Form Factor, TMDs, and PFDs of the Pion in the Light-Front Quark Model

Ho-Meoyng Choi (Kyungpook National Univ., Korea)

Work in progress (with Prof. Chueng-Ryong Ji)

2nd CENuM Workshop for Hadron Physics, Dec. 18-19, 2023, Inha University

Outline

1. Motivation

2. Why Light-Front?

3. Form Factors on the LF:
- New Development of self-consistent LFQM
- Pion Form Factor

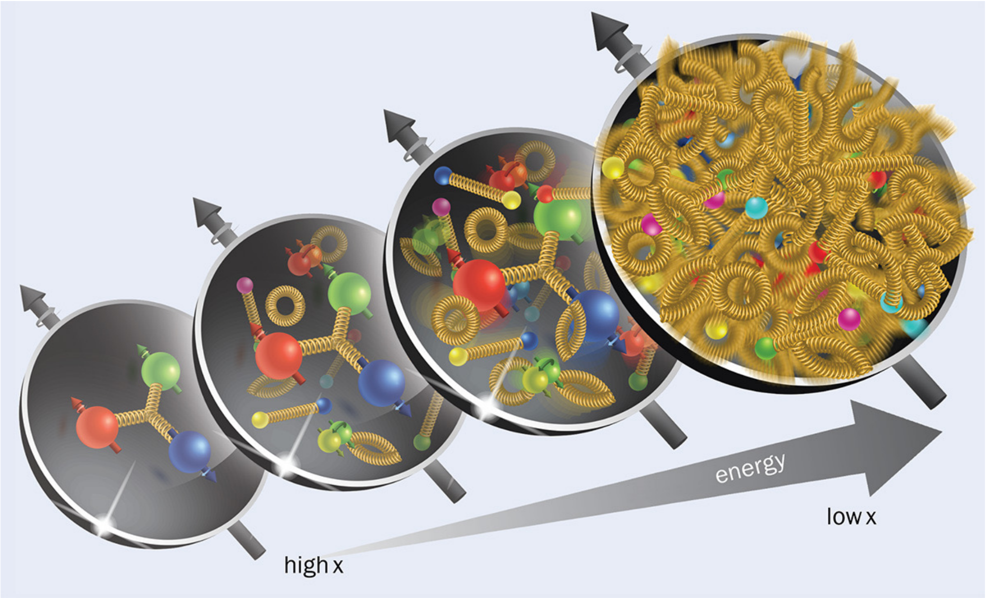
4. Unpolarized Transverse Momentum Distributions(TMDs) of pion

5. QCD evolution of Pion Parton Distribution Functions (PDFs)

6. Conclusions

The Electron-Ion Collider (EIC)

aims to provide detailed insights into the behavior of quarks and gluons…

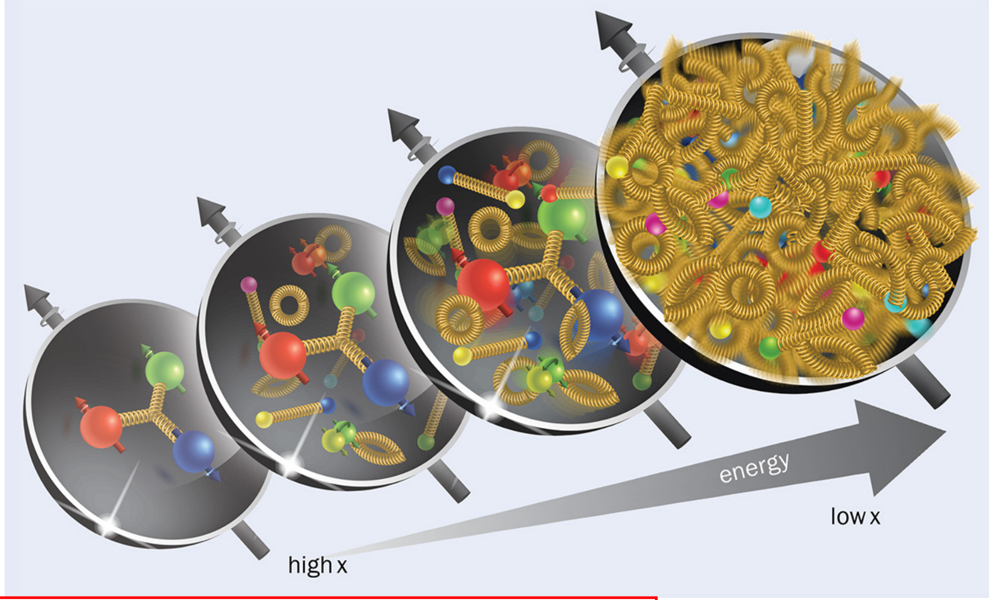


Physics Topics at the EIC

1. Precision 3D imaging of protons and nuclei.
2. Search for gluon saturation.
3. Solving the proton spin puzzle.

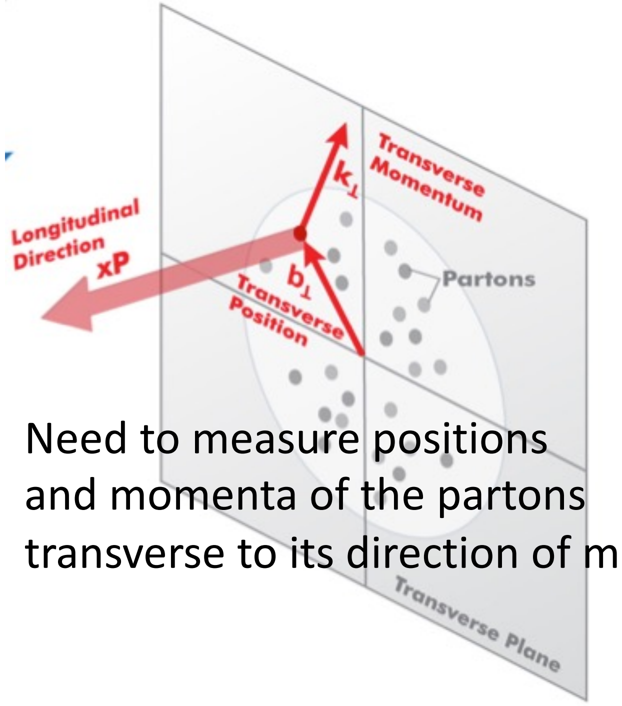
The Electron-Ion Collider (EIC)

aims to provide detailed insights into the behavior of quarks and gluons…



Physics Topics at the EIC

1. Precision 3D imaging of protons and nuclei.
2. Search for gluon saturation.
3. Solving the proton spin puzzle.



Need to measure positions and momenta of the partons transverse to its direction of motion.

