

Elliptic and triangular flow of charmonia in heavy ion collisions

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Incheon, Korea

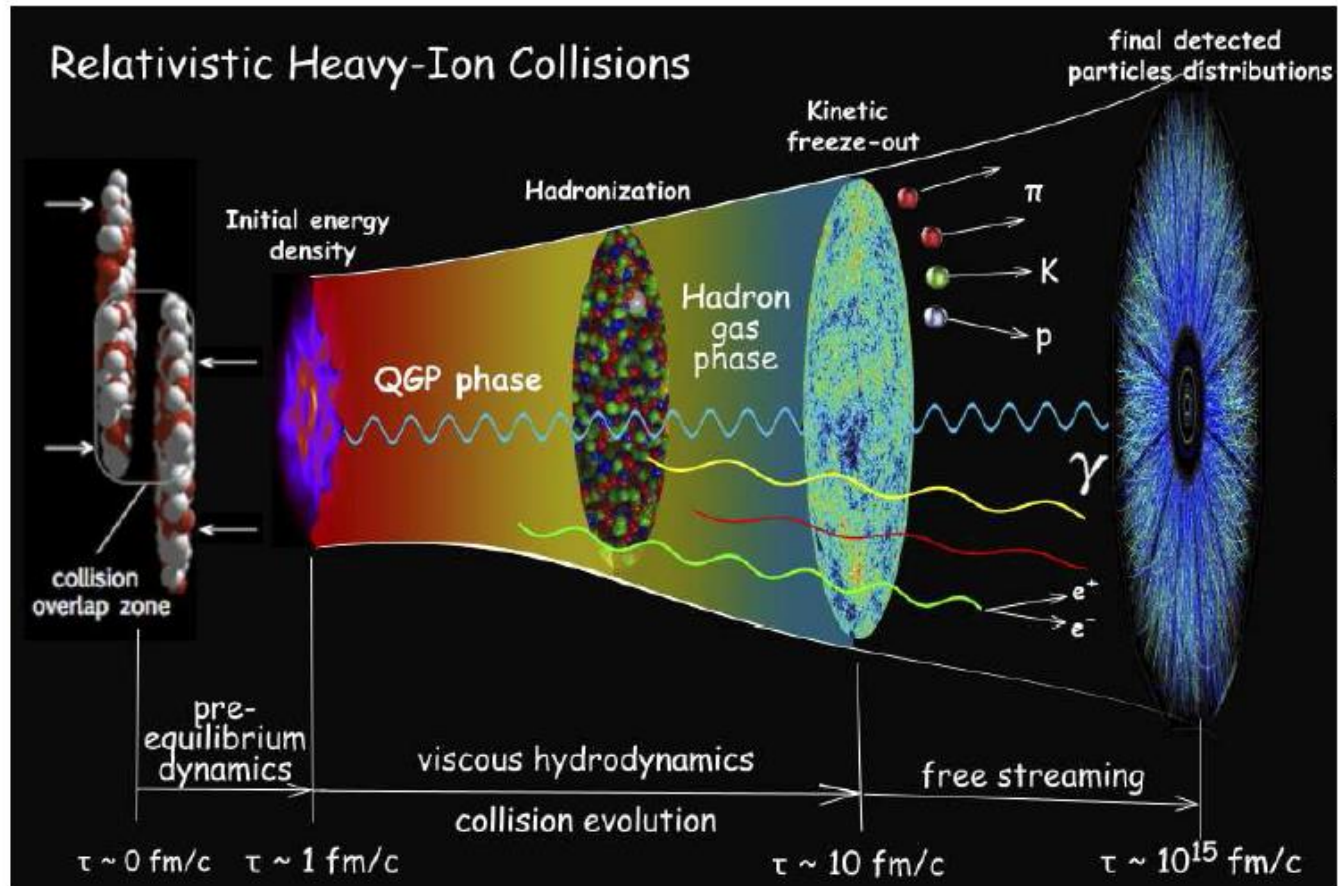


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Introduction

– Relativistic heavy ion collisions



U. W. Heinz, J. Phys. Conf. Ser. **455**, 012044 (2013)

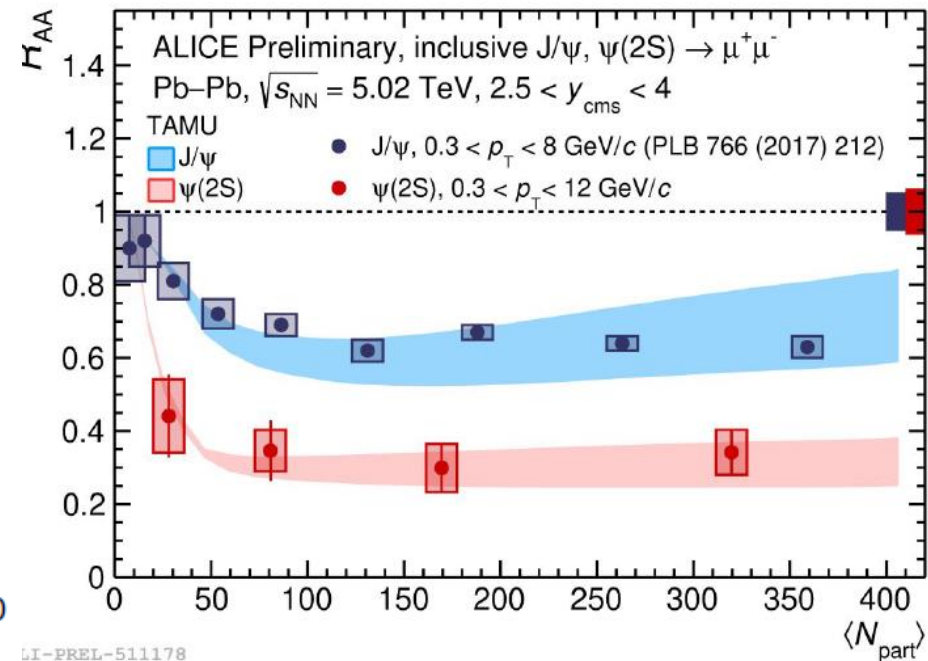
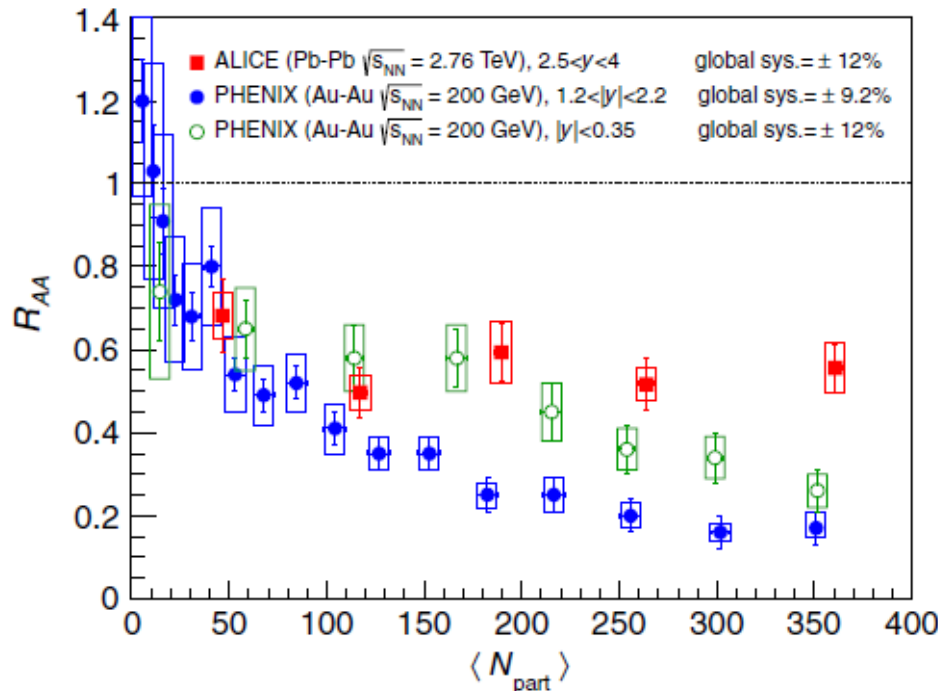
Charmonium states in heavy ion collisions

