The elastic $\pi^+ A$ scattering with density effects

Hyeon-dong Han[1,2], Parada Hutauruk[1], Seung-il Nam[1,2]

[1]Pukyong National University (PKNU)
[2]Center for Extreme Nuclear Matters (CENuM)

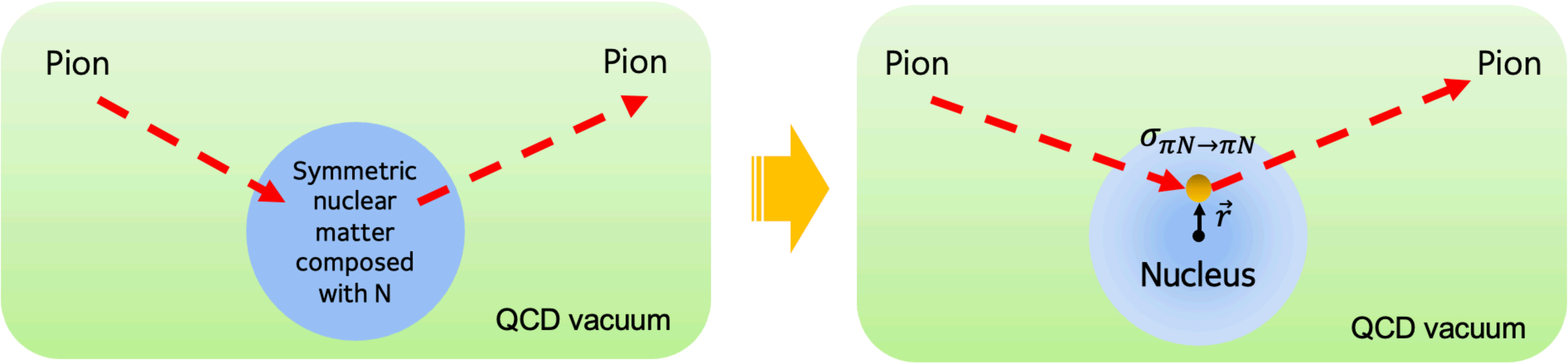
Table of contents

I. Motivation

We studied about the elastic $\pi^+ p$ scattering at finite baryon density.

[1] https://doi.org/10.48550/arXiv.2112.11060

And our next study is the elastic $\pi^+ A$ scattering at finite baryon density.

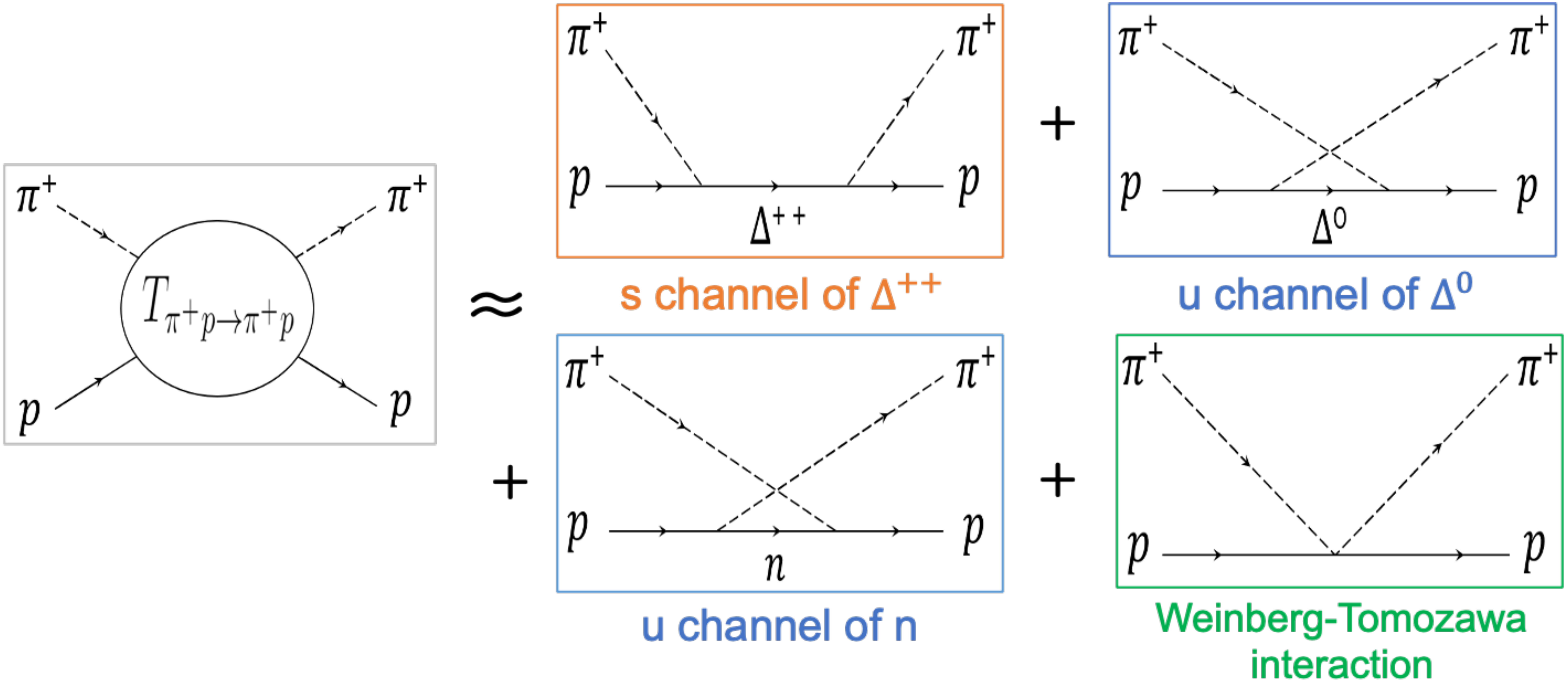


the elastic $\pi^+ p$ scattering at finite baryon density
Using Quark-Meson Coupling (QMC) Model
(Considered with nuclear matter)

the elastic $\pi^+ A$ scattering at finite baryon density
Using Eikonal Glauber Model
(Considered with finite nuclei)

II. Elastic $\pi^+ p$ scattering at finite baryon density

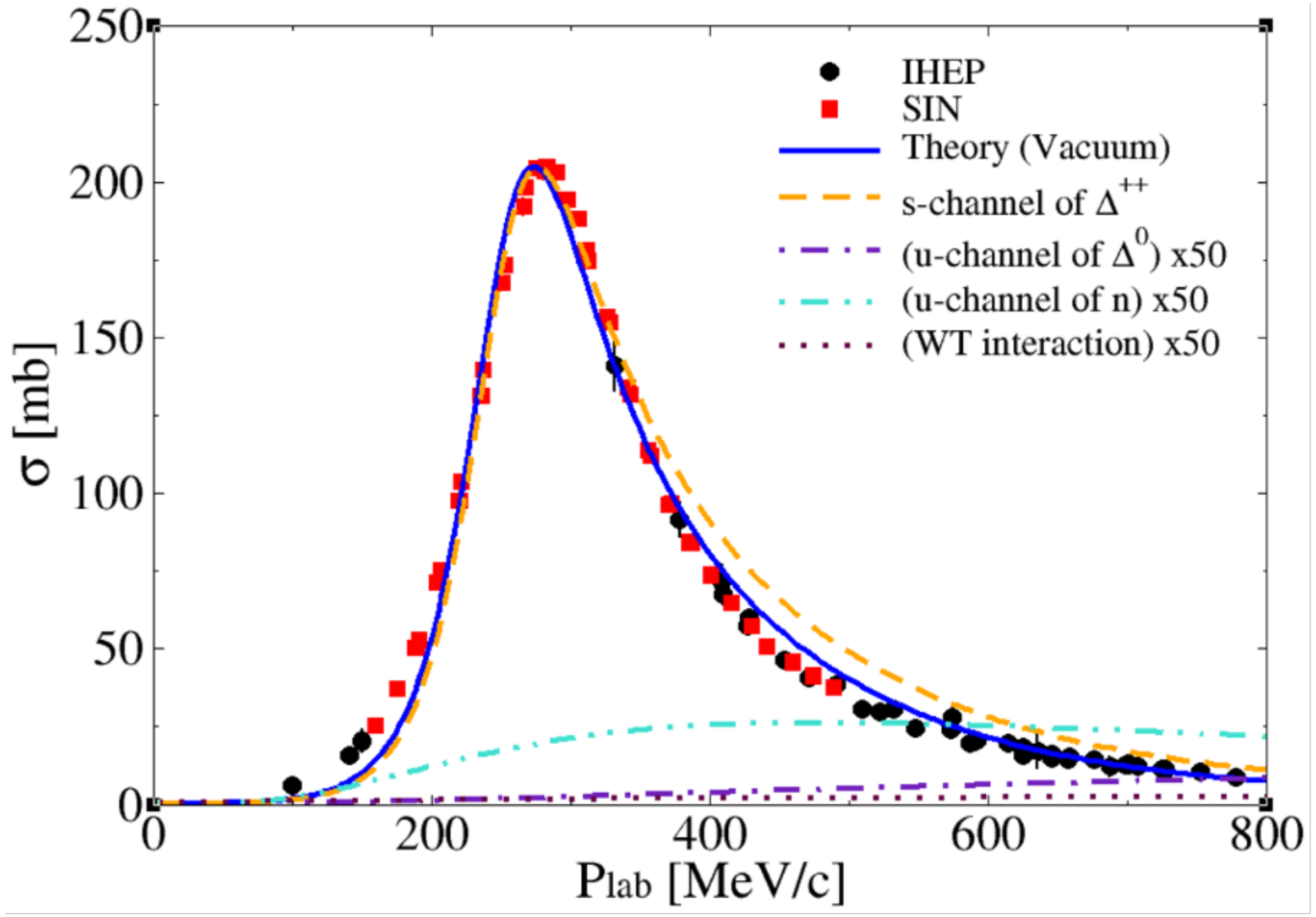
We used the effective Lagrangians for $\pi^+ p$ elastic scattering at tree level for each contributions.

