

# **Gluon and valence quark distributions for the pion and kaon in nuclear matter**

**Seung-il Nam**

Department of Physics, Pukyong National University (PKNU)  
Center for Extreme Nuclear Matters (CENuM), Korea University



In collaboration with

**Dr. Parada T.P. Hutauruk (PKNU, Korea)**

Contents based on  
**Physical Review D 105, 034021 (2022)**



# Introduction

- Feynman's partonic interpretations of high-E scattering: Bjorken scaling etc.

- Partonic interpretations of (bound state) hadron

~ light-front quantization (LFQ)

- Well-defined Fock space and frame-independent

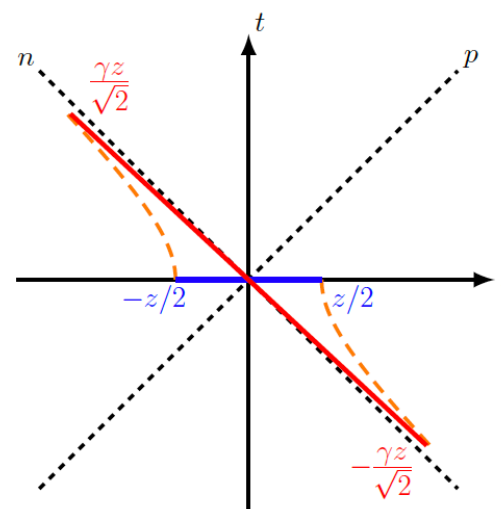
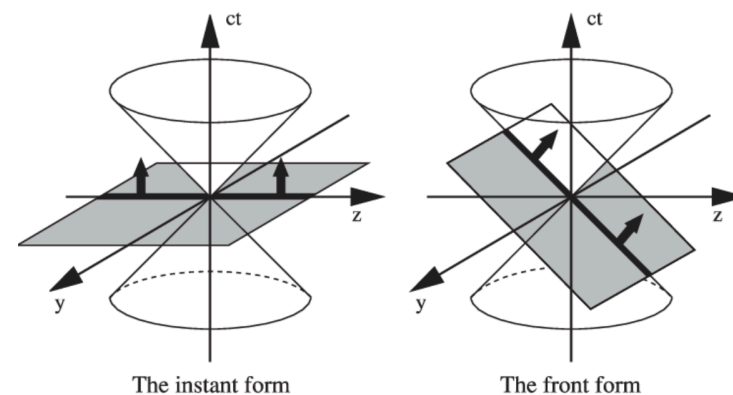
for hadron wave function

- Difficult in covariant Euclidean space time:

Moments calculation in LQCD

- Recent development from LFQ

to Large-momentum (LaM) frame for LQCD



# Introduction

- What are partonic structure functions??

1) Local operators or bilocal? 2) Momentum transferred or not? 3) Matter or vacuum?

$\langle H(k+q) | O(x,y) | H(k) \rangle$  : Generalized parton distribution (GPD)

$\langle H(k) | O(x,y) | H(k) \rangle$  : Parton distribution function (PDF)

$\langle H(k+q) | O(x) | H(k) \rangle$  : Form factor (FF)

$\langle H(k) | O(x) | H(k) \rangle$  : Coupling constant (CC)

$\langle 0 | O(x,y) | H(k) \rangle$  : Light-cone wave function (LCWF)

And more...

# Introduction

- What are partonic structure functions??

1) Local operators or bilocal? 2) Momentum transferred or not? 3) Matter or vacuum?

$\langle H(k+q) | O(x,y) | H(k) \rangle$  : Generalized parton distribution (GPD)

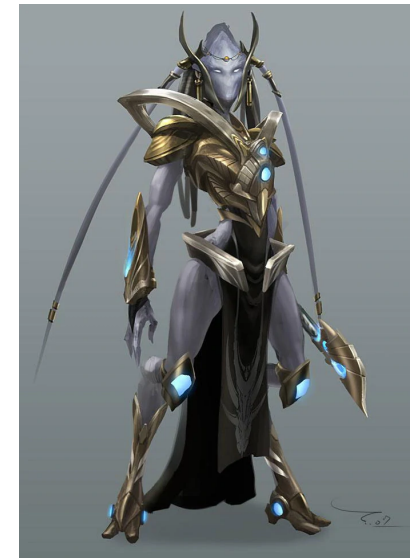
$\langle H(k) | O(x,y) | H(k) \rangle$  : Parton distribution function (PDF)

$\langle H(k+q) | O(x) | H(k) \rangle$  : Form factor (FF)

$\langle H(k) | O(x) | H(k) \rangle$  : Coupling constant (CC)

$\langle 0 | O(x,y) | H(k) \rangle$  : Light-cone wave function (LCWF)

And more... with the Great prototype "Parton correlation function"



# Introduction

- PDF is **UNIVERSAL** for processes depending on several scales (factorization & renormalization)

- Non-perturbative nature of QCD

- Effective models, LQCD, Global fit with Neural network (CTEQ)

