Instanton effects on charmonium spectrum and electromagnetic transitions

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Outline

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    - ❖ Instanton-induced heay-quark potential
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- Results
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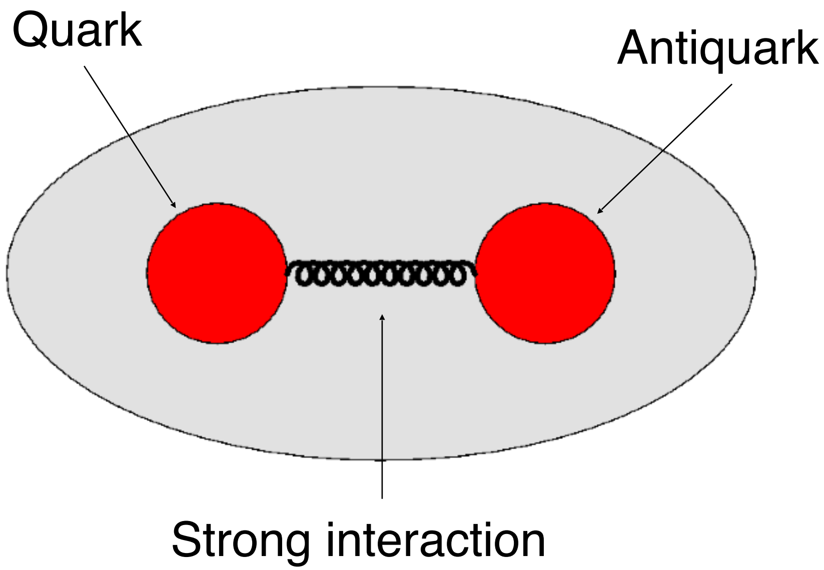
    - ❖ Decay rate of the E1 & M1 Radiative Transition

- Summary

Heavy-quark potential
&
Instanton

Heavy-quark Potential



- Light mesons

  $E_B \gg m_q$, E_B: Bound state energy

  - Light quarks inside of a meson are relativistic (quite complicated)

- Quarkonia

  $E_B < m_Q$

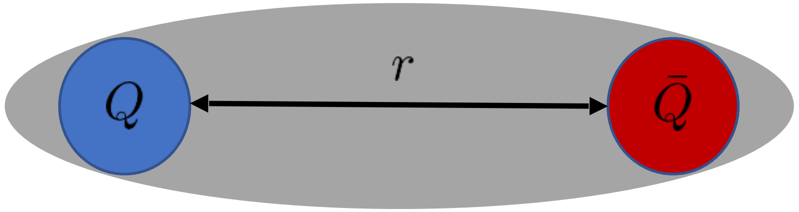
  - Relativistic effects are sufficiently small.
  - $1/m_Q$ is taken to be a small parameter.
  - A non-relativistic potential approach is theoretically valid.

- Usual heavy-quark potential : $V(r) = -\frac{\alpha}{r} + kr$

one-gluon exchange perturbative potential

Phenomenological confining potential

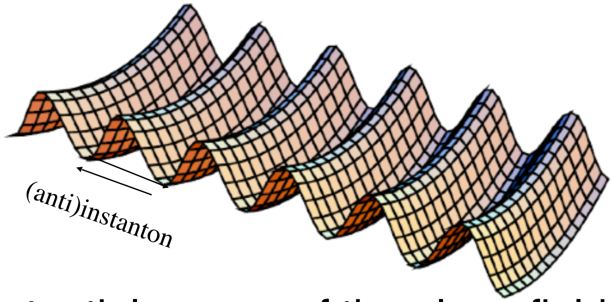
$r \to 0,\ \alpha(p^2 \to \infty) \to \epsilon$: Asymptotic freedom



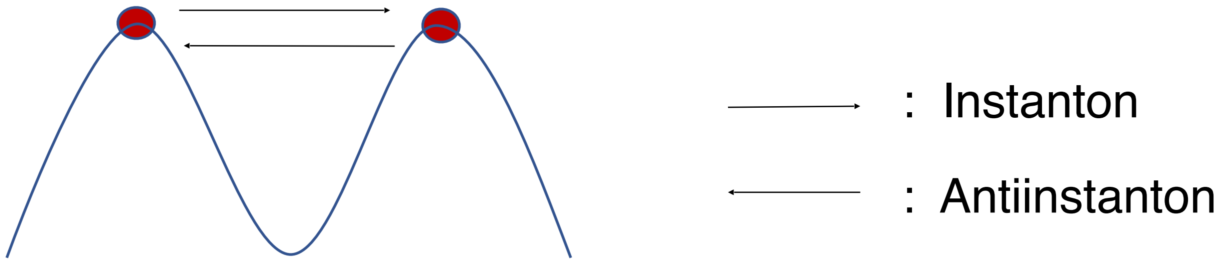
: confinement

Instanton

- In the Chern-Simon coordinate, the instanton is a large fluctuation of the gluon field corresponding to quantum tunneling from one vacuum(minimum of the potential energy) to the neighboring one:



Potential energy of the gluon field



Classical trajectory in Euclidean space

- To find the best tunneling trajectory having the largest amplitude one has thus to minimize the YM action, which becomes

$$S = \frac{1}{4g^2}\int d^4x\; F^a_{\mu\nu}F^a_{\mu\nu} = \frac{8\pi^2}{g^2}.$$

$F^a_{\mu\nu} = \tilde{F}^a_{\mu\nu}$: self-duality condition

- The singular gauge field satisfying the self-duality equation can be written as:

$$A_{I,\mu}(x, z_I) = \frac{\eta^{-a}_{\mu\nu}(x-z_I)_\mu \lambda^a \rho^2}{(x-z_I)^2((x-z_I)^2+\rho^2)},$$

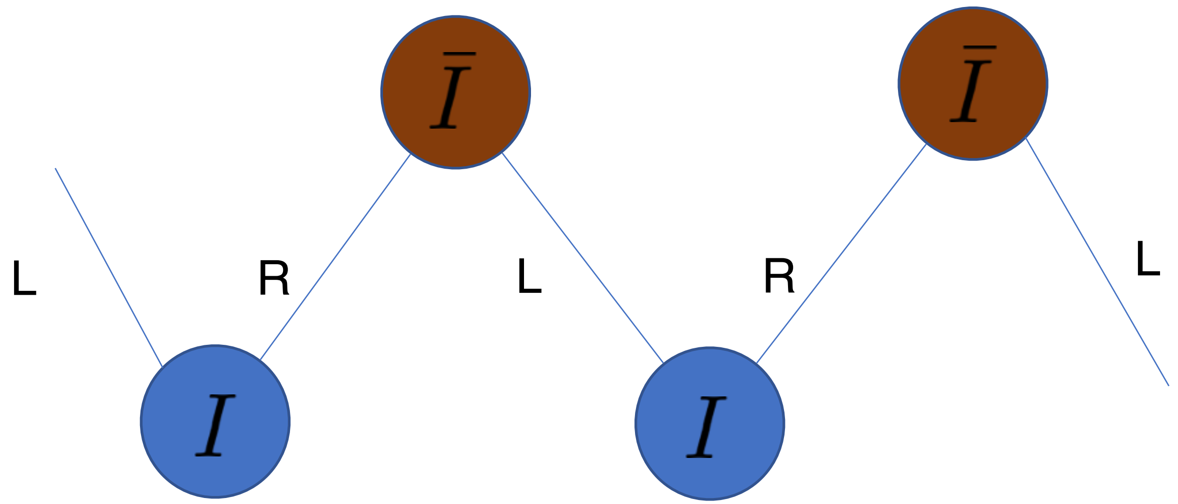
ρ : Average instanton size =1/3 fm

z_I : A position of instanton

Instanton in the light-quark system

- The Instanton is known to contribute to the interaction between light quarks more than that between heavy quarks.