3D nucleon structure from 5D tomography

$W(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$: Wigner distribution

$\int d\boldsymbol{b}_\perp$ $\qquad$ $\int d\boldsymbol{k}_\perp$

$f(x, \boldsymbol{k}_\perp)$

TMDs :semi-inclusive processes

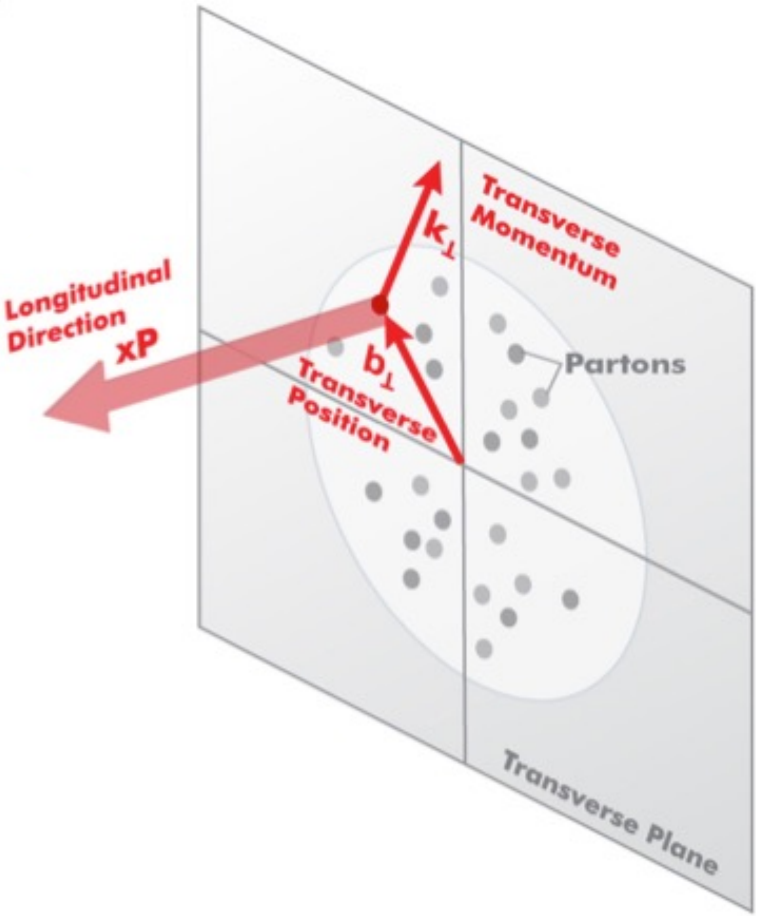
$f(x, \boldsymbol{b}_\perp)$

Impact parameter distributions

$\int d\boldsymbol{k}_\perp$ $\qquad$ $\int d\boldsymbol{b}_\perp$

$f(x)$

PDFs: inclusive and semi-inclusive processes



3D nucleon structure from 5D tomography

$W(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$: Wigner distribution

$\int d\boldsymbol{b}_\perp$ $\int d\boldsymbol{k}_\perp$

$f(x, \boldsymbol{k}_\perp)$

TMDs :semi-inclusive processes

$f(x, \boldsymbol{b}_\perp)$

Impact parameter distributions

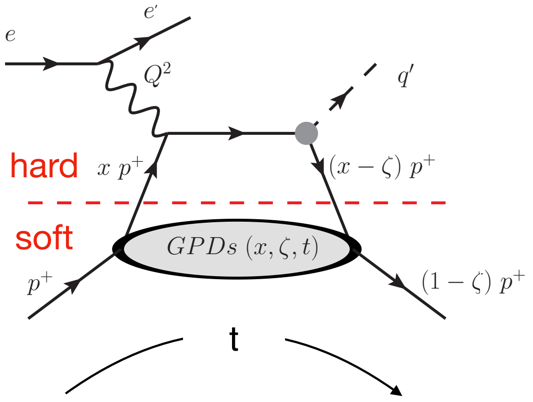
$\int d\boldsymbol{k}_\perp$ $\int d\boldsymbol{b}_\perp$

$f(x)$

PDFs: inclusive and semi-inclusive processes

FT

$\boldsymbol{b}_\perp \leftrightarrow \Delta$



$H(x, \zeta, t = -\Delta^2)$ $\zeta = \frac{(P - P')^+}{P^+}$

GPDs :exclusive processes

$\int dx$

$F(t)$

Form factors: elastic scattering

3D nucleon structure from 5D tomography

$W(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$: Wigner distribution

$\int d\boldsymbol{b}_\perp$ → $f(x, \boldsymbol{k}_\perp)$

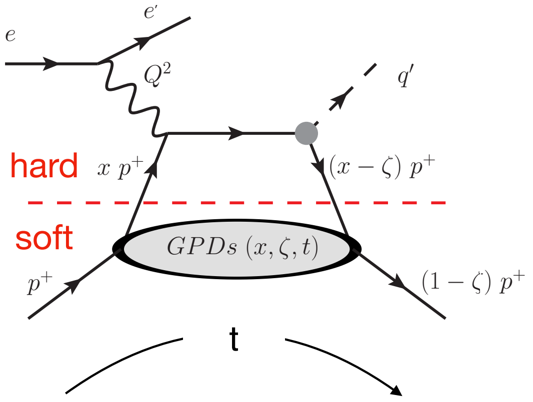
TMDs :semi-inclusive processes

$\int d\boldsymbol{k}_\perp$ → $f(x, \boldsymbol{b}_\perp)$

Impact parameter distributions

FT

$\boldsymbol{b}_\perp \leftrightarrow \Delta$



$H(x, \zeta, t = -\Delta^2)$ $\quad \zeta = \frac{(P - P')^+}{P^+}$

GPDs :exclusive processes

$\int dx$ → $F(t)$

Form factors: elastic scattering

$\int d\boldsymbol{k}_\perp$, $\int d\boldsymbol{b}_\perp$ → $f(x)$

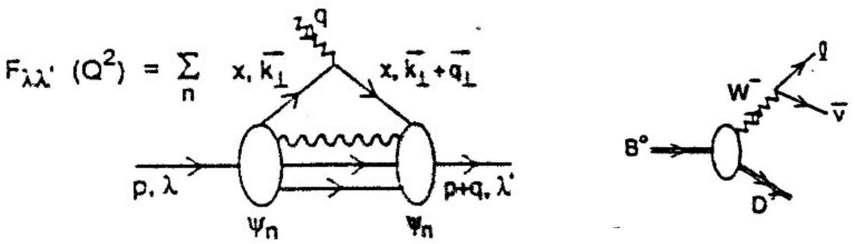
PDFs: inclusive and semi-inclusive processes

⇒ $(x, \boldsymbol{k}_\perp)$: Light front (LF)variables ...

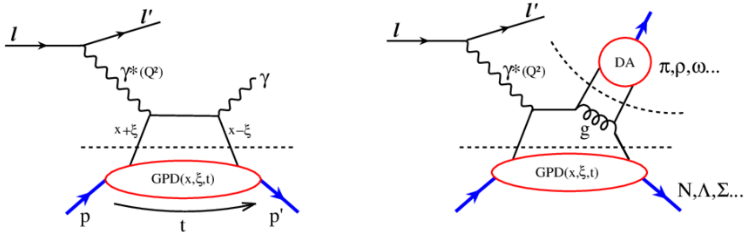
1. Motivation

• Light-Front Dynamics(LFD) has been quite successful in describing various hadron properties such as Decay constants, DAs, PDFs, Form Factors, GPDs and TMDs etc.

Electromagnetic & Weak decay Process



DVCS & DVMP



• We have developed a new self-consistent covariant LFQM for decay constants, DAs, and weak form factors(e.g. $B \to Dll$) of mesons:

PRD 89, 033011(14); PRD91, 014018(15); PRD95, 056002(17) by HMC and C.-R. Ji

PRD 103, 073004(21); Adv. High Energy Phys., 4277321(21) by HMC

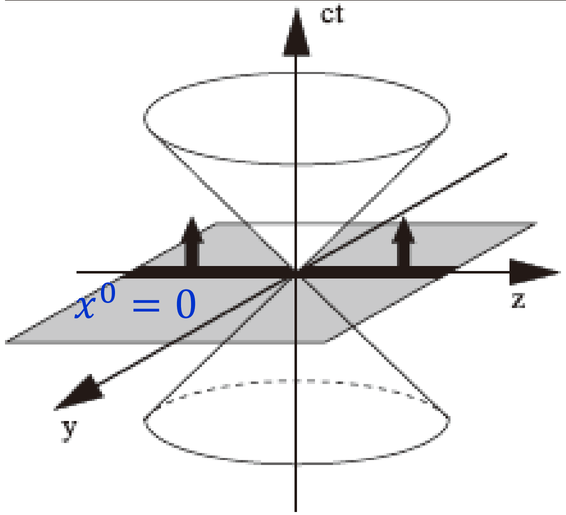
PRD 107, 053003(23); PRD108, 013006(23) by A. J. Arifi, HMC and C.-R. Ji

In this work, we apply our new developed LFQM to compute Form Factor, TMDs, and PDFs of Pion!

