The mass-radius relations of neutron stars in an pion mean-field approach

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In collaboration with

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Outline

- Motivation
- Pion-mean field approach
- Nuclear matter
- Neutron star
- Results
- Summary

Motivation

- In the large N_c limit, the nucleon can be viewed as a state of N_c valence quarks bound by the meson field[1].

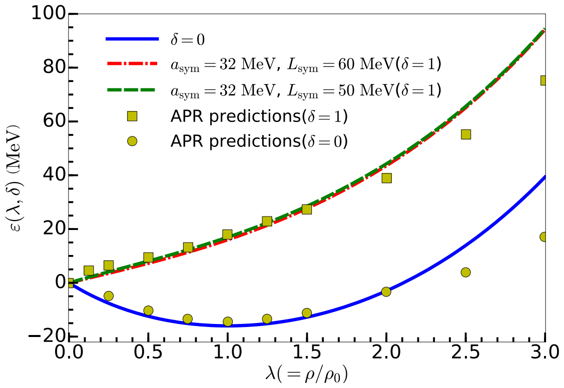
Mass [Mev]	Exp. [input]	Numerical results[2]
M_p	938.272 ± 0.00008	938.76 ± 3.65
M_n	939.565 ± 0.00008	940.27 ± 3.64
M_Λ	1115.683 ± 0.006	1109.61 ± 0.70
M_{Σ^+}	1189.37 ± 0.007	1188.75 ± 0.70
M_{Σ^0}	1192.642 ± 0.024	1190.20 ± 0.77
M_{Σ^-}	1197.449 ± 0.030	1195.48 ± 0.71
M_{Ξ^0}	1314.83 ± 0.20	1319.30 ± 3.43
M_{Ξ^-}	1321.31 ± 0.13	1324.52 ± 3.44

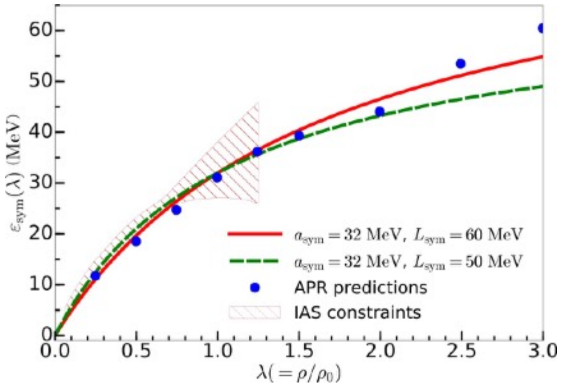
Mass [Mev]	Exp.	Numerical results[2]
$M_{\Delta^{++}}$	$1231 - 1233$	1248.54 ± 3.39
M_{Δ^+}	$1231 - 1233$	1249.36 ± 3.37
M_{Δ^0}	$1231 - 1233$	1251.53 ± 3.38
M_{Δ^-}	$1231 - 1233$	1255.08 ± 3.37
$M_{\Sigma^{*+}}$	1382.8 ± 0.4	1388.48 ± 0.34
$M_{\Sigma^{*0}}$	1383.7 ± 1.0	1390.66 ± 0.37
$M_{\Sigma^{*-}}$	1387.2 ± 0.5	1394.20 ± 0.34
$M_{\Xi^{*0}}$	1531.80 ± 0.32	1529.78 ± 3.38
$M_{\Xi^{*-}}$	1535.0 ± 0.6	1533.33 ± 3.37
M_{Ω^-}	1672.45 ± 0.29	Input

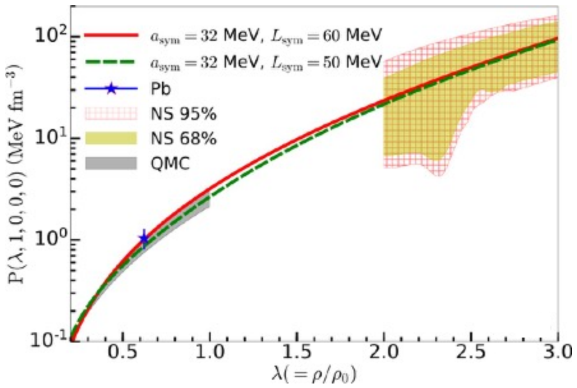
[1] E. Witten Nucl. Phys B 160, 57 (1979).
[2] G. S. Yang and H. Ch. Kim, Prog. Theor. Phys. **128**, 397 (2012).

Motivation

- In the previous work[3], We successfully describe the in-medium properties of nucleons based on linear-response approximation.







[3] N. Y. Ghim, G. S. Yang, H. Ch. Kim, U. Yakhshiev, Phys. Rev. C. **103**, 064306 (2021).

Pion mean-field approach

- Collective Hamiltonian[2][4]

$$H = M_{\mathrm{cl}} + \frac{1}{2I_1}\sum_{i=1}^{3}\hat{J}_i^2 + \frac{1}{2I_2}\sum_{p=4}^{7}\hat{J}_p^2$$

$$+ (m_{\mathrm{d}} - m_{\mathrm{u}})\left(\frac{\sqrt{3}}{2}\alpha D_{38}^{(8)}(\mathcal{A}) + \beta\hat{T}_3 + \frac{1}{2}\gamma\sum_{i=1}^{3}D_{3i}^{(8)}(\mathcal{A})\hat{J}_i\right)$$

$$+ (m_{\mathrm{s}} - \bar{m})\left(\alpha D_{88}^{(8)}(\mathcal{A}) + \beta\hat{Y} + \frac{1}{\sqrt{3}}\gamma\sum_{i=1}^{3}D_{8i}^{(8)}(\mathcal{A})\hat{J}_i\right) + H_{\mathrm{em}}$$