*Can we survive this? orZ*

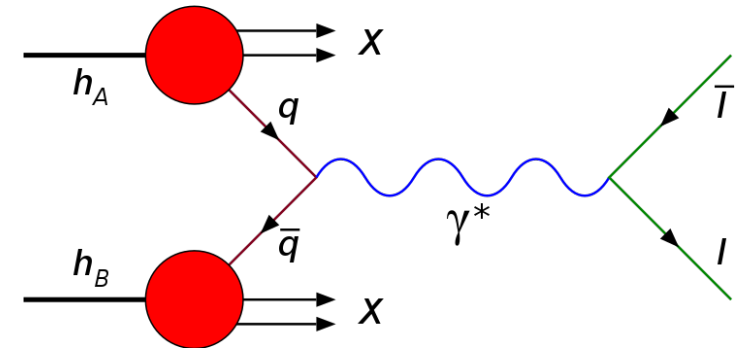
- Momentum distribution of partons inside hadrons  $x = k^+/P^+$

- Pseudo-scalar meson PDF defined in LFQ by

$$q_m(x) = \frac{p^+}{2\pi} \int d\xi^- \exp(ixp^+\xi^-) \langle m | \bar{q}(0) \gamma^+ q(\xi^-) | m \rangle_c,$$

- Two quarks on LF "coordinate (-)" with separation  $\xi$  evolving with LF "time (+)"

- What happens for PDF in medium?  $\Rightarrow$  Extension to nuclear PDF!?!



# Theoretical framework

- NJL Lagrangian density

$$\mathcal{L}_{\text{NJL}} = \bar{q}(i\rlap{/}\partial - \hat{m}_q)q + G_\pi [(\bar{q}\lambda_a q)^2 - (\bar{q}\lambda_a\gamma_5 q)^2] - G_\rho [(\bar{q}\lambda_a\gamma^\mu)^2 + (\bar{q}\lambda_a\gamma^\mu\gamma_5 q)^2].$$

- Constituent-quark mass via the gap equation with proper-time regularization (PTR)

$$M_q = m_q + \frac{3G_\pi M_q}{\pi^2} \int_{\tau_{\text{UV}}^2}^{\tau_{\text{IR}}^2} \frac{d\tau}{\tau^2} \exp(-\tau M_q^2),$$

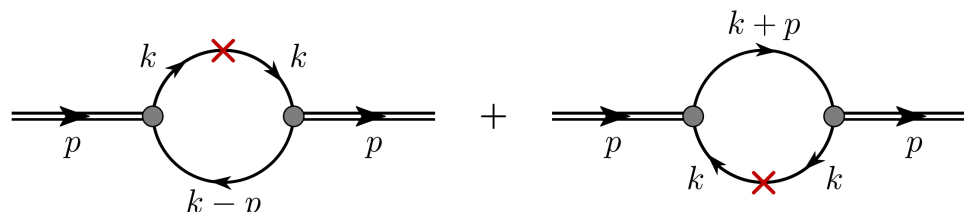
- Pseudo-scalar meson dressed propagators via Dyson-Schwinger Eq.

$$t_{\pi,K} = \frac{-2iG_\pi}{1 + 2G_\pi \Pi_{\pi,K}(p^2)} \quad \Pi_\pi(p^2) = 6i \int \frac{d^4k}{(2\pi)^4} [\gamma_5 S_l(k) \gamma_5 S_l(k+p)]$$

- Pseudo-scalar meson mass and coupling are the pole and residue of  $t$

# Theoretical framework

- PDF in vacuum NJL



$$q_{\pi,K}(x) = ig_{mq\bar{q}}^2 \int \frac{d^4k}{(2\pi)^4} \delta(k^+ - xp^+) \text{Tr}_{c,f,\gamma} [\gamma_5 \lambda_a^\dagger S_l(k) \gamma^+ \hat{P}_{u/d} S_l(k) \gamma_5 \lambda_a S_l(k-p)]$$

- Quarks are charged fermion  $\Rightarrow$  Gluon Bremsstrahlung  $\Rightarrow$  Energy loss
- How about inverse process?: PDF at low  $\Lambda \Rightarrow$  higher  $\Lambda$ : DGLAP at larger  $Q^2$
- Gluon radiation at local point described by **Splitting function  $P$**
- Flavor-singlet quark PDF (qPDF) and gluon PDF (gPDF) are inter-related

$$\frac{\partial}{\partial \ln Q^2} \begin{bmatrix} q_S(x, Q^2) \\ g(x, Q^2) \end{bmatrix} = \begin{bmatrix} \mathcal{P}_{qq} & \mathcal{P}_{qg} \\ \mathcal{P}_{gq} & \mathcal{P}_{gg} \end{bmatrix} \otimes \begin{bmatrix} q_S(x, Q^2) \\ g(x, Q^2) \end{bmatrix}.$$

- Once we know qPDF, by solving differential eq. At certain  $Q^2$ , we have gPDF

# Theoretical framework

- NJL modified in medium with iso-scalar vector potential (SNM)

$$\mathcal{L}_{\text{SNM-NJL}} = \bar{q}(i\not{\partial} - M_q - \not{V})q - \frac{(M_q - m_q)^2}{4G_\pi} + \frac{V_\mu V^\mu}{2G_\omega}$$

- Compute medium quark mass and iso-scalar potential V (not field)

$$M_q = m_q - 2G_\pi \langle \rho | \bar{q}q | \rho \rangle,$$

$$V^\mu = 2G_\omega \langle \rho | \bar{q}\gamma^\mu q | \rho \rangle = 2\delta^{0\mu} G_\omega \langle q^\dagger q \rangle,$$