- The dynamical light quark mass is generated from the instanton vacuum (spontaneous breakdown of chiral symmetry:



- Quarks hopping from instantons (I) to anti-instantons ($\bar{I}$) and vice versa flip helicity.

- An infinite number of such jumps generates the dynamical mass $M(k) = MF^2(k)$.

$$F(k) = -k\frac{d}{dk}\left[I_0\left(\frac{k\rho}{2}\right)K_0\left(\frac{k\rho}{2}\right) - I_1\left(\frac{k\rho}{2}\right)K_1\left(\frac{k\rho}{2}\right)\right]$$

$$M \approx 350\ \mathrm{MeV}\ \ (M \gg m_q)$$

: quark

Instanton in the heavy-quark system

- The gauge field of the instanton medium can be represented by linear superposition of separate instantons and anti-instantons A_I:

$$A(x) = \sum_I A_I(x) \qquad A_I(x) = U_I A(x - z_I, \rho_I) U_I^\dagger$$

- Each instanton field is characterized by its centre z_I, size ρ_I, and the color-orientation matrix U_I.

- Averaging over the quark propagator in the instanton vacuum[1]:

$$W = P\exp\left(i\int_C dx_\mu A_\mu\right) = \langle T| \left\langle\left\langle \left(\frac{d}{dt} - \sum_I a_I(t)\right)^{-1} \right\rangle\right\rangle |0\rangle$$

$a_I(t) = iA_{I\mu}[x(t)]\dot{x}_\mu(t)$: is a Yang-Mills field tangent

- The instanton ensemble $\langle\langle\rangle\rangle$ is composed of the instanton positions and the color-orientation.
- From the instanton ensemble one obtains

$$w^{-1} - \theta^{-1} = \frac{N}{2VN_c}\,\mathrm{Tr}_c\left\{\int d^4 z_I (w - a_I^{-1})^{-1} + I \to \bar{I}\right\}.$$

$$w = \left\langle\left\langle \left(\theta^{-1} - \sum_I a_I(t)\right)^{-1} \right\rangle\right\rangle$$

$$\theta^{-1} = d/dt$$

$\left\langle t \middle| \theta \middle| t' \right\rangle = \theta(t - t')$ gives the Heaviside function

[1] Diakonov et al, Phys. Lett. B 226, 372 (1989)

Instanton in the heavy-quark system

- To use the perturbation theory, we can choose a small parameter $\frac{\bar{\rho}^4 N}{V N_c} \simeq 0.004$ with the instanton density $\frac{N}{V} = \left(1 \text{ fm}\right)^{-4}$ and the averaged instanton size $\bar{\rho} = 1/3$ fm.

$$w^{-1} = \theta^{-1} - \frac{N}{2VN_c} \mathrm{Tr}_c \left\{ \int d^4 z_I \theta^{-1}(w_I - \theta)\theta^{-1} + I \to \bar{I} \right\} + \mathcal{O}((N/VN_c)^2).$$

$$w_I = (\theta^{-1} - a_I)^{-1}.$$

  - Here the operator w_I is a one-instanton operator:

- After the Fourier transformation, we can use the definition [1]

$$P \exp \left(i \int_0^T dx_4 A_4 \right) = \exp(-MT) = \exp\left(-\frac{N}{2VN_c} g(0) T \right),$$

$$g(0) = \int d^3 z_I \, \mathrm{Tr}_c \left[1 - P\exp\left(i \int_{-\infty}^{\infty} dx_4 A_{I4} \right)\Big|_{z_{I4}=0} \right] + I \to \bar{I}.$$

$$m_Q \gg M = 16\pi \int_0^\infty dz \left(z \cos \frac{\pi z}{2\sqrt{z^2+1}} \right)^2 \approx 70 \text{ MeV} : \text{ Instanton dynamical mass[1]}$$

- The instanton effects on quarkonia are small but still important because they allow one to use physical values of the parameters in the heavy-quark potential.

[1] Diakonov et al, Phys. Lett. B 226, 372 (1989)

Heavy-quark potential in the instanton vacuum

Instanton-induced heavy-quark potential

- In the static state, we can use a rectangular Wilson loop along a contour $T \times R$ with $T \to \infty$:
  - In this limit, we can neglect the short sides of the rectangle then we can write

$$W(L_1, L_2) = -\left\langle\left\langle \langle T| \left(\theta^{-1} - \sum_I a_I^{(1)}\right)^{-1} |0\rangle\langle 0| \left(\theta^{-1} - \sum_I a_I^{(2)}\right)^{-1} |T\rangle \right\rangle\right\rangle .$$

- The instanton-induced heavy-quark potential can be expressed in the leading order of N/VN_c:

$$V_I = \frac{4\pi\bar{\rho}^3 N}{VN_c}\mathcal{I}_{\rm NP}\left(r/\bar{\rho}\right) .$$

$$\mathcal{I}_{\rm NP}(x) = \int_0^\infty y^2 dy \int_{-1}^1 dt \left[1 - \cos\left(\frac{\pi y}{\sqrt{y^2+1}}\right)\cos\left(\pi\sqrt{\frac{y^2+x^2+2xyt}{y^2+x^2+2xyt+1}}\right) - \frac{y+xt}{\sqrt{y^2+x^2+2xyt}}\sin\left(\frac{\pi y}{\sqrt{y^2+1}}\right)\sin\left(\pi\sqrt{\frac{y^2+x^2+2xyt}{y^2+x^2+2xyt+1}}\right)\right]$$

Instanton-induced heavy-quark potential

$$\mathcal{I}_{\rm NP}(x) = \mathcal{I}_0^{\rm d} \left\{ 1 + \sum_{i=1}^{2} \left[a_i^d x^{2(i-1)} + a_3^d(-b_3^d x)^i \right] e^{-b_i^d x^2} + \frac{a_3^d}{x} \left(1 - e^{-b_3^d x^2} \right) \right\},$$

$$\mathcal{I}_0^d = 4.41625,$$

$$a^d = \begin{pmatrix} -1 \\ 0.128702 \\ -1.1047 \end{pmatrix},$$

$$b^d = \begin{pmatrix} 0.404875 \\ 0.453923 \\ 0.420733 \end{pmatrix},$$

$$V_I = \frac{4\pi\bar{\rho}^3 N}{V N_c} \mathcal{I}_{\rm NP}(\infty) \simeq 140\text{MeV}.$$