2. Why Light-Front?

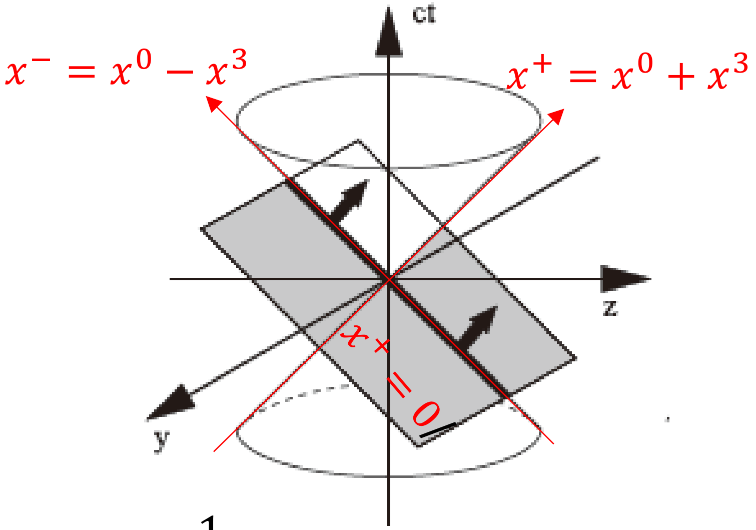
Light-Front Dynamics (LFD) (by Dirac in 1949)

Instant form ($x^0 = ct = 0$)



$$a \cdot b = a^0 b^0 - \vec{a} \cdot \vec{b}$$

Front form ($x^+ = x^0 + x^3 = 0$)



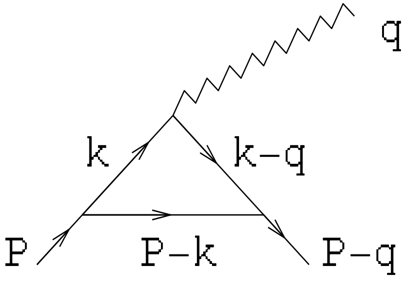
$$a \cdot b = \frac{1}{2}(a^+ b^- + a^- b^+) - \boldsymbol{a}_\perp \cdot \boldsymbol{b}_\perp$$

Hamiltonian	P^0	$P^- = P^0 - P^3$
Momentum	$\boldsymbol{P}_\perp = (P^1,\ P^2)$ P^3	$\boldsymbol{P}_\perp$ $P^+ = P^0 + P^3$
Energy-Momentum Dispersion Relation	$P^0 = \sqrt{M^2 + \vec{P}^2}$	$P^- = \dfrac{M^2 + \boldsymbol{P}_\perp^2}{P^+}$

Irrational vs. Rational

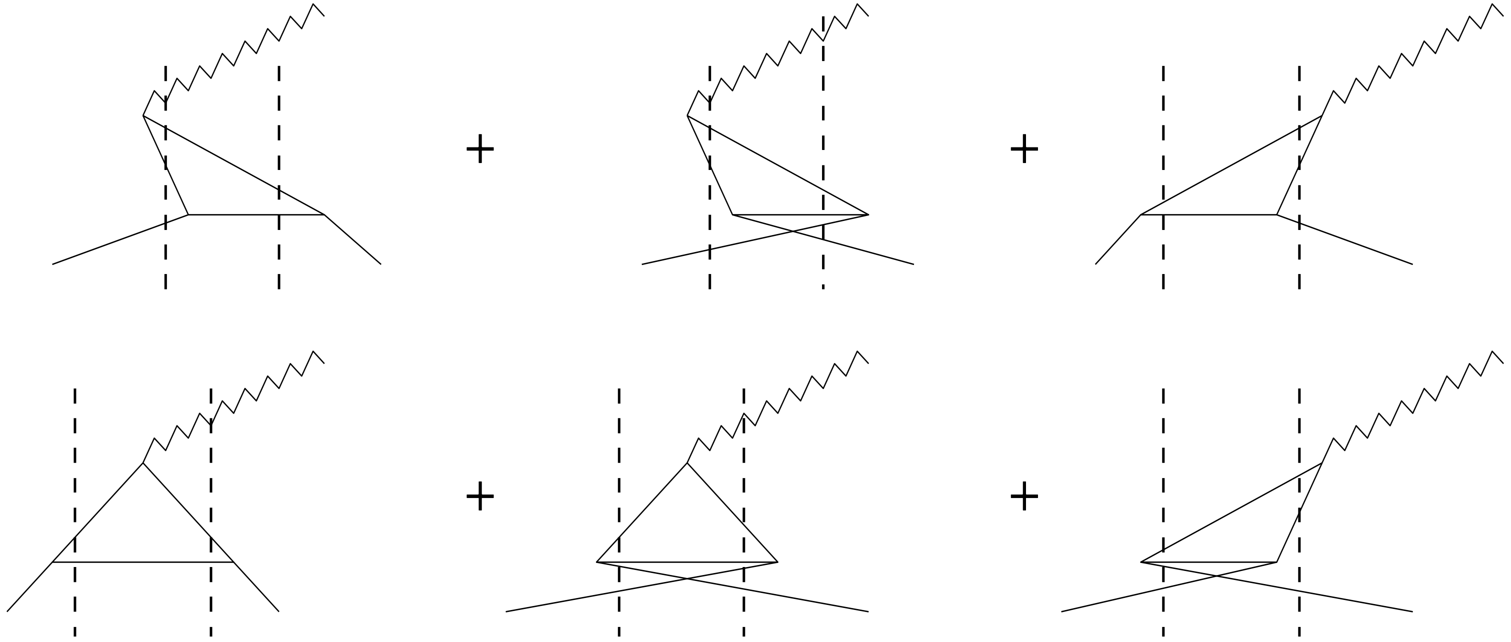
- Advantage of LFD in the calculation of Form Factors :

  Equal-t vs Equal Light-front τ formulations

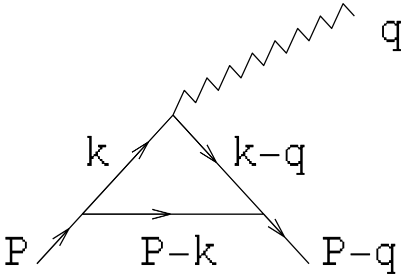


Equal t (Instant form)

=

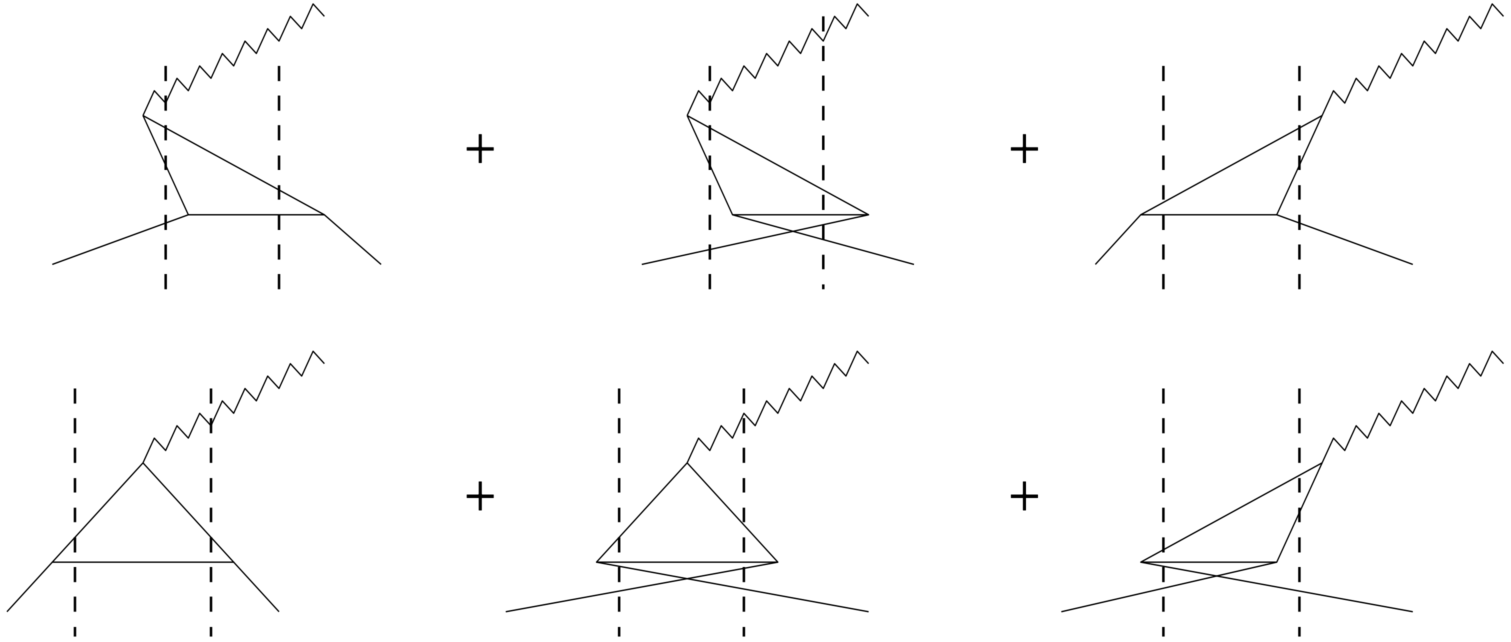


- Advantage of LFD in the calculation of Form Factors :
  Equal-t vs Equal Light-front τ formulations