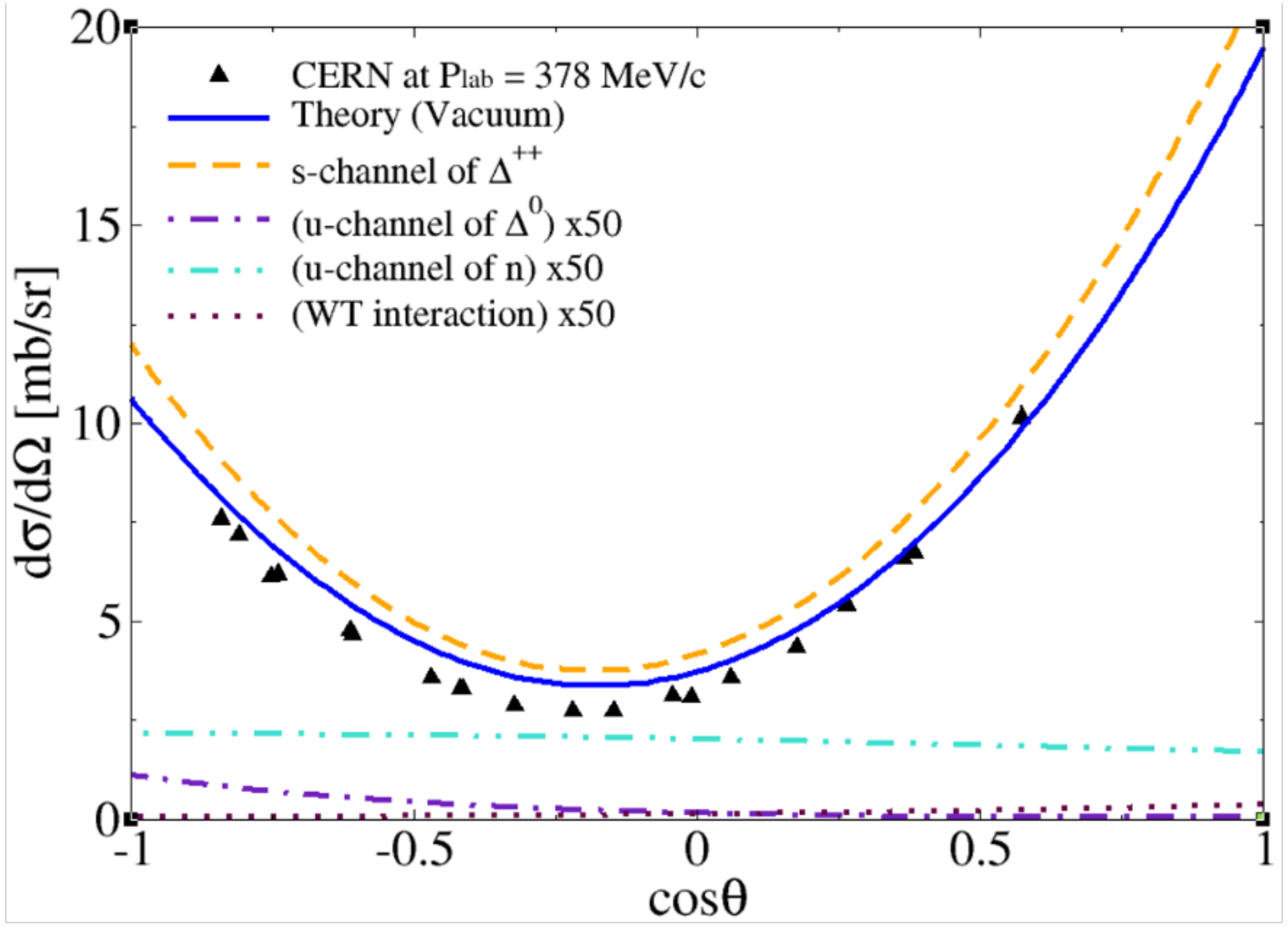


$$\boxed{\mathcal{M}_{\pi^+ p} \approx \mathcal{M}_s^{\Delta^{++}} + \mathcal{M}_u^{\Delta^0} + \mathcal{M}_u^{n} + \mathcal{M}_{WT}}$$

II. Elastic $\pi^+ p$ scattering at finite baryon density

O The TCS & DCS as a function of $cos\theta$ of the elastic $\pi^+ p$ scattering in vacuum





Delta mass (M_Δ) : 1215.5 MeV
Delta decay width (Γ_Δ^0) : 94.0 MeV

Delta Breit-Wigner mass (M_Δ^0) : 1232 MeV
Delta Breit-Wigner width (Γ_Δ^0) : 117 MeV

II. Elastic $\pi^+ p$ scattering at finite baryon density

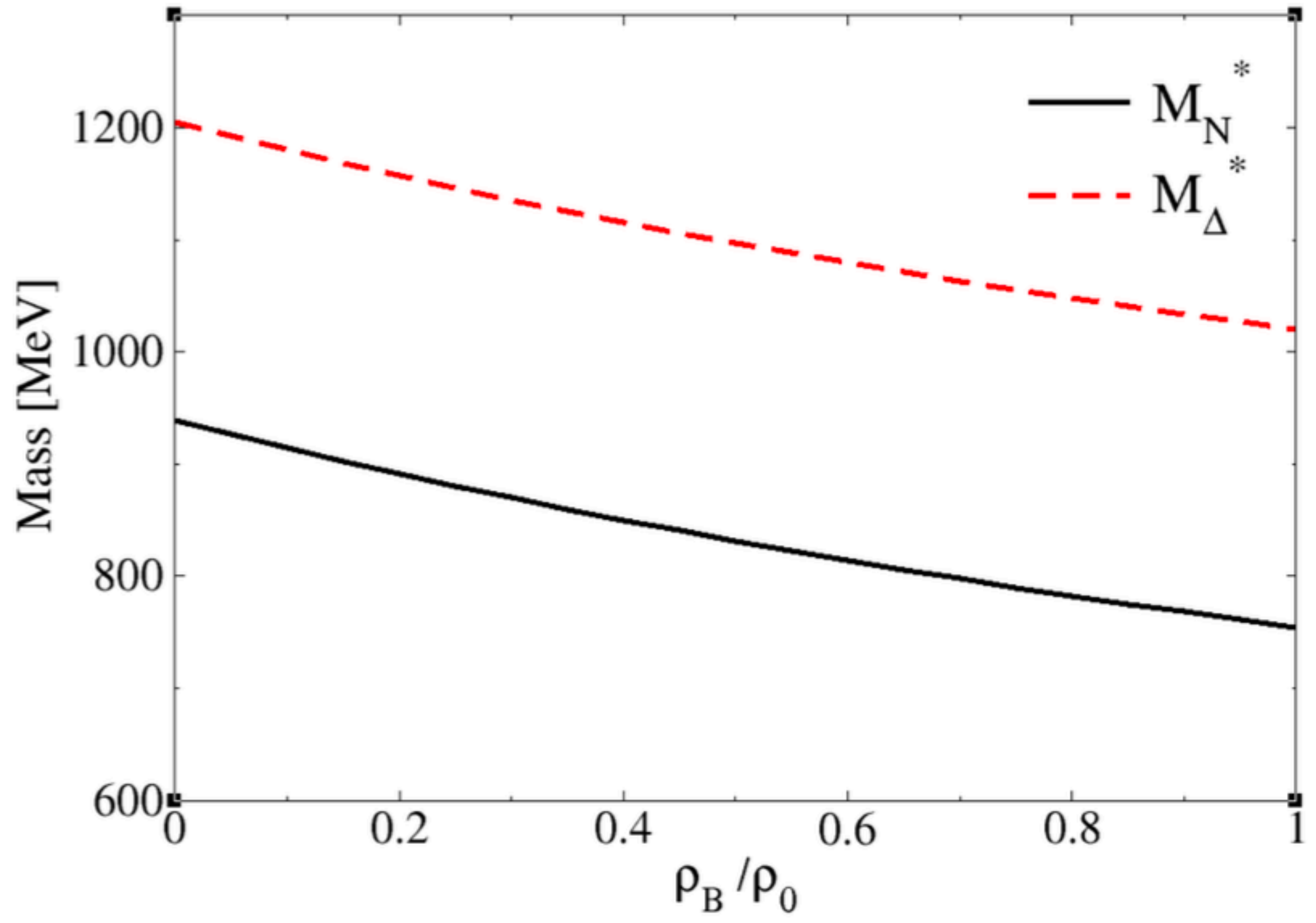
At finite density, hadron structure properties change (mass, decay width..)
so we apply the Quark-meson coupling (QMC) model to include medium effect.