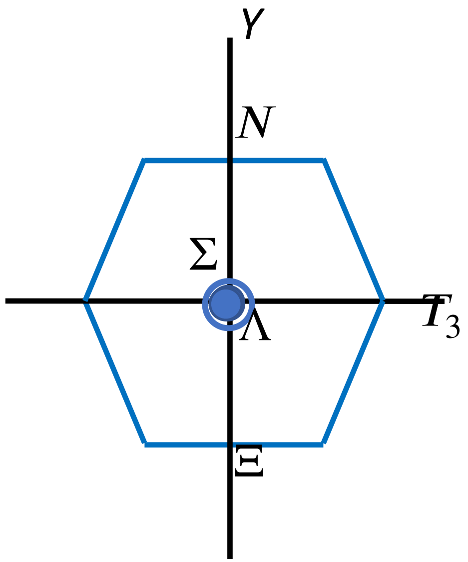
$$\alpha = -\left(\frac{2}{3}\frac{\Sigma_{\pi N}}{m_{\mathrm{u}} + m_{\mathrm{d}}} - Y'\frac{K_2}{I_2}\right) \qquad \beta = -\frac{K_2}{I_2} \qquad \gamma = 2\left(\frac{K_1}{I_1} - \frac{K_2}{I_2}\right)$$

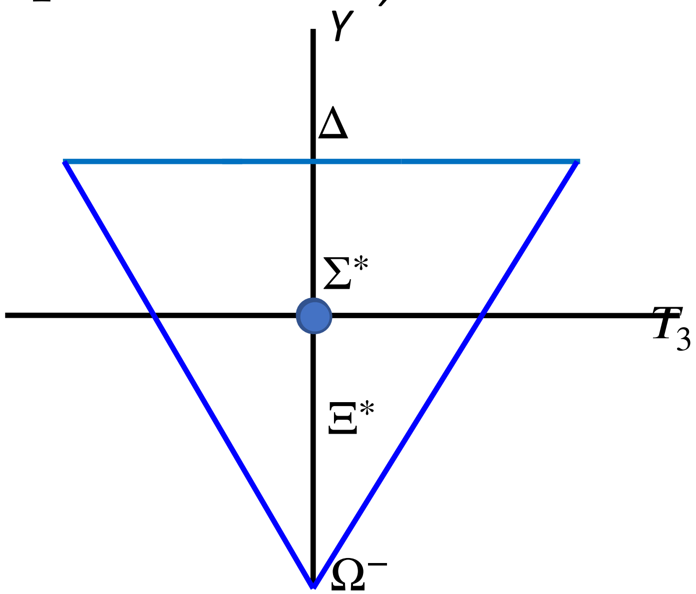
[2] G. S. Yang and H. Ch. Kim, Prog. Theor. Phys. 128, 397 (2012).
[4] G. S. Yang, H. Ch. Kim, Phys. Lett B. **808**, 13519(2020)

Pion mean-field approach

- SU(3) symmetry breaking term.

$$(m_{\rm s}-\bar{m})\left(\alpha D^{(8)}_{88}(\mathcal{A})+\beta\hat{Y}+\frac{1}{\sqrt{3}}\gamma\sum_{i=1}^{3}D^{(8)}_{8i}(\mathcal{A})\hat{J}_i\right)$$

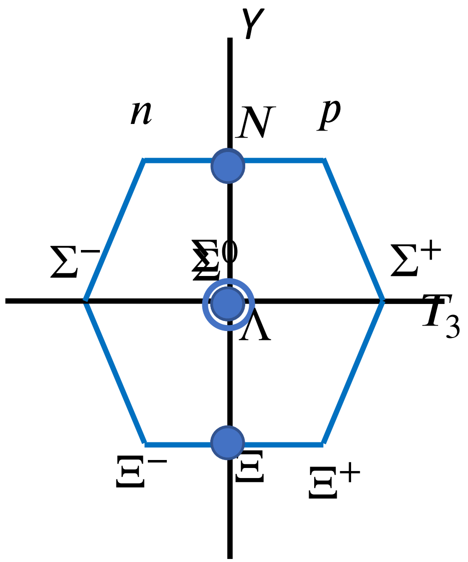


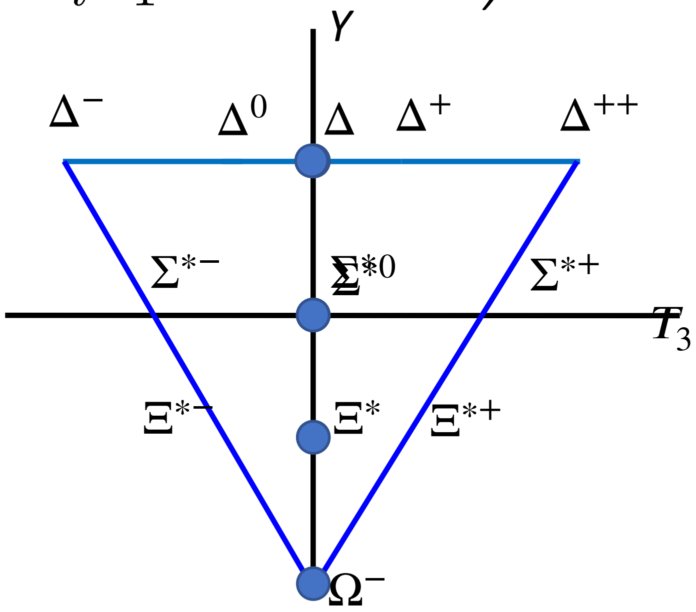


Pion mean-field approach

- Isospin symmetry breaking term.