- Effective potential by solving Faddeev eq. With quark-diquark bound state

$$\mathcal{E} = \mathcal{E}_V - \frac{V_0^2}{4G_\omega} + 4 \int \frac{d^3 p}{(2\pi)^3} \theta(p_F - |\mathbf{p}|) \epsilon_p,$$

Integration up to Fermi momentum

$$\epsilon_p = \sqrt{M_N^{*2} + \mathbf{p}^2} + 3V_0 \equiv E_p + 3V_0.$$

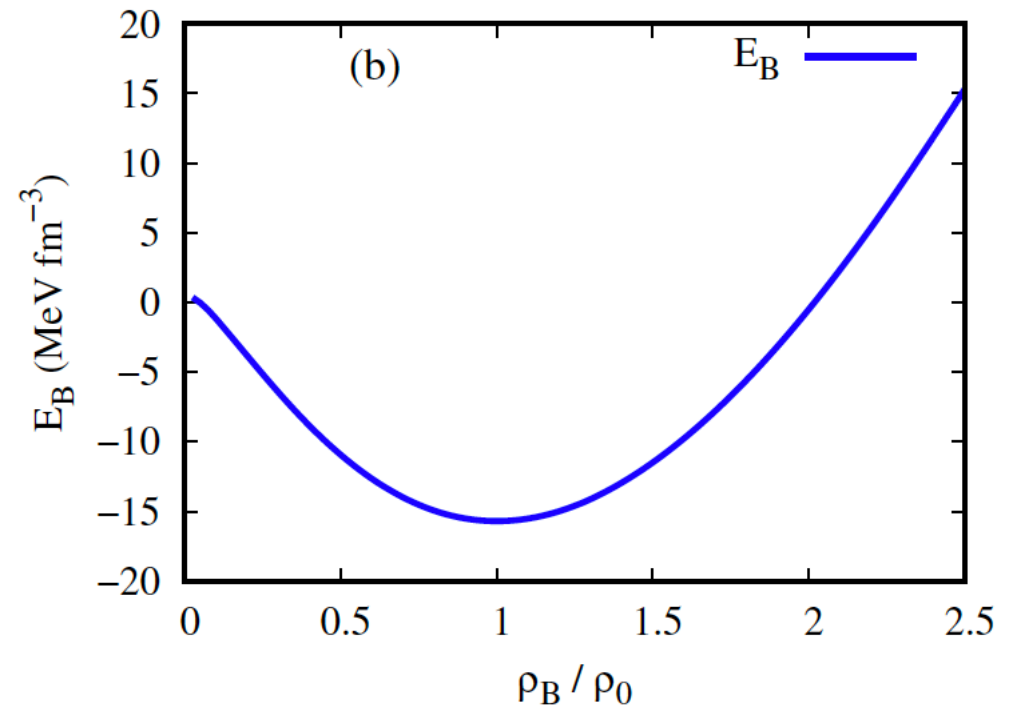
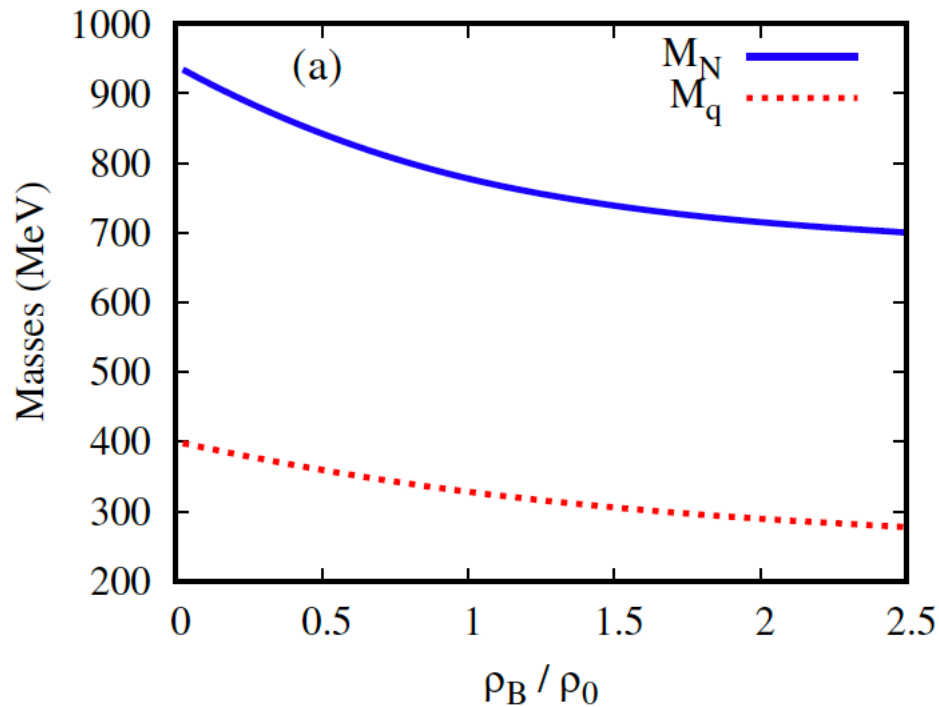