O Quark-Meson Coupling (QMC) Model

σ : The scalar mean field

$$\boxed{M^*_\Delta = M_\Delta - g_\sigma \sigma, \qquad M^*_N = M_N - g_\sigma \sigma}$$

Effective baryon masses are calculated by Dr. Parada.
(We fixed the pion mass & coupling constant in this work)



Nucleon and Δ baryon masses decrease as the baryon density increases.

II. Elastic $\pi^+ p$ scattering at finite baryon density

O In-medium Δ decay width Γ^*_Δ

$q(m, m_1, m_2)$: The c.m. momentum

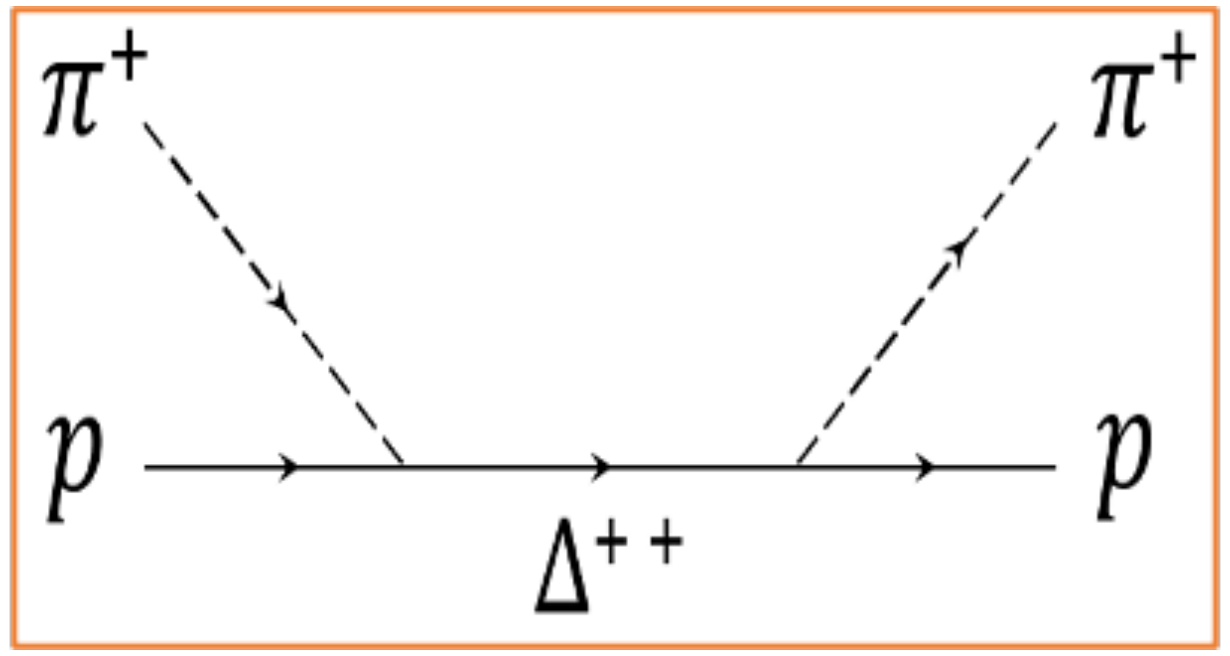
$$\Gamma^*_\Delta(\sqrt{s^*}) = \Gamma_{sp}\left(\frac{\rho_B}{\rho_0}\right) + \Gamma^0_\Delta \left[\frac{q(\sqrt{s^*}, m^*_N, m_\pi)}{q(m_\Delta, m_N, m_\pi)}\right]^3 \frac{m^*_\Delta}{\sqrt{s^*}} \frac{\beta_0^2 + q^2(m^*_\Delta, m^*_N, m_\pi)}{\beta_0^2 + q^2(\sqrt{s^*}, m^*_N, m_\pi)}$$

[2] A. B. Larionov and U. Mosel, The N N —>N Delta cross-section in nuclear matter, Nucl. Phys. A728, 135-164 (2003)

< Δ(1232) resonance contribution >

$$\mathscr{L}_{\pi N\Delta} = \frac{f_{\pi N\Delta}}{M_\pi}\bar{\Delta}^\mu \boldsymbol{S}^\dagger \partial_\mu \boldsymbol{\pi} N + h.c.$$

[3] A. M. Gasparyan, J. Haidenbauer, C. Hanhart, and J. Speth, Phys. Rev. C 68, 045207



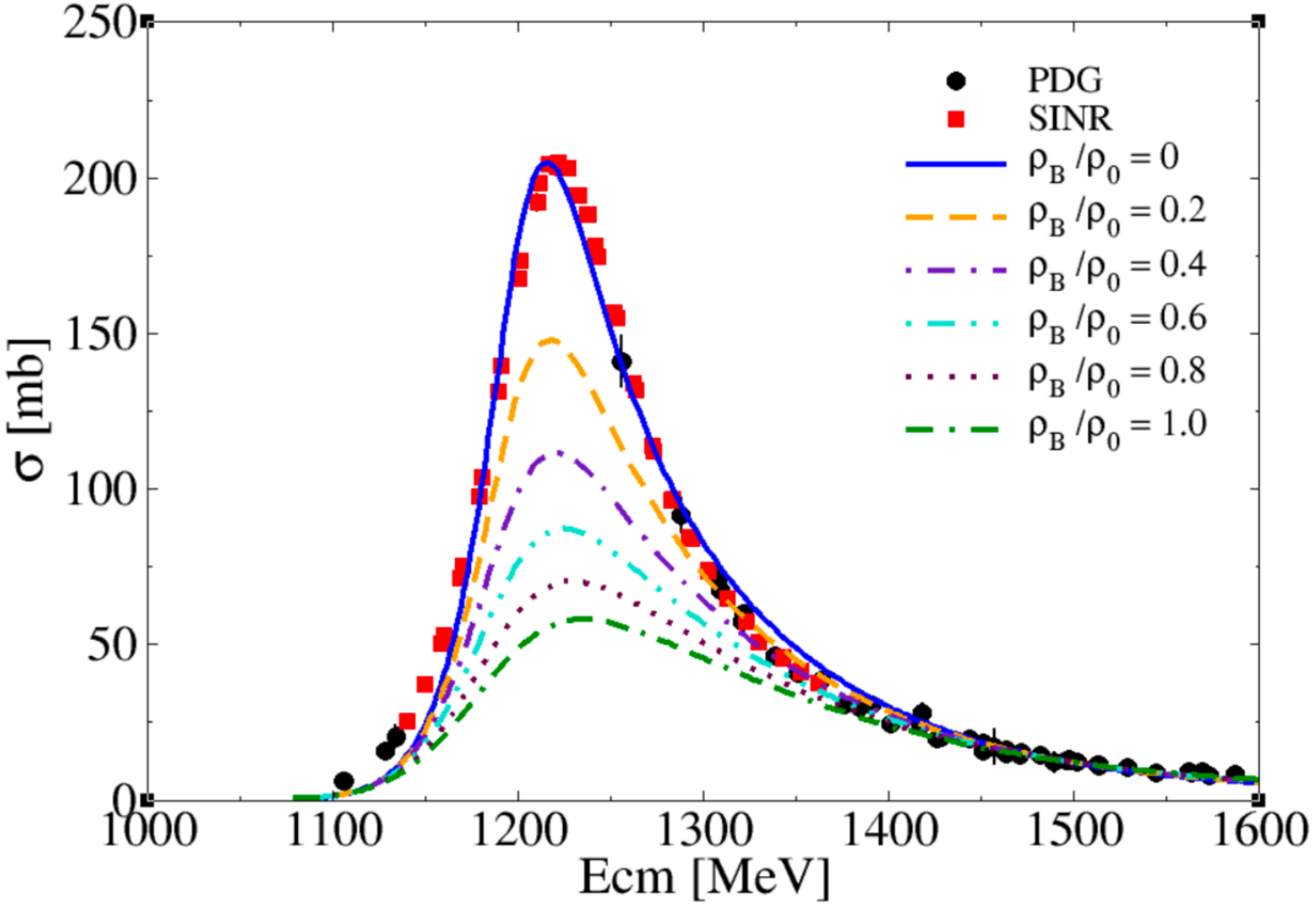
s channel of Δ^{++}

$$\therefore \mathcal{M}_s^{\Delta^{++}} = i\frac{f^2_{\pi N\Delta}}{M^2_\pi}\bar{u}(p')\underline{k'_\mu} G^{\mu\nu}(p+k)\underline{k_\nu} u(p)$$