$$(m_{\mathrm{d}} - m_{\mathrm{u}}) \left(\frac{\sqrt{3}}{2} \alpha D_{38}^{(8)}(\mathcal{A}) + \beta \hat{T}_3 + \frac{1}{2} \gamma \sum_{i=1}^{3} D_{3i}^{(8)}(\mathcal{A}) \hat{J}_i \right) + H_{\mathrm{em}}$$





Nuclear matter

- Binding energy per baryon[3]

$$\varepsilon = \frac{E^* - E}{A} = \frac{Z\Delta M_p + N\Delta M_n + \sum_{s=1}^{3} N_s \Delta M_s}{A}$$

$$= \Delta M_N \left(1 - \sum_{s=1}^{3} \delta_s\right) + \frac{1}{2}\delta \Delta M_{np} + \sum_{s=1}^{3} \delta_s \Delta M_s$$

$$M_{np} = M_n - M_p$$

$$\Delta M_N = M_N^* - M_N$$

$$\delta = \frac{N - Z}{A}$$

$$\delta_s = \frac{N_s}{A}$$

[3] N. Y. Ghim, G. S. Yang, H. Ch. Kim, U. Yakhshiev, Phys. Rev. C. **103**, 064306 (2021).

Nuclear matter

- The properties of symmetric nuclear matter

Volume energy: $$\varepsilon(\lambda, 0, 0, 0, 0) = \varepsilon_V(\lambda) = \Delta M_N(\lambda)$$

Pressure: $$P(\lambda) = \rho_0 \lambda^2 \frac{\partial \varepsilon_V(\lambda)}{\partial \lambda}$$

Compressibility: $$K(\lambda) = 9\lambda^2 \frac{\partial^2 \varepsilon_V(\lambda)}{\partial \lambda^2}$$

Nuclear matter

- Medium functions

$$M_{\mathrm{cl}}^{*} = M_{\mathrm{cl}}\left(1 + C_{\mathrm{cl}}\lambda\right)$$

$$I_{1}^{*} = I_{1}\left(1 + C_{1}\lambda\right)$$

$$I_{2}^{*} = I_{2}\left(1 + C_{2}\lambda\right)$$

Input data

$$\varepsilon_{V}(1) = -16\ \mathrm{MeV}$$

$$P(1) = 0\ \mathrm{MeVfm}^{-3}$$

$$K(1) = 240\ \mathrm{MeV}$$

Nuclear matter

- The properties of asymmetric nuclear matter

The nuclear symmetry energy: $\varepsilon_{\rm sym}(\lambda) = \frac{1}{2!}\frac{\partial^2 \varepsilon(\lambda, \delta, 0, 0, 0)}{\partial \delta^2}\Big|_{\delta=0}$

The slope parameter: $L_{sym} = 3\frac{\partial \varepsilon_{\rm sym}(\lambda)}{\partial \lambda}\Big|_{\lambda=1}$

Nuclear matter

- Medium functions

$$\frac{K^{I*}_{1,2}}{I^*_{1,2}} = \frac{K^{I}_{1,2}}{I_{1,2}} \left(1 + \frac{C_{\text{num}}\lambda\delta}{1 + C_{\text{den}}\lambda}\right)$$

Input data

$$\varepsilon_{\text{sym}}(1) = 32\ \text{MeV}$$
$$L_{sym} = 60\ \text{MeV}$$

Nuclear matter

- Medium functions

$$M_{\mathrm{cl}}^{*} = M_{\mathrm{cl}}\left(1 + C_{\mathrm{cl}}\lambda\right)$$

$$I_{1}^{*} = I_{1}\left(1 + C_{1}\lambda\right)$$

$$I_{2}^{*} = I_{2}\left(1 + C_{2}\lambda\right)$$

$$\frac{K_{1,2}^{I*}}{I_{1,2}^{*}} = \frac{K_{1,2}^{I}}{I_{1,2}}\left(1 + \frac{C_{\mathrm{num}}\lambda\delta}{1 + C_{\mathrm{den}}\lambda}\right)$$

