The axial-vector meson in coupled-channel approach

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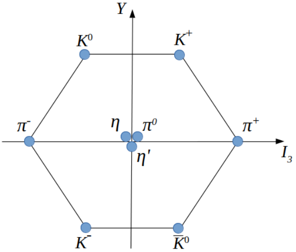
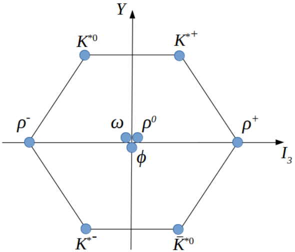
Outlines

- The axial-vector meson
  - It was proven experimentally 4 decades ago.
  - Until now the uncertainty of its mass and width are still large.
  - Its nature is still unclear. Composite? Elementary?
  - The quasi-bound state from pseudoscalar and vector meson interaction.





$I = 0$	$h_1(1170)0^-$	$f_1(1285)0^+$
$I = 1$	$b_1(1235)1^+$	$a_1(1260)1^-$
$I = 1/2$	$K_1(1270)$	

Motivation

- The axial-vector meson
  - It was proven experimentally 4 decades ago.
  - Until now the uncertainty of its mass and width are still large.
  - Its nature is still unclear. Composite? Elementary?
  - The quasi-bound state from pseudoscalar and vector meson interaction.
- The full off-shell T matrix of this interaction can then be applied to other processes as an elementary process.
  - $D\bar{D}$ and $D\bar{D}^*$ process
  - $N \to \Delta$ axial-vector form factor
  - Ω baryon radiative form factor
  - τ decay
  - etc

Formalism

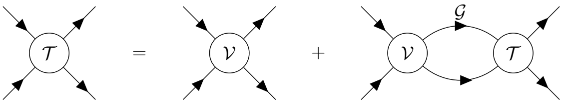
Blanckenbecler-Sugar Scheme

We start from the S matrix, $S = 1 + iT$, which can be written as

$$S_{fi} = \delta_{fi} - i(2\pi)^4\delta^4(P_f - P_i)\mathcal{T}_{fi}$$

The Bethe-Salpeter equation for two-body interaction expressed as

$$\mathcal{T}_{fi}(p', p) = \mathcal{V}_{fi}(p', p) + \frac{1}{(2\pi)^4}\int d^4q\mathcal{V}_{fg}(p', q)\mathcal{G}_g(q)\mathcal{T}_{gi}(q, p)$$



The unitarity requirement of S matrix bring us to Blanckenbecler-Sugar scheme. [R. Blankenbecler, PR**142**, 1051 (1966)]

$$\mathcal{G}_g(q) = \delta\left(q_0 - \frac{E_{1g}}{2} + \frac{E_{2g}}{2}\right)\frac{\pi}{E_{1g}E_{2g}}\frac{E_g}{s - E_g^2}$$

Blanckenbecler-Sugar Scheme

We start from the S matrix, $S = 1 + iT$, which can be written as

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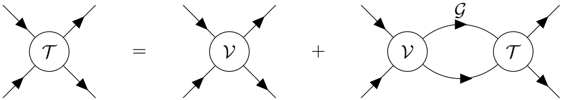
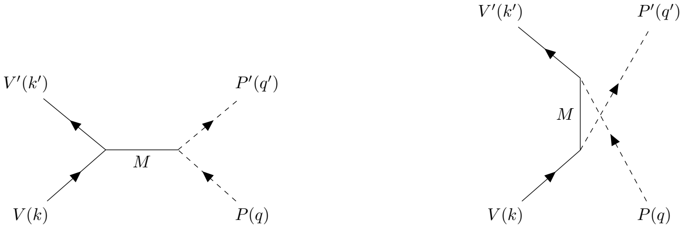


The unitarity requirement of S matrix bring us to Blanckenbecler-Sugar scheme. [R. Blankenbecler, PR**142**, 1051 (1966)]

$$\mathcal{T}_{fi}(\mathbf{p}, \mathbf{p}') = \mathcal{V}_{fi}(\mathbf{p}, \mathbf{p}') + \frac{1}{(2\pi)^3} \int \frac{d^3q}{2E_{1g}E_{2g}} \mathcal{V}_{fg}(\mathbf{p}, \mathbf{q}) \frac{E_g}{s - E_g^2} \mathcal{T}_{gi}(\mathbf{q}, \mathbf{p}')$$