Spin-dependent parts of the $Q\bar{Q}$ potential

- The spin-dependent parts of the heavy-quark potential are represented by the Breit-Fermi equation [3, 4]:

$$V_{SD} = V_{SS}\mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}} + V_{LS}\mathbf{L}\cdot\mathbf{S} + V_T\left[3(\mathbf{S}_Q\cdot\hat{\mathbf{n}})(\mathbf{S}_{\bar{Q}}\cdot\hat{\mathbf{n}}) - \mathbf{S}_Q\cdot\mathbf{S}_{\bar{Q}}\right]$$

$$V_{SS}(r) = \frac{2}{3m_Q^2}\nabla^2 V_V = \frac{32\pi\alpha_s}{9m_Q^2}\delta(r)$$

$$= \frac{32\alpha_s\sigma^3}{9m_Q^2\sqrt{\pi}}e^{-\sigma^2 r^2}$$

$$V_{LS}(r) = \frac{1}{2m_Q^2 r}\left(3\frac{dV_V}{dr} - \frac{dV_S}{dr}\right)$$

$$V_T(r) = \frac{1}{3m_Q^2}\left(\frac{1}{r}\frac{dV_V}{dr} - \frac{d^2V_V}{dr^2}\right)$$

$$\mathbf{S} = \mathbf{S}_Q + \mathbf{S}_{\bar{Q}}$$

$$V_V = -\frac{4\alpha_s}{3r}$$

$$V_S = kr$$

- The spin-dependent parts of the instanton case are derived in Ref. [3]:

$$V^I_{SD} = V^I_{SS}\mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}} + V^I_{LS}\mathbf{L}\cdot\mathbf{S} + V^I_T\left[3(\mathbf{S}_Q\cdot\hat{\mathbf{n}})(\mathbf{S}_{\bar{Q}}\cdot\hat{\mathbf{n}}) - \mathbf{S}_Q\cdot\mathbf{S}_{\bar{Q}}\right]$$

$$V^I_{SS}(r) = \frac{1}{3m_Q^2}\nabla^2 V_I^{(\mathrm{NP})}$$

$$V^I_{LS}(r) = \frac{1}{2m_Q^2 r}\frac{dV_I^{(\mathrm{NP})}}{dr}$$

$$V^I_T(r) = \frac{1}{3m_Q^2}\left(\frac{d^2V_I^{(\mathrm{NP})}}{dr^2} - \frac{1}{r}\frac{dV_I^{(\mathrm{NP})}}{dr}\right)$$

[3] E. Eichten and F. Feinberg, Phys. Rev. D173090 (1981)
[4] M. B. Voloshin, Progress in Particle and Nucl Phys. 61 (2008) 455-511

Eigenvalues of the Hamiltonian

- To obtain the bound state energy of the quarkonia, we solved the non-relativistic Schrödinger equation

$$\left[-\frac{\hbar^2}{m_Q}\frac{d^2}{dr^2}+\frac{\hbar^2 l(l+1)}{m_Q r^2}+V(r)\right]\chi(r)=E_B\chi(r)$$

$$\chi(r)=r\psi(r)$$

$$m_{Q\bar{Q}}=2m_Q+E_B$$

$m_{Q\bar{Q}}$: mass of the quarkonia
E_B: Bound state energy (eigenvalue of the Hamiltonian)

$$V(r)=-\frac{4\alpha_s}{3r}+V_{\rm s}+V_{\rm SD}+V_I+V_{\rm SD}^I.$$

$$V_{\rm s}=\frac{k(1-e^{-br^2})}{b}$$

- We have 5 fitting parameters: $\alpha_s,\ \sigma,\ k,\ b,\ m_c$

Results:
Charmonium spectrum

Results: Fitting parameters

Parameter	Linear potential model	Screened potential model
m_c (GeV)	1.4830	1.4110
α_s	0.5461	0.5070
b (GeV^2)	0.1425	0.2100
σ (GeV)	1.1384	1.1600
r_c (fm)	0.202	0.180
μ (GeV)	⋯	0.0979

- Linear (LP) and screened (SP) potential models [4]

$$V_s = \begin{cases} kr & \text{: Linear Potential (LP)} \\ \frac{k}{\mu}(1-e^{-\mu r}) & \text{: Screened potential (SP)} \end{cases}$$

- SP model has the advantage that it gives the smaller running coupling constant and the heavy quark mass than LP.
- However, SP model is not easy to optimize in the instanton model because of the additional dynamical mass from the instanton vacuum.

- We made the modified screened potential to include the induced heavy-quark potential.

$$V_s = \frac{k(1-e^{-br^2})}{b}$$

Model	α_s(-)	k(GeV^3)	σ(GeV)	m_c(GeV)	b(GeV^2)
Model I	0.5336	0.0299	1.1704	1.5704	0.0202
Model II	0.4930	0.0269	1.1969	1.5433	0.0187

- Model I is not including the instanton effects

$$V(r) = -\frac{4\alpha_s}{3r} + V_s + V_{SD}.$$

- Model II is including the instanton effects

$$V(r) = -\frac{4\alpha_s}{3r} + V_s + V_{SD} + V_I + V_{SD}^I.$$

[4] Wei-Jun Deng et al, Phys. Rev. D 95. 034026 (2017)

Results: Charmonium spectrum

State	Exp	Model I	Model II	NR [28]	GI [28]	LP [29]	SP [29]
$J/\psi(1^3S_1)$	$3096.900 \pm 0.006^*$	3097	3097	3090	3098	3097	3097
$\eta_c\ (1^1S_0)$	$2983.9 \pm 0.4^*$	2984	2984	2982	2975	2984	2984
$\psi\ (2^3S_1)$	$3686.097 \pm 0.025^*$	3687	3687	3672	3676	3679	3679
$\eta_c\ (2^1S_0)$	$3637.6 \pm 1.1^*$	3637	3637	3630	3623	3635	3637
$\psi\ (3^3S_1)$	4039 ± 1	4115	4113	4072	4100	4078	4030
$\eta_c\ (3^1S_0)$		4084	4080	4043	4064	4048	4004
$\psi\ (4^3S_1)$	$4421 \pm 4^*$	4421	4421	4406	4450	4412	4281
$\eta_c\ (4^1S_0)$		4402	4401	4384	4425	4388	4264
$\chi_{c2}\ (1^3P_2)$	$3556.17 \pm 0.07^*$	3557	3556	3556	3550	3552	3553
$\chi_{c1}\ (1^3P_1)$	3510.67 ± 0.05	3509	3510	3505	3510	3516	3521
$\chi_{c0}\ (1^3P_0)$	3414.71 ± 0.30	3415	3415	3424	3445	3415	3415
$h_c\ (1^1P_1)$	$3525.38 \pm 0.11^*$	3525	3526	3516	3517	3522	3526
$\chi_{c2}\ (2^3P_2)$	3922.5 ± 1.0	4012	4008	3972	3979	3967	3937
$\chi_{c1}\ (2^3P_1)$	3871.65 ± 0.06	3968	3966	3925	3953	3937	3914
$\chi_{c0}\ (2^3P_0)$	3862^{+26+40}_{-32-13}	3897	3894	3852	3916	3869	3848
$h_c\ (2^1P_1)$		3980	3975	3934	3956	3940	3916
$\psi_3\ (1^3D_3)$	3842.7 ± 0.2	3824	3824	3806	3849	3811	3808
$\psi_2\ (1^3D_2)$	3823.7 ± 0.5	3818	3818	3800	3838	3807	3807
$\psi\ (1^3D_1)$	3773.7 ± 0.4	3798	3774	3785	3819	3787	3792
$\eta_{c2}\ (1^1D_2)$		3818	3818	3799	3837	3806	3805
$\psi_3\ (2^3D_3)$		4215	4212	4167	4217	4172	4112
$\psi_2\ (2^3D_2)$		4208	4205	4158	4208	4165	4109
$\psi\ (2^3D_1)$	4191 ± 5	4184	4035	4142	4194	4144	4095
$\eta_{c2}\ (2^1D_2)$		4208	4205	4158	4208	4164	4108

- NR and GI models are nonrelativistic model and Godfrey-Isgur (relativized quark) model.
- Nonrelativistic model: Model I(II), NR, LP, SP
- Number of input data

  This work: 7

  Ref. [28]: 11

  Ref. [29]: 12
- Our model requires relatively less data than another models.

[28] T. Barnes and S. Godfrey, Phys. Rev. D 69, 054008 (2004)
[29] Wei-Jun Deng et al, Phys. Rev. D 95. 034026 (2017)

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- The modified screened potential model well describes each ground state.