- Density-dependent parameters

$$C_{\mathrm{cl}} = -0.056$$

$$C_{1} = 0.6434$$

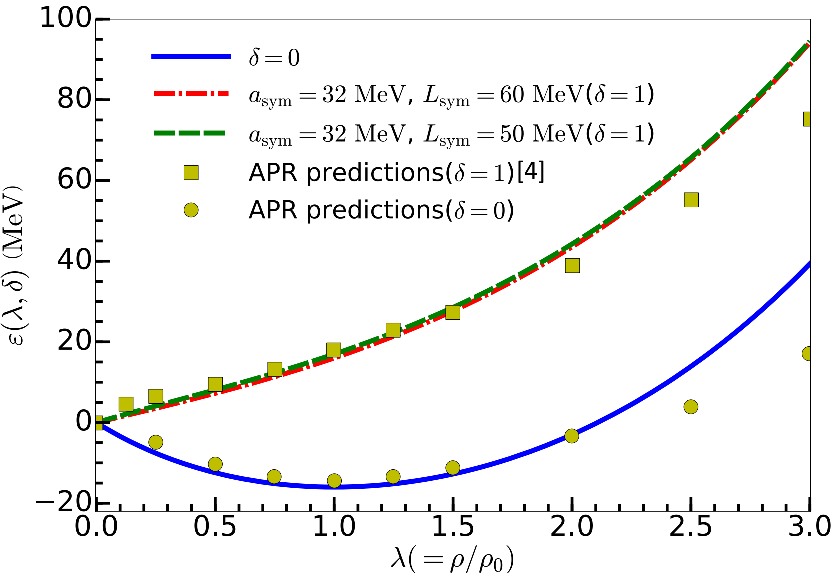
$$C_{2} = -0.1218$$

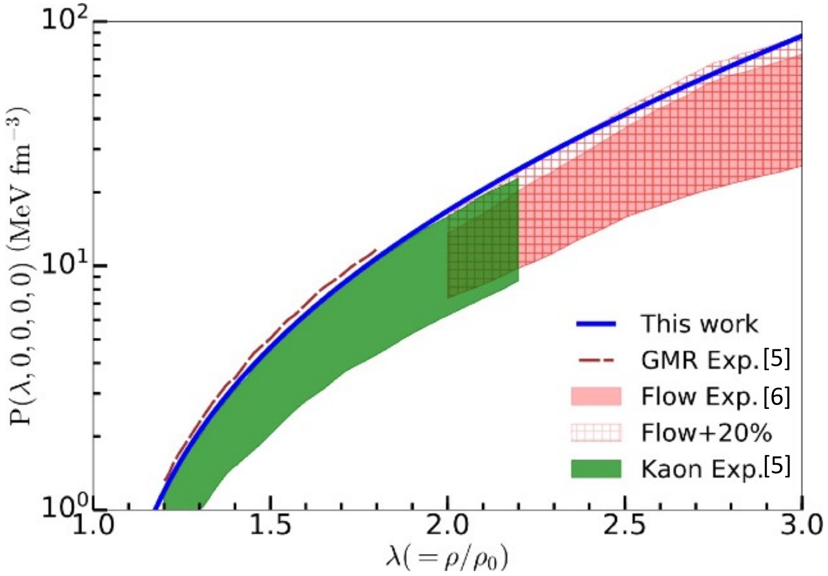
$$C_{\mathrm{num}} = 65.60$$

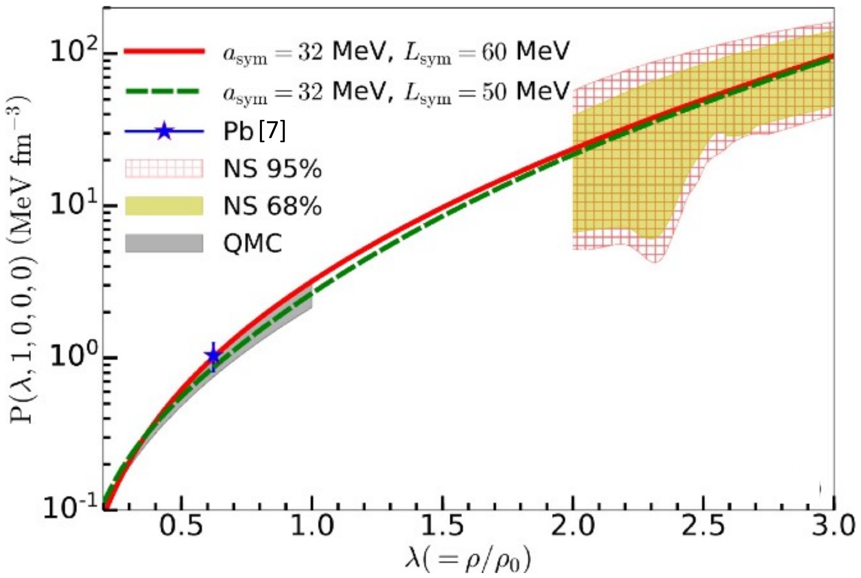
$$C_{\mathrm{den}} = 0.60$$

Nuclear matter E.O.S

- Equations of state







[4] A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys.Rev.C 58, 1804 (1998)
[5] W. G. Lynch, M. B. Tsang, Y. Zhang, P. Danielewicz, M. Famiano, Z. Li, and A. W. Steiner, Prog. Part. Nucl. Phys. 62, 427 (2009)
[6] A. W. Steiner, J. M. Lattimer, and E. F. Brown, Astrophys. J.Lett. 765, L5 (2013).
[7] M. B. Tsang et al., Phys. Rev. C 86, 015803 (2012).

Neutron star

- Spherical symmetry.

$$\mathcal{M}(r) = 4\pi \int_0^r \mathrm{d}r \; r^2 \mathcal{E}(r)$$

- TOV Equation

$$-\frac{d\ P(r)}{dr} = \frac{G\mathcal{E}(r)\mathcal{M}(r)}{r^2}\left(1 - \frac{2G\mathcal{M}(r)}{r}\right)^{-1}\left(1 + \frac{P(r)}{\mathcal{E}(r)}\right)\left(1 + \frac{4\pi r^3 P(r)}{\mathcal{M}(r)}\right)$$