# Theoretical framework

- Variation of potential wrt  $V$  and  $M_q$

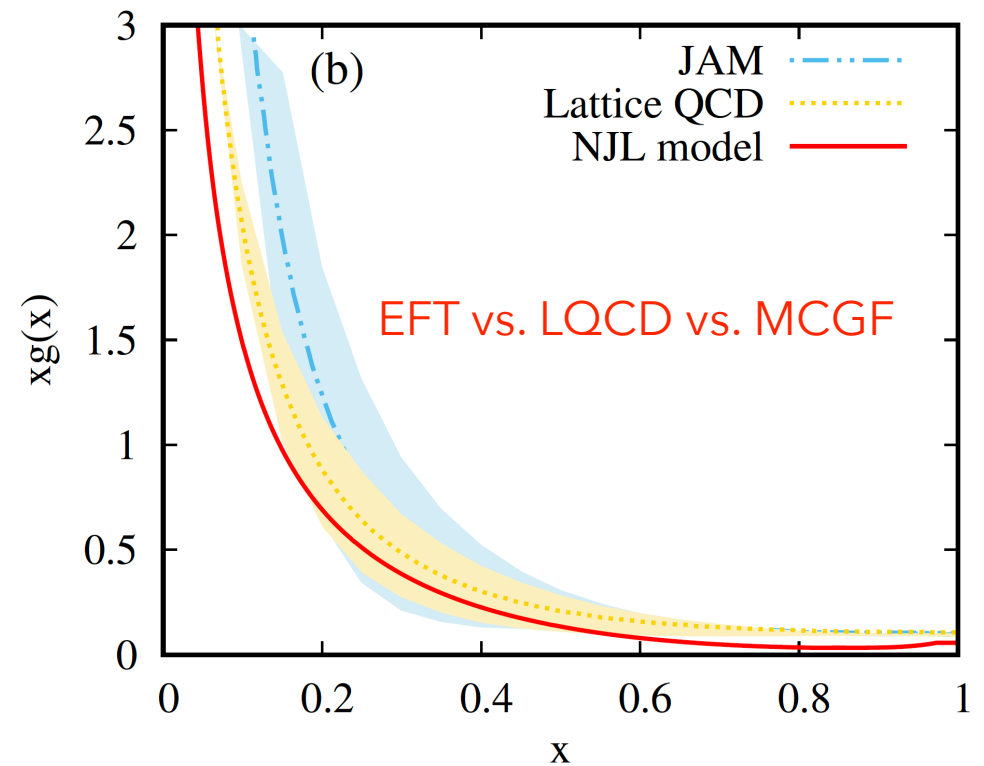
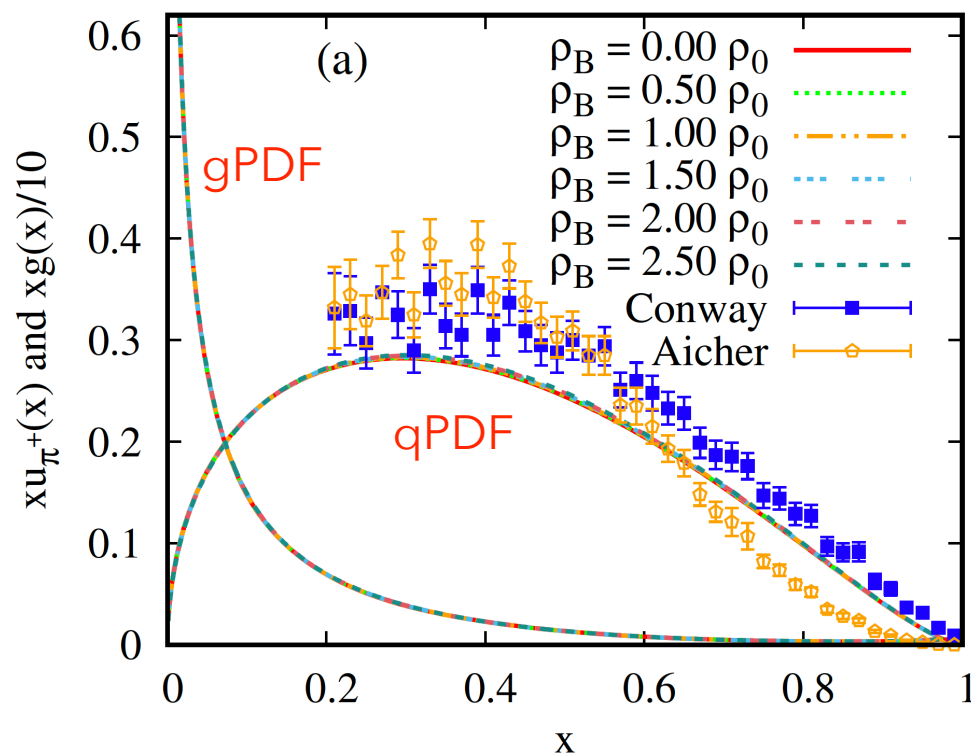
$$\frac{\partial \mathcal{E}}{\partial V_0} = 0, \quad V_0 = 6G_\omega \rho_B, \quad \rho_B = 2p_F^3/3\pi^2 \quad \frac{\partial \mathcal{E}}{\partial M_q} = 0$$

- By combining them, we obtain  $M_q$  as a function of density!!!!