The ground state of each wave

Most accurate prediction

- Model II (instanton model) is the best prediction model.
- However, it still cannot describe the excited states of P and D wave because the modified screened potential cannot explain the confinement completely.

[28] T. Barnes and S. Godfrey, Phys. Rev. D 69, 054008 (2004)
[29] Wei-Jun Deng et al, Phys. Rev. D 95. 034026 (2017)

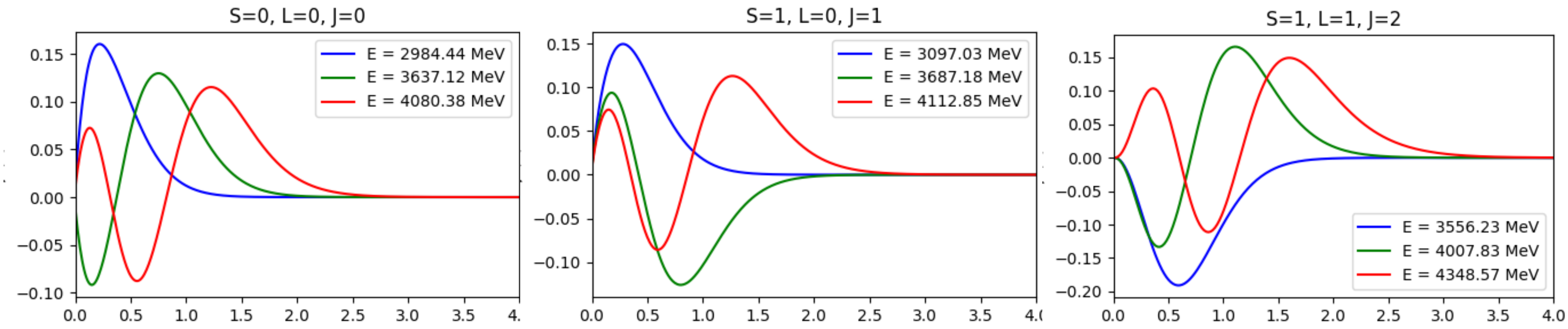
Results:
Decay width of the E1 and M1 radiative transitions

Decay width of the electromagnetic transitions

- To see the electromagnetic transitions, we need to know the initial and final state because we have to calculate the matrix element of the EM hamiltonian.

- From the previous results, we get also each wavefunction of state.
  - The wavefunctions ($\chi(r) = r\psi(r)$) are obtained by the eigenvalue problem.

Example:



Decay width of the electromagnetic transitions

- The interaction Hamiltonian of the quark-photon EM coupling at the tree level is given by

$$H_{\rm EM} = -e_c|e|\bar{\psi}\gamma^\mu A_\mu \psi.$$

$e_c = 2/3$: charge of the charm quark

- It gives the electric and magnetic dipole interaction:

$$H_{\rm E1} = e_c|e|\vec{r}\cdot\vec{E}, \quad H_{\rm M1} = -\frac{e_c|e|}{2m_Q}\vec{\sigma}\cdot\vec{B}$$

- The initial and final Fock states are represented by

$$|nL\rangle \to |n;L,m_l;S,m_s;J,m_J;0\rangle = |n;L,m_l;S,m_s;J,m_J\rangle \otimes |0\rangle$$
$$|n'L'\rangle \to |n';L',m_l';S',m_{s'};J,m_{J'};\gamma\rangle = |n';L',m_{l'};S',m_{s'};J',m_{J'}\rangle \otimes |\gamma\rangle$$

Decay width of the electromagnetic transitions

- Using these states, we get the E1 and M1 decay width[28]

$$\Gamma_{\mathrm{E1}}(n^{2S+1}L_J \to n'^{2S'+1}L'_{J'} + \gamma) = \frac{4}{3} C_{fi}\delta_{SS'}e_c^2\alpha|\langle\psi_f|r|\psi_i\rangle|^2 E_\gamma^3 \frac{E_f^{(c\bar{c})}}{M_i^{(c\bar{c})}}$$

$$C_{fi} = \max(L, L')(2J'+1)\left\{ \begin{array}{ccc} L' & J' & S \\ J & L & 1 \end{array} \right\}^2$$

$$\Gamma_{\mathrm{M1}}(n^{2S+1}L_J \to n'^{2S'+1}L'_{J'} + \gamma) = \frac{4}{3}\frac{2J'+1}{2L+1}\delta_{LL'}\delta_{S,S'\pm1}e_c^2\frac{\alpha}{m_c^2}|\langle\psi_f|\psi_i\rangle|^2 E_\gamma^3 \frac{E_f^{(c\bar{c})}}{M_i^{(c\bar{c})}}$$

- E_γ is a final photon energy, $E_f^{\bar{c}c}$ denotes a total energy of the final $c\bar{c}$ state, $M_i^{c\bar{c}}$ stands for a mass of the initial $c\bar{c}$ state and $\alpha = 4\pi|e|^2$ is known as the fine-structure constant.

[28] T. Barnes and S. Godfrey, Phys. Rev. D 69, 054008 (2004)

Results: E1 & M1 Radiative Transitions

2S→1P transitions (keV)

E1 transition								
Initial	Final	Model I	Model II	[28] NR	[28] GI	[29] LP	[29] SP	PDG Exp.
$\psi(2^3S_1)$	$\chi_{c2}(1^3P_2)$	48	40	38	24	36	44	28 ± 1
	$\chi_{c1}(1^3P_1)$	45	44	54	29	45	48	29 ± 1
	$\chi_{c0}(1^3P_0)$	29	29	63	26	27	26	29 ± 1
$\eta_c(2^1S_0)$	$h_c(1^1P_1)$	44	44	49	36	49	52	-

- The instanton effects tend to correct the mass spectrum from Model I.
- $\psi \to \chi_{c1}$ and $\psi \to \chi_{c2}$ are still big, which are a common problem of the nonrelativistic models.
- The confinement potentials of these models are not enough to reproduce the excited states.
- Nevertheless, the instanton effects offer the smaller values than the no instanton models.

1P → 1S transitions (keV)

E1 transition								
Initial	Final	Model I	Model II	[28] NR	[28] GI	[29] LP	[29] SP	PDG Exp.
$\chi_{c2}(1P)$	$J/\psi(1S)$	387	393	424	313	327	338	374 ± 19
$\chi_{c1}(1P)$		305	311	314	239	269	278	288 ± 16
$\chi_{c0}(1P)$		150	152	152	114	141	146	151 ± 12
$h_c(1P)$	$\eta_c(1S)$	442	449	498	352	361	373	350 ± 210

- Overall, the modified screened potential model gives excellent results in this transition compared to other models.
- However, $\chi_{c1} \to J/\psi$ (may be χ_{c2} also) littile big.
- We are guessing that is from the overestimated excited states(2P).

