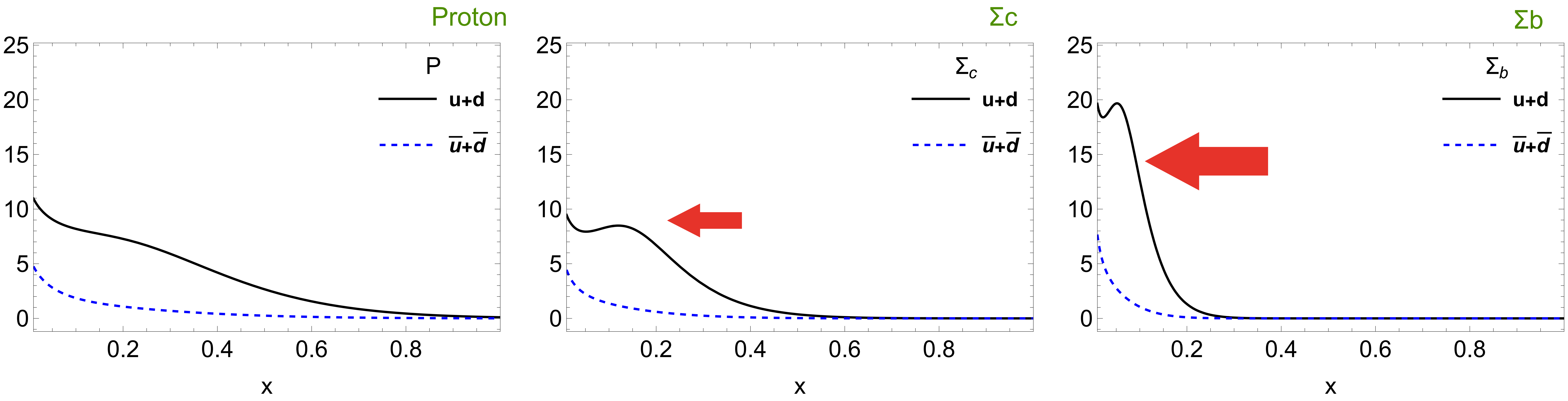
$$H\Phi_n(\vec{x})=E_n\Phi_n(\vec{x})$$

Sum-rules: heavy baryon PDFs

		Heavy quark
Baryon number	$\int_{-1}^{1} dx\, u(x) + d(x) = (N_c - 1)B$	$1B$
Momentum	$\int_{-1}^{1} dx\, x\, (u(x) + d(x)) = M_{sol}/M_h$	M_Q/M_h
Spin	$\int_{-1}^{1} dx\, (\Delta u(x) - \Delta d(x)) = (2T_3)g_A^{(3)}$	

Numerical results and discussions

$u(x) + d(x)$



Light quarks inside a heavy baron are more concentrated at small x region

More probable to find a quark with small momentum fraction

Momentum sum-rule: light quarks are less energetic in a heavy baryon (M_{sol}/M_h)

δ-like heavy quark distribution function $c(x) = \delta(x - M_Q/M_h)$

[HDS, H.-Ch. Kim, manuscript under preparation]

$u(x) + d(x)$: naive quark limit

Mean-field size →0, the model exhibit the properties of the naive quark limit

No interaction: **naive parton model**

Proton:

$u(x) + d(x) = N_c \delta(x - M/M_N)$, M: constituent quark mass ($M_N = N_c M$)

Momentum sum-rule:

$$\int_0^1 dx\ x\ u(x) + d(x) = N_c M/M_N = 1$$

Heavy baryon:

$u(x) + d(x) = (N_c - 1)\delta(x - M/M_h)$, $M_h = (N_c - 1)M + M_Q$

→ The distribution is squeezed to small x as M_Q grows

Momentum sum-rule:

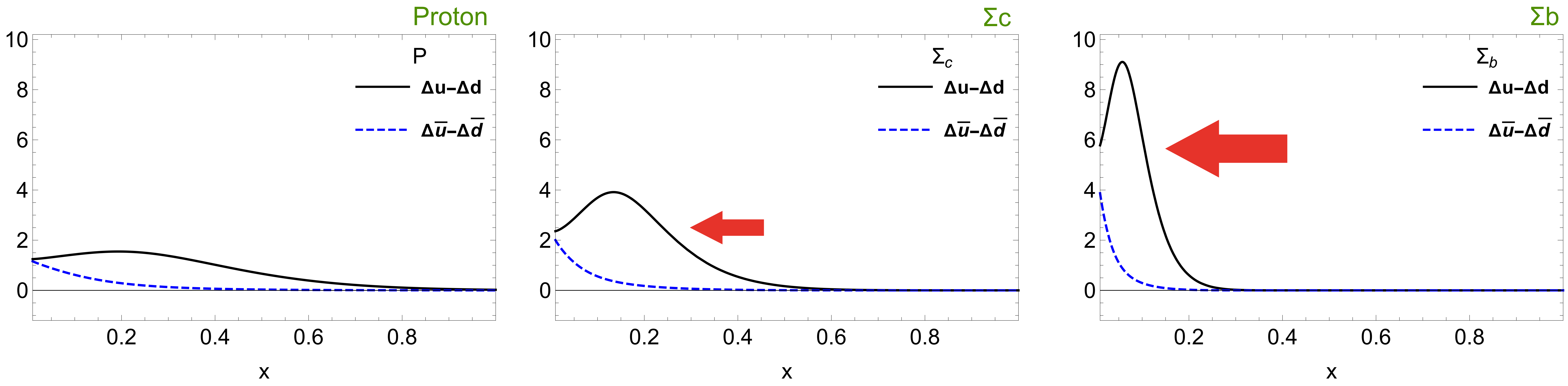
$$\int_0^1 dx\ x\ u(x) + d(x) = (N_c - 1)M/M_h$$ goes to 0 in the limit $M_Q \to \infty$

Momentum sum-rule

$$\int_0^1 dxx\ [\underbrace{u(x) + d(x) + \bar{u}(x) + \bar{d}(x)}_{M_{sol}/M_h} + \underbrace{Q(x)}_{M_Q/M_h}] = 1$$

[HDS, H.-Ch. Kim, manuscript under preparation]

$\Delta u(x) - \Delta d(x)$



Similar behavior as the isoscalar unpolarized distribution, squeezed into small x

Spin sum-rule $\int_0^1 dx[\Delta u(x) - \Delta d(x) + \Delta \bar{u}(x) - \Delta \bar{d}(x)]$ is **identical for Σ_c and Σ_b**

Numerically, $\int_0^1 dx[\Delta u(x) - \Delta d(x) + \Delta \bar{u}(x) - \Delta \bar{d}(x)]$ = **1.4** (T_3=+1). (Δc=-1/3, NR)

[HDS, H.-Ch. Kim, manuscript under preparation]

Positivity and inequality

Quark PDF

$$q^{\uparrow a}$$

Spin (parallel ↑ /antiparallel ↓)

Flavor (singlet: u, d, s, …)

Twist-2 Quark distribution functions (singlet)