– Regeneration of charmonium states

1) The nuclear modification factor of charmonium states

B. Abelev et al, (ALICE Collaboration), Phys. Rev. Lett. **109**, 072301

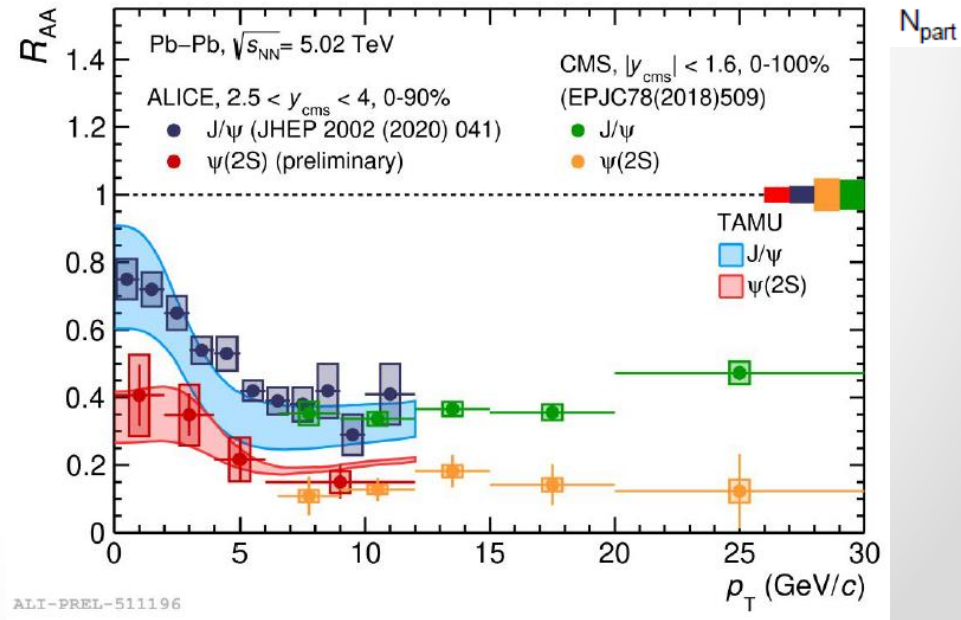
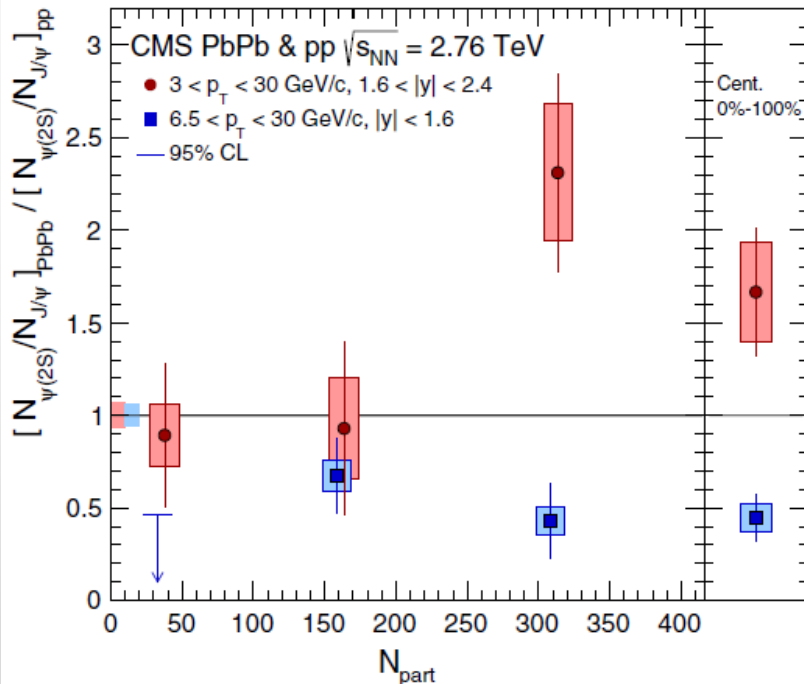
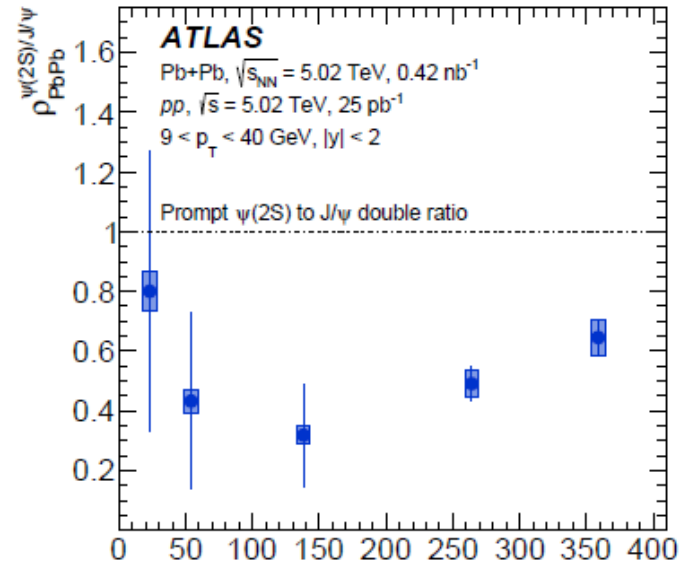
Jon-Are Saetre (Univ. of Bergen), Quark Matter 2022, Krakow, April 4-10