[28] T. Barnes and S. Godfrey, Phys. Rev. D 69, 054008 (2004)
[29] Wei-Jun Deng et al, Phys. Rev. D 95. 034026 (2017)

Results: E1 & M1 Radiative Transitions

1D → 1P transitions (keV)

E1 transition								
Initial	Final	Model I	Model II	[28] NR	[28] GI	[29] LP	[29] SP	PDG Exp.
$\psi_3(1^3D_3)$	$\chi_{c2}(1^3P_2)$	330	336	272	296	377	393	-
$\psi_2(1^3D_2)$	$\chi_{c2}(1^3P_2)$	78	79	64	66	79	82	-
	$\chi_{c1}(1^3P_1)$	311	314	307	268	281	291	-
$\psi(1^3D_1)$	$\chi_{c2}(1^3P_2)$	6.9	4.6	4.9	3.3	5.4	5.7	≤ 17
	$\chi_{c1}(1^3P_1)$	145	105	125	77	115	111	68 ± 7
	$\chi_{c0}(1^3P_0)$	311	250	403	213	243	232	188 ± 18

- One can see the problems of the nonrelativistic models.
- In the higher states, they cannot well describe the transitions.
- However, the instanton effects give better results than Model I.
- Especially, $\psi \rightarrow \chi_{c1}$ is remarkable for the best result of the nonrelativistic models.

(keV)

M1 transition								
Initial	Final	Model I	Model II	[28] NR	[28] GI	[29] LP	[29] SP	PDG Exp.
$J/\psi(1^3S_0)$	$\eta_c(1^1S_0)$	2.2	2.3	2.9	2.4	2.39	2.44	1.7 ± 0.4
$\psi(2^3S_1)$	$\eta_c(2^1S_0)$	0.2	0.2	0.21	0.17	0.19	0.19	0.2 ± 0.2
	$\eta_c(1^1S_0)$	4.8	5.0	4.6	9.6	8.08	7.80	1.0 ± 0.1
$\eta_c(2^1S_0)$	$J/\psi(1^3S_1)$	8.5	9.0	7.9	5.6	2.64	2.29	< 158

- The modified screened potential models predict the first two transitions very well.
- On the other hands, we cannot explain the last two transitions. (Homework!)

[28] T. Barnes and S. Godfrey, Phys. Rev. D 69, 054008 (2004)
[29] Wei-Jun Deng et al, Phys. Rev. D 95. 034026 (2017)

Summary

- We make the modified screened potential to include the instanton effects on SP model.
- We obtained the charmonium spectrum from the heavy-quark potential with the instanton effects.
- The instanton model (Model II) gives revised results than the others.
- We also evaluated the radiative transition width of E1 and M1 transitions.
- Although the instanton effect in the heavy-quark system is known to have little contribution, it gives remarkable results from E1 and M1 transitions.
- So, we can say that the instanton is meaningful in the heavy-quark system also.

Thank you for your attention

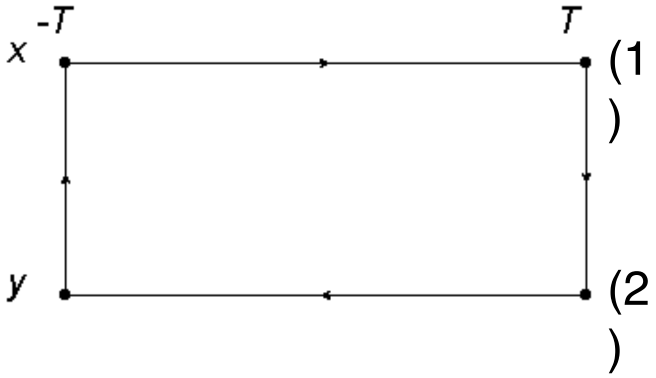
Back Up

Heavy-quark potential in the instanton vacuum (Including perturbative corrections) [2]

$\theta^{-1} = \frac{d}{dt}$

$D = \theta^{-1} - g\sum_I A_I$

- In Ref. [2], they considered the one-gluon exchange(OGE) perturbation part.
- They used the instanton packing parameter $\lambda = \frac{\rho^4}{R^4} \sim 0.01$ as the running coupling constant $\alpha_s \sim \lambda^{\frac{1}{2}}$.
- Averaged Wilson loop($Q\bar{Q}$ correlator) can be written as



$$W = \int D\xi \exp\left[\frac{1}{2}\sum_{i\neq j=1}^{2}\left(\frac{\delta}{\delta a_a^{(i)}} S_{ab}^{(ij)} \frac{\delta}{\delta a_b^{(j)}}\right)\right] \frac{1}{D^{(1)} - g a^{(1)}} \frac{1}{D^{(2)} - g\bar{a}^{(2)}},$$

order of $\alpha_s (\propto g^2)$ → $W^{-1} = \int D\xi \left(D^{(1)} D^{(2)} - g^2 \frac{\lambda_a}{2}\frac{\bar{\lambda}_b}{2} S_{ab}\right)$

- Using the Fourier transform of W^{-1}: $W^{-1}(\omega) = i\omega + f(\omega) + g(\omega)$

$$\langle t_1 | W | t_2 \rangle = \int \frac{d\omega}{2\pi} e^{i\omega(t_1 - t_2)} \frac{1}{W^{-1}(\omega)}$$

- Correlation function from the Fourier transformation :

$$\exp\left(-V_I^1 T\right) = \exp\left[-(V_I^{1,(\mathrm{NP})} + V_I^{1,(\mathrm{P})})T\right] = \exp[-(f(0) + g(0))T]$$

[1] Diakonov et al, Phys. Lett. B 226, 372 (1989)
[2] M.Musakhanov et al, PhysRevD.102.076022

Heavy-quark potential in the instanton vacuum (Including perturbative corrections) [2]

$$V_I^{1,(\mathrm{NP})} = \frac{N}{2VN_c}\sum_{\pm}\int d_3 z_{\pm}\,\mathrm{Tr}_c\left[1 - P\exp\left(i\int_{-\infty}^{\infty} dx_4 A_{\pm,4}\right)P\exp\left(-i\int_{-\infty}^{\infty} dy_4 A_{\pm,4}\right)\right]$$

$$V_I^{1,(\mathrm{P})} = g^2\frac{\lambda_a}{2}\frac{\bar{\lambda}_a}{2}\int\frac{d^3q}{(2\pi)^3}\frac{e^{i\vec{q}\cdot\vec{r}}}{q^2 + M_g^2(q)}$$

$$M_g(q) = \frac{2\pi}{\rho}\left(\frac{6\lambda}{N_c^2 - 1}\right)^{1/2} q\rho K_1(q\rho)$$: Momentum dependent gluon mass

K_1 : Modified Bessel function of the second type

$\rho = 1/3$ fm

$R = 1$ fm

$$\left(\frac{\lambda_a}{2}\frac{\bar{\lambda}_a}{2}\right)_S = -\frac{N_c^2 - 1}{2N_c}I,$$

$$\left(\frac{\lambda_a}{2}\frac{\bar{\lambda}_a}{2}\right)_A = \frac{1}{2N_c}I.$$

In the color-singlet state

$$= -\frac{4\alpha_s}{3r}\left(1 - \frac{2r}{\pi}\int_0^{\infty} dq\, j_0(qr)\frac{3\pi^2\lambda K_1^2(q\rho)}{1 + 3\pi^2\lambda K_1^2(q\rho)}\right)$$

$$= V_C^{1,(\mathrm{P})}(r) + V_I^{1,(\mathrm{SCR})}(r),$$

$$V_C^{1,(\mathrm{P})}(r) \equiv -\frac{4\alpha_s}{3r},$$

$$V_I^{1,(\mathrm{SCR})}(r) \equiv \frac{8\alpha_s}{3\pi}\int_0^{\infty} dq\, j_0(qr)\frac{3\pi^2\lambda K_1^2(q\rho)}{1 + 3\pi^2\lambda K_1^2(q\rho)}$$

Outlooks

- For the case of the hadronic transition amplitude [5] of the two-pion transition between n^3S_1 states:

$$A(\psi' \to \pi^+\pi^- J/\psi) = \frac{1}{2}\langle \pi^+\pi^- | E_i^a E_j^a | 0\rangle \alpha_{ij}^{(12)}$$

- The $\psi' \to J/\psi$ transition in the chromo-electric field is described by the effective Hamiltonian

$$H_{\text{eff}} = -\frac{1}{2}\alpha_{ij}^{(12)} E_i^a E_j^a,$$

with the chromo-polarizability given by

$$\alpha^{(12)} = \frac{1}{48}\langle 1S|\xi^a r_i G r_i \xi^a|2S\rangle = \frac{1}{9}\langle 1S| r_i \frac{1}{H_o - E_{2S}} r_i |2S\rangle, \qquad \xi^a = \frac{\lambda^a}{2} - \frac{\bar{\lambda}^a}{2}$$

where G is the Green's function of the heavy quark pair in the color octet state.