$$G^{\mu\nu}(q) = i\frac{(\not{q} + M_\Delta)}{q^2 - M^2_\Delta + iM_\Delta\Gamma_\Delta} \times \left[-g^{\mu\nu} + \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{2q^\mu q^\nu}{3M^2_\Delta} - \frac{q^\mu\gamma^\nu - q^\nu\gamma^\mu}{3M_\Delta}\right]$$

II. Elastic π^+p scattering at finite baryon density

O The π^+p total cross-section at finite baryon density



[4] Snigdha Ghosh, Sourav Sarkar, and Sukanya Mitra
Phys. Rev. D 95, 056010

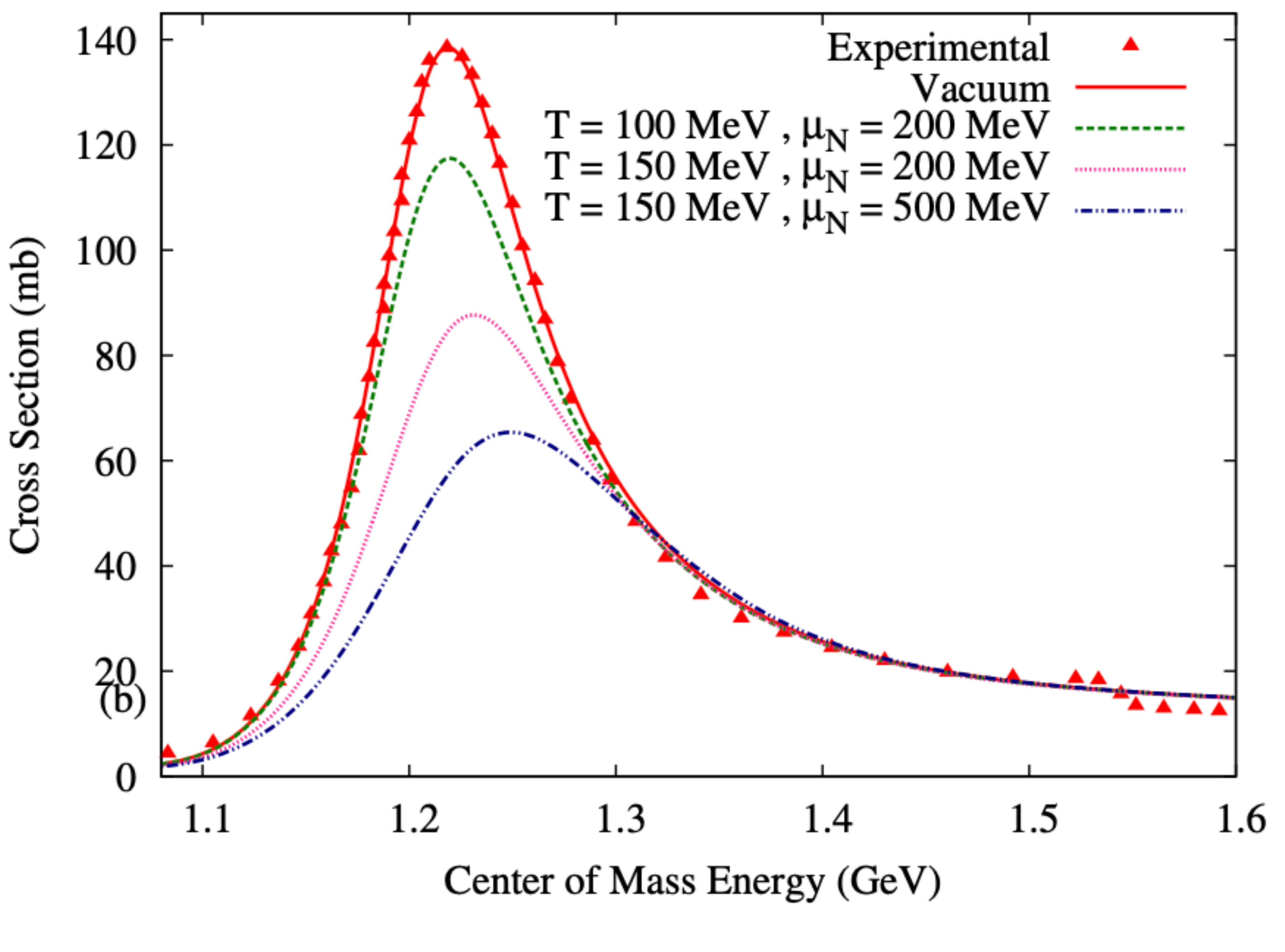
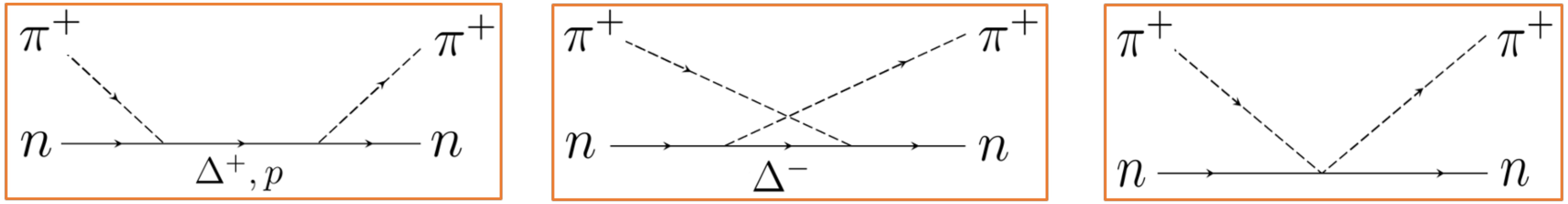


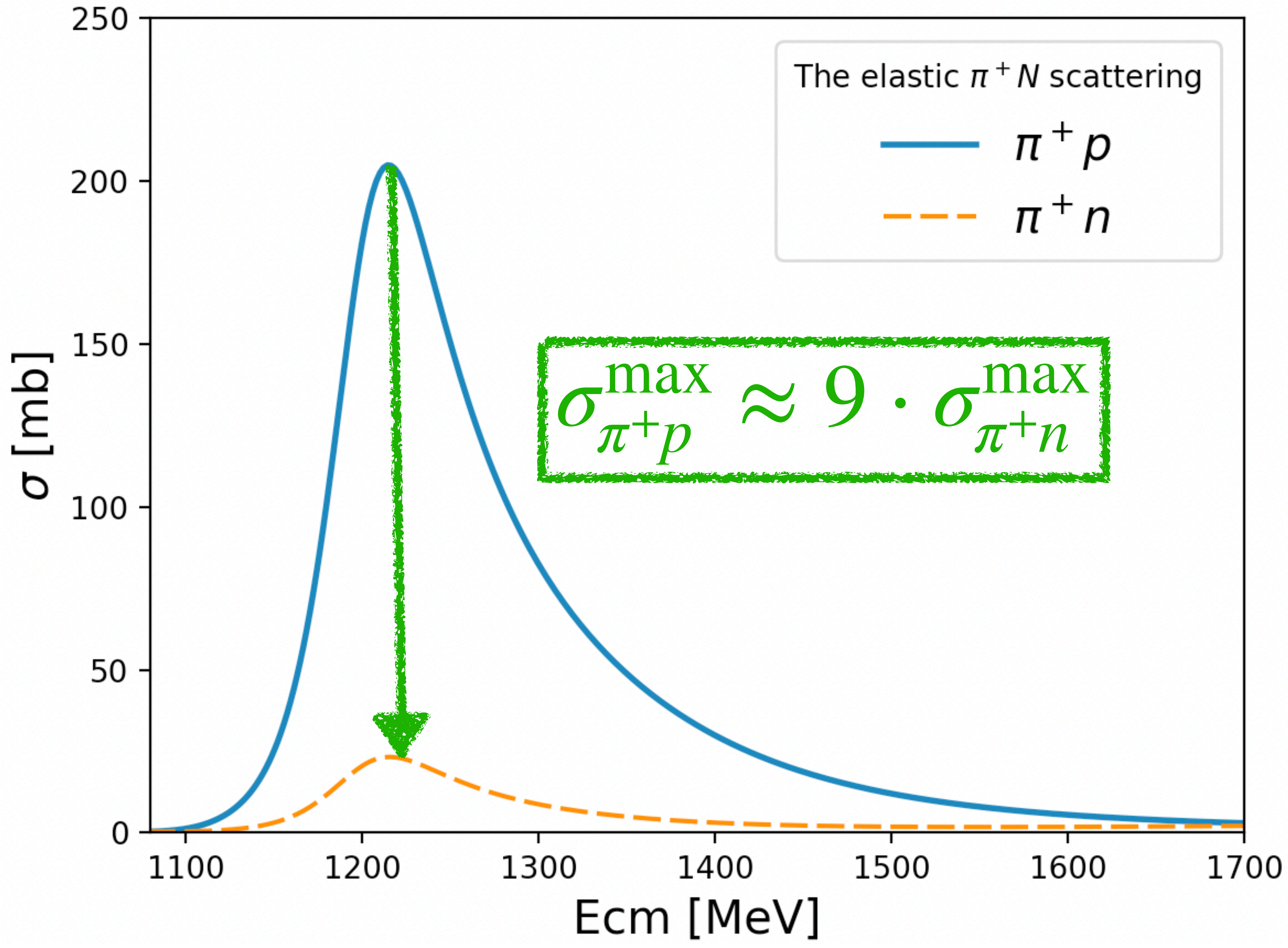
FIG. 8. The π-N elastic scattering cross section with medium effects.