Potential

The potential is modeled by one meson exchange diagram



Each diagram gives

$$\mathcal{V} = IS \times F^2 \times \Gamma_1 \times \mathcal{P} \times \Gamma_2$$

We use the SU(3) symmetric Lagrangian given by

$$\mathcal{L}_{PPV} = g_1 \,\mathrm{Tr}\left([P, \partial_\mu P]_- \, V^\mu\right)$$
$$\mathcal{L}_{VVV} = -\frac{1}{2} g_1 \,\mathrm{Tr}\left[(\partial_\mu V_\nu - \partial_\nu V_\mu) V^\mu V^\nu\right]$$
$$\mathcal{L}_{PVV} = \frac{g_2}{m_V}\, \varepsilon^{\mu\nu\alpha\beta} \mathrm{Tr}\left(\partial_\mu V_\nu \partial_\alpha V_\beta P\right)$$

with

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$V_\mu = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0_\mu + \frac{1}{\sqrt{2}}\omega_\mu & \rho^+_\mu & K^{*+}_\mu \\ \rho^-_\mu & -\frac{1}{\sqrt{2}}\rho^0_\mu + \frac{1}{\sqrt{2}}\omega_\mu & K^{*0}_\mu \\ K^{*-}_\mu & \bar{K}^{*0}_\mu & \phi_\mu \end{pmatrix}$$

Lagrangian

We use the SU(3) symmetric Lagrangian given by

$$\mathcal{L}_{PPV} = g_1\,\mathrm{Tr}\left([P,\partial_\mu P]_-\,V^\mu\right)$$
$$\mathcal{L}_{VVV} = -\frac{1}{2}g_1\,\mathrm{Tr}\left[(\partial_\mu V_\nu - \partial_\nu V_\mu)V^\mu V^\nu\right]$$
$$\mathcal{L}_{PVV} = \frac{g_2}{m_V}\,\varepsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\left(\partial_\mu V_\nu \partial_\alpha V_\beta P\right)$$

where the coupling constants are

$$g_1^2/4\pi = 0.71 \quad \text{and} \quad g_2^2/4\pi = 1.88$$

* G.Janssen, PRC**49**, 2763 (1994)

Propagator and Form factor

- We use static propagator,

$$\frac{1}{(k'-k)^2-m^2} \Longrightarrow \frac{-1}{\left(\vec{k}'-\vec{k}\right)^2+m^2}$$

- Since hadron has a finite size, we need to introduce form factor in each vertex in the diagrams.

$$F(n,k,k') = \left(\frac{n\Lambda^2-m^2}{n\Lambda^2+k^2+k'^2}\right)^n$$

As for cut-off mass Λ,

$$\Lambda = \lambda + m.$$

One-dimensional integral

- Through the partial wave decomposition, the BS equation becomes

$$\mathcal{T}^{fi}_{\lambda'\lambda}(\mathrm{p},\mathrm{p}') = \mathcal{V}^{fi}_{\lambda'\lambda}(\mathrm{p},\mathrm{p}') + \frac{1}{(2\pi)^3}\sum_{g,\,\lambda_g}\int \frac{\mathrm{q}^2 d\mathrm{q}}{2E_{g1}E_{g2}}\mathcal{V}^{fg}_{\lambda'\lambda_g}(\mathrm{p},\mathrm{q})\frac{E_g}{s-E_g^2}\mathcal{T}^{gi}_{\lambda_g\lambda}(\mathrm{q},\mathrm{p}')$$

  where

$$\mathcal{V}^{fi}_{\lambda'\lambda}(\mathrm{p}',\mathrm{p}) = 2\pi\int \mathrm{d}(\cos\theta)\, d^J_{\lambda'\lambda}(\theta)\mathcal{V}^{fi}_{\lambda'\lambda}(\mathrm{p}',\mathrm{p},\theta),$$