Unpolarized $$f_1^a = (q^{\uparrow a} + q^{\downarrow a})/2$$

Longitudinally polarized $$g_1^a = (q^{\uparrow a} - q^{\downarrow a})/2$$

$$f_1^a + g_1^a = q^{\uparrow a}$$

$$f_1^a - g_1^a = q^{\downarrow a}$$

Probability to find a quark with spin parallel / antiparallel to the target

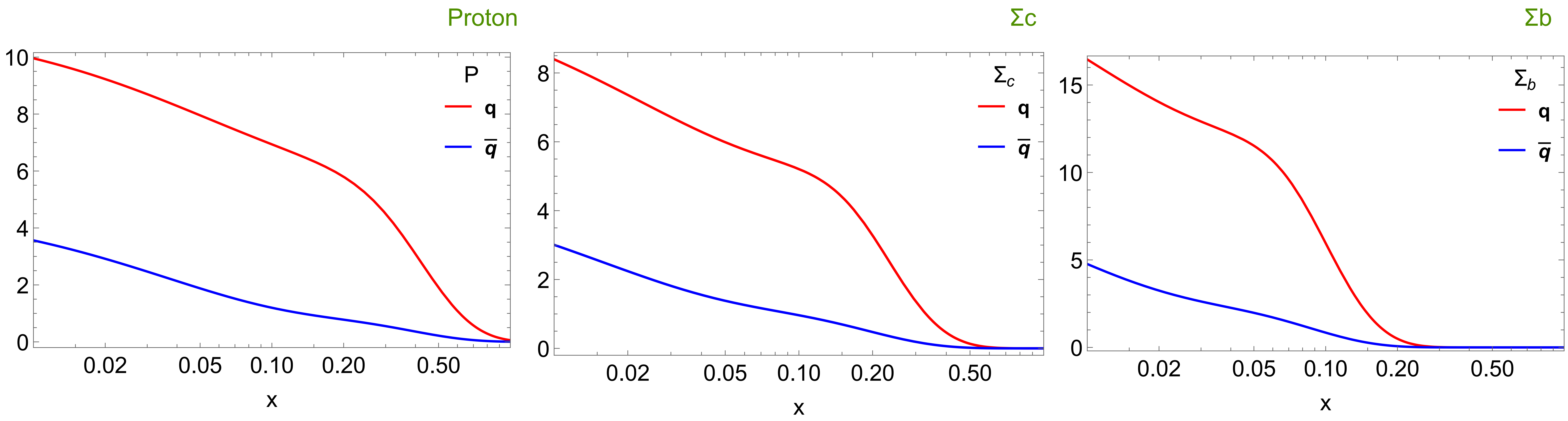
→ Positive

$$f_1^a \geq |g_1^a|$$

[HDS, H.-Ch. Kim, manuscript under preparation]

Positivity and inequality

$f_1^a \geq |g_1^a|$ In the large Nc, u-d and Δu + Δd are small → $u + d - |\Delta u - \Delta d| \geq 0$



[HDS, H.-Ch. Kim, manuscript under preparation]

Antiquark flavor asymmetry

Antiquark asymmetries in the proton

Unpolarized antiquarks: $\bar{d} > \bar{u}$ [Glück, Reya, Vogt, ZPC (1995)]

PDFs from polarized DIS: assumed $\Delta\bar{u} - \Delta\bar{d} = 0$ [Glück, Reya, Volgesang, PLB 359 (1995)] [Glück et al., PRD 53 (1996)]

χQSM prediction: $\Delta\bar{u} - \Delta\bar{d}$ **is large and positive** [Diakonov et al., NPB (1996) / PRD (1997)]

DIS is insensitive to the antiquark flavor asymmetry, but Drell-Yan is! [Dressler et al, EPJC 14 (2000), EPJC 18 (2001)] [Kumano and Miyama, PLB 479 (2000)]

Analyses using DIS + SIDIS, Drell-Yan [Glück et al., PRD 63 (2001)] [De Florian et al, PRD 80 (2009)] [Nocera et al. (NNPDF), NPB 887 (2014)]

Single spin asymmetry (W-boson) in polarized PP collision is used to study the asymmetry

(STAR collaboration) [L. Adamczyk et al. PRL 113 (2014)] [A. Adare et al. PRD 98 (2018)] [J. Adam et al. PRD 99 (2019)]

Global analyses updates:

[De Florian et al. PRD 100 (2019)]
[Cocuzza et al. (JAM) arXiv:2202.03371 (2022)]

Antiquark asymmetries in the proton: new results

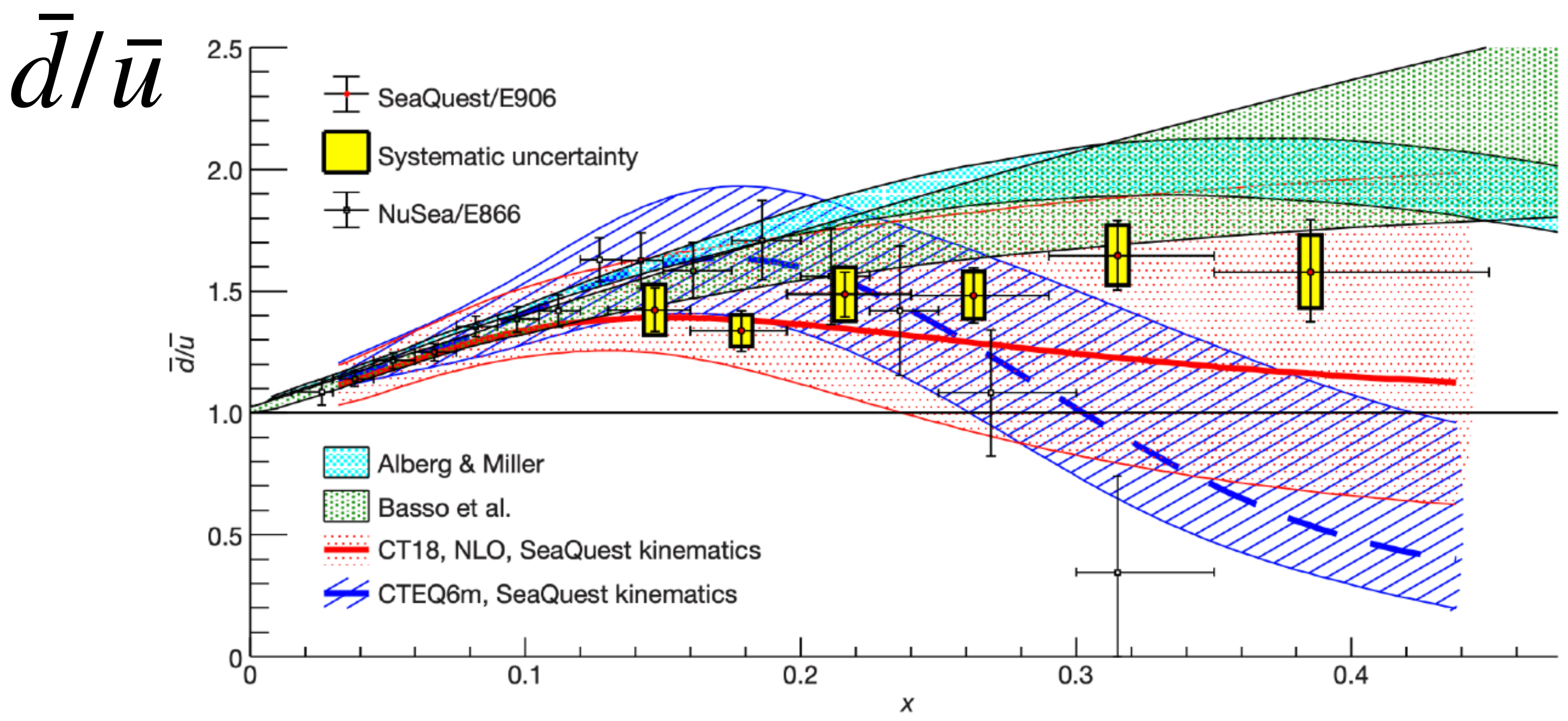


Fig. 2 | Ratios $\bar{d}(x)/\bar{u}(x)$. Ratios $\bar{d}(x)/\bar{u}(x)$ in the proton (red filled circles) with their statistical (vertical bars) and systematic (yellow boxes) uncertainties extracted from the present data based on NLO calculations of the Drell–Yan cross-sections. Also shown are the results obtained by the NuSea experiment (open black squares) with statistical and systematic uncertainties added in quadrature[4]. The cyan band shows the predictions of the meson–baryon model of Alberg & Miller[25] and the green band shows the predictions of the statistical parton distributions of Basso et al.[21]. The red solid (blue dashed) curves show the ratios $\bar{d}(x)/\bar{u}(x)$ calculated with CT18[29] (CTEQ6[35]) parton distributions at the scales of the SeaQuest results. The horizontal bars on the data points indicate the width of the bins.