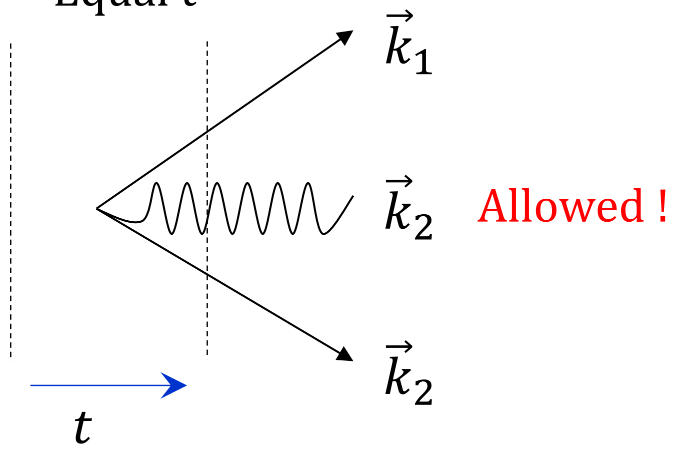
Equal t (Instant form)

=



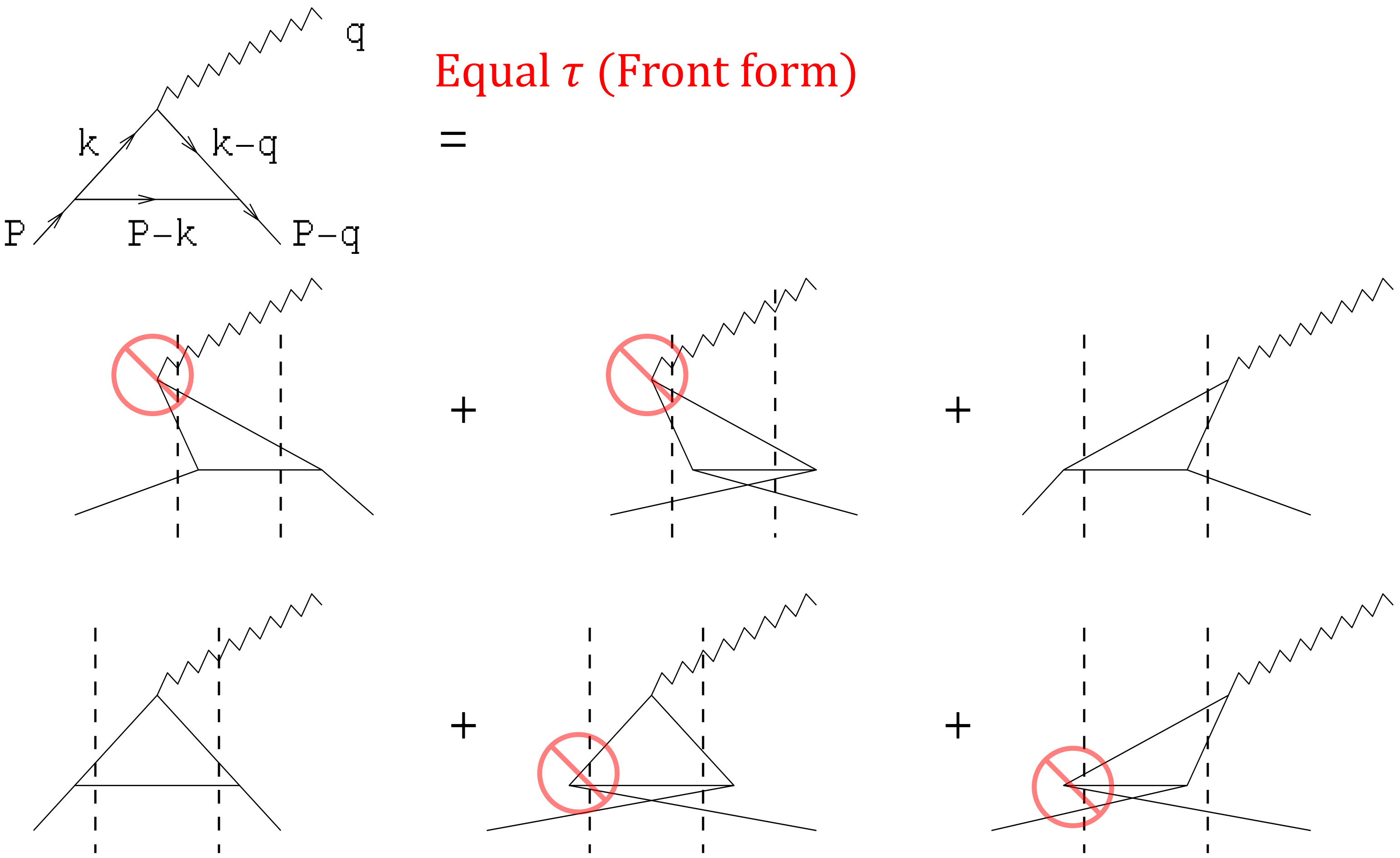
$$k^0 = \sqrt{m^2 + \vec{k}^2}$$

Equal t

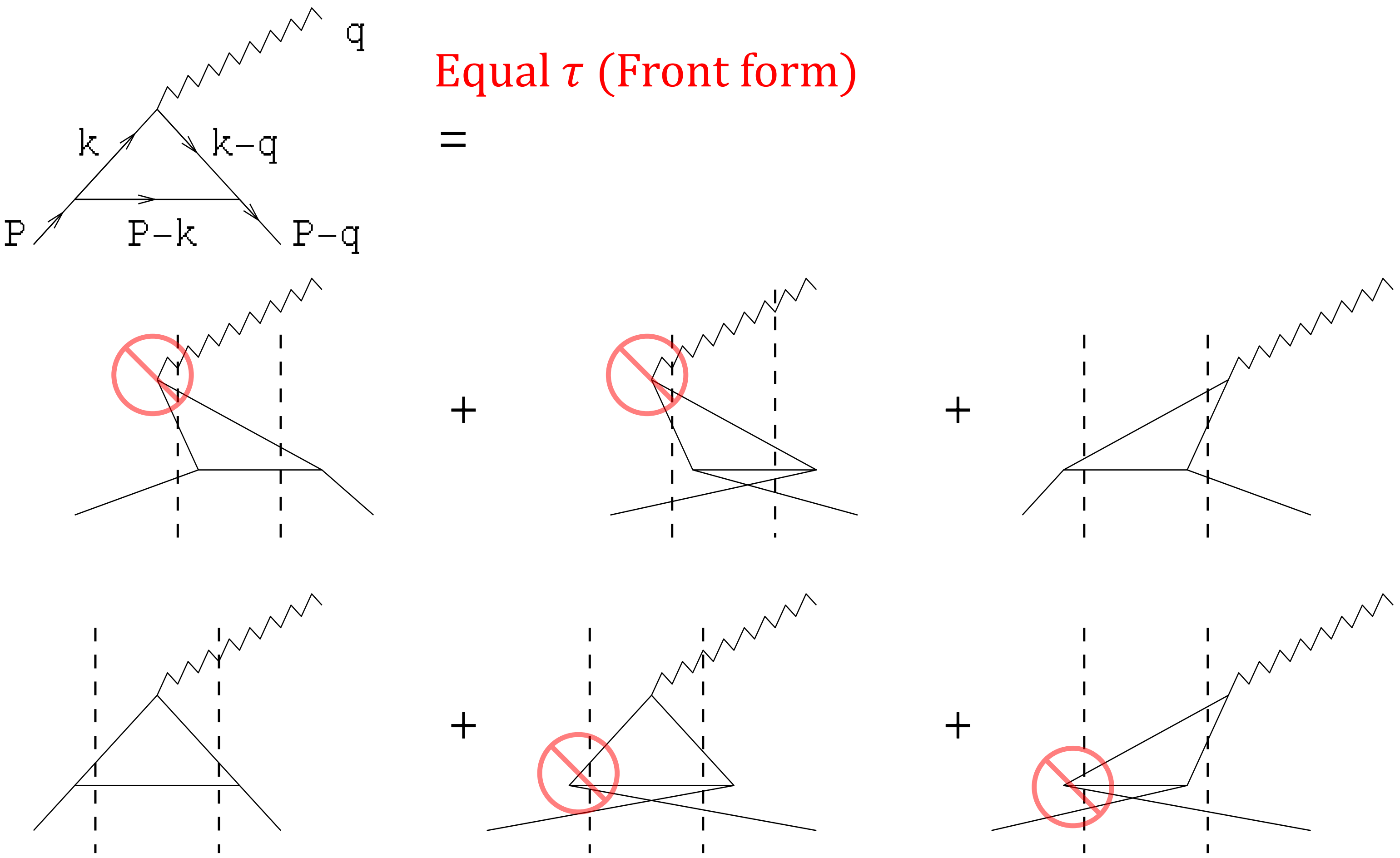


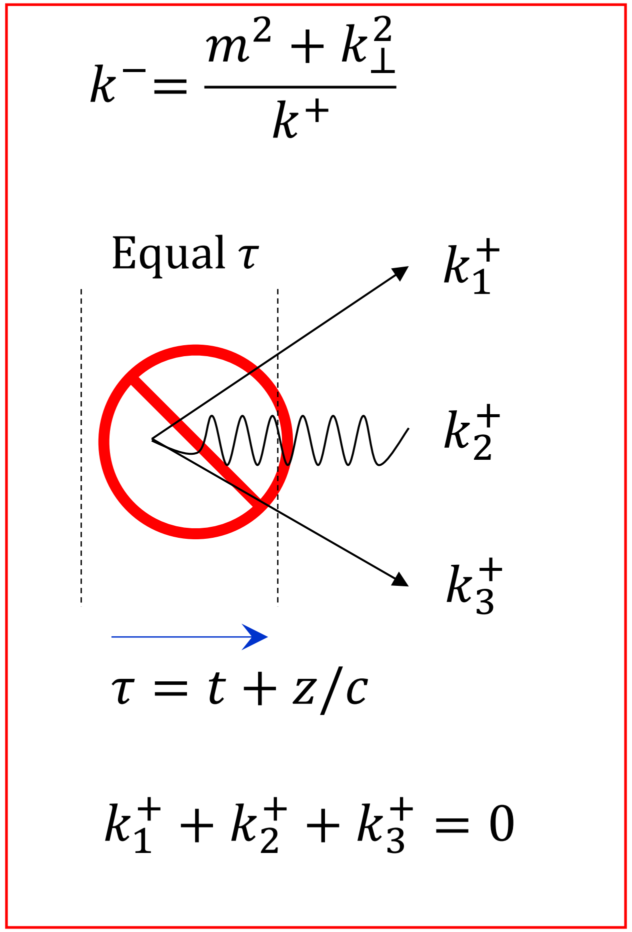
$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$$