- Boundary condition

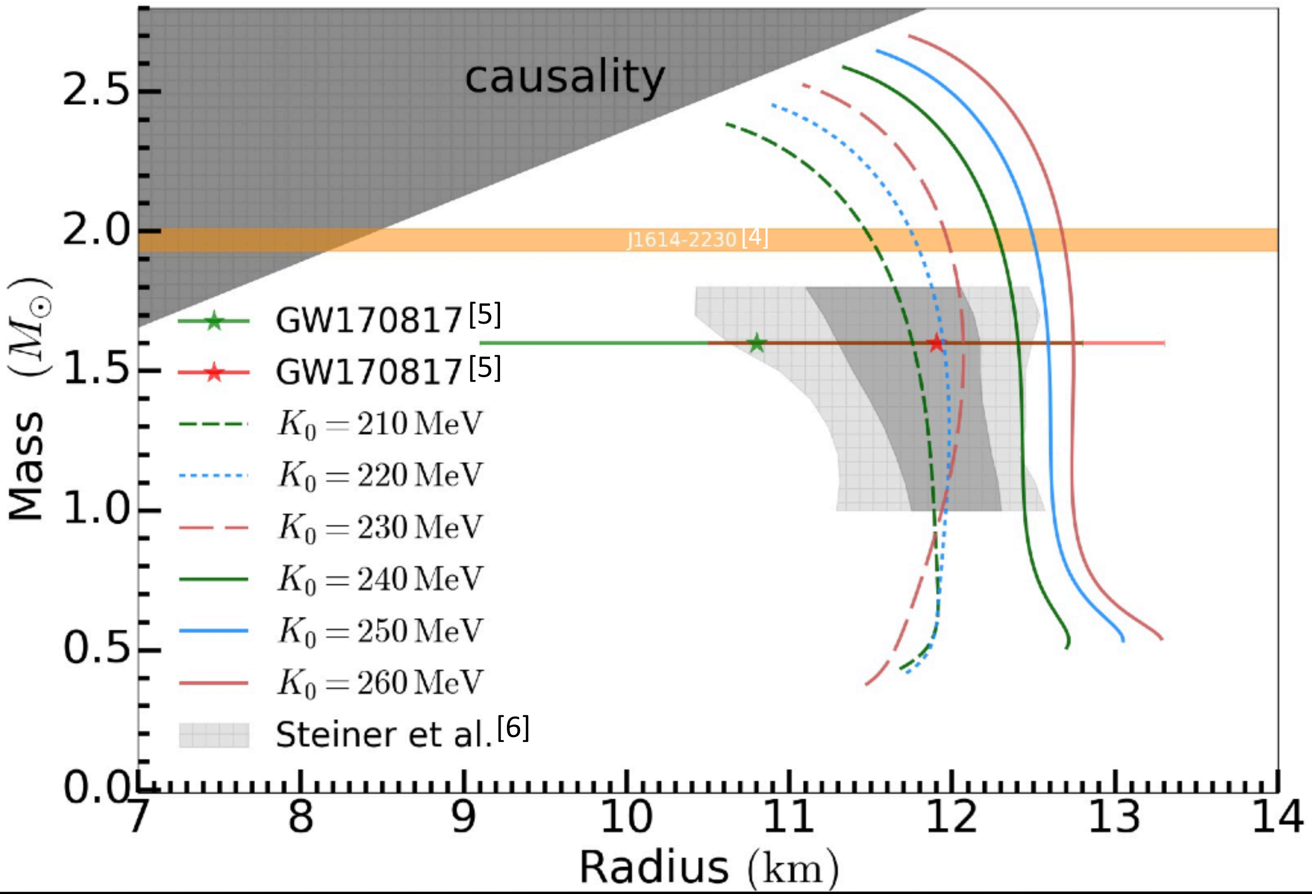
$$\mathcal{M}(0) = 0, \quad \mathcal{E}(0) = \mathcal{E}_{\mathrm{cent}}, \quad P(r = R) = 0$$

Results

- The mass-radius relation

Pure neutron matter

$a_{\mathrm{sym}} = 32\ \mathrm{MeV}$, $L_{\mathrm{sym}} = 60$ Me



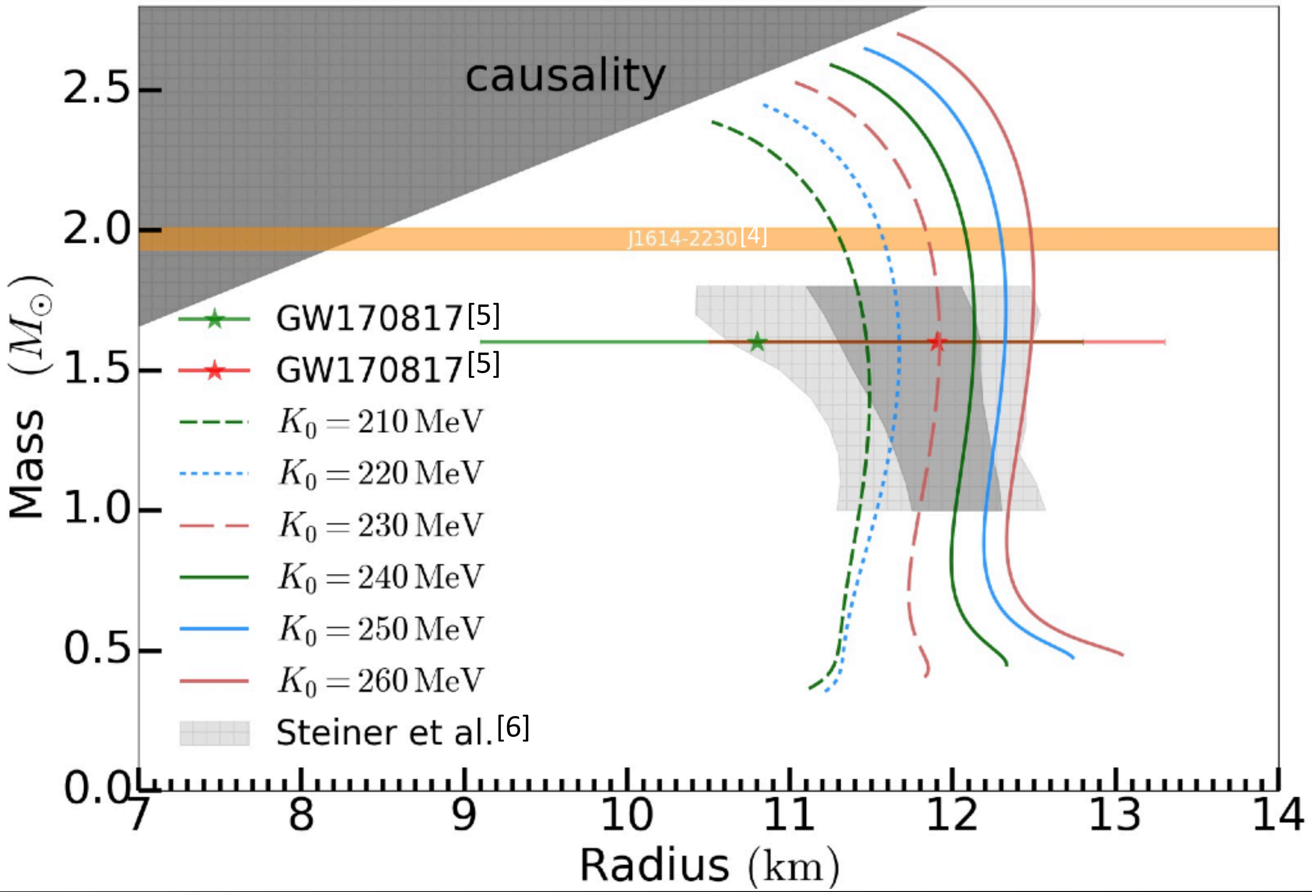
[4] Demorest, P., Pennucci, T., Ransom, S. et al. A two-solar-mass neutron star measured using Shapiro delay. Nature 467, 1081–1083 (2010).
[5] B. P. Abbott *et al.* (The LIGO Scientific Collaboration and the Virgo Collaboration) Phys. Rev. Lett. **121**, 161101
[6] Andrew W. Steiner *et al.* 2010 *ApJ* **722.** 33