[SeaQuest, Nature 590 (2021) 7847, 561-565]

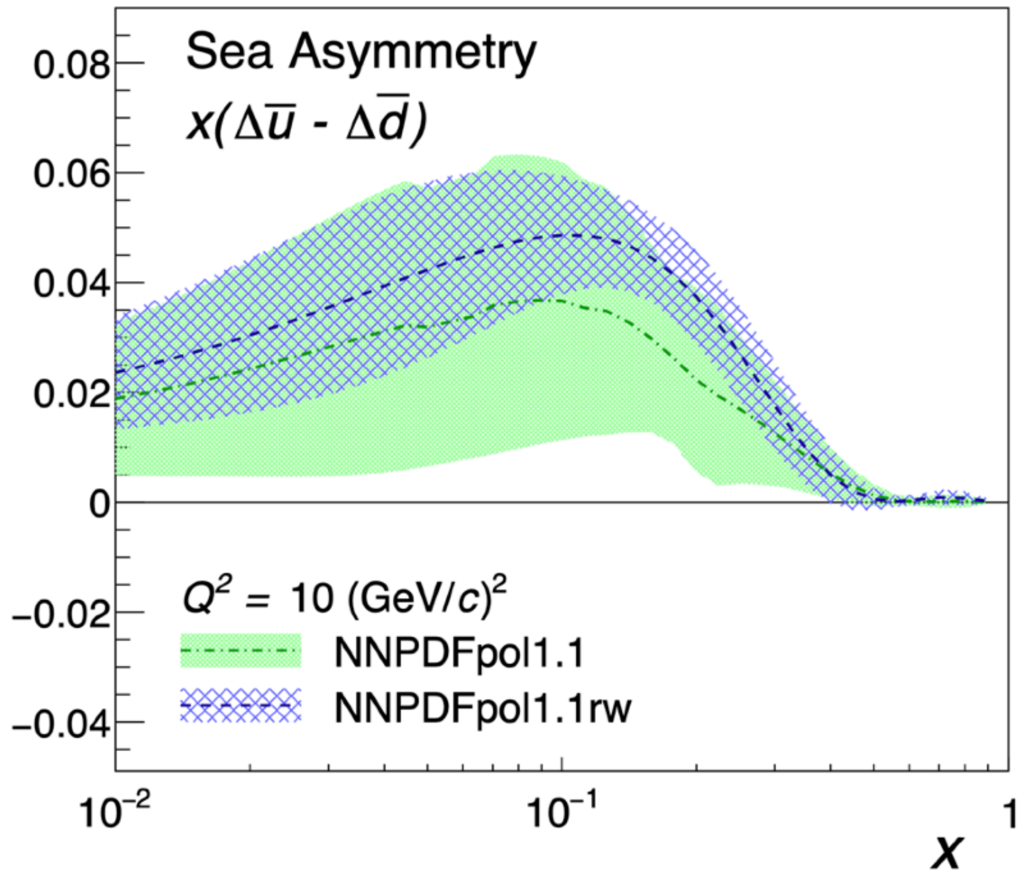
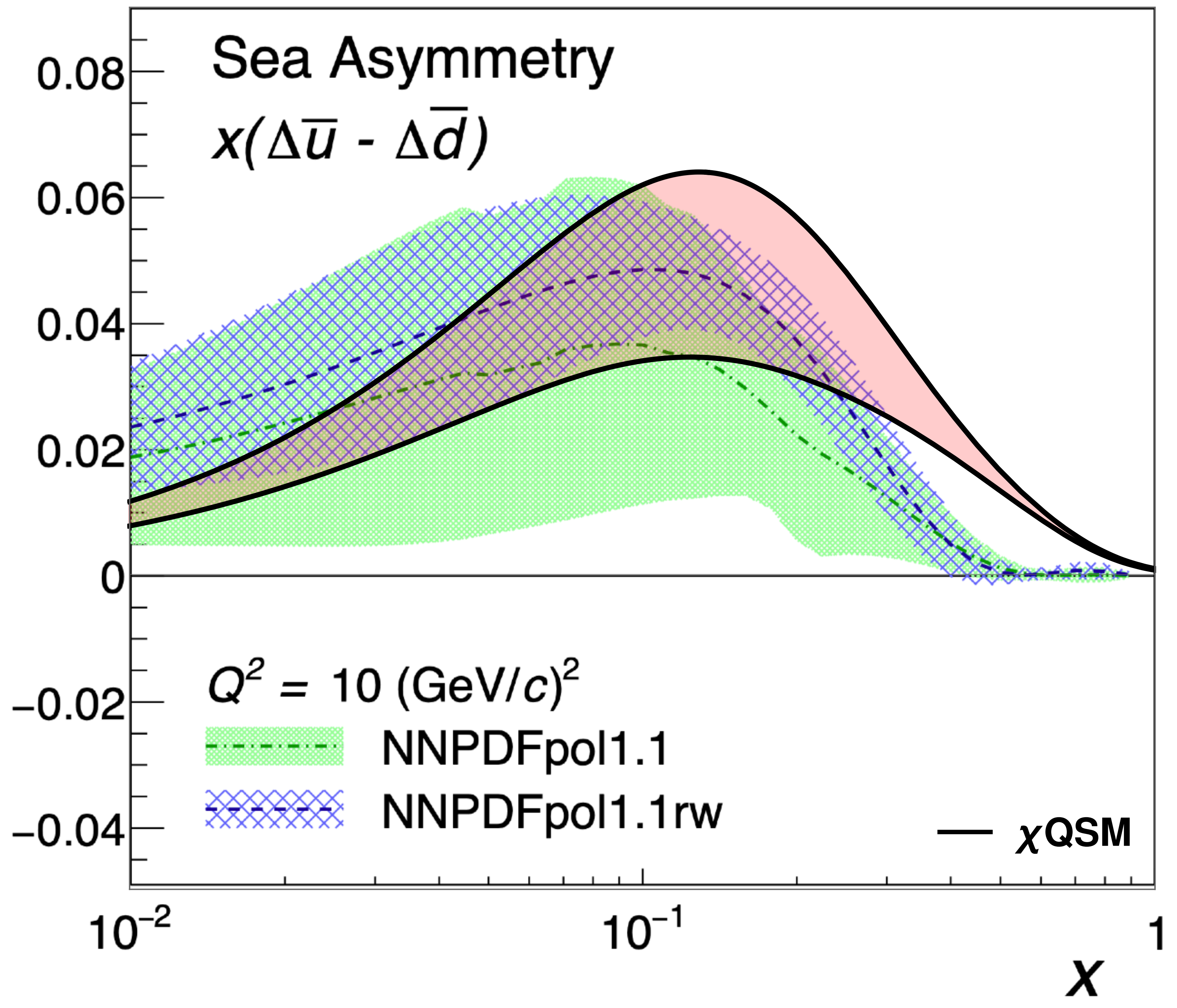


FIG. 6. The difference of the light sea-quark polarizations as a function of x at a scale of $Q^2 = 10\,(\mathrm{GeV}/c)^2$. The green band shows the NNPDFpol1.1 results [1] and the blue hatched band shows the corresponding distribution after the STAR 2013 $W^{\pm}$ data are included by reweighting.

[STAR collaboration, Phys.Rev.D 99 (2019) 5, 051102]

Polarized antiquark flavor asymmetry: model case



[STAR collaboration, Phys.Rev.D 99 (2019) 5, 051102]

FIG. 6. The difference of the light sea-quark polarizations as a function of x at a scale of $Q^2 = 10\,(\mathrm{GeV}/c)^2$. The green band shows the NNPDFpol1.1 results [1] and the blue hatched band shows the corresponding distribution after the STAR 2013 $W^{\pm}$ data are included by reweighting.