- Advantage of LFD in the calculation of Form Factors :
  Equal-t vs Equal Light-front τ formulations



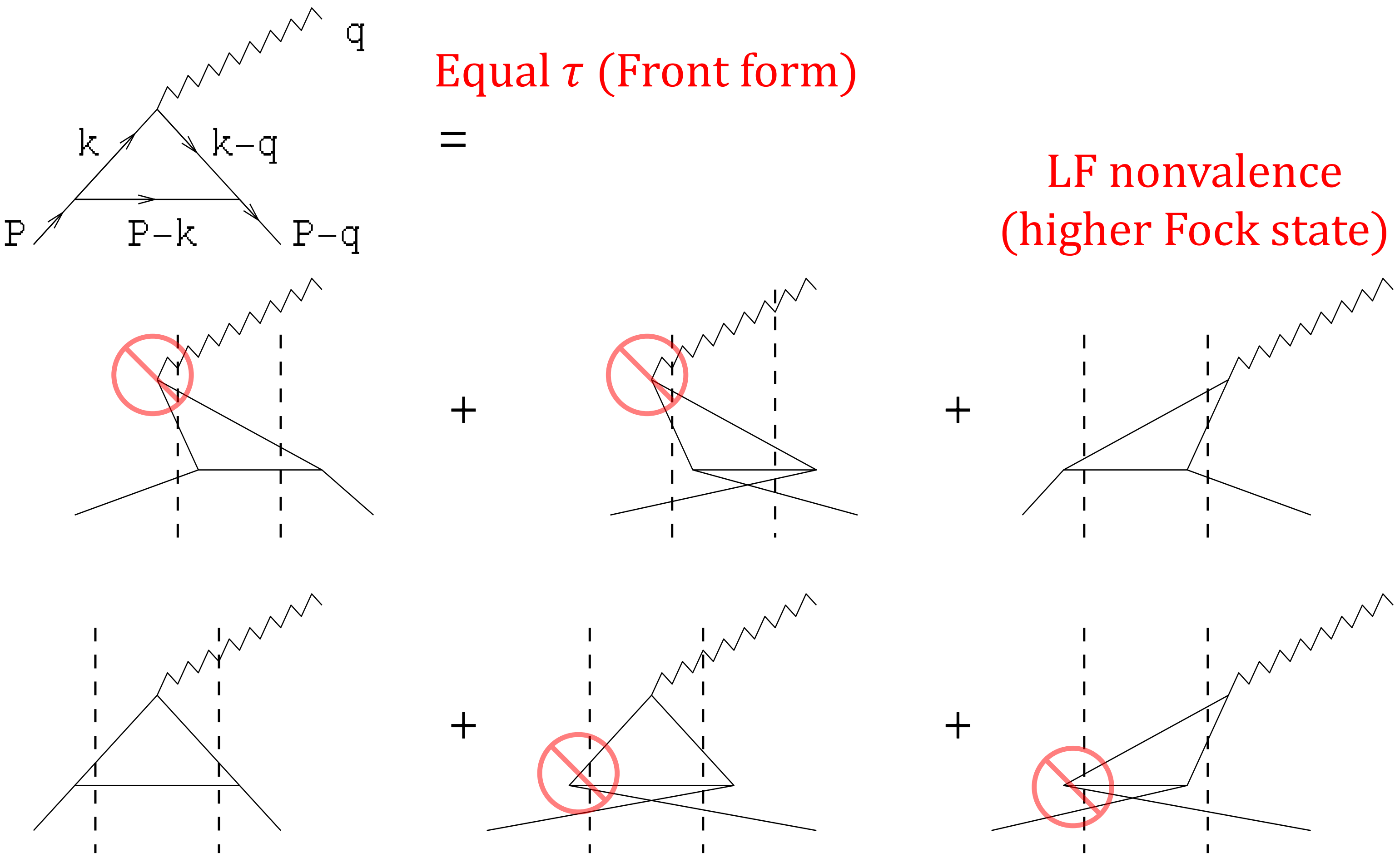
- Advantage of LFD in the calculation of Form Factors :
  Equal-t vs Equal Light-front τ formulations