[5] M. B. Voloshin, Progress in Particle and Nucl Phys. 61 (2008) 455-511

Results: Charmonium spectrum

SetA	ρ_I (fm)	R (fm)	α_s(-)	k(GeV^2)	σ(GeV)	m_Q(GeV)	V_0(GeV)
SetIa	1/3	1	0.5141	0.1432	1.136	1.3634	0
SetIIa	0.36	0.89	0.4783	0.1375	1.174	1.3251	0
SetB	**ρ_I (fm)**	**R (fm)**	**α_s(-)**	**k(GeV^2)**	**σ(GeV)**	**m_Q(GeV)**	**V_0(GeV)**
SetIb	1/3	1	0.5098	0.1444	1.166	1.3932	-0.0563
SetIIb	0.36	0.89	0.4773	0.1375	1.174	1.3211	0.0062

State	Exp	SetIa	SetIIa	SetIb	SetIIb
χ_{c2} (3^3P_2)		4318	4313	4318	4313
χ_{c1} (3^3P_1)		4286	4287	4287	4287
χ_{c0} (3^3P_0)		4236	4235	4235	4235
h_c (3^1P_1)		4290	4287	4290	4287
ψ_3 (1^3D_3)		3810	3806	3810	3806
ψ_2 (1^3D_2)	3822.2 ± 1.2	3808	3806	3809	3806
ψ (1^3D_1)	3778.1 ± 1.2	3787	3791	3789	3791
η_{c2} (1^1D_2)		3806	3804	3807	3804
ψ_3 (2^3D_3)		4172	4169	4172	4168
ψ_2 (2^3D_2)		4168	4166	4168	4165
ψ (2^3D_1)	4191 ± 5	4140	4148	4143	4148
η_{c2} (2^1D_2)		4167	4164	4167	4164

Parameter	Linear potential model	Screened potential model
m_c (GeV)	1.4830	1.4110
α_s	0.5461	0.5070
b (GeV^2)	0.1425	0.2100
σ (GeV)	1.1384	1.1600
r_c (fm)	0.202	0.180
μ (GeV)	...	0.0979

Fitting parameters

[4]

$n^{2S+1}L_J$	name	J^{PC}	Exp. [6]	LP	SP
3^3P_2	$\chi_{c2}(3P)$	2^{++}		4310	4211
3^3P_1	$\chi_{c1}(3P)$	1^{++}		4284	4192
3^3P_0	$\chi_{c0}(3P)$	0^{++}		4230	4146
3^1P_1	$h_c(3P)$	1^{+-}		4286	4193
1^3D_3	$\psi_3(1D)$	3^{--}		3811	3808
1^3D_2	$\psi_2(1D)$	2^{--}	3823	3807	3807
1^3D_1	$\psi_1(1D)$	1^{--}	3778	3787	3792
1^1D_2	$\eta_{c2}(1D)$	2^{-+}		3806	3805
2^3D_3	$\psi_3(2D)$	3^{--}		4172	4112
2^3D_2	$\psi_2(2D)$	2^{--}		4165	4109
2^3D_1	$\psi_1(2D)$	1^{--}	4191?	4144	4095
2^1D_2	$\eta_{c2}(2D)$	2^{-+}		4164	4108

$$V_s = \begin{cases} kr & \text{: LP} \\ \frac{k}{\mu}(1 - e^{-\mu r}) & \text{: SP} \end{cases}$$

[4] Wei-Jun Deng et al, Phys. Rev. D 95. 034026 (2017)

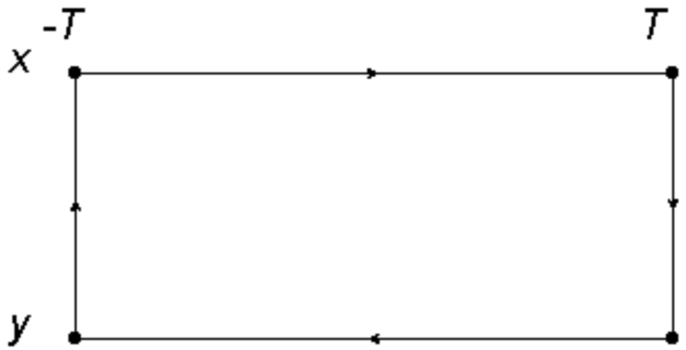
Potential from Wilson Loop

- The $Q\bar{Q}$ state evaluates in time T and can be represented as

$$|\Phi(\vec{x},T;\vec{y},T)\rangle = \bar{Q}(\vec{x})U(x,y)Q(\vec{y})|0\rangle$$

- $U(x,y) = P\exp\left(ig\int_x^y \frac{\lambda_a}{2}A_\mu^a(z)dz_\mu\right)$: Wilson line

- We assume that the heavy quark and antiquark masses $m_{Q,\bar{Q}} \to \infty$ and they are in static state during $T \to \infty$.



$$T \gg r\ (= x - y)$$

- Heavy quark potential from the correlation function:

$$W(C) = U\left(\vec{x}, -T; \vec{x}, T\right)U(\vec{y}, T; \vec{x}, -T)$$

$$\langle\Phi(\vec{y},-T;\vec{x},-T)|\Phi(\vec{x},T;\vec{y},T)\rangle = \langle e^{-HT}\rangle \sim e^{-VT} = \langle P\exp\left(\oint A_4(z)dz_4\right)\rangle \longrightarrow V = -\lim_{T\to\infty}\frac{1}{T}\ln\langle W(C)\rangle$$

$$V_I^{1,(\mathrm{NP})}(\infty) = 2\Delta M = 0.664789\ \mathrm{fm}^{-1} = 0.131\ \mathrm{GeV} = C$$

Results

$$V^1 = V_C^{1,(\mathrm{P})} + \sigma r + C, \qquad V^{T^a} = V_C^{T^a,(\mathrm{P})} + \sigma r + C$$