The effective baryon mass m_B^* & in-medium delta decay width Γ_Δ^* make the total cross section decreased in nuclear medium.

III. Elastic $\pi^+ A$ scattering at finite baryon density

O The total cross-section of the elastic $\pi^+ n$ scattering





O Isospin factor of $\pi^+ n$ channel

s channel of Δ^+ : 1/3
s channel of p : 2/3
u channel of Δ^- : 1

III. Elastic $\pi^+ A$ scattering at finite baryon density

From the Glauber model, total cross section of the elastic $\pi^+ A$ scattering is given by

$$\boxed{\underline{\sigma_{\pi^+ A \to \pi^+ A}(\sqrt{s})} = \int d^2\boldsymbol{b} \left[1 - \exp[-\underline{\tilde{\sigma}_{\pi^+ N \to \pi^+ N}(\sqrt{s}, \rho_A^{(n,p)})} \cdot T_A^{(n,p)}(\boldsymbol{b})]\right]}$$

III. Elastic $\pi^+ A$ scattering at finite baryon density

From the Glauber model, total cross section of the elastic $\pi^+ A$ scattering is given by

$$\boxed{\underline{\sigma_{\pi^+A\to\pi^+A}(\sqrt{s})} = \int d^2\boldsymbol{b}\left[1 - \exp[-\underline{\tilde{\sigma}_{\pi^+N\to\pi^+N}(\sqrt{s}, \rho_A^{(n,p)})} \cdot T_A^{(n,p)}(\boldsymbol{b})]\right]}$$

$$T_A^{(n,p)}(\boldsymbol{b}) = \int_{-\infty}^{\infty} \rho_A^{(n,p)}(\sqrt{|\boldsymbol{b}|^2 + z^2})dz, \quad \boxed{\rho_A^{(n,p)}(r) = \frac{\rho_0^A\left[1 + c(r/r_A)^2\right]}{1 + \exp[(r - r_A)/d]}} \cdot \frac{(A - Z, Z)}{A} \quad \left(r = \sqrt{|\boldsymbol{b}|^2 + z^2}\right)$$

The Wood-Saxon density profile

Nucleus	A	Z	r_A [fm]	d [fm]	ρ_0^A [fm^{-3}]	c
He	4	2	1.01	0.327	0.2381	0.445
C	12	6	2.36	0.522	0.1823	−0.149
Au	197	79	6.38	0.535	0.1772	0
Pb	208	82	6.62	0.549	0.1700	0

[5] ATOMIC DATA AND NUCLEAR DATA TABLES 14, 479-508 (1974)

III. Elastic $\pi^+ A$ scattering at finite baryon density

From the Glauber model, total cross section of the elastic $\pi^+ A$ scattering is given by

$$\boxed{\underline{\sigma_{\pi^+A\to\pi^+A}(\sqrt{s})} = \int d^2\boldsymbol{b}\left[1-\exp[-\underline{\tilde{\sigma}_{\pi^+N\to\pi^+N}(\sqrt{s},\rho_A^{(n,p)})}\cdot T_A^{(n,p)}(\boldsymbol{b})]\right]}$$

$$T_A^{(n,p)}(\boldsymbol{b}) = \int_{-\infty}^{\infty}\rho_A^{(n,p)}(\sqrt{|\boldsymbol{b}|^2+z^2})dz,\quad \boxed{\rho_A^{(n,p)}(r) = \frac{\rho_0^A\left[1+c(r/r_A)^2\right]}{1+\exp[(r-r_A)/d]}}\cdot\frac{(A-Z,Z)}{A}\quad \left(r=\sqrt{|\boldsymbol{b}|^2+z^2}\right)$$

The Wood-Saxon density profile

Nucleus	A	Z	r_A [fm]	d [fm]	ρ_0^A [fm^{-3}]	c
He	4	2	1.01	0.327	0.2381	0.445
C	12	6	2.36	0.522	0.1823	−0.149
Au	197	79	6.38	0.535	0.1772	0
Pb	208	82	6.62	0.549	0.1700	0

[5] ATOMIC DATA AND NUCLEAR DATA TABLES 14, 479-508 (1974)

$$\boxed{{}^*\ \tilde{\sigma}_{\pi^+N\to\pi^+N} = \sum_{N=n,p}\bar{\sigma}_{\pi^+N\to\pi^+N} = \left(\frac{A-Z}{A}\right)\cdot\sigma_{\pi^+n\to\pi^+n} + \left(\frac{Z}{A}\right)\cdot\sigma_{\pi^+p\to\pi^+p}}$$

III. Elastic $\pi^+ A$ scattering at finite baryon density

From the Glauber model, total cross section of the elastic $\pi^+ A$ scattering is given by

$$\boxed{\underline{\sigma_{\pi^+A\to\pi^+A}(\sqrt{s})} = \int d^2\boldsymbol{b}\left[1 - \exp[-\underline{\tilde{\sigma}_{\pi^+N\to\pi^+N}(\sqrt{s}, \rho_A^{(n,p)})} \cdot T_A^{(n,p)}(\boldsymbol{b})]\right]}$$

$$T_A^{(n,p)}(\boldsymbol{b}) = \int_{-\infty}^{\infty} \rho_A^{(n,p)}(\sqrt{|\boldsymbol{b}|^2 + z^2})dz, \quad \boxed{\rho_A^{(n,p)}(r) = \frac{\rho_0^A\left[1 + c(r/r_A)^2\right]}{1 + \exp[(r - r_A)/d]}} \cdot \frac{(A-Z, Z)}{A} \quad \left(r = \sqrt{|\boldsymbol{b}|^2 + z^2}\right)$$

The Wood-Saxon density profile

Nucleus	A	Z	r_A [fm]	d [fm]	ρ_0^A [fm^{-3}]	c
He	4	2	1.01	0.327	0.2381	0.445
C	12	6	2.36	0.522	0.1823	−0.149
Au	197	79	6.38	0.535	0.1772	0
Pb	208	82	6.62	0.549	0.1700	0

[5] ATOMIC DATA AND NUCLEAR DATA TABLES 14, 479-508 (1974)