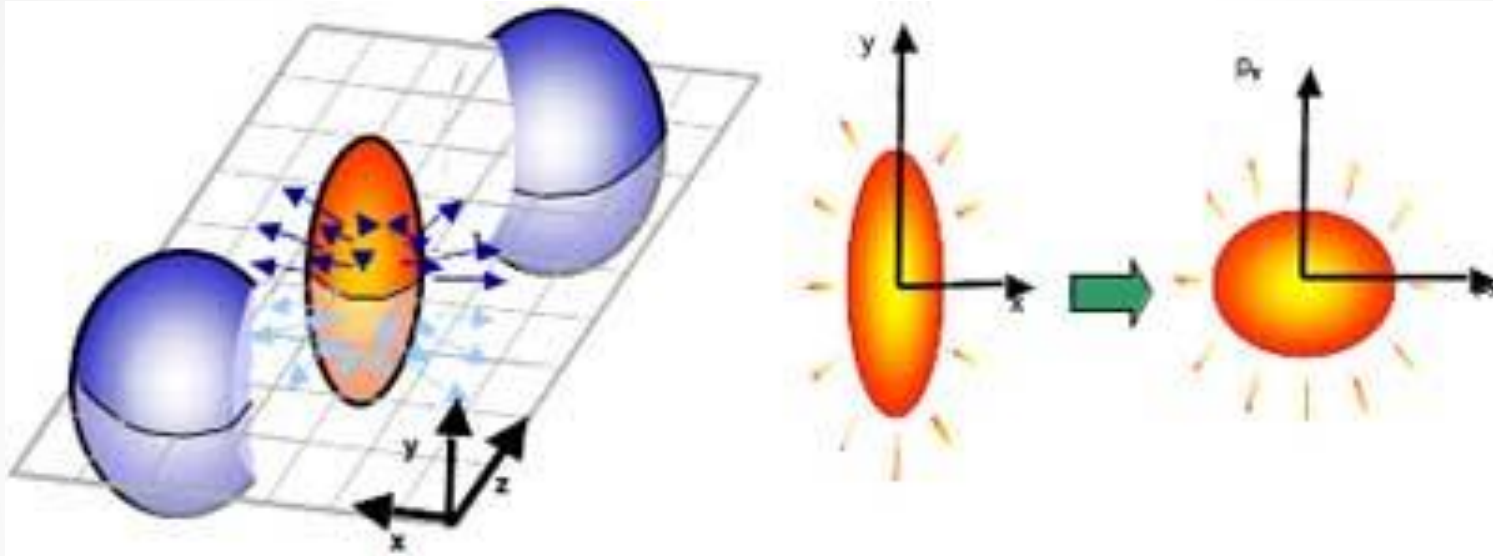
Nuclear modification factors of the J/ψ and $\psi(2S)$

1) The nuclear modification factor ratio between the J/ψ and $\psi(2S)$

V. Khachatryan et al, Phys. Rev. Lett. **113**, 262301 (2014)
 M. Aaboud et al, Eur. Phys. J. C **78**, 762 (2018)



- Non-central collisions, anisotropic flows



$$\begin{aligned}
 E \frac{d^3 N_q}{d^3 p} &= \frac{dN_q}{p_T dp_T d\varphi dy} = \frac{1}{2\pi} \frac{dN_q}{p_T dp_T dy} \left[1 + \sum_{n=1} 2v_{n,q}(p_T) \cos(n\varphi) \right] \\
 &= \frac{1}{2\pi} \frac{dN_q}{p_T dp_T dy} \left[1 + 2v_{1,q}(p_T) \cos(\varphi) + 2v_{2,q}(p_T) \cos(2\varphi) + \dots \right]
 \end{aligned}$$