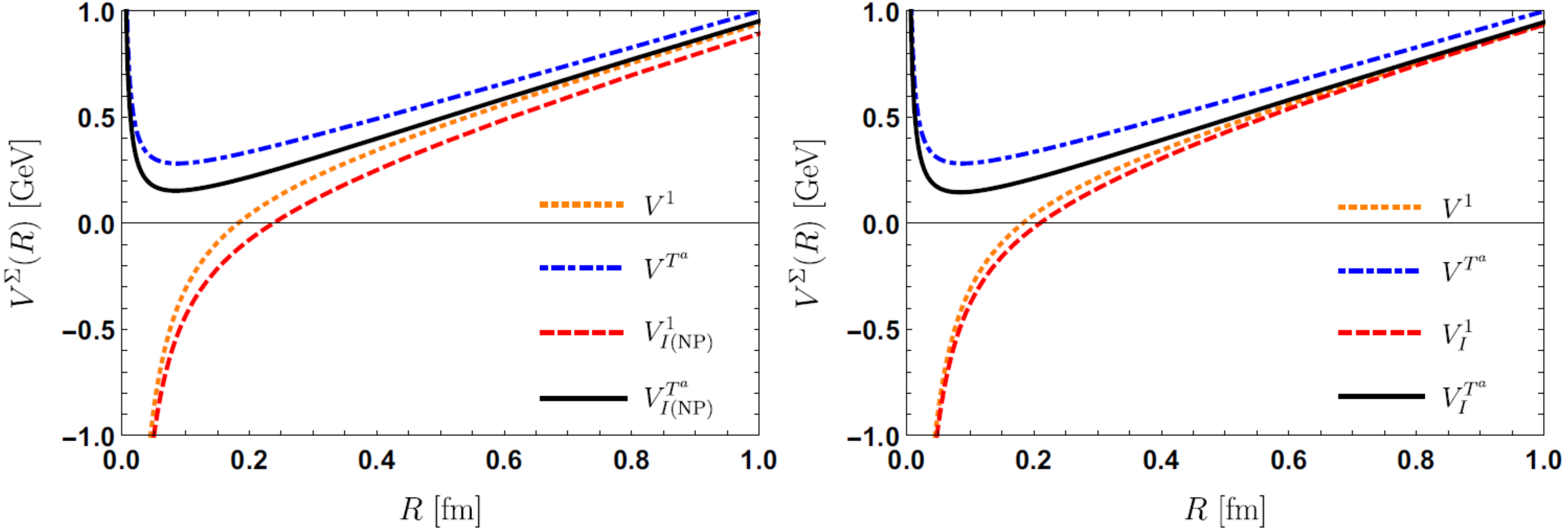
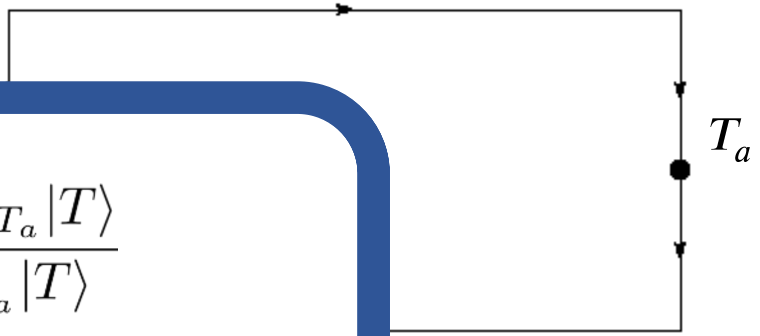


FIG. 2. The left panel is the total color siglet and octet potential without considering the instanton effect of the one gluon exchange. Otherwise, in the right panel, we considered all of instanton effects. Orange dotted lines are color singlet potential and the blue dot-dashed lines are color-octet potential without the instanton effects. The red lines are $V_{1,\mathrm{Ins}}$ and $V_{1,\mathrm{Ins,ge}}$, respectively. The black solid lines represent $V_{T^a,\mathrm{Ins}}$ and $V_{T^a,\mathrm{Ins,ge}}$. Here we set the parameters $\alpha_s = 0.2$, $\sigma = 0.17\ \mathrm{GeV}^2$, $C = 0.131183\ \mathrm{GeV}$, $\rho = 0.33\ \mathrm{fm}$ and $R = 1\ \mathrm{fm}$.

Color-octet heavy-quark potential (NP correction)

- Color-octet Wilson loop W_{T^a} can be represented by inserting the color exchange operator T_a



$$W_{T_a} = \int D\xi \exp\left[\frac{1}{2} \cdots\right.$$

$$V_I^{T_a} = -\lim_{T\to\infty}\frac{1}{T}\ln\langle T|W_{T_a}|T\rangle = -\lim_{T\to\infty}\frac{\frac{d}{dt}\langle T|W_{T_a}|T\rangle}{\langle T|W_{T_a}|T\rangle}$$

$$= \lim_{T\to\infty}\frac{2N_c^2\Delta M e^{-(2\Delta M - V_I^1)T} - V_I^1}{N_c^2 e^{-(2\Delta M - V_I^1)T} - 1} = V_I^1$$

$$W_{T_a} = \frac{1}{2(N_c^2-1)}\langle \mathrm{Tr} \cdots$$

$$= \frac{N_c^2}{2(N_c^2-1)} w^{(1)} w^{(2)} - \frac{1}{2(N_c^2-1)} W_1,$$

$\langle\langle \mathcal{W}(\vec{x},\Delta t)\rangle\rangle$

$\langle\langle \mathcal{W}^\dagger(\vec{y},\Delta t)\rangle\rangle$

$\langle\langle \mathcal{W}\rangle\langle \mathcal{W}^\dagger\rangle\rangle = \langle\langle \mathcal{W}\rangle\rangle\langle\langle \mathcal{W}^\dagger\rangle\rangle$:cluster decomposition for one-instanton calculation

$$\langle T|W_{T_a}|T\rangle = \frac{N_c^2}{2(N_c^2-1)}\langle T|w^{(1)}|-T\rangle\langle -T|w^{(2)}|T\rangle - \frac{1}{2(N_c^2-1)}\langle T|W_1|T\rangle$$

$$= \frac{N_c^2}{2(N_c^2-1)} e^{-2\Delta M T} - \frac{1}{2(N_c^2-1)} e^{-V_I^{1,(\mathrm{NP})}T}.$$

Eigenvalues of the Color-octet Hamiltonian

State	Exp	SetIa	SetIIa	SetIb	SetIIb
1^3S_1		3328	3330	3380	3328
1^1S_0		3332	3334	3384	3332
2^3S_1		3796	3797	3826	3796
2^1S_0		3800	3800	3829	3800
3^3S_1		4165	4165	4186	4165
3^1S_0		4168	4168	4189	4168
4^3S_1		4485	4485	4503	4485
4^1S_0		4488	4488	4506	4488
1^3P_2		3589	3590	3623	3589
1^3P_1		3617	3618	3651	3617
1^3P_0		3636	3637	3668	3636
1^1P_1		3604	3605	3638	3604
2^3P_2		3988	3989	4011	3988
2^3P_1		4012	4012	4034	4012
2^3P_0		4027	4028	4049	4027
2^1P_1		4001	4002	4024	4001
3^3P_2		4326	4327	4345	4326
3^3P_1		4347	4348	4365	4347
3^3P_0		4361	4362	4379	4361
3^1P_1		4339	4339	4357	4339

Bound state energy (E_B)

Unobservable quantities.
Physical implications are yet unknown.

Why we use the instanton?

- In the pQCD, the running coupling constant α_s at the one loop level is given by the expression [3-6]:

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0} \frac{1}{\ln(\mu^2/\Lambda^2_{QCD})}$$

$$\beta_0 = (11N_c - 2N_f)/3, \qquad \Lambda_{QCD} = 0.217 \text{ GeV}$$

Δm_I (GeV)	μ (GeV)	α_s (GeV)
0	1.275	0.4258
0.067	1.343	0.4137
0.1357	1.411	0.4029

$$\mu = m_c + \Delta m_I$$

Δm_I: Dynamical mass (Instanton mass) [1]
m_c: charm-quark mass=1275 MeV

The model	ρ (fm)	R (fm)	Δm_I (GeV)	α_s (GeV)
MWOI	Not applicable	Not applicable	Not applicable	0.2068
M-I	0.33	1.00	0.0676	0.3447
M-IIb	0.36	0.89	0.1357	0.4588

Table 1 [8]. The result using Cornell potential gives a 51% difference from one of pQCD. On the other hand, the instanton effects as in M-I and M-Iib give the difference 17% and 14%, respectively.