- Evaluating this integral by using principal value

$$\mathcal{T}^{fi}_{\lambda'\lambda}(\mathrm{p},\mathrm{p}') = \mathcal{V}^{fi}_{\lambda'\lambda}(\mathrm{p},\mathrm{p}') + \frac{1}{(2\pi)^3}\sum_{g,\,\lambda_g}\Bigg[\int dE_g \frac{\mathcal{F}(E_g)-\mathcal{F}(\sqrt{s})}{s-E_g^2} - \frac{\mathcal{F}(\sqrt{s})}{2\sqrt{s}}\left(\ln\left|\frac{\sqrt{s}+E_g^{\mathrm{thr}}}{\sqrt{s}-E_g^{\mathrm{thr}}}\right| + i\pi\right)\Bigg]$$

  with

$$\mathcal{F}(E_g) = \frac{1}{2}\mathrm{q}\,\mathcal{V}^{fg}_{\lambda'\lambda_g}(\mathrm{p},\mathrm{q})\,\mathcal{T}^{gi}_{\lambda_g\lambda}(\mathrm{p},\mathrm{q}),$$

- We solved this integral by utilizing matrix inversion method.

Results and Discussion

$G = -1$ and $I = 1$

a_1 resonance

Channels involved : $\pi\rho$ and $K\bar{K}^*$

Since K and K^* have no definite G-parity,

$$|KK^*(-)\rangle = \frac{1}{\sqrt{2}}\left(|K\bar{K}^*\rangle - |\bar{K}K^*\rangle\right)$$

Potential :

$$V^{fi} = \begin{pmatrix} V^{\pi\rho\to\pi\rho} & V^{K\bar{K}^*\to\pi\rho} \\ V^{\pi\rho\to K\bar{K}^*} & V^{K\bar{K}^*\to K\bar{K}^*} \end{pmatrix}$$

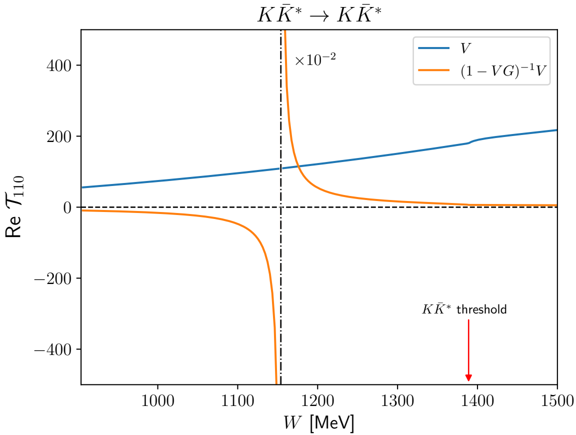
Channels involved : $\pi\rho$ and $K\bar{K}^*$

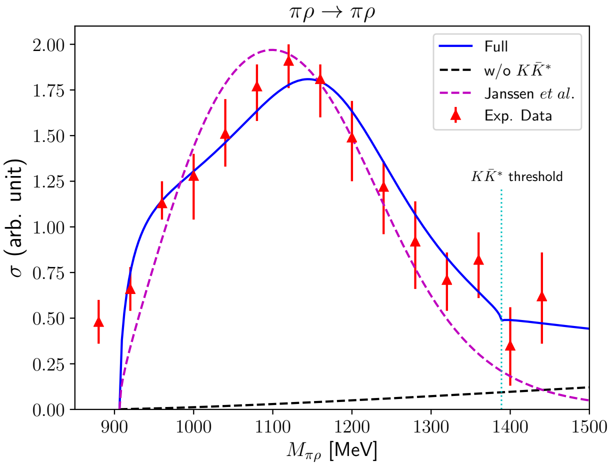
	meson	channel	IS	λ (MeV)	n
$\pi\rho \to \pi\rho$	π	u	4	600	1
	ρ	t	-4	600	1
	ω	u	-4	600	2
$\pi\rho \to K\bar{K}^*(\bar{K}K^*)$	K	u	$-2(2)$	1500	1
	K^*	t	$2(-2)$	1110	1
$K\bar{K}^* \to K\bar{K}^*$	ρ	t	1	1050	1
	ω	t	-1	1050	1
	ϕ	t	-2	1700	1
$K\bar{K}^* \to \bar{K}K^*$	π	u	1	1050	1
	η	u	-3	1050	1
	ρ	u	-1	1050	2
	ω	u	1	1050	2
	ϕ	u	2	1050	2

*Note that for ϕ-exchange diagram the coupling g differ by about 14%

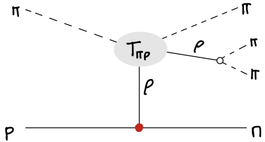
a_1 resonance

The singularities arises as a result of integral equation in the region below $K\bar{K}^*$ threshold.