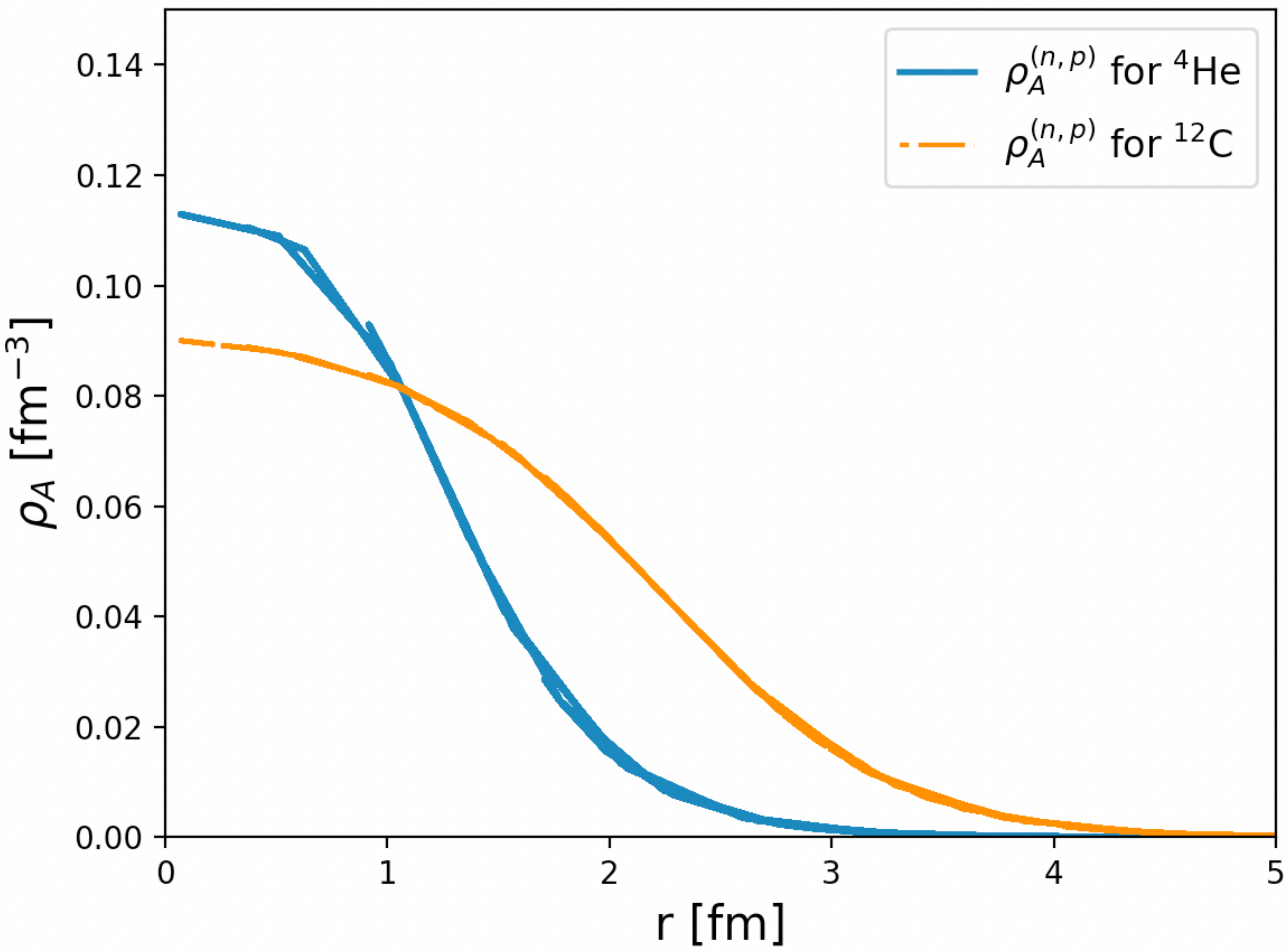
$$\boxed{{}^{*}\ \tilde{\sigma}_{\pi^+N\to\pi^+N} = \sum_{N=n,p} \bar{\sigma}_{\pi^+N\to\pi^+N} = \left(\frac{A-Z}{A}\right) \cdot \sigma_{\pi^+n\to\pi^+n} + \left(\frac{Z}{A}\right) \cdot \sigma_{\pi^+p\to\pi^+p}}$$

And $\rho_A^{(n,p)}(r)$ is normalized with the atomic mass number A by

$$A = \sum_{N=n,p} \int dz d^2\boldsymbol{b}\rho_A^{(n,p)}\left(\sqrt{|\boldsymbol{b}|^2 + z^2}\right) = \sum_{N=n,p} 2\pi \int_{-\infty}^{\infty} dz \int_0^{\infty} bdb\rho_A^{(n,p)}\left(\sqrt{|\boldsymbol{b}|^2 + z^2}\right)$$

III. Elastic $\pi^+ A$ scattering at finite baryon density

O Nuclear density distribution ρ_A as a function of r



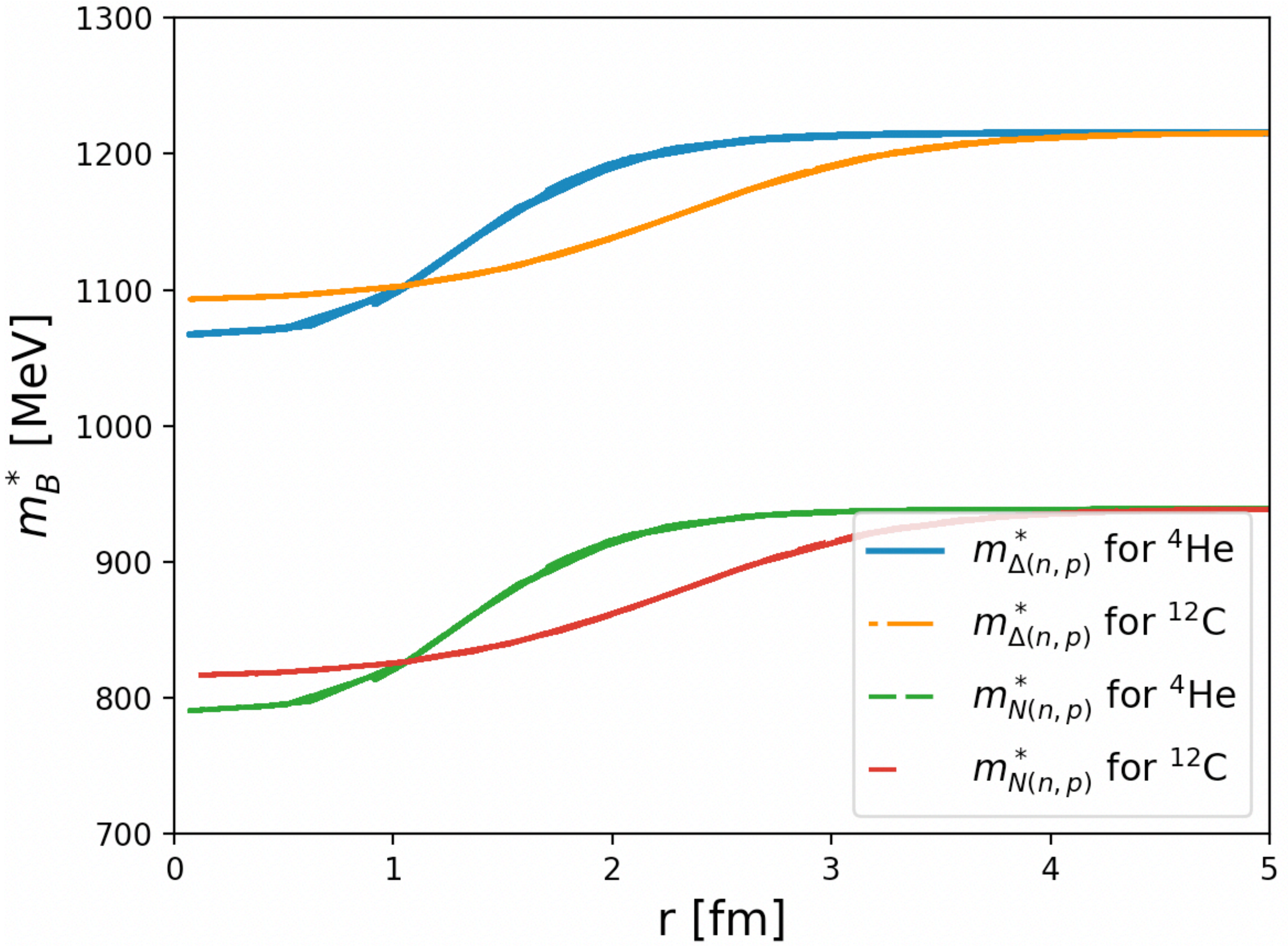
$$\rho_A^{(n,p)}(r) = \frac{\rho_0^A\left[1 + c(r/r_A)^2\right]}{1 + \exp[(r - r_A)/d]} \cdot \frac{(A - Z, Z)}{A}$$

Nucleus	A	Z	r_A [fm]	d [fm]	ρ_0^A [fm^{-3}]	c
He	4	2	1.01	0.327	0.2381	0.445
C	12	6	2.36	0.522	0.1823	−0.149
Au	197	79	6.38	0.535	0.1772	0
Pb	208	82	6.62	0.549	0.1700	0

[5] ATOMIC DATA AND NUCLEAR DATA TABLES 14, 479-508 (1974)