# Numerical results

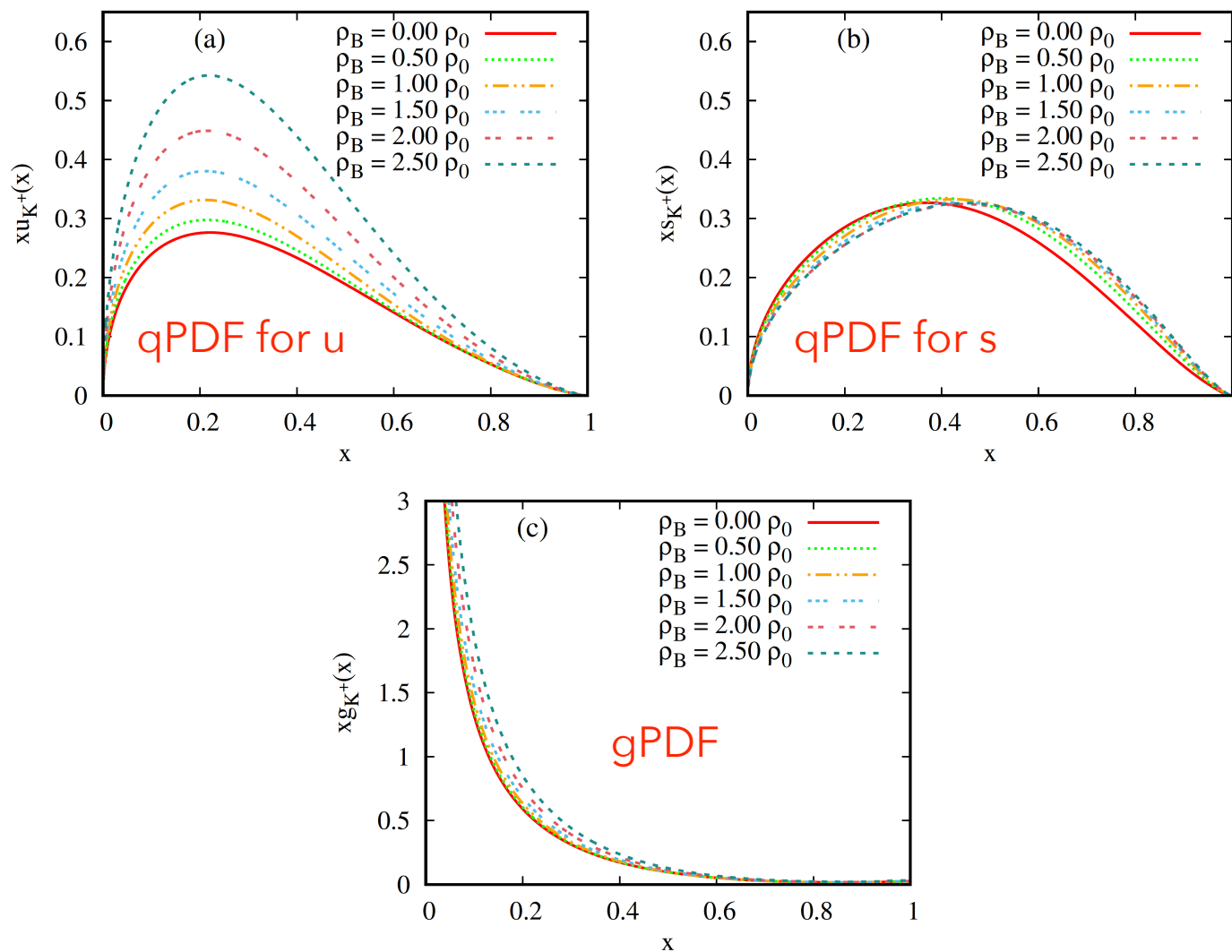
- qPDF and gPDF for pion (u-d) in symmetric nuclear matter



- DGLAP evolution up to  $|Q| = 4$  GeV
- Density effects are almost negligible: Why?