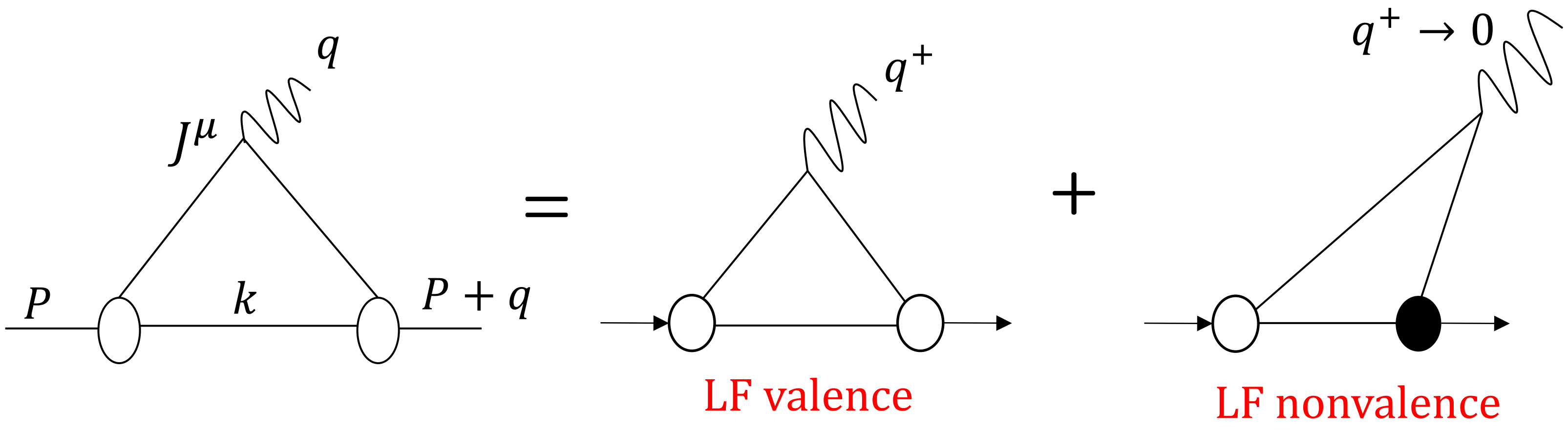
- Advantage of LFD in the calculation of Form Factors :

  Equal-t vs Equal Light-front τ formulations

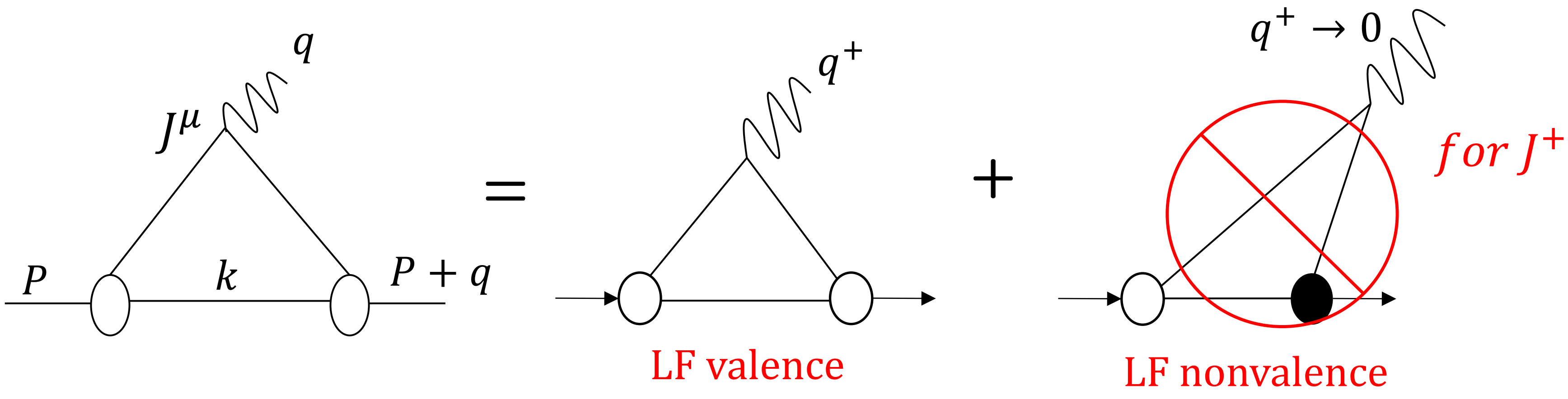


LF valence

3. Form Factors on the Light-Front

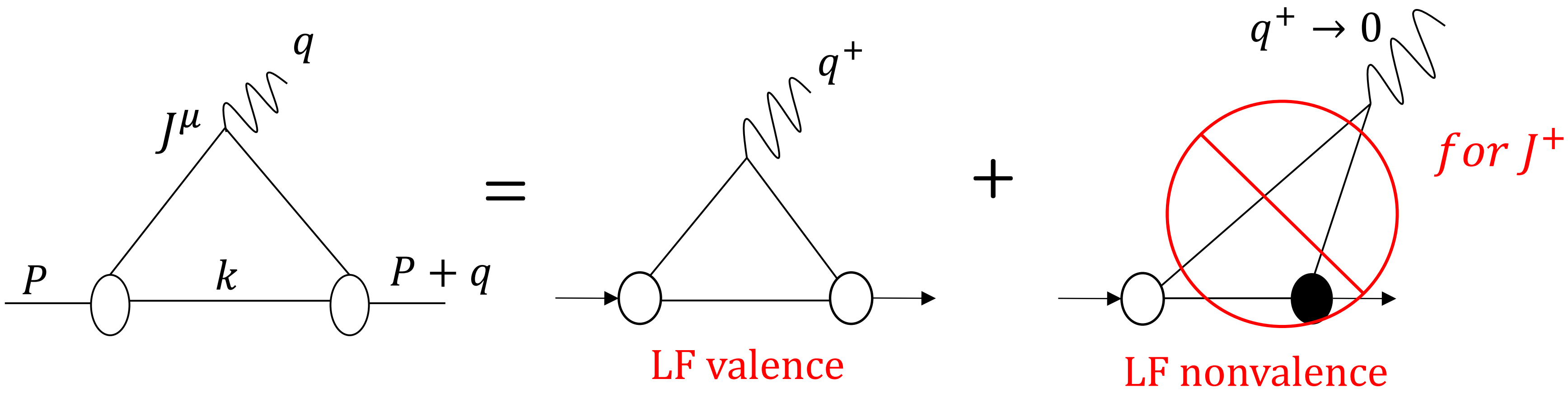


3. Form Factors on the Light-Front



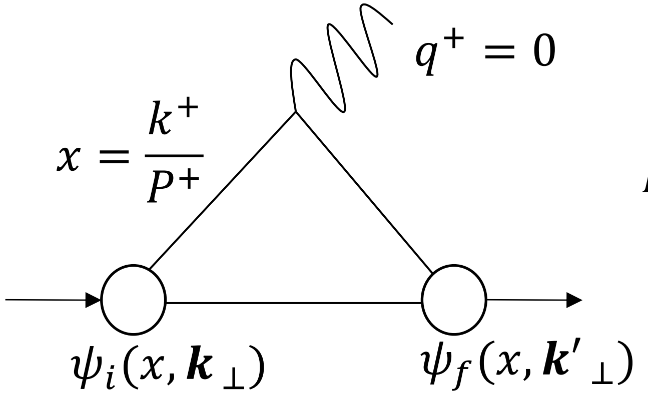
$\Longrightarrow$ facilitates the partonic interpretation of the amplitude!

3. Form Factors on the Light-Front



⟹ facilitates the partonic interpretation of the amplitude!