Band: Model systematic uncertainty

fixed $\rho \sim 1/(600\mathrm{MeV})$, in the chiral limit

M [MeV]	330	420
M_N [MeV]	1161	1077
ρ/R	0.32	0.37
F_π [MeV]	77	90

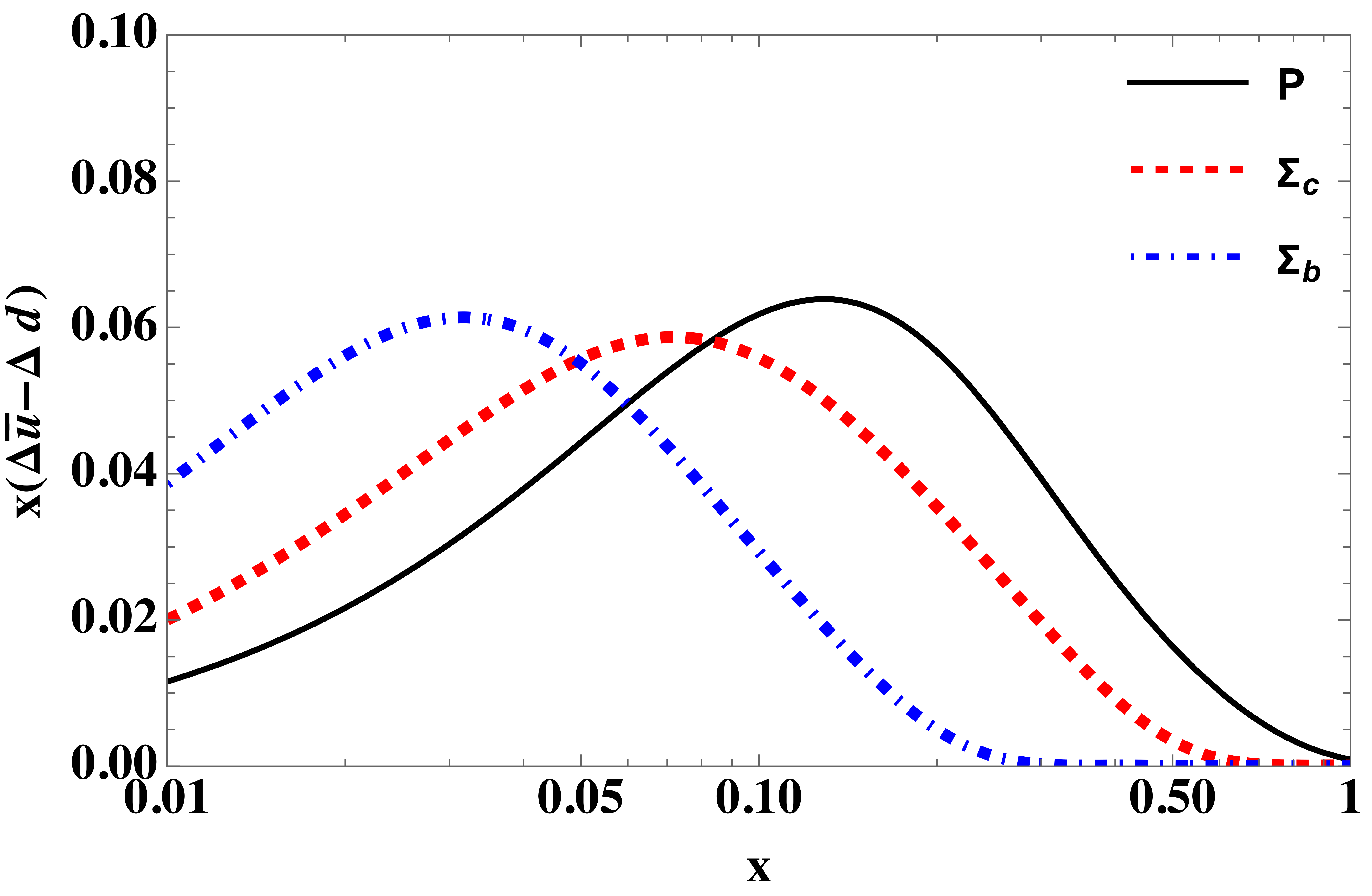
Continuum contribution (Polarized vacuum) is crucial

Softness: quark virtuality (momentum dep. mass)

1/Nc correction can enhance the PDF ~30%

Scale evolution

Antiquark flavor asymmetry: heavy baryon



Closing remarks

Summary and outlook

- **Light-quark distribution functions in a heavy baryon**
- **Light quarks in a heavy baryon are much less energetic than those in a proton**

- **Can this be measured? Or at least studied indirectly? eg. Fragmentation functions**

  : suitable reaction? (exclusive) Decay of heavy baryon(b), heavy production in e+e-, …
- **$1/M_Q$ corrections**

  Smearing of the heavy quark distribution

  Heavy-quark $\longleftrightarrow$ mean field, small? Stability?
- **Can be computed in the LaMET framework on the lattice (but P is not enough!)**

  Moments can be studied (eg. Momentum ratio of Heavy / light quarks)
- **$SU(3)_f$ extension: (sea) strange quark distribution in nucleon/heavy baryon**

Thank you very much!

Backup sides

Quasi parton distribution function

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

$$q(x,\mu,P^z) = \int \frac{dz}{4\pi} e^{-ixP^z z} \langle P|\bar{\psi}(0)\gamma^z \exp\left[-ig\int_0^z dz' A^z(z')\right]\psi(z)|P\rangle + \mathcal{O}(\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}, \frac{M_N^2}{(P^z)^2})$$

$x \in (-\infty, +\infty)$

μ: renormalization scale
P_z: nucleon momentum

La**r**ge **M**omentum **E**ffective **T**heory

Spacelike matrix element → can be calculated on the Lattice

No unique definition → **Γ=γ³** or **Γ=γ⁰**

Approaches the PDFs in the limit Pz →∞, or v → 1.

Quasi parton distribution function

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

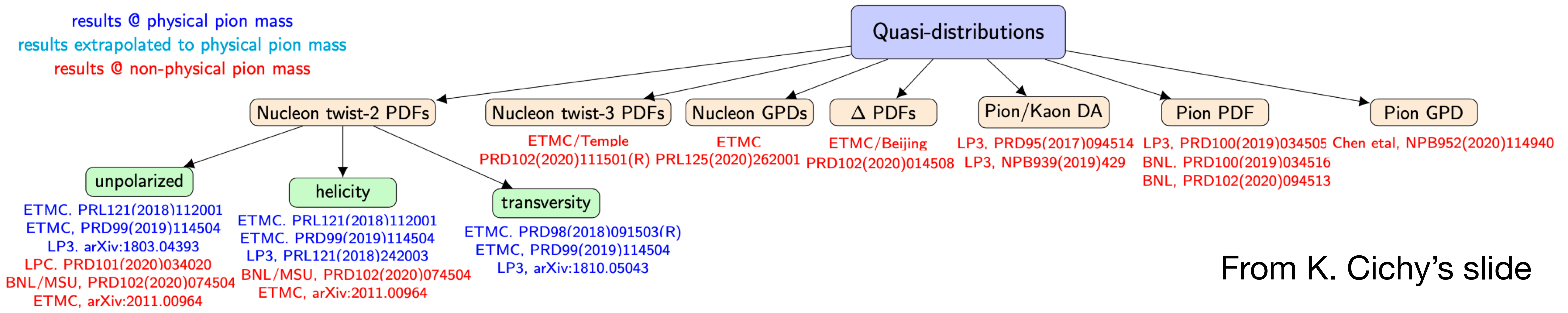
$$q(x, \mu_R, P^z) = \int_{-1}^{1} \frac{dy}{|y|} \underline{C(\frac{x}{y}, \frac{\mu_R}{\mu}, \frac{\mu}{p^z})} q(y, \mu) + \mathcal{O}(\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}, \frac{M_N^2}{(P^z)^2})$$

Perturbative matching coefficients

Extensively studied for the Lattice calculation

Market results $P_z \sim$ 2-3 GeV

N, π, K / PDFs, DAs, GPDs



From K. Cichy's slide

Quasi parton distribution function

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

$$q(x,\mu_R,P^z) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\mu_R}{\mu},\frac{\mu}{p^z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(P^z)^2},\frac{M_N^2}{(P^z)^2}\right)$$