[3] M. Peter, Phys. Rev. Lett. 78, 602 (1997).
[4]M. Peter, Nucl. Phys. B501, 471 (1997).
[5] Y. Schroder, Phys. Lett. B 447, 321 (1999).
[6] A. V. Smirnov, V. A. Smirnov, and M. Steinhauser, Phys. Rev. Lett. 104, 112002 (2010).
[7] C. Anzai, Y. Kiyo, and Y. Sumino, Phys. Rev. Lett. 104, 112003 (2010).
[8] Yakhshiev et al, PhysRevD.98.114036

Color-singlet & octet $Q\bar{Q}$ potential

A. $\rho = 0.33$ **fm,** $R = 1$ **fm**

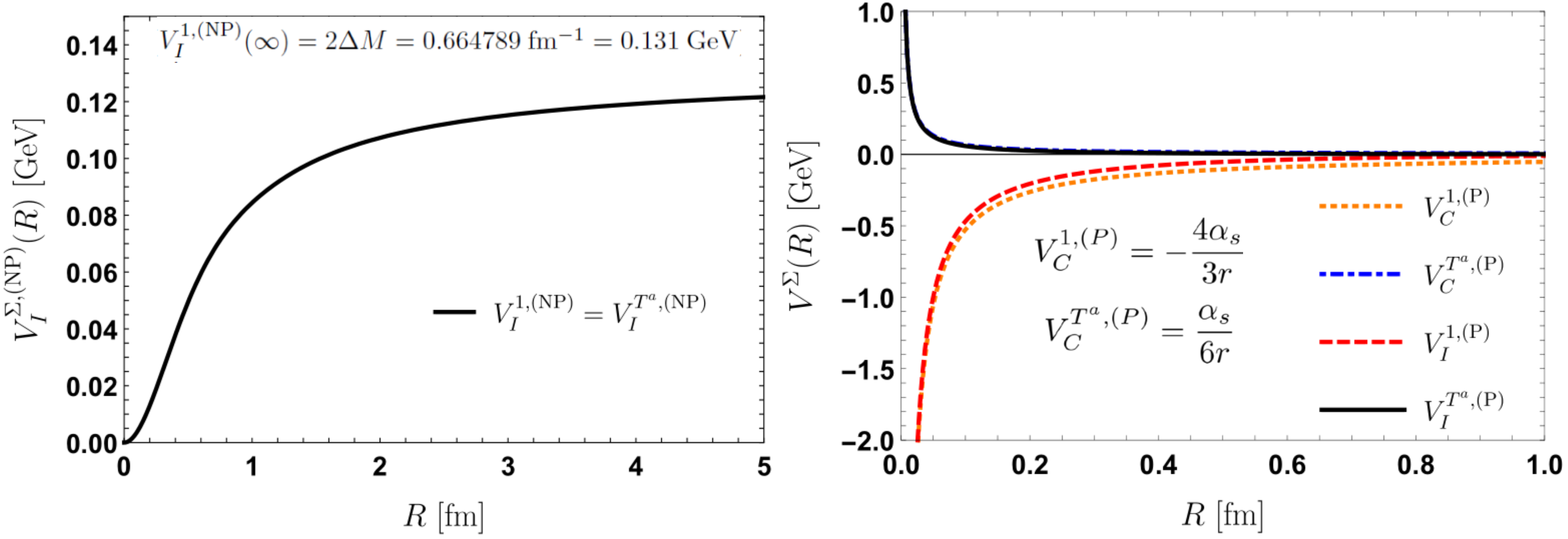


FIG. 1. Left panel(Non-perturbative potential) : The Non-perturbative instanton potential is not affected by color-state, which is black solid line. Right panel(Perturbative potential) : $V_C^{1,(P)} = -4\alpha_s/3r$ and $V_C^{T^a,(P)} = \alpha_s/6r$ are the perturbative color singlet and octet potential without instanton effect, respectively. $V_I^{1,(P)}$ and $V_I^{T^a,(P)}$ are including the instanton effects. We set $\alpha_s = 0.2$, $\rho = 0.33$ fm and $R = 1$ fm.

Results: Charmonium spectrum

Method 1

State	Exp	Instanton effects Off	Instanton effects On	Method 2
$J/\psi(1^3S_1)$	3096.900 ± 0.006	3099	3098	3121
$\eta_c\ (1^1S_0)$	2983.9 ± 0.5	2984	2984	2996
$\psi\ (2^3S_1)$	3686.097 ± 0.025	3682	3684	3682
$\eta_c\ (2^1S_0)$	3637.6 ± 1.2	3637	3639	3619
$\psi\ (3^3S_1)$	4039 ± 1	4085	4085	4084
$\eta_c\ (3^1S_0)$		4054	4053	4034
$\psi\ (4^3S_1)$	4421 ± 4	4422	4421	4424
$\eta_c\ (4^1S_0)$		4397	4396	4382
$\chi_{c2}\ (1^3P_2)$	3556.17 ± 0.07	3557	3552	3526
$\chi_{c1}\ (1^3P_1)$	3510.67 ± 0.05	3510	3510	3500
$\chi_{c0}\ (1^3P_0)$	3414.71 ± 0.30	3415	3415	3415
$h_c\ (1^1P_1)$	3525.38 ± 0.17	3520	3520	3508
$\chi_{c2}\ (2^3P_2)$	3927.2 ± 2.6	3976	3971	3950
$\chi_{c1}\ (2^3P_1)$		3933	3934	3925
$\chi_{c0}\ (2^3P_0)$	3862^{+26+40}_{-32-13}	3874	3874	3859
$h_c\ (2^1P_1)$		3942	3942	3931
$\chi_{c2}\ (3^3P_2)$		4323	4318	4303
$\chi_{c1}\ (3^3P_1)$		4281	4283	4278
$\chi_{c0}\ (3^3P_0)$		4236	4236	4222
$h_c\ (3^1P_1)$		4290	4290	4284

- For the case of the instanton effects off: $V_I = 0,\ V_{SD}^I = 0$
- We used the instanton parameters $\rho = 1/3$ fm and $R = 1$ fm

Old version : Method 1

Instanton	m_Q	α_s	k	σ
Off	1.4796	0.5426	0.1444	1.1510
On	1.36530	0.51770	0.14280	1.12900

New version : Method 2

Instanton	m_Q	α_s	k	σ
On	1.3353	0.41495	0.14679	1.79984

fitting parameters

The running coupling constant at the one-loop level:

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0}\frac{1}{\log(\mu^2/\Lambda_{\rm QCD}^2)}$$

$$\beta_0 = \frac{11N_c - 2N_f}{3}, \quad \Lambda_{\rm QCD} = 0.217\ \text{GeV}$$

$$\mu = m_Q = m_Q^0 + \Delta m_I^{\rm pert} = 1.275 + 0.1454\alpha_s\ [\text{GeV}]$$

Summary & Outlook

- We showed the non-perturbative color-octet heavy quark potential in the instanton vacuum:

$$V_I^{T^a,(\mathrm{NP})}(r) = V_I^{1,(\mathrm{NP})}(r)$$

- The perturbative one gluon exchange instanton effects make the color-singlet(octet) heavy quark potential a little weaker → Instanton makes the screening effects.

- We obtained the charmonium spectrum and the bound energies in the color-octet states.

- Using the color-octet potential derived in the present work, we are going to calculate chromo-polarizabilities of quarkonia.

- From this chromo-polarizability, we will show hadronic transition between charmonium resonances.