A. M. Poskanzer and S. A. Voloshin, Phys. Rev. C **58**, 1671 (1998)

1) Quark number scaling of the elliptic flow

D. Molnar and S. A. Voloshin, Phys. Rev. Lett **91**, 092301 (2003)

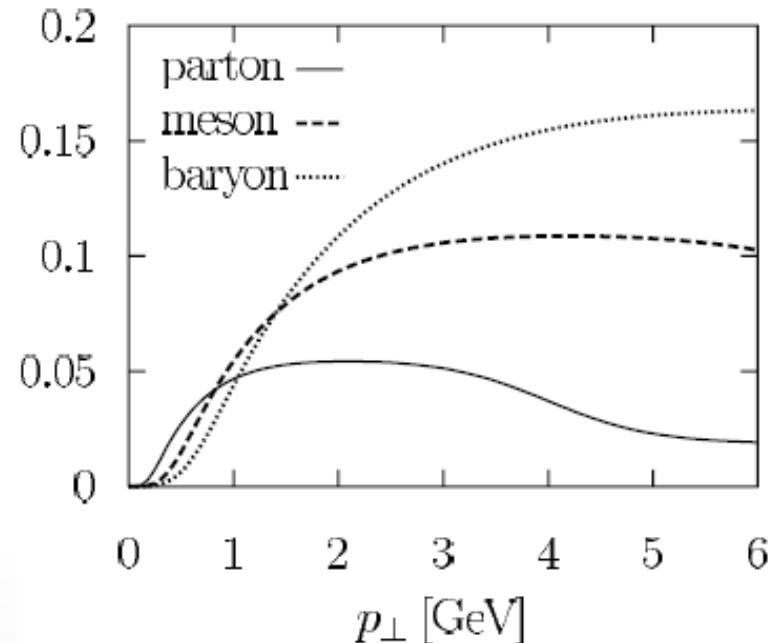
$$v_2(p_T) = \frac{\int d\phi \cos 2\phi \frac{d^2 N}{dp_T^2}}{\int d\phi \frac{d^2 N}{dp_T^2}} \quad \frac{dN_q}{p_T dp_T d\phi} = \frac{1}{2\pi} \frac{dN_q}{p_T dp_T} \left[1 + 2v_{2,q}(p_T) \cos(2\phi) \right]$$

Coalescence model predictions

$$v_{2,M}(p_T) = \frac{2v_{2,q}(p_T/2)}{1 + 2v_{2,q}^2(p_T/2)}$$

$$v_{2,B}(p_T) = \frac{3v_{2,q}(p_T/3) + 3v_{2,q}^3(p_T/3)}{1 + 6v_{2,q}^2(p_T/3)}$$

$$v_{2,h}(p_T) \approx nv_{2,q}\left(\frac{1}{n} p_T\right)$$



Yields and transverse momentum distributions of charmonium states

– Yields of hadrons in the coalescence model

V. Greco, C. M. Ko, and P. Levai, Phys. Rev. C **68**, 034904 (2003)

R. J. Freis, B. Muller, C. Nonaka, and S. Bass, Phys. Rev. C **68**, 044902 (2003)

$$N^{Coal} = g \int \left[\prod_{i=1}^n \frac{1}{g_i} \frac{p_i \cdot d\sigma_i}{(2\pi)^3} \frac{d^3 p_i}{E_i} f(x_i, p_i) \right] f^W(x_1, \dots, x_n : p_1, \dots, p_n)$$

1) The Wigner function, the coalescence probability function

$$f^W(x_1, \dots, x_n : p_1, \dots, p_n) = \int \prod_{i=1}^n dy_i e^{p_i y_i} \psi^* \left(x_1 + \frac{y_1}{2}, \dots, x_n + \frac{y_n}{2} \right) \psi \left(x_1 - \frac{y_1}{2}, \dots, x_n - \frac{y_n}{2} \right)$$

2) A Lorentz-invariant phase space integration of a space-like hyper-surface constraints the number of particles in the system

$$\int p_i \cdot d\sigma_i \frac{d^3 p_i}{(2\pi)^3 E_i} f(x_i, p_i) = N_i$$

– Production of charmonium states by recombination

S. Cho, Phys. Rev. C **91**, 054914 (2015)

1) Coalescence production of charmonium states

$$N_\psi = g_\psi \int p_c \cdot d\sigma_c p_{\bar{c}} \cdot d\sigma_{\bar{c}} \frac{d^3 \vec{p}_c}{(2\pi)^3 E_c} \frac{d^3 \vec{p}_{\bar{c}}}{(2\pi)^3 E_{\bar{c}}} f_c(r_c, p_c) f_{\bar{c}}(r_{\bar{c}}, p_{\bar{c}}) W_\psi(r_c, r_{\bar{c}}; p_c, p_{\bar{c}}),$$

The transverse momentum distribution of the charmonium yield

$$\frac{dN_\psi}{d^2 \vec{p}_T} = \frac{g_\psi}{V} \int d^3 \vec{r} d^2 \vec{p}_{cT} d^2 \vec{p}_{\bar{c}T} \delta^{(2)}(\vec{p}_T - \vec{p}_{cT} - \vec{p}_{\bar{c}T}) \frac{dN_c}{d^2 \vec{p}_{cT}} \frac{dN_{\bar{c}}}{d^2 \vec{p}_{\bar{c}T}} W_\psi(\vec{r}, \vec{k})$$

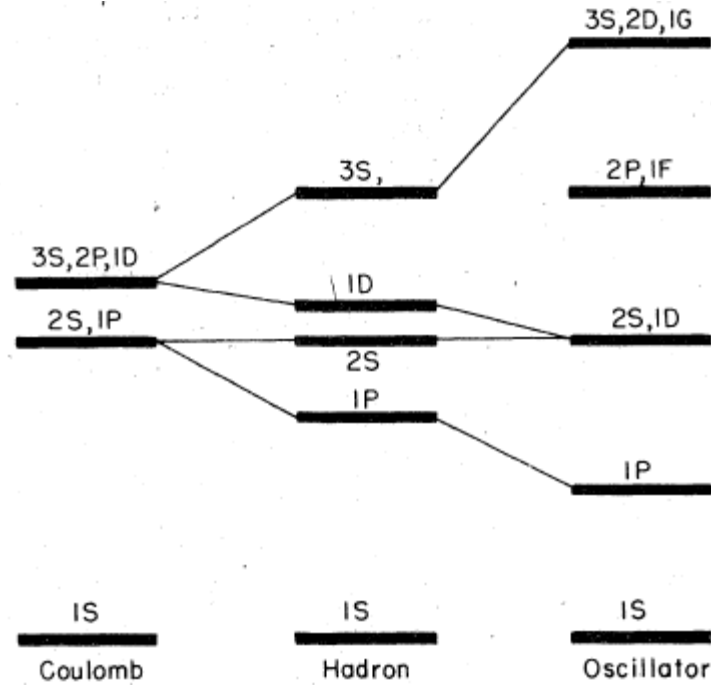
2) Gaussian Wigner functions for different charmonium states

$$W_s(\vec{r}, \vec{k}) = 8e^{-\frac{r^2}{\sigma^2} - k^2 \sigma^2}$$

$$W_p(\vec{r}, \vec{k}) = \left(\frac{16}{3} \frac{r^2}{\sigma^2} - 8 + \frac{16}{3} \sigma^2 k^2 \right) e^{-\frac{r^2}{\sigma^2} - k^2 \sigma^2}$$

$$W_{\psi_{10}}(\vec{r}, \vec{k}) = \frac{16}{3} \left(\frac{r^4}{\sigma^4} - 2 \frac{r^2}{\sigma^2} + \frac{3}{2} - 2\sigma^2 k^2 + \sigma^4 k^4 - 2r^2 k^2 + 4(\vec{r} \cdot \vec{k})^2 \right) e^{-\frac{r^2}{\sigma^2} - k^2 \sigma^2}.$$