Perturbative matching coefficients

Extensively studied for the Lattice calculation

Market results P_z ~ 2-3 GeV

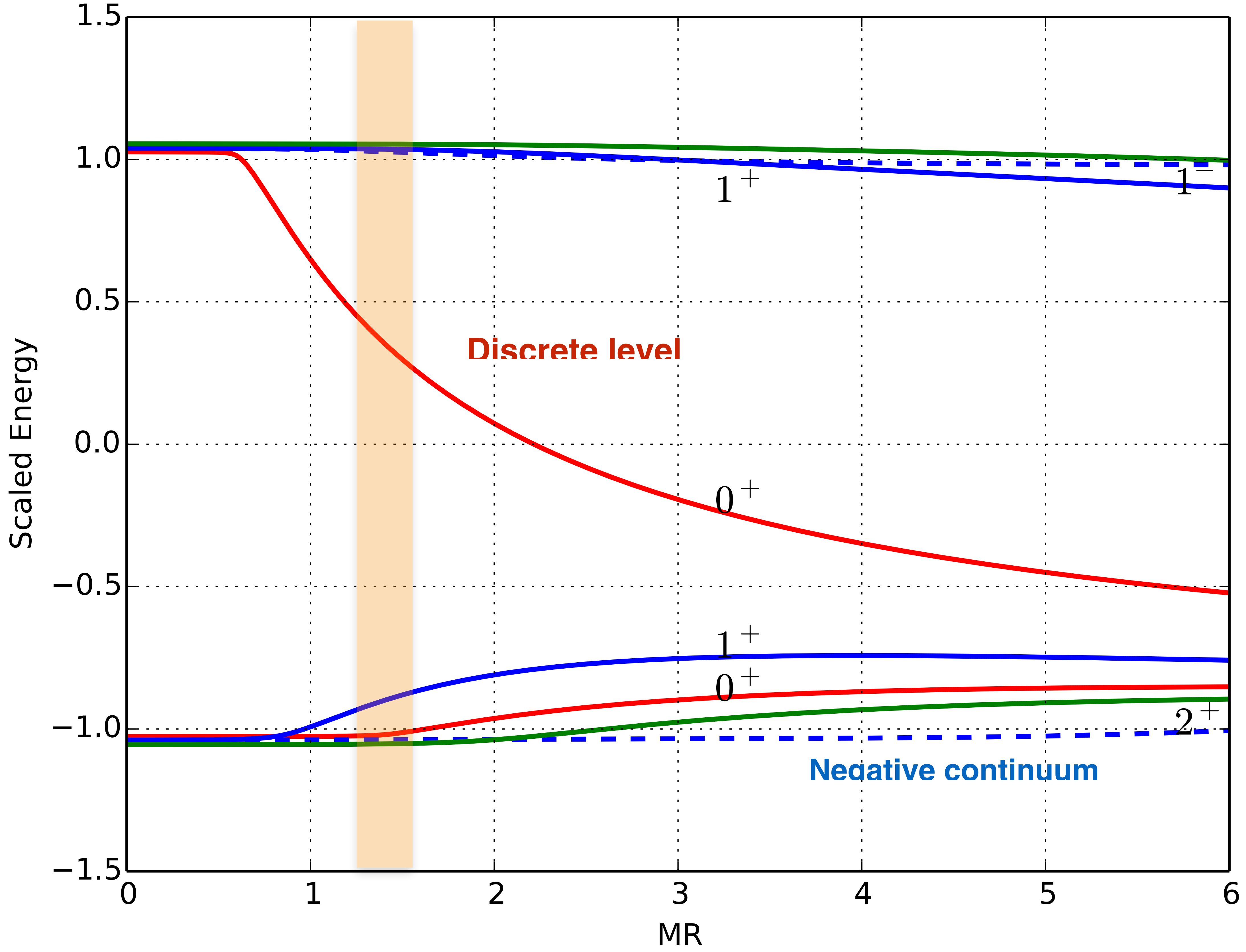
N, π, K / PDFs, DAs, GPDs

Enough accuracy and uncertainty for actual application?

Reliable model computations on quasi-PDFs is needed

Review: K. Cichy and M. Constantinou, Adv.High Energy Phys. 2019 (2019) 3036904
Community report: M. Constantinou et al, Prog.Part.Nucl.Phys. 121 (2021) 103908
and many more..

Hedgehog Ansatz: $U_{\mathrm{SU}(2)} = \exp\left[i\gamma_5 \mathbf{n}\cdot\boldsymbol{\tau} P(r)\right]$



Quantum Numbers:

G = J + τ

P = $(-1)^{G,G+1}$

Quarks are bound by the pion mean-field

Quasi-PDFs in the χQSM

Nucleon at rest → Lorentz boost to a inertial frame with velocity v in the z direction

Quark and antiquark quasi number densities $x \in (-\infty, \infty)$

$$D_f(x,v) = \frac{1}{2E_N}\int \frac{d^3k}{(2\pi)^3}\delta\left(x - \frac{k^3}{P_N}\right)\int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}}\langle N_v|\bar{\psi}_f\left(-\frac{\mathbf{x}}{2},t\right)\Gamma\psi_f\left(\frac{\mathbf{x}}{2},t\right)|N_v\rangle$$

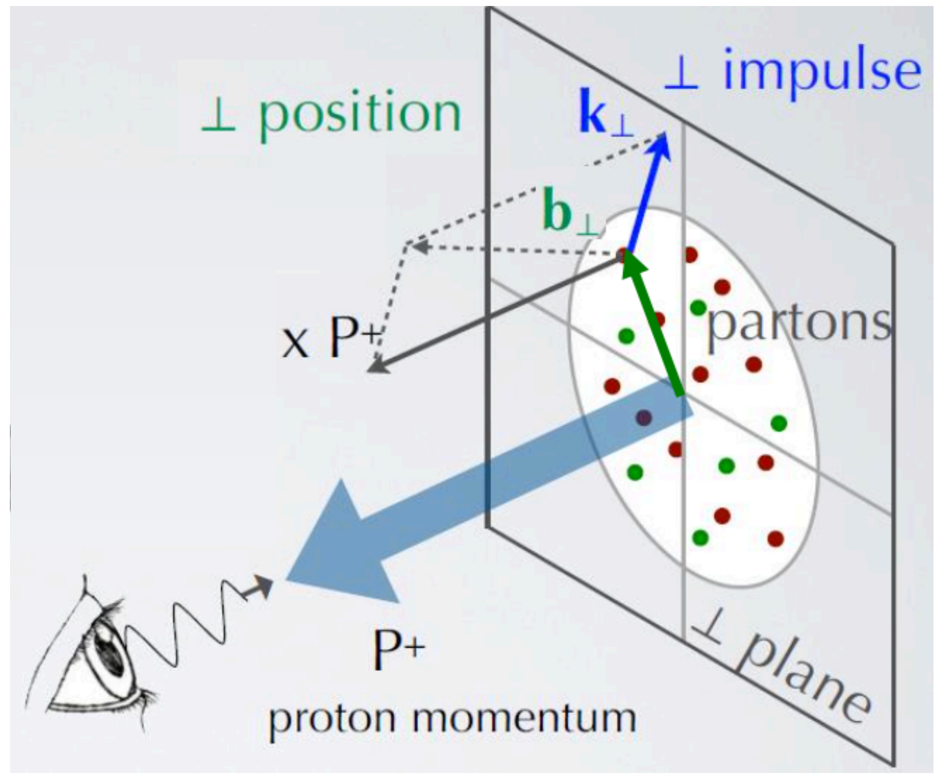
$$\bar{D}_f(x,v) = \frac{1}{2E_N}\int \frac{d^3k}{(2\pi)^3}\delta\left(x - \frac{k^3}{P_N}\right)\int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}}\langle N_v|\mathrm{Tr}\left[\Gamma\psi_f\left(-\frac{\mathbf{x}}{2},t\right)\bar{\psi}_f\left(\frac{\mathbf{x}}{2},t\right)\right]|N_v\rangle$$