We compare the model to experimental data from charge exchange reaction $\pi p \to 3\pi n$.



And define:

$$\sigma \equiv -C\,\mathrm{Im}\left[T_{\pi\rho}(M_{\pi\rho})\right]$$

Exp.Data : J.A.Dankowych, PRL**46** (1981). Model : G.Janssen, PRC**54** (1996)

a_1 properties

- The $a_1(1260)$ pole position, $\sqrt{s_R} = 1170.1 - i104.1$
- Residue and coupling strength can be defined as

$$\lim_{s \to s_R} (s - s_R)\, T_{a,b} = R_{a,b}, \qquad g_a = \sqrt{R_{a,a}}$$

- The elastic residue and coupling strength of $a_1(1260)$ resonance for S-wave and D-wave are

	S-wave	D-wave	Unit
$R_{\pi\rho}$	$26.68 - i6.54$	$0.31 + i0.02$	GeV^2
$R_{K\bar{K}^*}$	$37.17 - i4.74$	$0.09 + i0.16$	GeV^2
$g_{\pi\rho}$	$5.20 - i0.63$	$0.56 + i0.02$	GeV
$g_{K\bar{K}^*}$	$6.11 - i0.39$	$0.37 + i0.22$	GeV

$G = -1$ and $I = 0$

h_1 resonance

Channels involved : $\pi\rho$, $\eta\omega$, $K\bar{K}^*$ and $\eta\phi$.

Potential :

$$V^{fi} = \begin{pmatrix} V^{\pi\rho\to\pi\rho} & V^{\eta\omega\to\pi\rho} & V^{K\bar{K}^*\to\pi\rho} & V^{\eta\phi\to\pi\rho} \\ V^{\pi\rho\to\eta\omega} & V^{\eta\omega\to\eta\omega} & V^{K\bar{K}^*\to\eta\omega} & V^{\eta\phi\to\eta\omega} \\ V^{\pi\rho\to K\bar{K}^*} & V^{\eta\omega\to K\bar{K}^*} & V^{K\bar{K}^*\to K\bar{K}^*} & V^{\eta\phi\to K\bar{K}^*} \\ V^{\pi\rho\to\eta\phi} & V^{\eta\omega\to\eta\phi} & V^{K\bar{K}^*\to\eta\phi} & V^{\eta\phi\to\eta\phi} \end{pmatrix}$$

	$\pi\rho$	$\eta\omega$	$K\bar{K}^*$	$\eta\phi$
$\pi\rho$	O	O	O	$\times$
$\eta\omega$	O	O	O	$\times$
$K\bar{K}^*$	O	O	O	O
$\eta\phi$	$\times$	$\times$	O	O

Diagrams and Parameters

Channels involved : $\pi\rho$, $\eta\omega$, $K\bar{K}^*$ and $\eta\phi$.

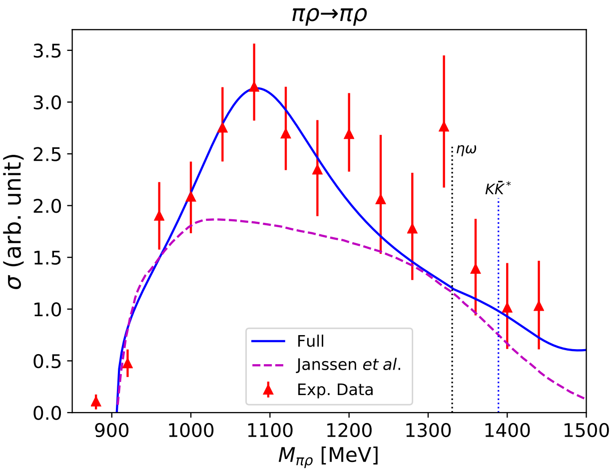
	meson	channel	IS	λ (MeV)	n
$\pi\rho \to \pi\rho$	π	u	-8	600	1
	ρ	t	-8	600	1
	ω	u	4	600	2
$\pi\rho \to K\bar{K}^*(\bar{K}K^*)$	K	u	$\sqrt{6}(-\sqrt{6})$	1500	1
	K^*	t	$\sqrt{6}(-\sqrt{6})$	1110	1
$K\bar{K}^* \to K\bar{K}^*$	ρ	t	-3	1050	1
	ω	t	-1	1050	1
	ϕ	t	-2	1700	1
$K\bar{K}^* \to \bar{K}K^*$	π	u	3	1050	1
	η	u	3	1050	1
	ρ	u	-3	1050	2
	ω	u	-1	1050	2
	ϕ	u	-2	1050	2