III. Elastic $\pi^+ A$ scattering at finite baryon density

O Effective baryon mass m_B^* as a function of r



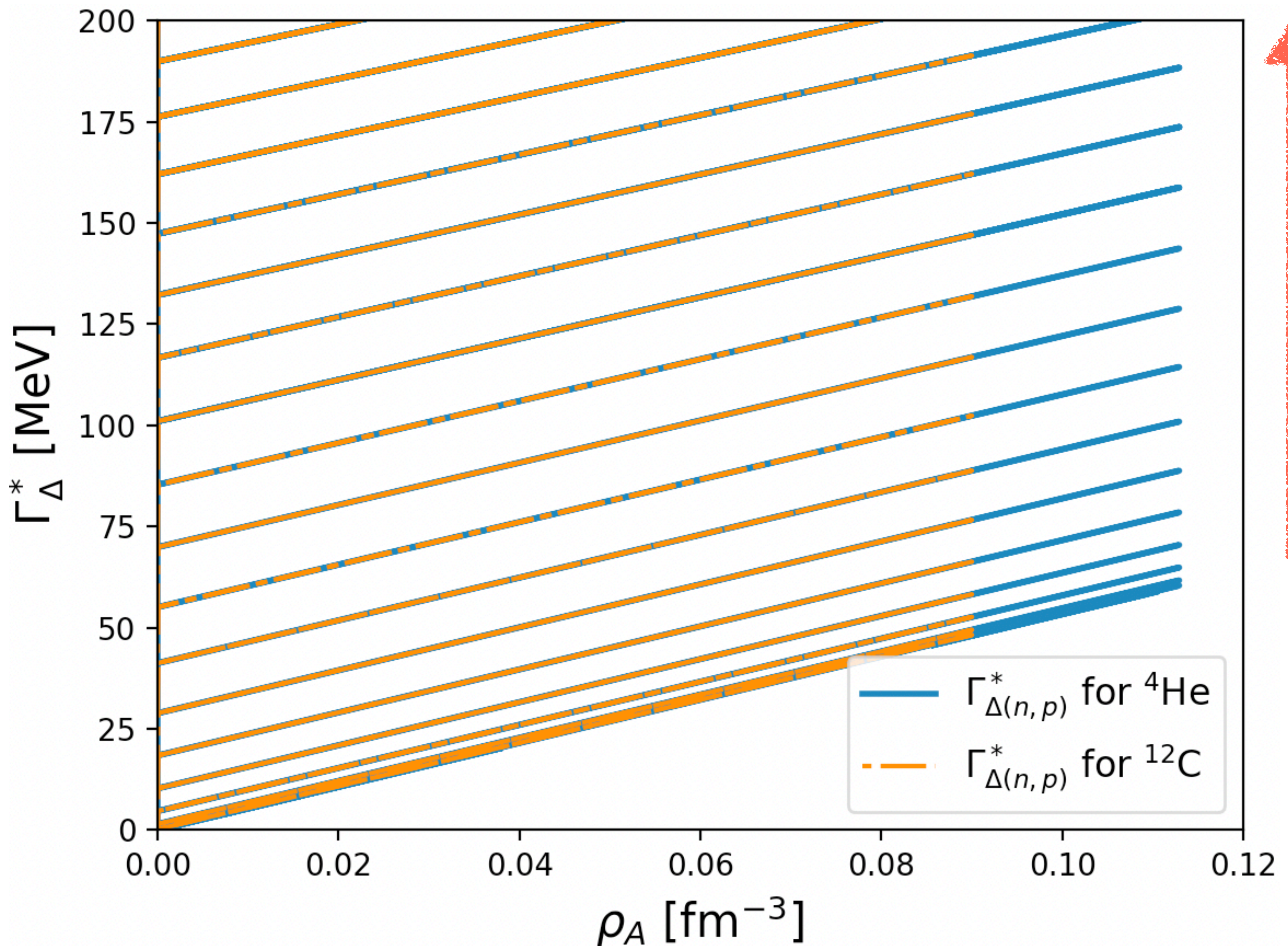
$$m_B^* = m_B + C_1 \rho_A^{(n,p)} + C_2 (\rho_A^{(n,p)})^2 \ (B = \Delta, N)$$

$$C_1 = -1543.08,\ C_2 = 2036.14$$

(C_1, C_2 are parameterized by the QMC model)

III. Elastic $\pi^+ A$ scattering at finite baryon density

O In-medium Δ decay width Γ^*_Δ as a function of ρ_A



Γ^*_Δ increases as $\sqrt{s^*}$ increases.

$$\Gamma^*_\Delta = \Gamma_{\rm sp}\left(\frac{\rho_A}{\rho_0}\right) + \Gamma^0_\Delta \left[\frac{q(m^*_N, m_\pi, \sqrt{s^*})}{q(m^*_N, m_\pi, m^*_\Delta)}\right]^3 \frac{m^*_\Delta}{\sqrt{s^*}} \frac{\beta_0^2 + q^2(m^*_N, m_\pi, m^*_\Delta)}{\beta_0^2 + q^2(m^*_N, m_\pi, \sqrt{s^*})}$$

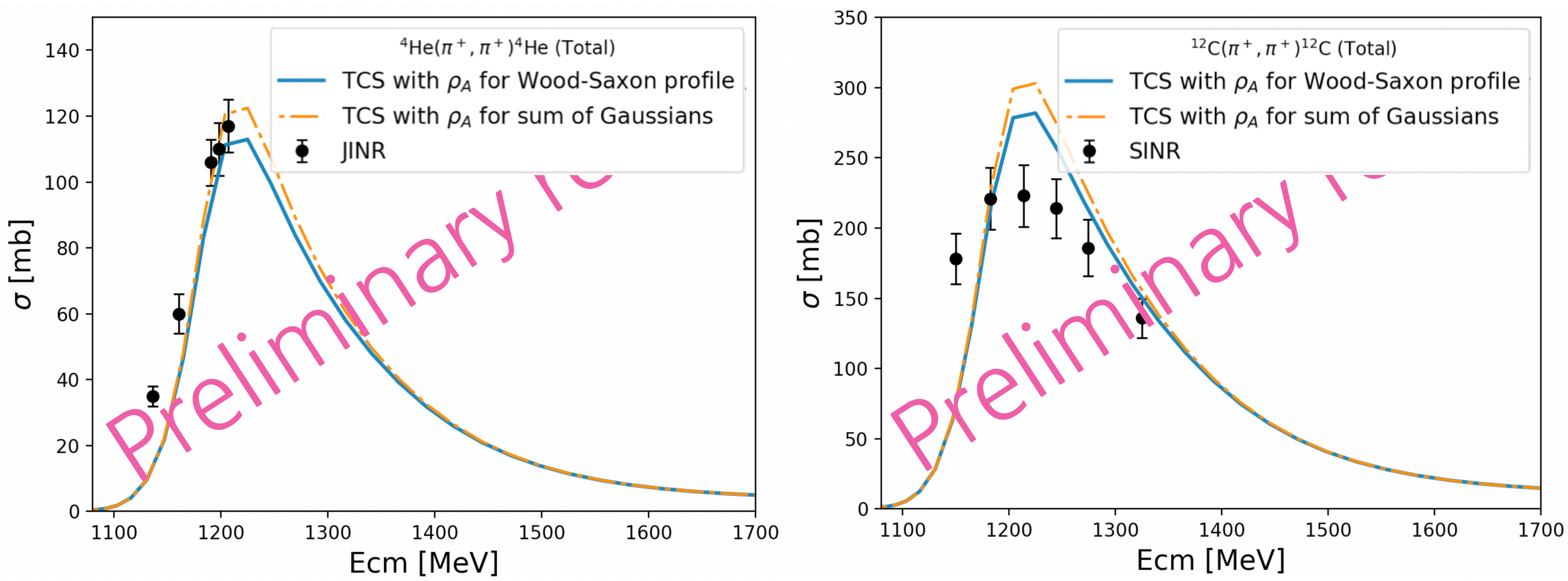
IV. Preliminary Results

Sum of Gaussians : Model-independent analysis by means of an expansion for ρ_A

$$\rho(r) = \sum_i A_i \left\{ \exp(-[(r-R_i)/\gamma]^2) + \exp(-[(r+R_i)/\gamma]^2) \right\} \text{ with } A_i = ZeQ_i/[2\pi^{3/2}\gamma^3(1+2R_i^2/\gamma^2)]\,.$$

[4] ATOMIC DATA AND NUCLEAR DATA TABLES 36, 495-536 (1987)

O The TCS of the elastic $\pi^+ A$ scattering for ^{4}He, ^{12}C



Summary

I. We studied about the elastic $\pi^+ p$ scattering at finite baryon density in symmetric nuclear matter using QMC model.

II. The elastic $\pi^+ A$ scattering at finite baryon density has been investigated using Glauber model with $\tilde{\sigma}_{\pi^+ N}(\sqrt{s}, \rho_A)$ which is the result of previous study.

III. We are planning to consider in-medium coupling constant such as $f^*_{\pi N \Delta}$ using QMC model and improve our preliminary results.