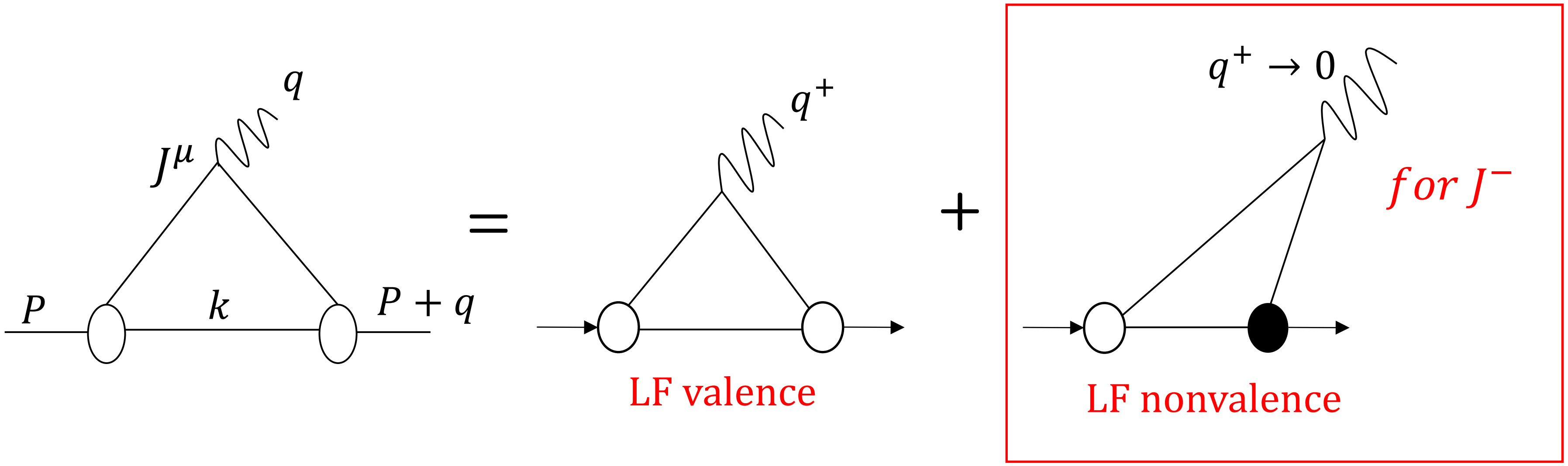
(e.g.) E&M form factors of pseudoscalar and vector mesons



$$F(Q^2) = \int [dx][d^2\boldsymbol{k}_\perp]\, \psi_f^*(x, \boldsymbol{k}'_\perp)\psi_i(x, \boldsymbol{k}_\perp)$$

for J^+ & $\mathbf{J}^\perp$

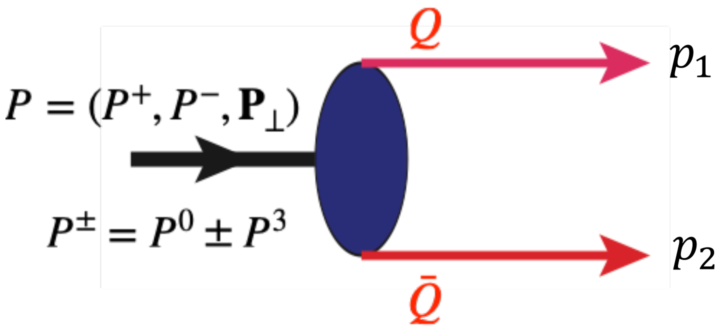
3. Form Factors on the Light-Front



Nonvanishing : LF Zero-Mode !

☞ One has to take into account of the zero mode in using J^{-} current!

Light-Front Quark Model(LFQM)



$$P^- = p_Q^- + p_{\bar{Q}}^-\text{, i.e. } M^2 \to M_0^2 = \frac{m_Q^2 + \boldsymbol{k}_\perp^2}{x} + \frac{m_{\bar{Q}}^2 + \boldsymbol{k}_\perp^2}{1-x}$$

Meson state: Noninteracting "on-mass" shell Q & $\bar{Q}$ representation
consistent with Bakamjian-Thomas(BT) constuction!

The interaction is added to the mass operator

$$M_{Q\bar{Q}} = \langle \Psi | H_{Q\bar{Q}} | \Psi \rangle$$

$$H_{Q\bar{Q}} = \sqrt{m_Q^2 + \vec{k}^2} + \sqrt{m_{\bar{Q}}^2 + \vec{k}^2} + V_{Q\bar{Q}} \qquad V_{Q\bar{Q}} = a + br - \frac{4\kappa}{3r} + \frac{2\vec{S}_Q \cdot \vec{S}_{\bar{Q}}}{3m_Q m_{\bar{Q}}} \nabla^2 V_{\text{Coul}}$$

Refs.) PRD59, 074015(99); PLB460, 461(99) by HMC and CRJ; PRC92, 055203(2015) by HMC, CRJ, Z. Li, and H. Ryu
PRD106, 014009(2022) by A. J. Arifi, HMC, and CRJ

New Development of including the LF Zero-Mode in the LFQM

General structure for $P(P) \rightarrow P(P')$ transition:

$$\langle P'|\bar{q}\ \gamma^{\mu}\ q|\mathrm{P}\rangle = \wp^{\mu}\ F(q^2)\ + q^{\mu}\ \frac{(M^2 - M'^2)}{q^2} H(q^2), \qquad \wp^{\mu} = (P + P')^{\mu} - q^{\mu}\ \frac{(M^2 - M'^2)}{q^2}$$

$$q^{\mu} = (P - P')^{\mu}$$

New Development of including the LF Zero-Mode in the LFQM

General structure for $P(P) \to P(P')$ transition:

$$\langle P'|\bar{q}\ \gamma^\mu\ q|\mathrm{P}\rangle = \wp^\mu\ F(q^2)\ + q^\mu\ \frac{(M^2 - M'^2)}{q^2} H(q^2), \qquad \wp^\mu = (P + P')^\mu - q^\mu\ \frac{(M^2 - M'^2)}{q^2}$$

$$q^\mu = (P - P')^\mu$$

For elastic process$(M = M')$,
only gauge invariant form factor $F(q^2)$ survives!

$$\langle P'|\bar{q}\ \gamma^\mu\ q|\mathrm{P}\rangle = \wp^\mu\ F_P(q^2) \qquad \wp \cdot q = 0$$

New Development of including the LF Zero-Mode in the LFQM

$$\langle P'|\bar{q}\ \gamma^\mu\ q|\mathrm{P}\rangle = \wp^\mu\ F_P(q^2), \qquad \wp^\mu = (P+P')^\mu - q^\mu\ \frac{(M^2 - M'^2)}{q^2}$$

In $q^+ = 0$ frame,

$$\langle P'|\bar{q}\ \gamma^\mu\ q|\mathrm{P}\rangle = \int_0^1 \mathrm{d}p_1^+ \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\ \phi'(x,\mathbf{k}'_\perp)\phi(x,\mathbf{k}_\perp) \sum_{\lambda' s} \mathcal{R}^\dagger_{\lambda_2\bar{\lambda}} \left[\frac{\bar{u}_{\lambda_2}(p_2)}{\sqrt{x_2}}\gamma^\mu \frac{u_{\lambda_1}(p_1)}{\sqrt{x_1}}\right] \mathcal{R}_{\lambda_1\bar{\lambda}},$$

$$F_P^{(\mu)}(Q^2) = \int_0^1 \mathrm{d}p_1^+ \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\ \phi'(x,\mathbf{k}'_\perp)\phi(x,\mathbf{k}_\perp)\ \frac{1}{\wp^\mu} \sum_{\lambda' s} \mathcal{R}^\dagger_{\lambda_2\bar{\lambda}} \left[\frac{\bar{u}_{\lambda_2}(p_2)}{\sqrt{x_2}}\gamma^\mu \frac{u_{\lambda_1}(p_1)}{\sqrt{x_1}}\right] \mathcal{R}_{\lambda_1\bar{\lambda}},$$

New Development of including the LF Zero-Mode in the LFQM

$$\langle P'|\bar{q}\ \gamma^\mu\ q|\mathrm{P}\rangle = \wp^\mu\ F_P(q^2), \qquad \wp^\mu = (P+P')^\mu - q^\mu \frac{(M^2 - M'^2)}{q^2}$$