*All same as in a_1 channel except SIF values

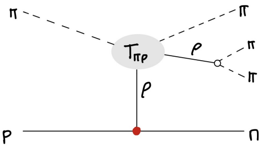
Channels involved : $\pi\rho$, $\eta\omega$, $K\bar{K}^*$ and $\eta\phi$.

	meson	channel	IS	λ (MeV)	n
$\pi\rho \to \eta\omega$	ρ	u	-4	600	2
$\eta\omega \to \eta\omega$	ω	u	$4/3$	600	2
$\eta\omega \to K\bar{K}^*(\bar{K}K^*)$	K	u	$-\sqrt{6}(\sqrt{6})$	940	1
	K^*	t	$-\sqrt{6}(\sqrt{6})$	940	1
$K\bar{K}^*(\bar{K}K^*) \to \eta\phi$	K	u	$2\sqrt{3}(-2\sqrt{3})$	940	1
	K^*	t	$2\sqrt{3}(-2\sqrt{3})$	940	1
$\eta\phi \to \eta\phi$	ϕ	u	$16/3$	1400	2

*Note that for ϕ-exchange diagram the coupling g_2 differ by about 11%



We compare the model to experimental data from charge exchange reaction $\pi p \to 3\pi n$.



And define:

$$\sigma \equiv -C\,\mathrm{Im}\left[T_{\pi\rho}(M_{\pi\rho})\right]$$

Exp.Data : J.A.Dankowych, PRL**46** (1981). Model : G.Janssen, PRC**54** (1996)

- The $h_1(1170)$ pole position, $\sqrt{s_R} = 1152.3 - i162.7$
- Residue and coupling strength can be defined as

$$\lim_{s \to s_R} (s - s_R)\, T_{a,b} = R_{a,b}, \qquad g_a = \sqrt{R_{a,a}}$$

- The elastic residue and coupling strength of $h_1(1170)$ resonance for S-wave and D-wave are

	S-wave	D-wave	Unit
$R_{\pi\rho}$	$17.84 - i10.75$	$0.31 + i0.01$	GeV^2
$R_{\eta\omega}$	$16.57 + i6.40$	$0.08 + i0.04$	GeV^2
$R_{K\bar{K}^*}$	$24.86 + i2.26$	$-0.02 + i0.22$	GeV^2
$R_{\eta\phi}$	$70.88 - i12.89$	$0.96 + i0.10$	GeV^2

$G = +1$ and $I = 1$

b_1 resonance

Channels involved : $\pi\omega$, $\pi\phi$, $\eta\rho$ and $K\bar{K}^*$.

$$|KK^*(+)\rangle = \frac{1}{\sqrt{2}}\left(|K\bar{K}^*\rangle + |\bar{K}K^*\rangle\right)$$

Potential :

$$V^{fi} = \begin{pmatrix} V^{\pi\omega\to\pi\omega} & V^{\pi\phi\to\pi\omega} & V^{\eta\rho\to\pi\omega} & V^{K\bar{K}^*\to\pi\omega} \\ V^{\pi\omega\to\pi\phi} & V^{\pi\phi\to\pi\phi} & V^{\eta\rho\to\pi\phi} & V^{K\bar{K}^*\to\pi\phi} \\ V^{\pi\omega\to\eta\rho} & V^{\pi\phi\to\eta\rho} & V^{\eta\rho\to\eta\rho} & V^{K\bar{K}^*\to\eta\rho} \\ V^{\pi\omega\to K\bar{K}^*} & V^{\pi\phi\to K\bar{K}^*} & V^{\eta\rho\to K\bar{K}^*} & V^{K\bar{K}^*\to K\bar{K}^*} \end{pmatrix}$$

	$\pi\omega$	$\pi\phi$	$\eta\rho$	$K\bar{K}^*$
$\pi\omega$	O	$\times$	O	O
$\pi\phi$	$\times$	$\times$	$\times$	O
$\eta\rho$	O	$\times$	O	O
$K\bar{K}^*$	O	O	O	O

Channels involved : $\pi\omega$, $\pi\phi$, $\eta\rho$ and $K\bar{K}^*$.