Results

- The mass-radius relation

Pure neutron matter

$a_{\text{sym}} = 32\ \text{MeV}$, $L_{\text{sym}} = 50\ \text{Me}$



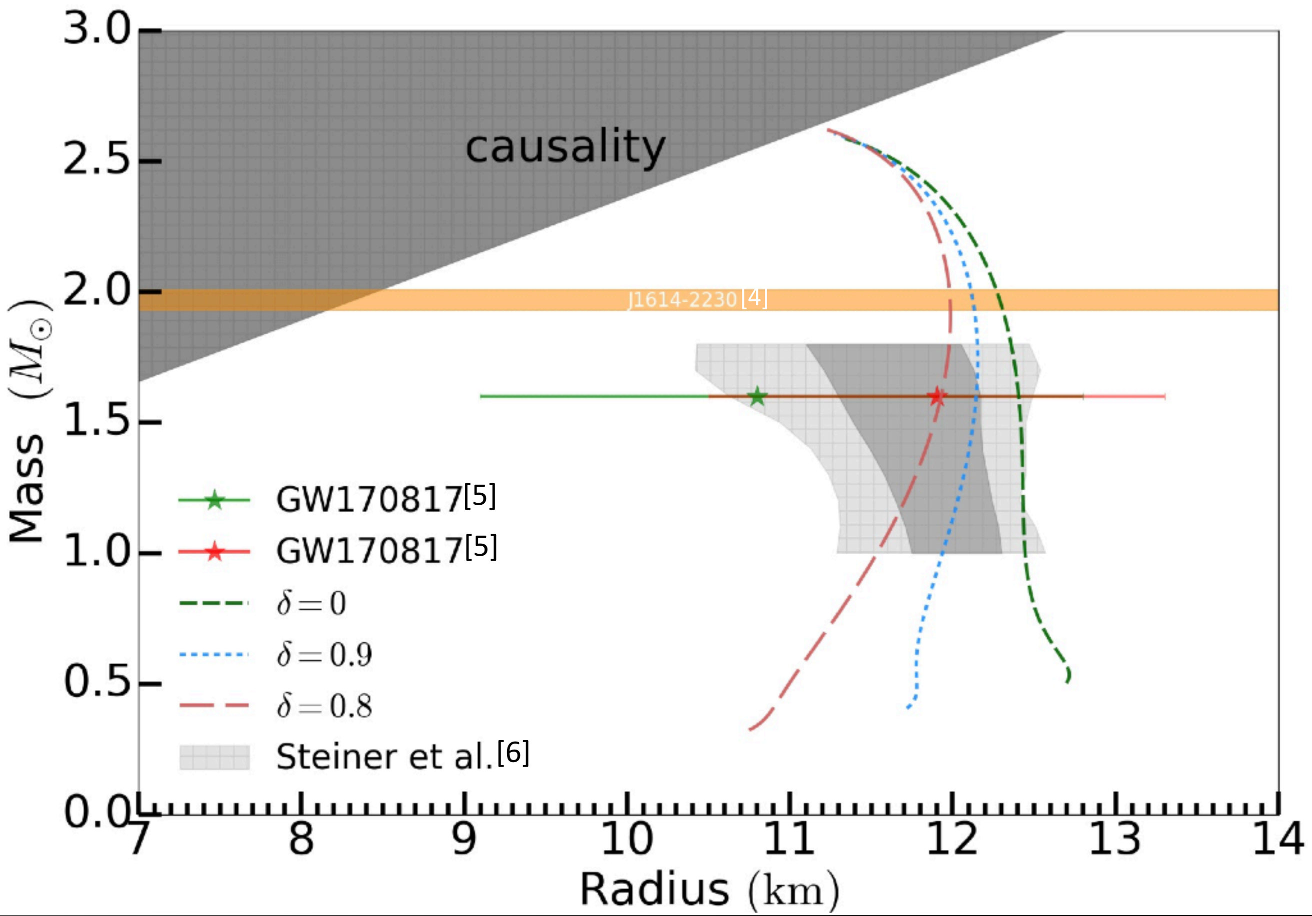
[4] Demorest, P., Pennucci, T., Ransom, S. et al. A two-solar-mass neutron star measured using Shapiro delay. Nature 467, 1081–1083 (2010).
[5] B. P. Abbott *et al.* (The LIGO Scientific Collaboration and the Virgo Collaboration) Phys. Rev. Lett. **121**, 161101
[6] Andrew W. Steiner *et al.* 2010 *ApJ* **722.** 33