3) Charmonium wave functions



E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, T-M, Yan, Phys. Rev. D **17**, 3090 (1978)

4) Integration of the Wigner function over the spatial coordinates

$$\int d^3\vec{r} W_\psi(\vec{r}, \vec{k}) = \begin{cases} (2\sqrt{\pi}\sigma)^3 e^{-k^2\sigma^2} & \psi_s^G; J/\psi \\ \frac{2}{3}(2\sqrt{\pi}\sigma)^3 e^{-k^2\sigma^2} \sigma^2 k^2 & \psi_p^G; \chi_c \\ \frac{2}{3}(2\sqrt{\pi}\sigma)^3 e^{-k^2\sigma^2} \left(\sigma^2 k^2 - \frac{3}{2}\right)^2 & \psi_{10}^G; \psi(2S) \\ 64\pi \frac{a_0^3}{(a_0^2 k^2 + 1)^4} & \psi_{1S}^C; J/\psi \\ 8\pi a_0^3 \frac{(a_0^2 k^2 - 1/4)^2}{(a_0^2 k^2 + 1/4)^6} & \psi_{2S}^C; \psi(2S) \end{cases}$$

$$\int d^3\vec{r} W(\vec{r}, \vec{k}) = |\tilde{\psi}(\vec{k})|^2$$

M. Hillery, R. F. O'Connell, M. O. Scully and E. P. Wigner, Phys. Rept. **106**, 121 (1984)

$$\frac{dN_\psi}{d\vec{p}_T} = \frac{g_\psi}{V} \int d\vec{p}_{cT} d\vec{p}_{\bar{c}T} \delta(\vec{p}_T - \vec{p}_{cT} - \vec{p}_{\bar{c}T}) \frac{dN_c}{d\vec{p}_{cT}} \frac{dN_{\bar{c}}}{d\vec{p}_{\bar{c}T}} |\tilde{\psi}(\vec{k})|^2$$

5) Transverse momentum distributions of charm and light quarks

$$\frac{dN_c}{d^2p_T} = \begin{cases} a_0 \exp[-a_1 p_T^{a_2}] & p_T \leq p_0 \\ a_0 \exp[-a_1 p_T^{a_2}] + a_3(1 + p_T^{a_4})^{-a_5} & p_T \geq p_0 \end{cases}$$

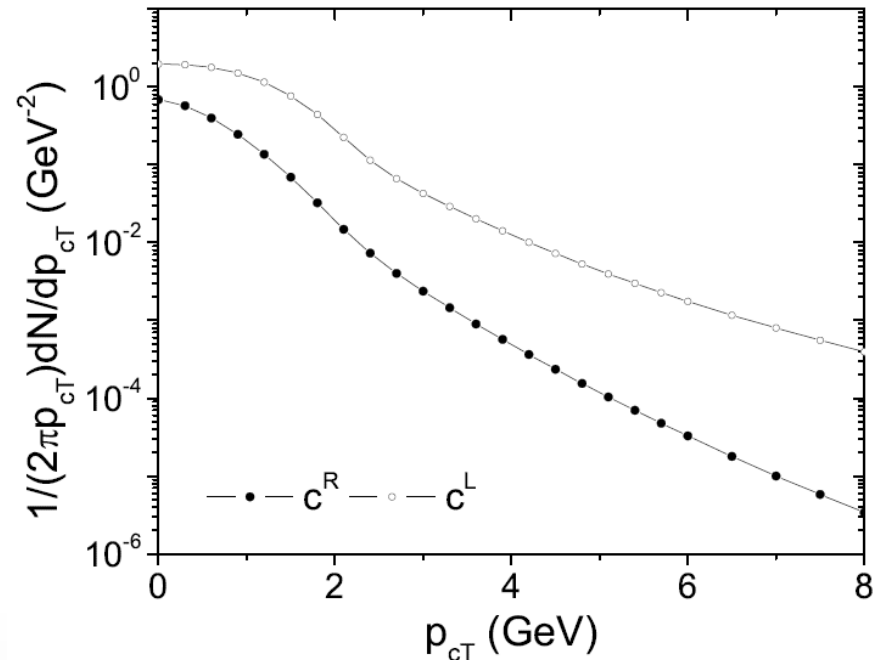
	a_0	a_1	a_2	a_3	a_4	a_5
RHIC						
$p_T \leq p_0$	0.69	1.22	1.57			
$p_T \geq p_0$	1.08	3.04	0.71	3.79	2.02	3.48
LHC						
$p_T \leq p_0$	1.97	0.35	2.47			
$p_T \geq p_0$	7.95	3.49	3.59	87335	0.5	14.31