Thank you for your attention!

II. Elastic $\pi^+ p$ scattering at finite baryon density

O In-medium Δ decay width Γ^*_Δ

$$\Gamma^*_\Delta(\sqrt{s^*}) = \Gamma_{sp}\left(\frac{\rho_B}{\rho_0}\right) + \Gamma^0_\Delta \left[\frac{q(\sqrt{s^*}, m^*_N, m_\pi)}{q(m_\Delta, m_N, m_\pi)}\right]^3 \frac{m^*_\Delta}{\sqrt{s^*}} \frac{\beta_0^2 + q^2(m^*_\Delta, m^*_N, m_\pi)}{\beta_0^2 + q^2(\sqrt{s^*}, m^*_N, m_\pi)}$$

[2] A. B. Larionov and U. Mosel, The N N —>N Delta cross-section in nuclear matter, Nucl. Phys. A728, 135-164 (2003)

$\Gamma_{sp} = \Delta$-spreading width (=80MeV)

$\beta_0 =$ cut-off parameter (=200MeV/c)

$\rho_B =$ baryon density

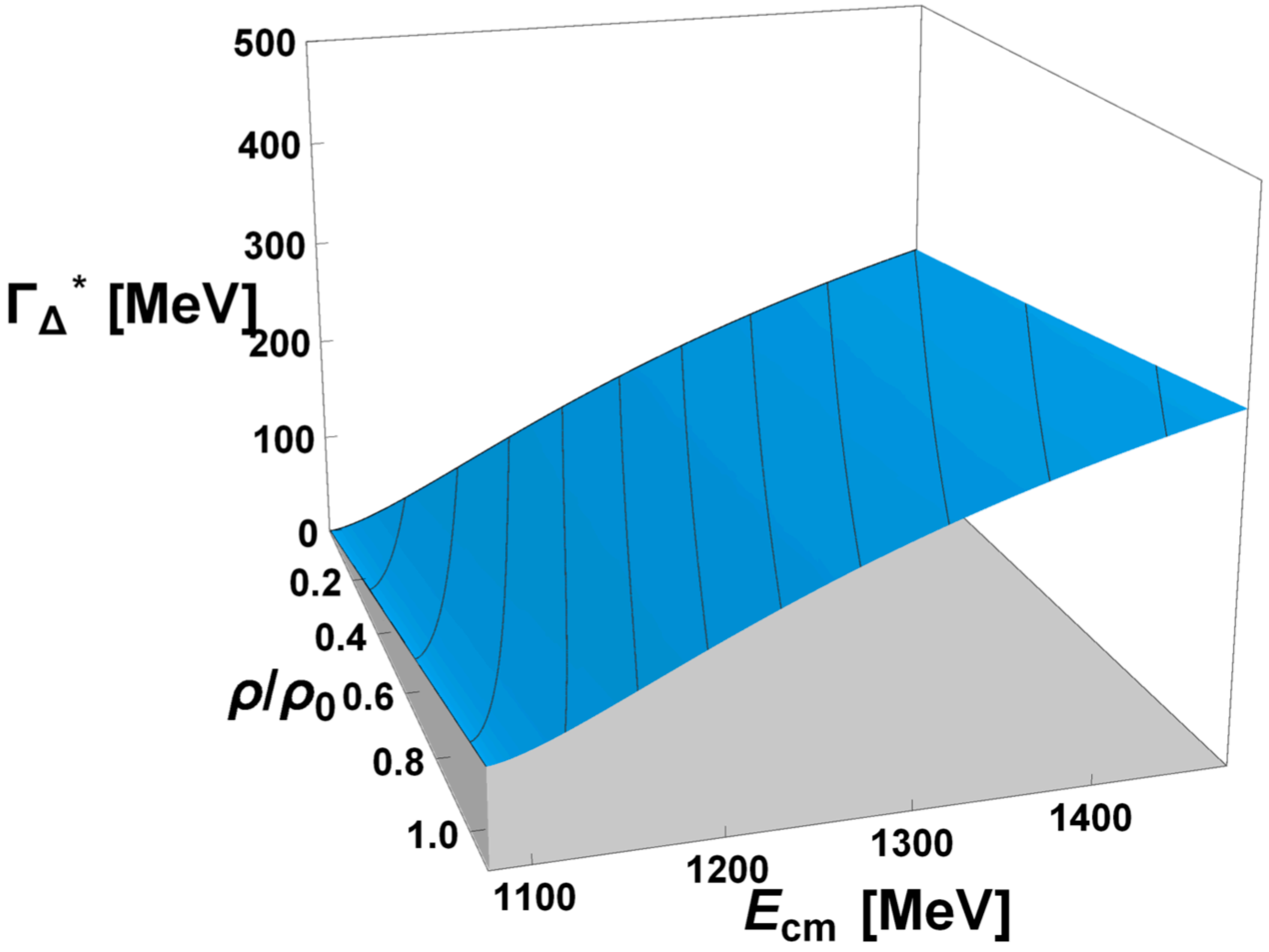
$\rho_0 =$ nuclear saturation density (=0.15fm^{-3})

$$q(m, m_1, m_2) = \sqrt{\frac{(m^2 + m_1^2 - m_2^2)^2}{4m^2} - m_1^2}\ :$$

the c.m. momentum of outgoing particles with masses m_1 and m_2 from the decay of a particle with mass m.

II. Elastic $\pi^+ p$ scattering at finite baryon density

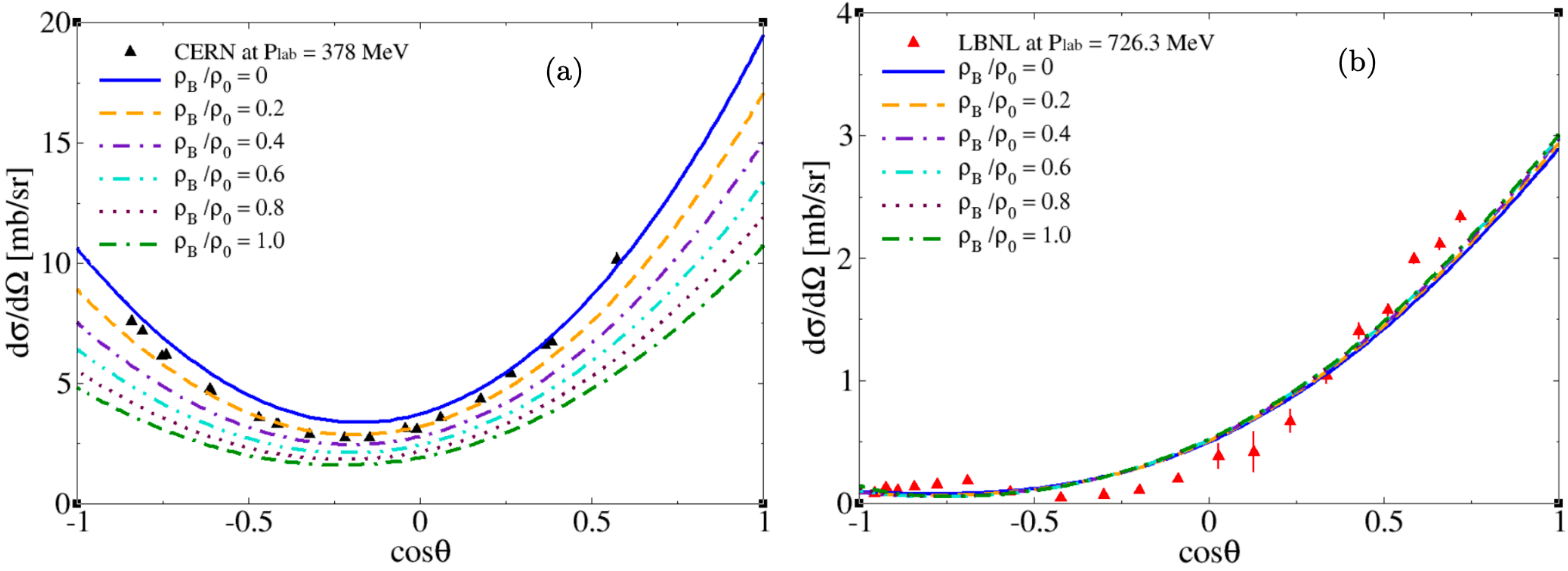
O In-medium Δ decay width Γ^*_Δ



The in-medium Δ decay width increases as the baryon density increases.

II. Elastic $\pi^+ p$ scattering at finite baryon density

O The differential cross-section of elastic $\pi^+ p$ scattering at finite baryon density



The differential cross-section of elastic $\pi^+ p$ scattering decreases as the baryon density increases.

(But in high $\mathrm{P_{lab}}$ region, the DCS increases a bit.)