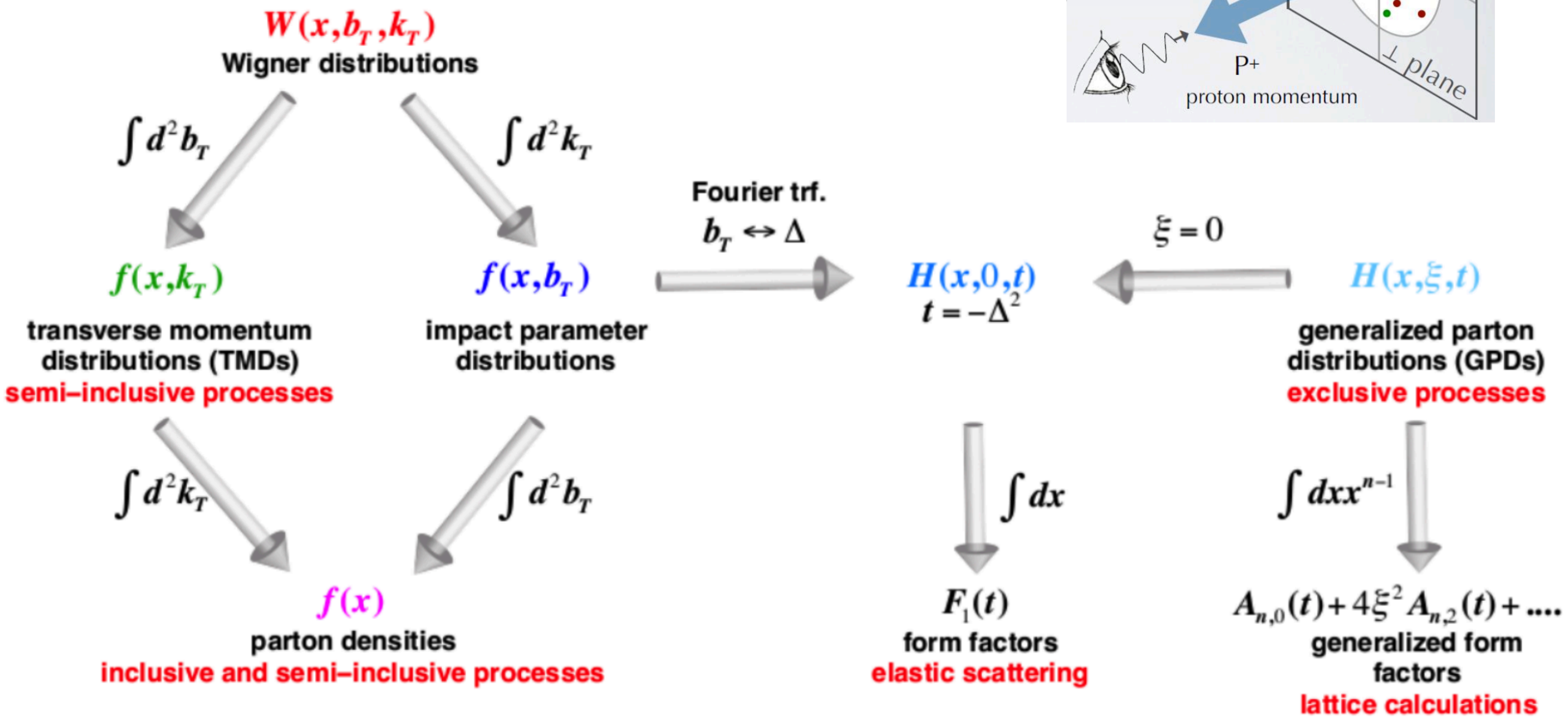
become exact number density in the limit $v \to 1$

Both the $\Gamma = \gamma^0$ and $\Gamma = \gamma^3$ define quasi-PDFs

A representation for the Green's function in the χQSM

$$\langle N_v|\mathrm{T}\left\{\psi(\vec{x}_1,t_1)\bar{\psi}(\vec{x}_2,t_2)\right\}|N_v\rangle = -\,S[\vec{v}]\Bigg[\Theta(t_2-t_1)\sum_{occ}\Phi_n(\vec{x}_1)\Phi_n^\dagger(\vec{x}_2)\gamma_0\exp(-iE_n(t_1-t_2))$$
$$-\Theta(t_1-t_2)\sum_{nocc}\Phi_n(\vec{x}_1)\Phi_n^\dagger(\vec{x}_2)\gamma_0\exp(-iE_n(t_1-t_2))\Bigg]S^{-1}[\vec{v}]$$


⊥ position
⊥ impulse
k⊥
b⊥
x P+
partons
P+
proton momentum
⊥ plane


$W(x,b_T,k_T)$
Wigner distributions
$\int d^2b_T$
$\int d^2k_T$
$f(x,k_T)$
transverse momentum distributions (TMDs)
semi–inclusive processes
$f(x,b_T)$
impact parameter distributions
Fourier trf.
$b_T \leftrightarrow \Delta$
$H(x,0,t)$
$t=-\Delta^2$
$\xi=0$
$H(x,\xi,t)$
generalized parton distributions (GPDs)
exclusive processes
$\int d^2k_T$
$\int d^2b_T$
$\int dx$
$\int dx x^{n-1}$
$f(x)$
parton densities
inclusive and semi–inclusive processes
$F_1(t)$
form factors
elastic scattering
$A_{n,0}(t)+4\xi^2A_{n,2}(t)+....$
generalized form factors
lattice calculations

$u(x,v) + d(x,v)$

[HDS, H.-Ch. Kim, manuscript under preparation]

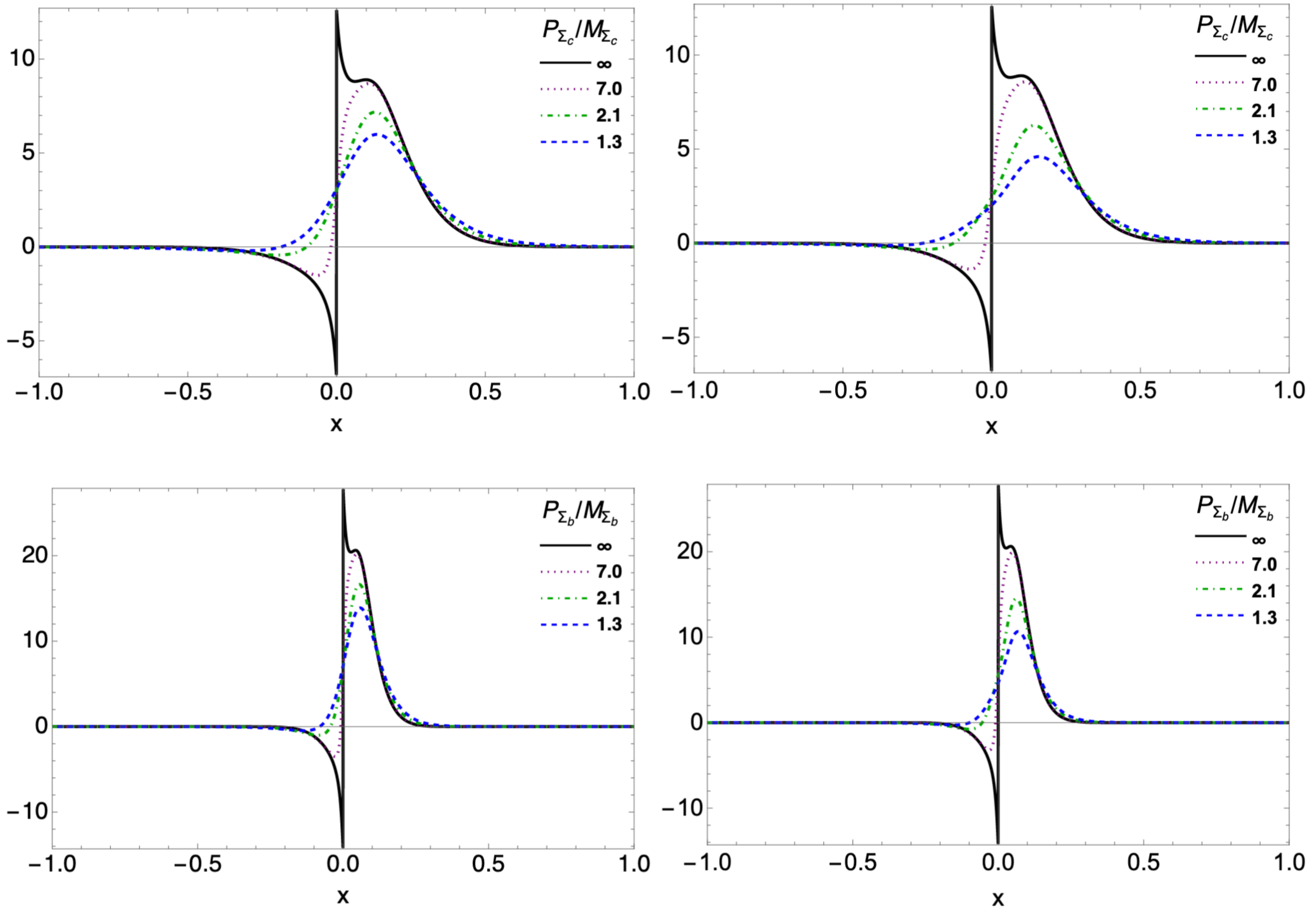


FIG. 5. Light-quark quasi distributions $u(x,P) + d(x,P)$ in Σ_c and Σ_b.