# Numerical results

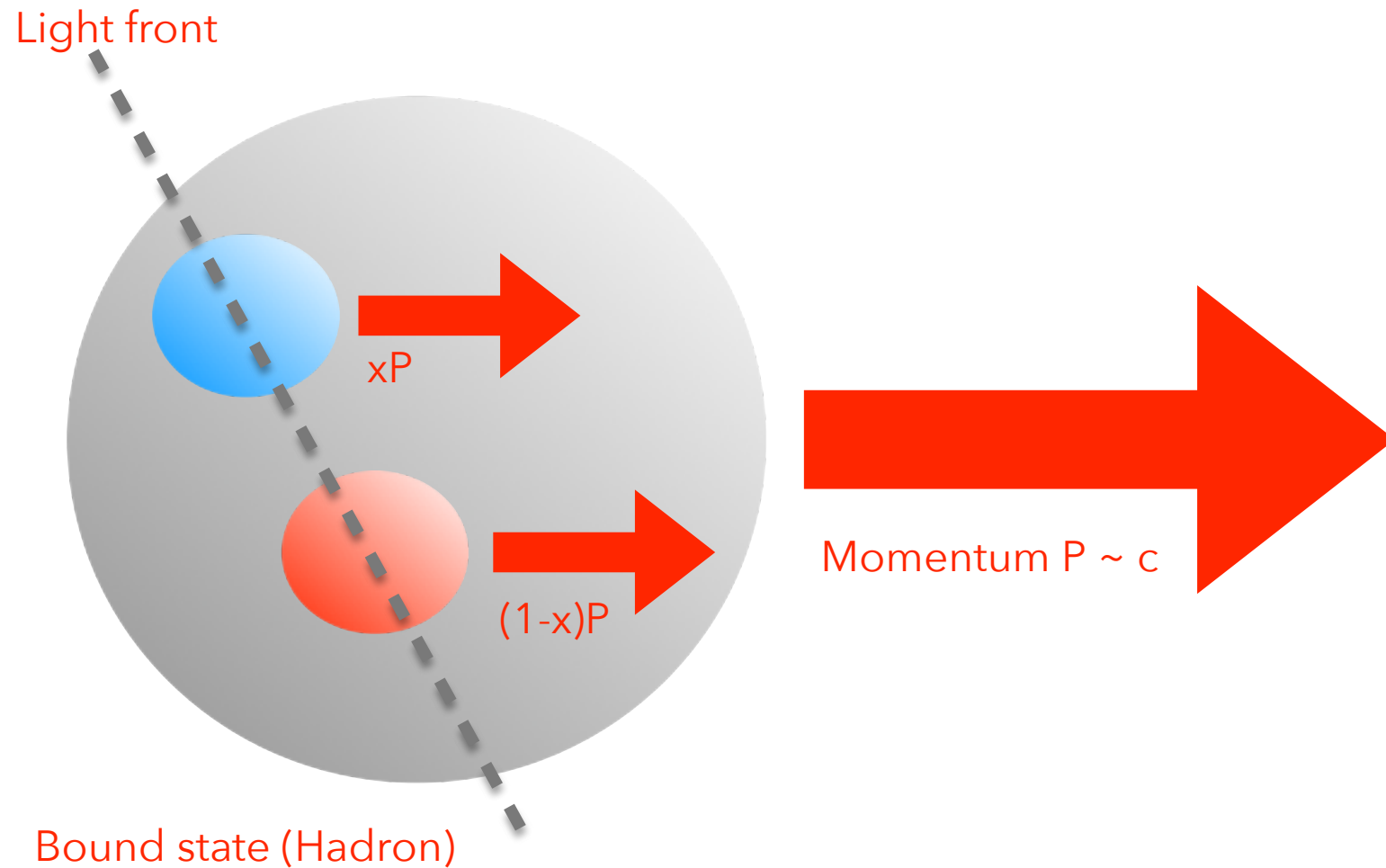
- What about Kaon ( $u$ - $s$ )???: Considerable changes in light quark  $u$ , not  $s$ !!!



- Density effects are obvious: Why?