S. Plumari, V. Minissale, S. K. Das, G. Coci and V. Greco, Eur. Phys. J. C **78**:348 (2017)

Y. Oh, C. M. Ko, S.-H. Lee, and S. Yasui, Phys. Rev. C **79** 044905 (2009)

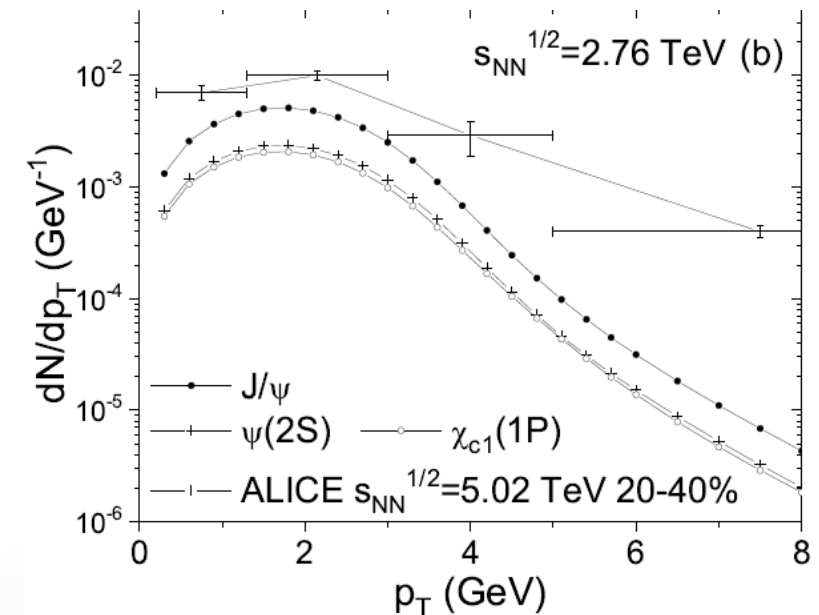
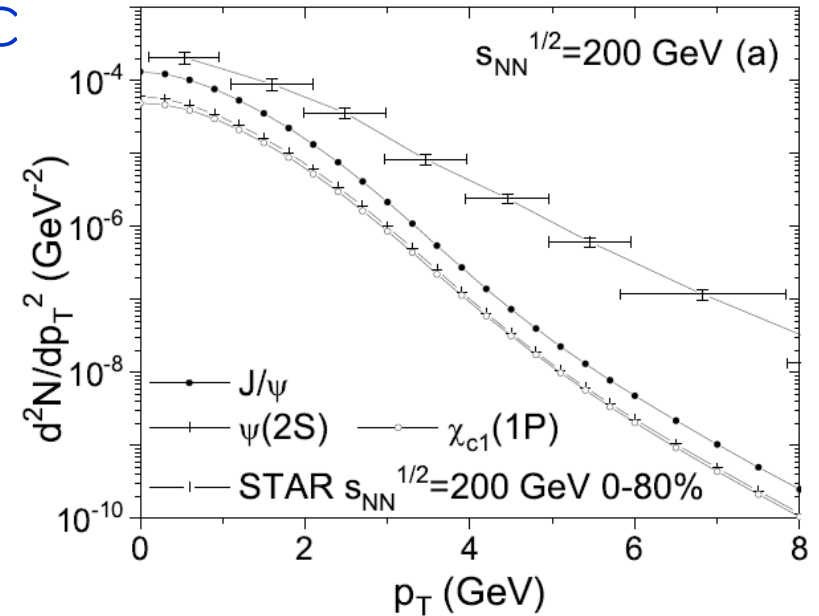
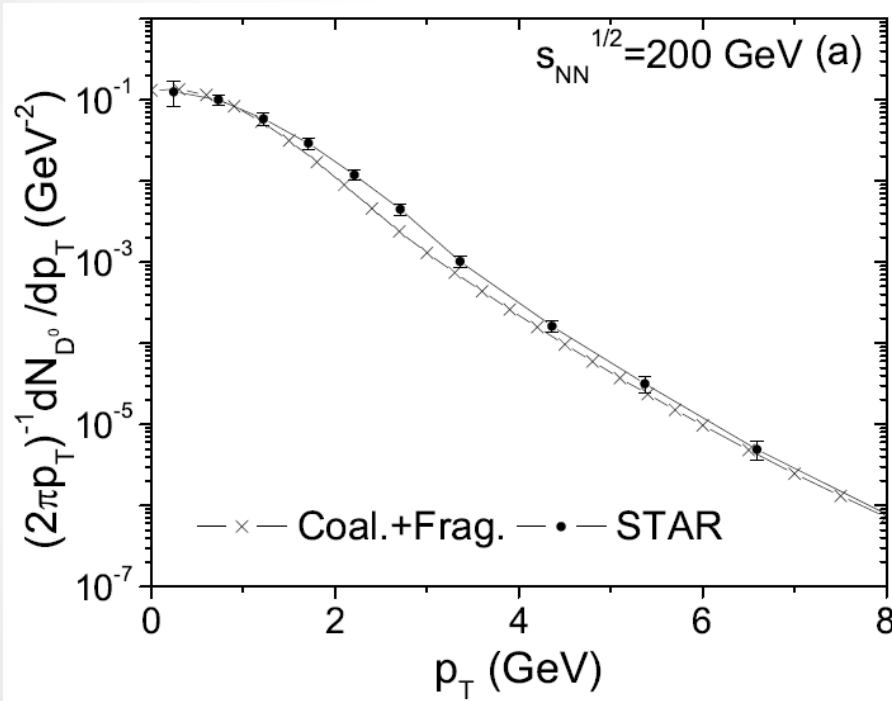
S. Cho *et al.* (EXHIC Collaboration), Prog. Part. Nucl. Phys. **95**, 279 (2017)

	RHIC		LHC (2.76 TeV)	
	Sc. 1	Sc. 2	Sc. 1	Sc. 2
T_H (MeV)		162		156
V_H (fm ³)		2100		5380
μ_B (MeV)		24		0
μ_s (MeV)		10		0
γ_c				39
γ_b		4.0×10^7		8.6×10^8
T_C (MeV)	162	166	156	166
V_C (fm ³)	2100	1791	5380	3533
$N_u = N_d$	320	302	700	593
$N_s = N_{\bar{s}}$	183	176	386	347
$N_c = N_{\bar{c}}$		4.1		11
$N_b = N_{\bar{b}}$		0.03		0.44



6) Transverse momentum distributions of charmonium states in 0-10% centralities at RHIC and LHC