$\Delta u(x,v) - \Delta d(x,v)$

[HDS, H.-Ch. Kim, manuscript under preparation]

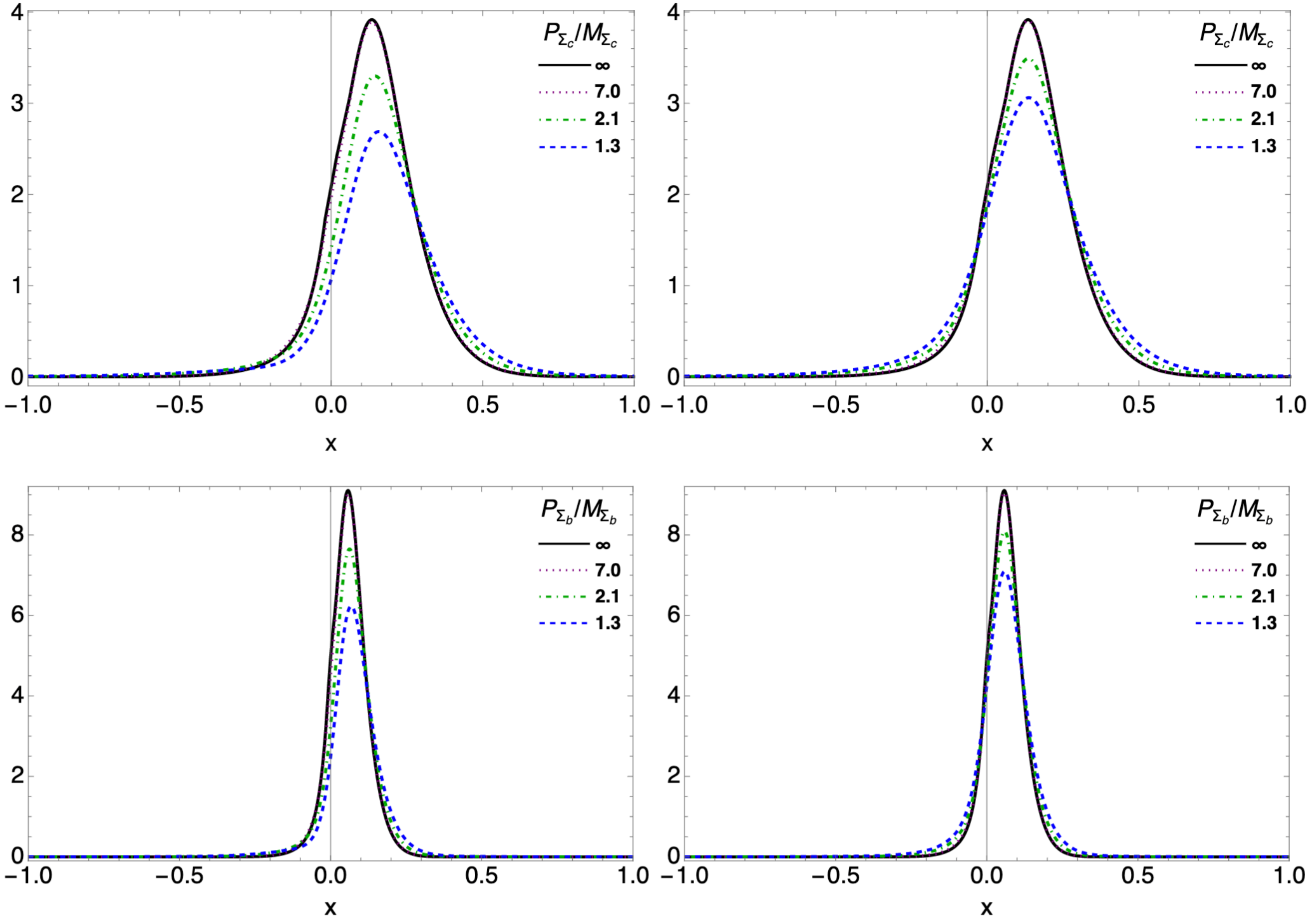


FIG. 6. Light-quark quasi distributions $\Delta u(x,P) - \Delta d(x,P)$ in Σ_c and Σ_b.

Sum-rules: heavy baryon PDFs

Heavy quark

Baryon number

$$\int_{-\infty}^{\infty} dx\, q(x,v) = \begin{cases} (N_c-1)B, & \Gamma=\gamma^0 \\ v(N_c-1)B, & \Gamma=\gamma^3 \end{cases}$$

Heavy quark: $1B$

Momentum

$$\int_{-\infty}^{\infty} dx\, xq(x,v) = \begin{cases} M_{sol}/M_h, & \Gamma=\gamma^0 \\ vM_{sol}/M_h, & \Gamma=\gamma^3 \end{cases}$$