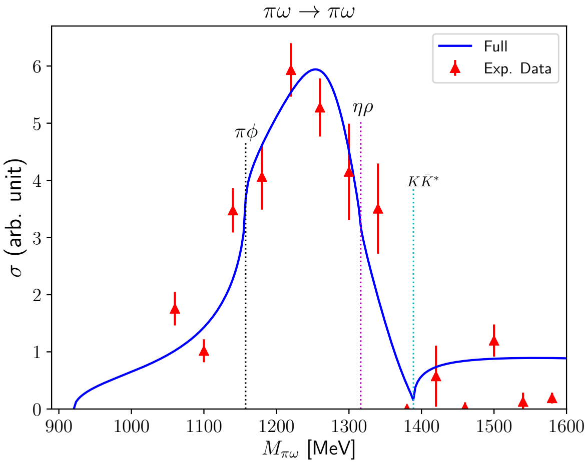
	meson	channel	IS	λ (MeV)	n
$K\bar{K}^* \to K\bar{K}^*$	ρ	t	1	1050	1
	ω	t	-1	1050	1
	ϕ	t	-2	1700	1
$K\bar{K}^* \to \bar{K}K^*$	π	u	1	1050	1
	η	u	-3	1050	1
	ρ	u	-1	1050	2
	ω	u	1	1050	2
	ϕ	u	2	1050	2

*All same as in a_1 channel

Channels involved : $\pi\omega$, $\pi\phi$, $\eta\rho$ and $K\bar{K}^*$.

	meson	channel	IS	λ (MeV)	n
$\pi\omega \to \pi\omega$	ρ	u	4	600	2
$\pi\omega \to \eta\rho$	ω	u	$4/\sqrt{3}$	600	2
$\pi\omega \to K\bar{K}^*(\bar{K}K^*)$	K	u	$\sqrt{2}(\sqrt{2})$	680	1
	K^*	t	$\sqrt{2}(\sqrt{2})$	680	1
$\pi\phi \to K\bar{K}^*(\bar{K}K^*)$	K	u	$-2(-2)$	660	1
	K^*	t	$-2(-2)$	660	1
$\eta\rho \to \eta\rho$	ρ	u	$4/3$	600	2
$\eta\rho \to K\bar{K}^*(\bar{K}K^*)$	K	u	$\sqrt{6}(\sqrt{6})$	530	1
	K^*	t	$\sqrt{6}(\sqrt{6})$	530	1

b_1 resonance



We compare the model to experimental data from charge exchange reaction $\pi p \to \omega\pi n$.

And define:

$$\sigma \equiv -C\,\mathrm{Im}\left[T_{\pi\omega}(M_{\pi\omega})\right]$$

Exp.Data : S.Fukui, PLB**257** (1991)

- The $b_1(1235)$ pole position, $\sqrt{s_R} = 1341.7 - i71.1$
- Residue and coupling strength can be defined as

$$\lim_{s \to s_R} (s - s_R)\, T_{a,b} = R_{a,b}, \qquad g_a = \sqrt{R_{a,a}}$$

- The elastic residue and coupling strength of $b_1(1235)$ resonance for S-wave and D-wave are

	S-wave	D-wave	Unit
$R_{\pi\omega}$	$4.35 + i8.65$	$-0.02 + i0.05$	GeV^2
$R_{\pi\phi}$	$12.34 + i14.07$	$0.01 + i0.01$	GeV^2
$R_{\eta\rho}$	$5.87 + i10.45$	$0.00 + i0.00$	GeV^2
$R_{K\bar{K}^*}$	$153.21 + i102.73$	$-0.08 + i0.04$	GeV^2

Summary and Conclusion

- We investigated the axial-vector meson resonance from pseudoscalar and vector meson interaction based on the fully off-mass-shell coupled channel formalism.
- By doing so, we generate a_1, h_1 and b_1 axial-vector meson dynamically.
- We also present the comparison of the model calculation to the experimental data from charge exchange reaction and we extracted the resonance properties of a_1, h_1 and b_1 axial-vector meson.
- For the future project, we will investigate $S = \pm 1$ channel.

Thank You