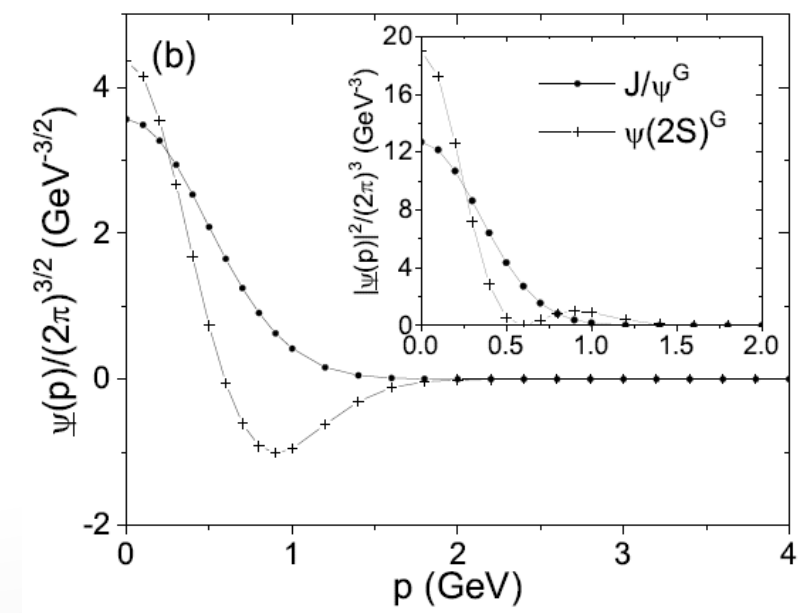
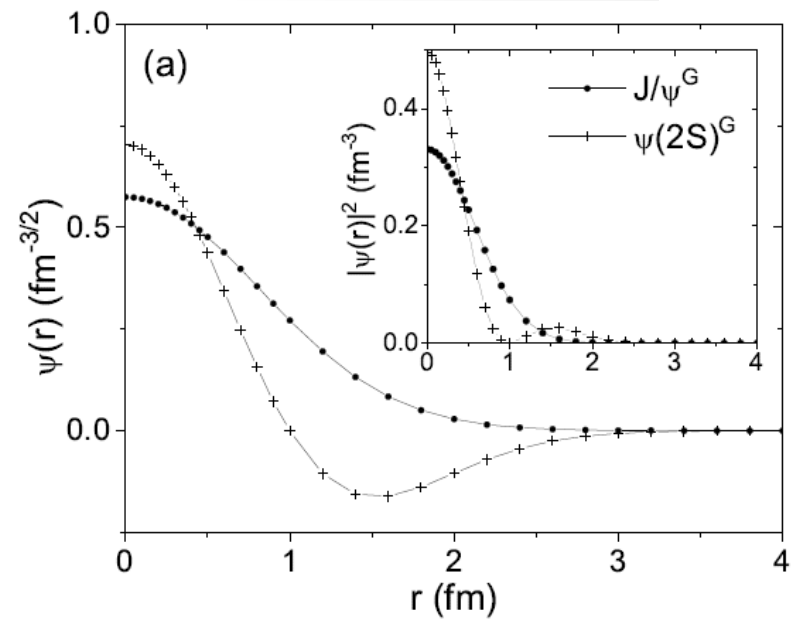
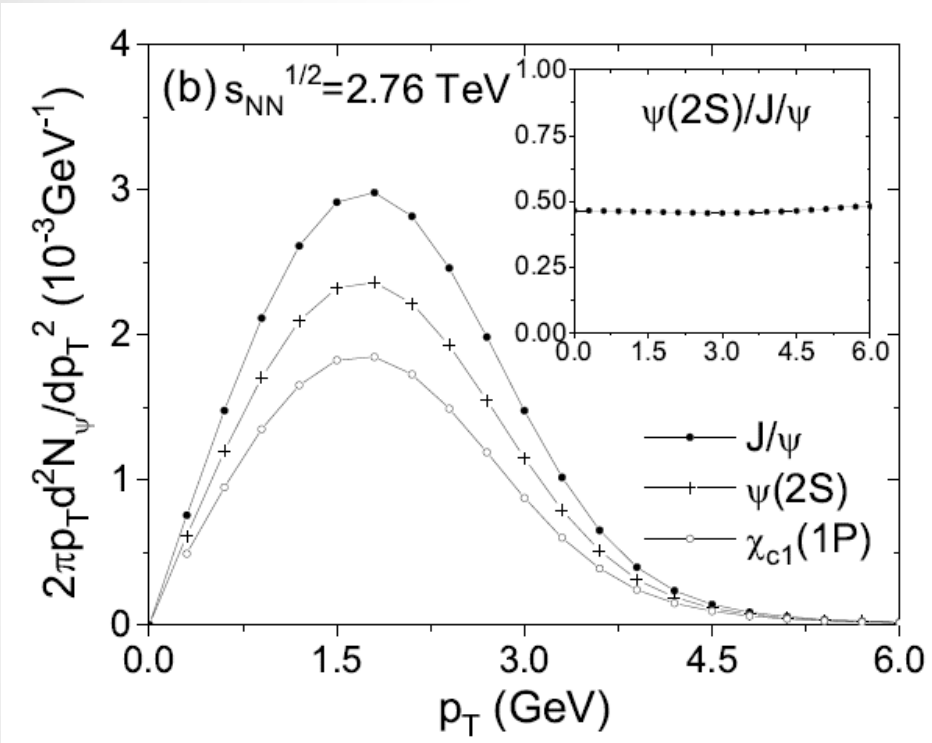
S. Cho and S. H. Lee, Phys. Rev. C **101**, 024902 (2020)



J. Adam et al. [STAR Collaboration], Phys. Lett. B **797**, 134917 (2019).

S. Acharya et al. [ALICE Collaboration], Phys. Lett. B **805**, 135434 (2020).

7) Wave function distributions of charmonium states



Elliptic and triangular flow of charmonium states

– Flow harmonics of charmonium states

$$v_n(p_T) = \langle \cos(n(\psi - \Psi_n)) \rangle$$

$$= \frac{\int d\psi \cos(n(\psi - \Psi_n)) \frac{d^2 N}{dp_T^2}}{\int d\psi \frac{d^2 N}{dp_T^2}}, \quad \Psi_n = \frac{1}{n} \tan^{-1} \left(\frac{\langle p_T \sin(n\psi) \rangle}{\langle p_T \cos(n\psi) \rangle} \right),$$