Results

- The mass-radius relation

Proton mixed matter

$a_{\mathrm{sym}} = 32$ MeV, $L_{\mathrm{sym}} = 60$ Me



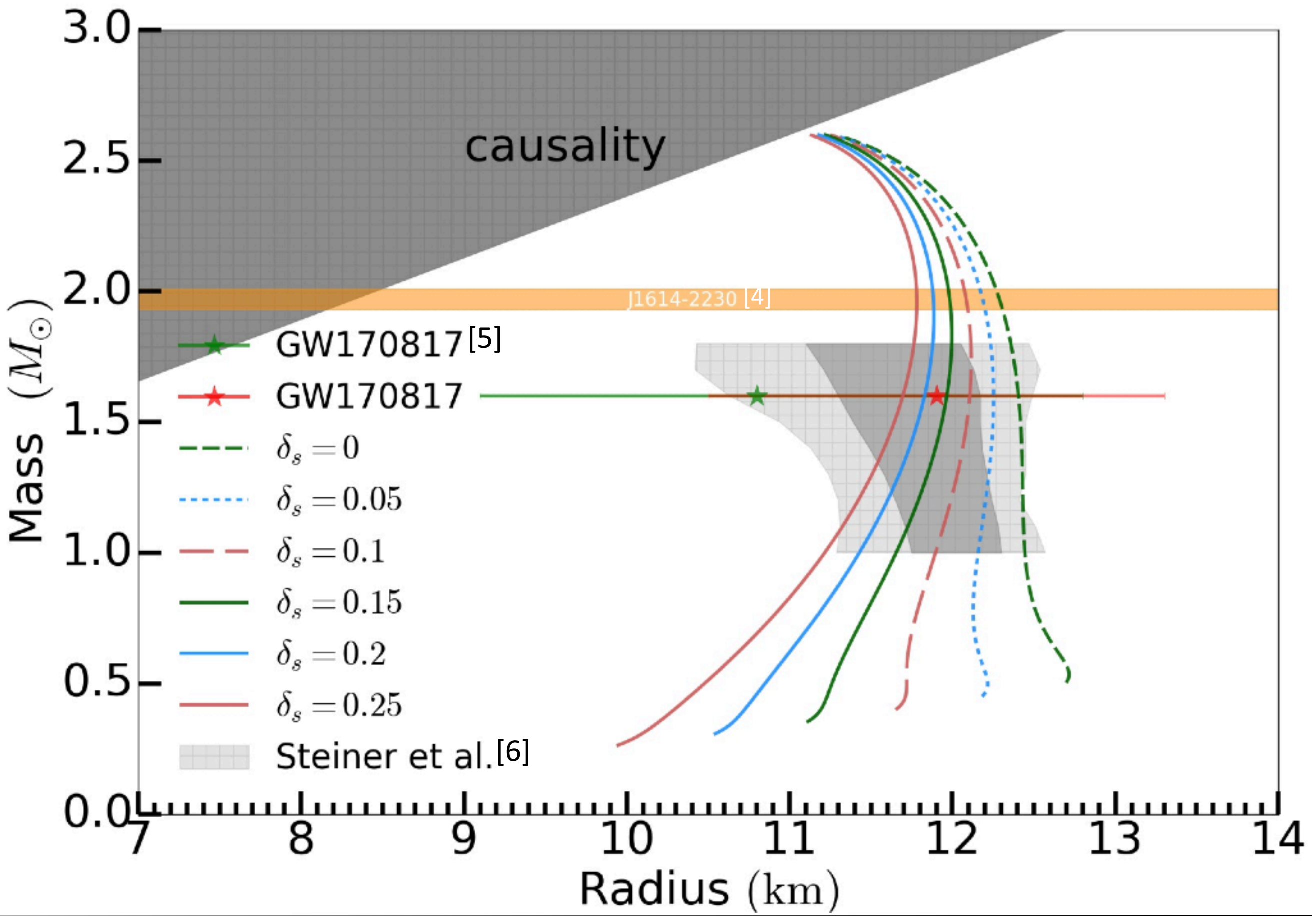
[4] Demorest, P., Pennucci, T., Ransom, S. et al. A two-solar-mass neutron star measured using Shapiro delay. Nature 467, 1081–1083 (2010).
[5] B. P. Abbott *et al.* (The LIGO Scientific Collaboration and the Virgo Collaboration) Phys. Rev. Lett. **121**, 161101
[6] Andrew W. Steiner *et al.* 2010 *ApJ* **722.** 33