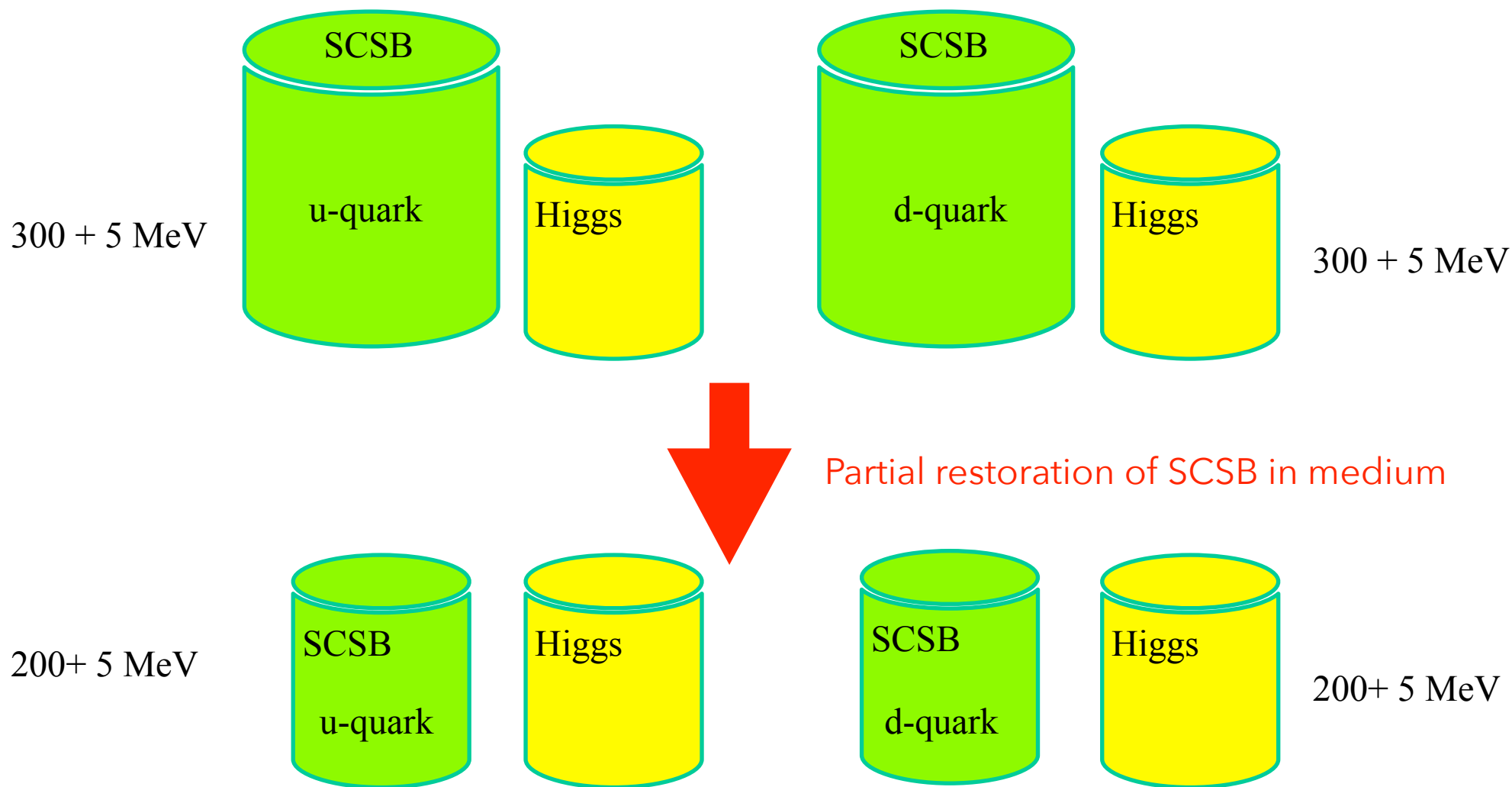
# Numerical results

- PDF describes fractional longitudinal momentum ( $x$ ) distribution of patron



# Numerical results

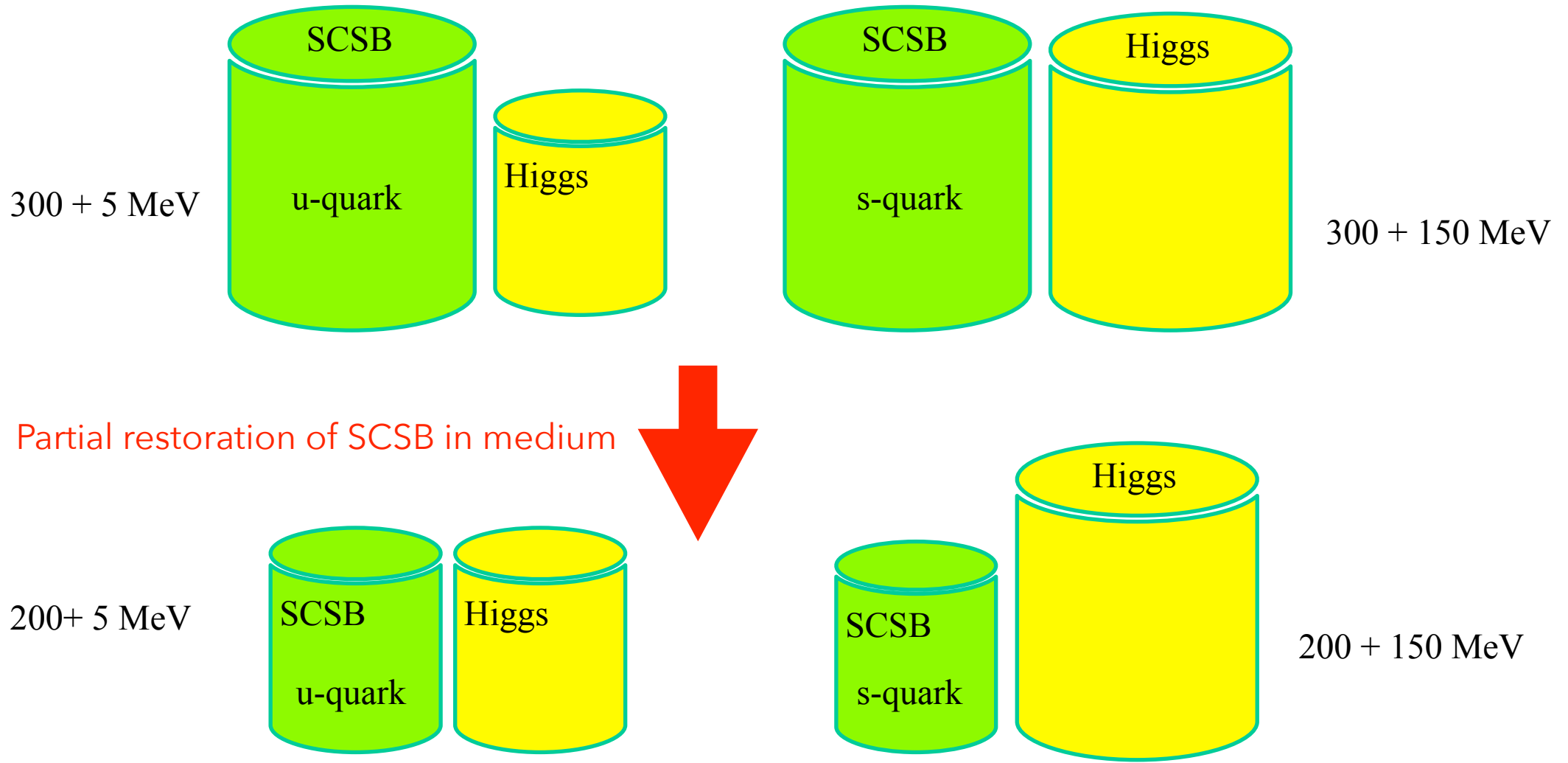
- Intuitive explanation: Pion



Same total mass-reduction rate for u (33%) and d (33%)