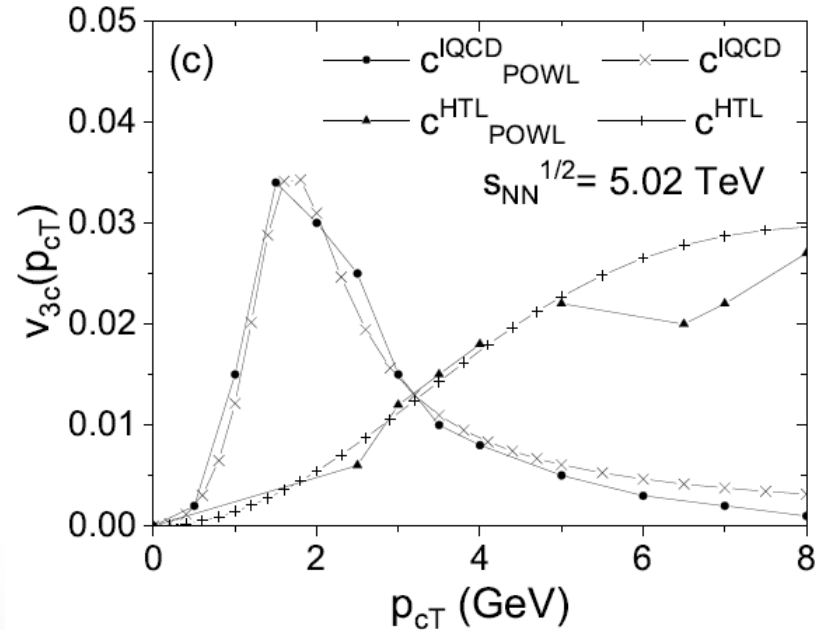
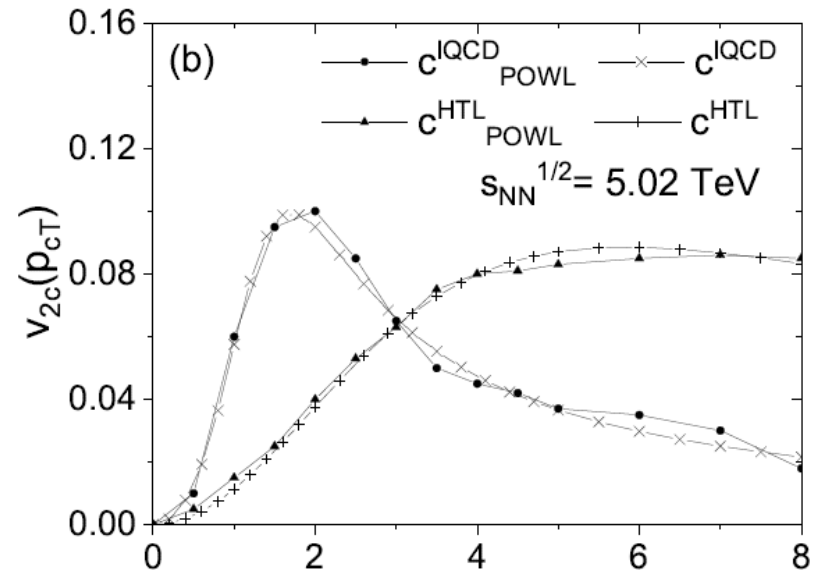
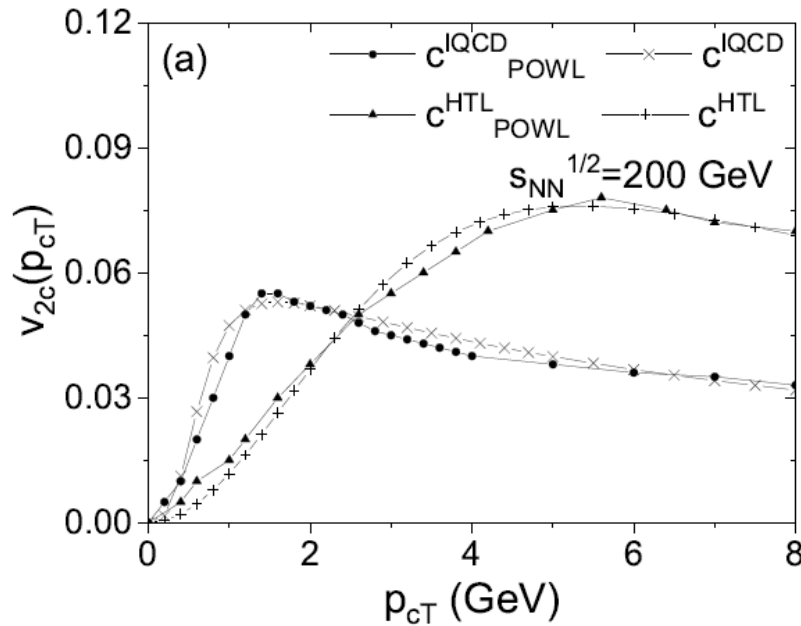
1) Transverse momentum distribution of charm quarks with flow harmonics

$$\frac{d^2 N_c}{dp_{cT}^2} = \frac{1}{2\pi p_{cT}} \frac{dN_c}{dp_{cT}} \left(1 + \sum_{n=1}^{\infty} 2v_{nc}(p_{cT}) \cos(n(\phi_c - \Psi_n)) \right),$$

2) Pade approximation of charm quark flow harmonics

$$v_{nc}(p_{cT}) = \frac{a_3 p_{cT}^3 + a_2 p_{cT}^2 + a_1 p_{cT}}{b_4 p_{cT}^4 + b_3 p_{cT}^3 + b_2 p_{cT}^2 + b_1 p_{cT} + 1},$$

3) Charm quark flow harmonics from POWLANG



A. Beraudo, A. De Pace, M. Monteno,
M. Nardi and F. Prino, JHEP **02**, 043 (2018).

Conclusion

- Elliptic and triangular flow of charmonia in heavy ion collisions
- 1) The production of heavy quarks hadrons, or charmonium states can be understood in the coalescence model.
- 2) The transverse momentum distribution is dependent on the internal structure of the hadron
- 3) The enhanced transverse momentum distribution of $\psi(2S)$ mesons, compared to that of J/ψ mesons, is originated from intrinsic wave function distributions between $\psi(2S)$ and J/ψ mesons.
- 4) The elliptic and triangular flow of charmonium states are also affected by wave function distributions of charmonium states.
- 5) Studying charmonium states in heavy ion collisions will help us to understand many aspects in heavy ion collisions experiments.



Thank you for your attention!