Results

- The mass-radius relation

Hyperon mixed matter

$a_{\text{sym}} = 32\text{ MeV}$, $L_{\text{sym}} = 50\text{ MeV}$



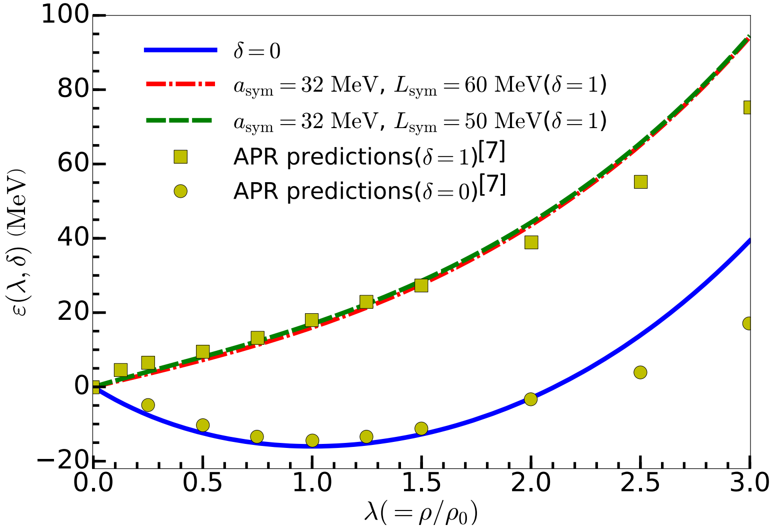
[4] Demorest, P., Pennucci, T., Ransom, S. et al. A two-solar-mass neutron star measured using Shapiro delay. Nature 467, 1081–1083 (2010).
[5] B. P. Abbott *et al.* (The LIGO Scientific Collaboration and the Virgo Collaboration) Phys. Rev. Lett. **121**, 161101
[6] Andrew W. Steiner *et al.* 2010 *ApJ* **722.** 33

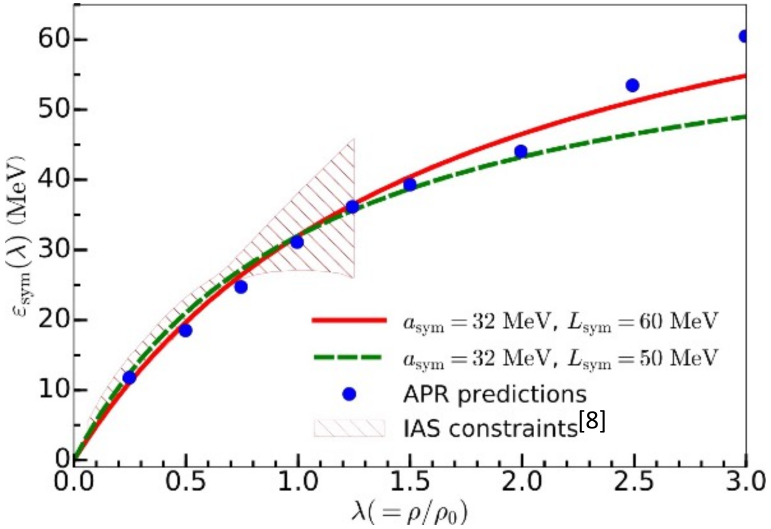
summary

- We investigated the masses and radii of neutron stars based on a pion mean-field approach and linear-response approximation.
- We determined the density-dependent parameters using the empirical data related to nuclear matter. The EOS well explain the data extracted from the phenomenological and experimental data.
- The results are in good agreement with the GW710817 measurements and Shapiro delay data.

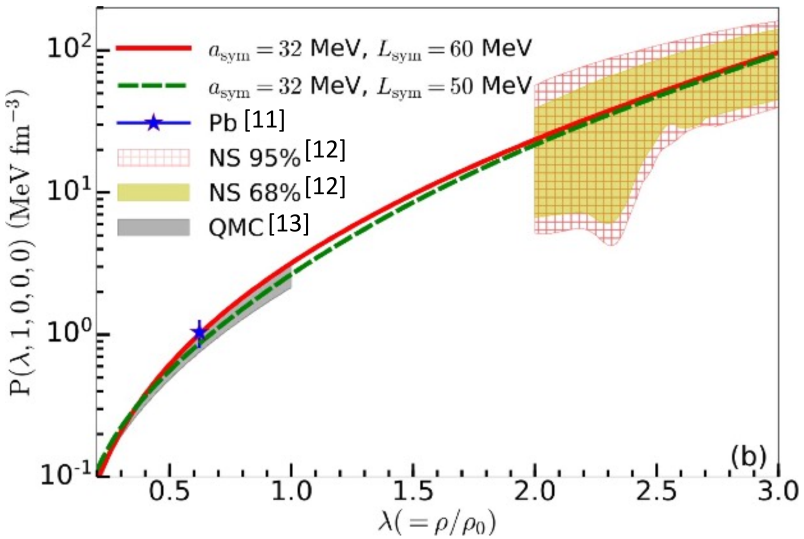
Thank you very much

Back up


$\delta=0$
$a_{sym}=32$ MeV, $L_{sym}=60$ MeV$(\delta=1)$
$a_{sym}=32$ MeV, $L_{sym}=50$ MeV$(\delta=1)$
APR predictions$(\delta=1)$[7]
APR predictions$(\delta=0)$[7]
$\varepsilon(\lambda,\delta)$ (MeV)
$\lambda(=\rho/\rho_0)$


$a_{sym}=32$ MeV, $L_{sym}=60$ MeV
$a_{sym}=32$ MeV, $L_{sym}=50$ MeV
APR predictions
IAS constraints[8]
$\varepsilon_{sym}(\lambda)$ (MeV)
$\lambda(=\rho/\rho_0)$

[9]
[10]


$a_{sym}=32$ MeV, $L_{sym}=60$ MeV
$a_{sym}=32$ MeV, $L_{sym}=50$ MeV
Pb [11]
NS 95% [12]
NS 68% [12]
QMC [13]
(b)
$P(\lambda,1,0,0,0)$ (MeV fm^{-3})
$\lambda(=\rho/\rho_0)$