# Numerical results

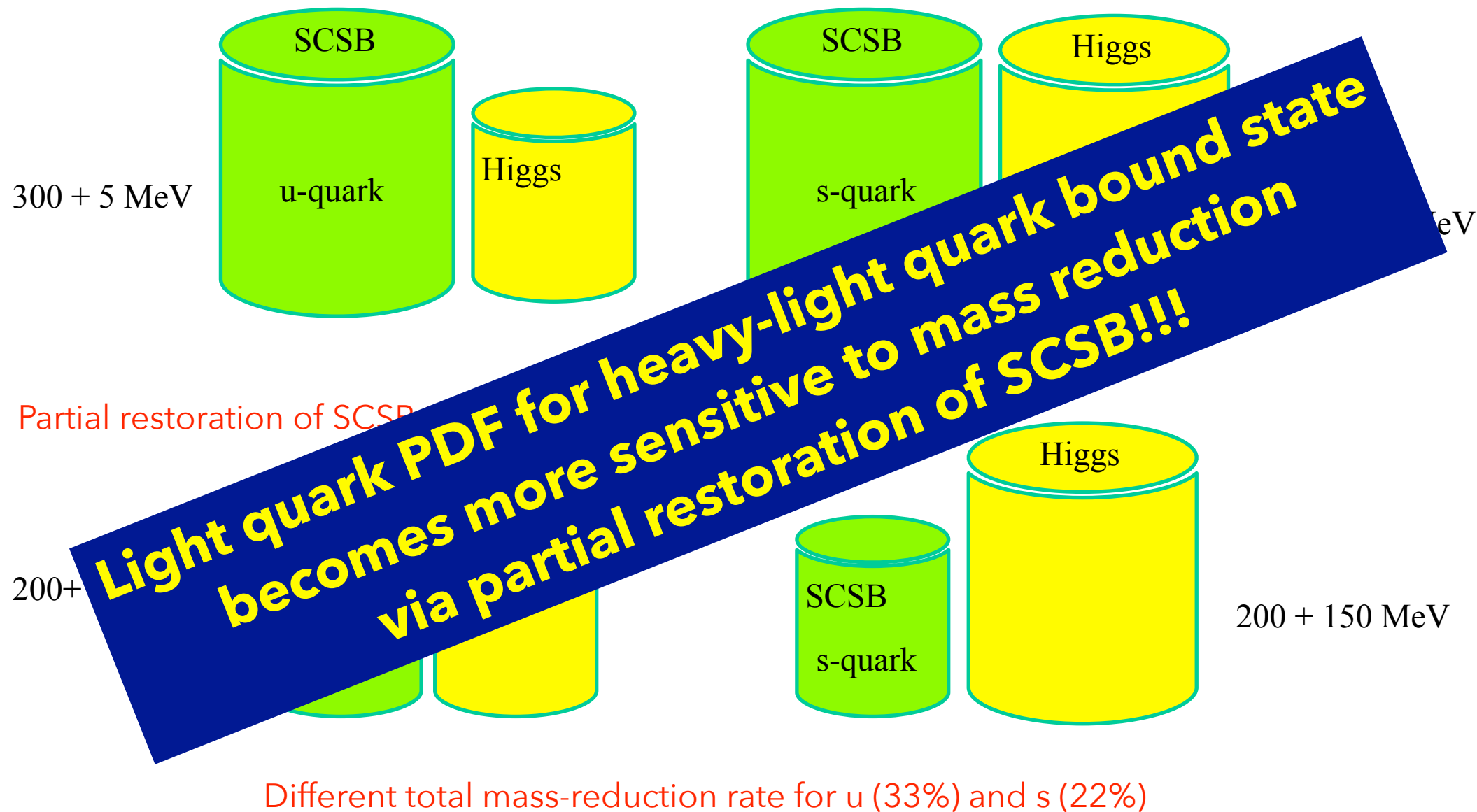
- Intuitive explanation: Kaon



Different total mass-reduction rate for u (33%) and s (22%)

# Numerical results

- Intuitive explanation: Kaon



# Summary and outlook

- qPDF and gPDF for pseudo-scalar meson investigated in medium
- Medium-modified NJL model employed: NJL-SNM in LFO
- gPDF from NS qPDF  $\otimes$  gPDF combined diff. Eq.
- (u,d)PDF-SNM and gPDF-SNM  $\sim$  vacuum ones for pion
- For kaon, sPDF-SNM and gPDF-SNM  $\sim$  vacuum ones, not uPDF-SNM
- Same and different mass reduction via SCSB for pion and kaon
- Competition between Higgs and SCSB mechanisms in PDF
- Important implications in heavy-light bound state: More obvious for  $B^{(*)}, D^{(*)}$  ?

More works on the way, please, stay tuned!!!



# Thank you for your attention!!

Supported by the National Research Foundation of Korea (NRF) grants:  
No. 2018R1A5A1025563 and No. 2022R1A2C1003964