Heavy quark: M_Q/M_h

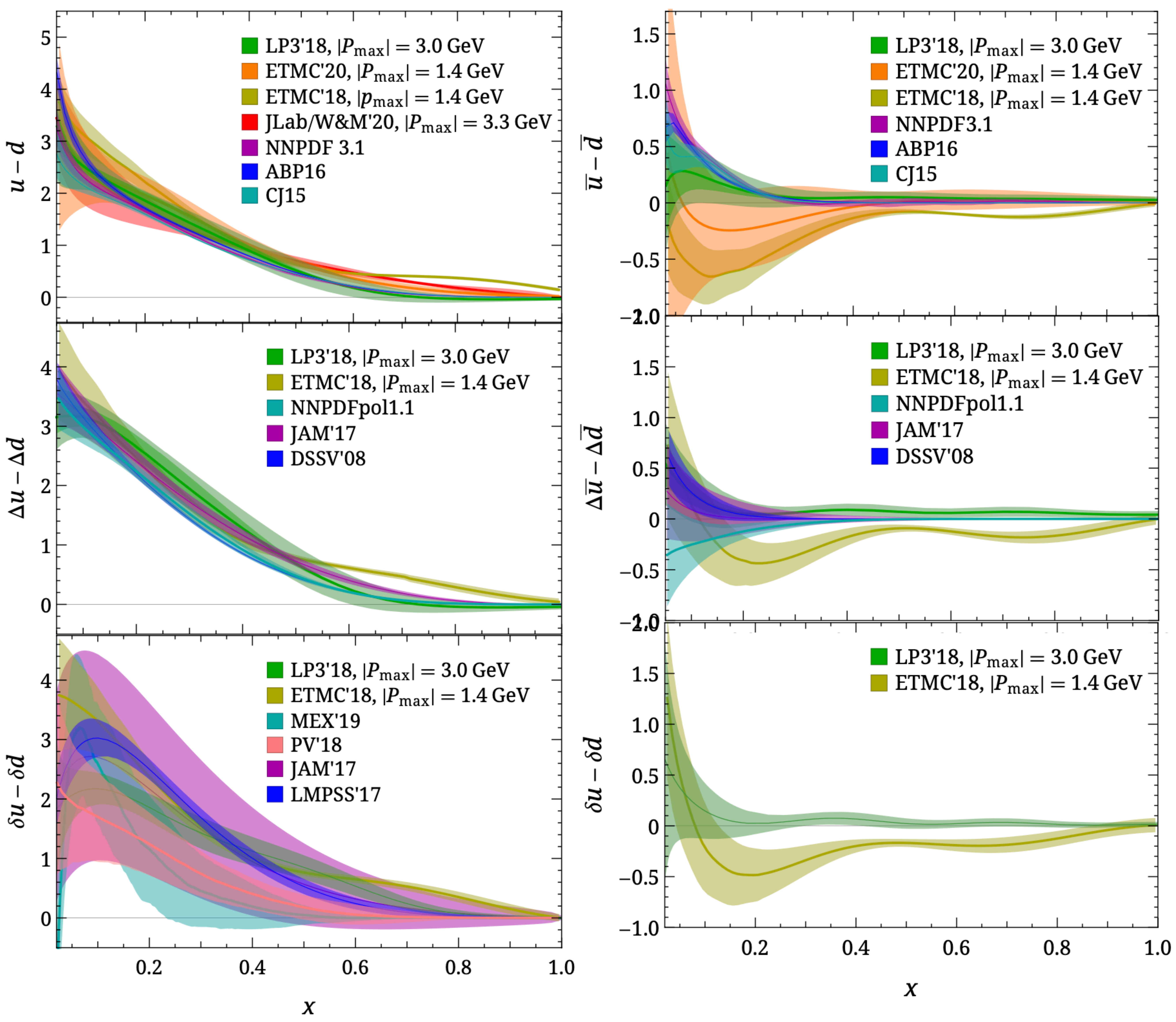
Spin

$$\int_{-\infty}^{\infty} dx\, (\Delta u(x,v)-\Delta d(x,v)) = \begin{cases} v(2T_3)g_A^{(3)}, & \Gamma=\gamma^0 \\ (2T_3)g_A^{(3)}. & \Gamma=\gamma^3 \end{cases}$$

Isovector PDFs

M. Constantinou's slide @ Spin 2021, Japan

State-of-the-art results



No continuum extrapolation yet

M. Constantinou *et al.* (2020) 2007.08636

CENuM

Copied from [STAR collaboration, Phys.Rev.D 99 (2019) 5, 051102]

The leptonic $W^+ \to e^+\nu$ and $W^- \to e^-\bar{\nu}$ decay channels provide sensitivity to the helicity distributions of the quarks, Δu and Δd, and antiquarks, $\Delta\bar{u}$ and $\Delta\bar{d}$, that is free of uncertainties associated with non-perturbative fragmentation. The cross-sections are well described [18]. The primary observable is the longitudinal single-spin asymmetry $A_L \equiv (\sigma_+ - \sigma_-)/(\sigma_+ + \sigma_-)$ where $\sigma_{+(-)}$ is the cross-section when the helicity of the polarized proton beam is positive (negative). At leading order,

$$A_L^{W^+}(y_W) \propto \frac{\Delta\bar{d}(x_1)u(x_2) - \Delta u(x_1)\bar{d}(x_2)}{\bar{d}(x_1)u(x_2) + u(x_1)\bar{d}(x_2)}, \qquad (1)$$

$$A_L^{W^-}(y_W) \propto \frac{\Delta\bar{u}(x_1)d(x_2) - \Delta d(x_1)\bar{u}(x_2)}{\bar{u}(x_1)d(x_2) + d(x_1)\bar{u}(x_2)}, \qquad (2)$$

where x_1 (x_2) is the momentum fraction carried by the colliding quark or antiquark in the polarized (unpolarized) beam. $A_L^{W^+}$ ($A_L^{W^-}$) approaches $-\Delta u/u$ ($-\Delta d/d$) in the very forward region of W rapidity, $y_W \gg 0$, and $\Delta\bar{d}/\bar{d}$ ($\Delta\bar{u}/\bar{u}$) in the very backward region of W rapidity, $y_W \ll 0$. The observed positron and electron pseudorapidities, η_e, are related to y_W and to the decay angle of the positron and electron in the W rest frame [19]. Higher-order corrections to $A_L(\eta_e)$ are known [20–22] and have been incorporated into the aforementioned global analyses.

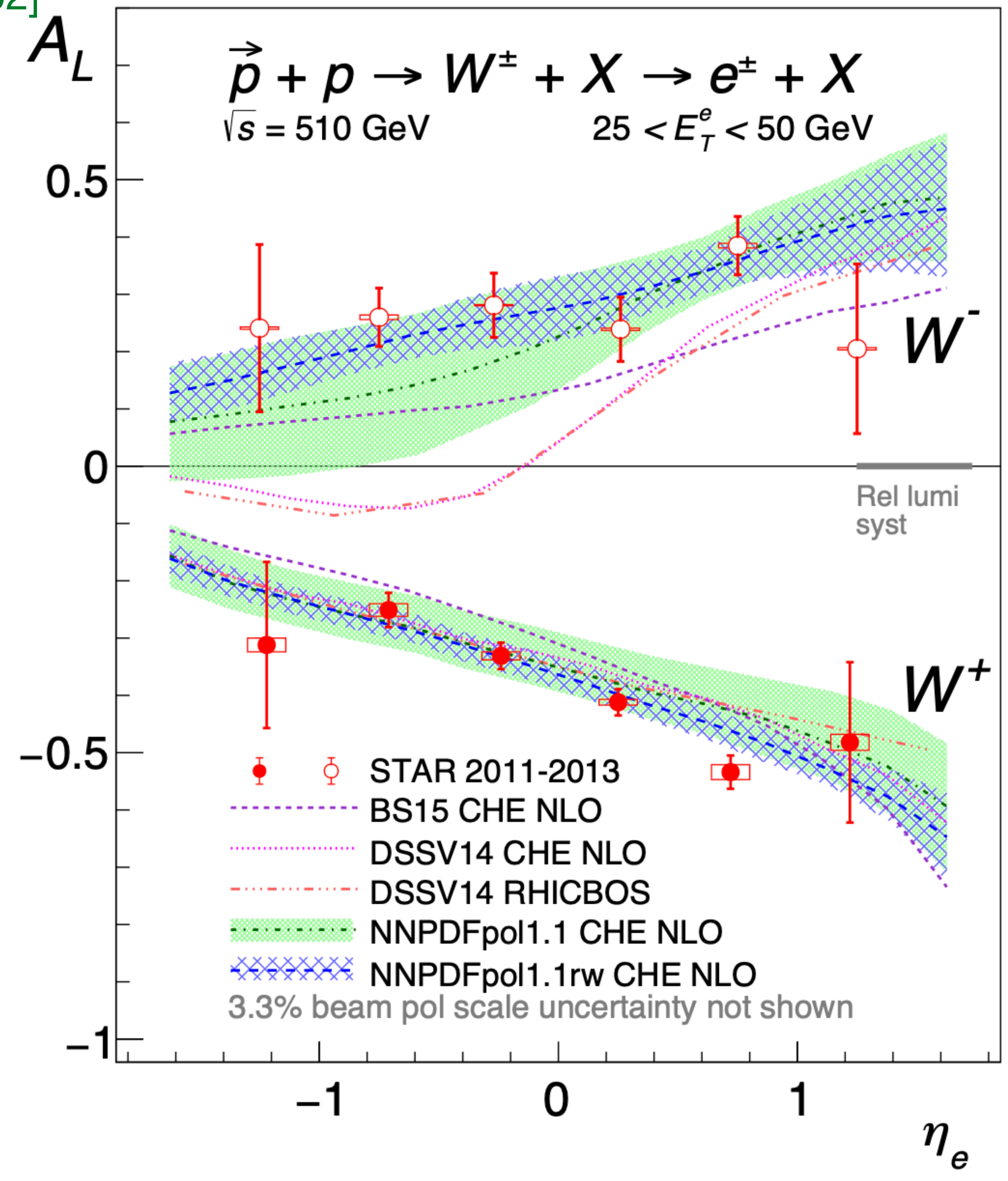


FIG. 5. Longitudinal single-spin asymmetries, A_L, for $W^\pm$ production as a function of the positron or electron pseudorapidity, η_e, for the combined STAR 2011+2012 and 2013 data samples for $25 < E_T^e < 50$ GeV (points) in comparison to theory expectations (curves and bands) described in the text.