In $q^+ = 0$ frame,

$$\langle P'|\bar{q}\ \gamma^\mu\ q|\mathrm{P}\rangle = \int_0^1 \mathrm{d}p_1^+ \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\ \phi'(x, \mathbf{k}'_\perp)\phi(x, \mathbf{k}_\perp) \sum_{\lambda' s} \mathcal{R}^\dagger_{\lambda_2\bar{\lambda}} \left[\frac{\bar{u}_{\lambda_2}(p_2)}{\sqrt{x_2}}\gamma^\mu \frac{u_{\lambda_1}(p_1)}{\sqrt{x_1}}\right] \mathcal{R}_{\lambda_1\bar{\lambda}},$$

Apply $P^- = p_q^- + p_{\bar{q}}^-$ (i. e. $M^2 \to M_0^2$)

New Effective Method

$$F_P^{(\mu)}(Q^2) = \int_0^1 \mathrm{d}p_1^+ \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\ \phi'(x, \mathbf{k}'_\perp)\phi(x, \mathbf{k}_\perp)\ \frac{1}{\wp^\mu} \sum_{\lambda' s} \mathcal{R}^\dagger_{\lambda_2\bar{\lambda}} \left[\frac{\bar{u}_{\lambda_2}(p_2)}{\sqrt{x_2}}\gamma^\mu \frac{u_{\lambda_1}(p_1)}{\sqrt{x_1}}\right] \mathcal{R}_{\lambda_1\bar{\lambda}},$$

Then we get $F_\pi^{(+)}(Q^2) = F_\pi^{(\perp)}(Q^2) = F_\pi^{(-)}(Q^2)$

Preliminary

$$F_\pi^{\mathrm{SLF}(\mu)}(Q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{\phi(x,\mathbf{k}_\perp)\phi'(x,\mathbf{k}'_\perp)}{\sqrt{\mathbf{k}_\perp^2+m^2}\sqrt{\mathbf{k}'^2_\perp+m^2}} \mathcal{O}^{(\mu)}_{\mathrm{LFQM}}$$

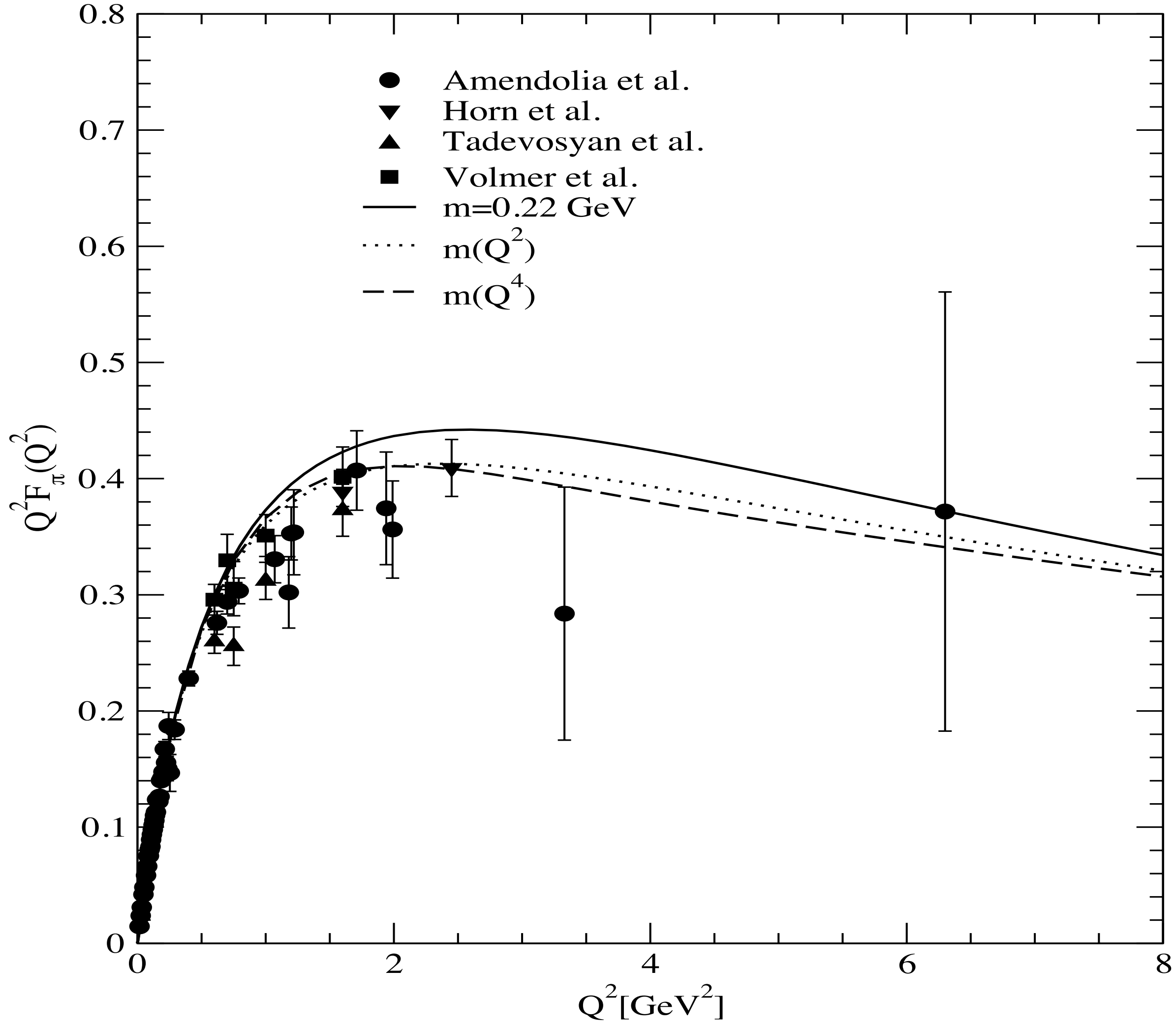
TABLE II: The operators $\mathcal{O}^{(\mu)}_{\mathrm{LFQM}}$ and their helicity contributions to the pion form factor in the standard LFQM.

$F_\pi^{(\mu)}$	$\mathcal{O}^{(\mu)}_{\mathrm{LFQM}}$	$\mathcal{H}^{(\mu)}_{(\uparrow\to\uparrow)+(\downarrow\to\downarrow)}$	$\mathcal{H}^{(\mu)}_{(\uparrow\to\downarrow)+(\downarrow\to\uparrow)}$
$F_\pi^{(+)}$	$\mathbf{k}_\perp\cdot\mathbf{k}'_\perp + m^2$	$\mathbf{k}_\perp\cdot\mathbf{k}'_\perp + m^2$	0
$F_\pi^{(\perp)}$	$\mathbf{k}_\perp\cdot\mathbf{k}'_\perp + m^2$	$\mathbf{k}_\perp\cdot\mathbf{k}'_\perp + m^2$	0
$F_\pi^{(-)}$	$\frac{2(1-x)\mathbf{q}_\perp^2 M_0^2(\mathbf{k}_\perp\cdot\mathbf{k}'_\perp+m^2+\mathbf{q}_\perp\cdot\mathbf{k}'_\perp)}{x[2M_0'^2\mathbf{q}_\perp^2+\mathbf{q}_\perp^4+(M_0^2-M_0'^2)^2]}$	$\frac{2\mathbf{q}_\perp^2\{(\mathbf{k}_\perp\cdot\mathbf{k}'_\perp+m^2)(\mathbf{k}_\perp^2+\mathbf{k}_\perp\cdot\mathbf{q}_\perp+m^2)+(1-x)(\mathbf{k}_\perp\times\mathbf{q}_\perp)^2\}}{x^2[2M_0'^2\mathbf{q}_\perp^2+\mathbf{q}_\perp^4+(M_0^2-M_0'^2)^2]}$	$\frac{2\mathbf{q}_\perp^2\{(1-x)m^2\mathbf{q}_\perp^2\}}{x^2[2M_0'^2\mathbf{q}_\perp^2+\mathbf{q}_\perp^4+(M_0^2-M_0'^2)^2]}$

$$\mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$$

$$F_\pi^{(+)}(Q^2) = F_\pi^{(\perp)}(Q^2) = F_\pi^{(-)}(Q^2)$$

"The first proof of the pion form factor's independence from current components in the LFQM!"



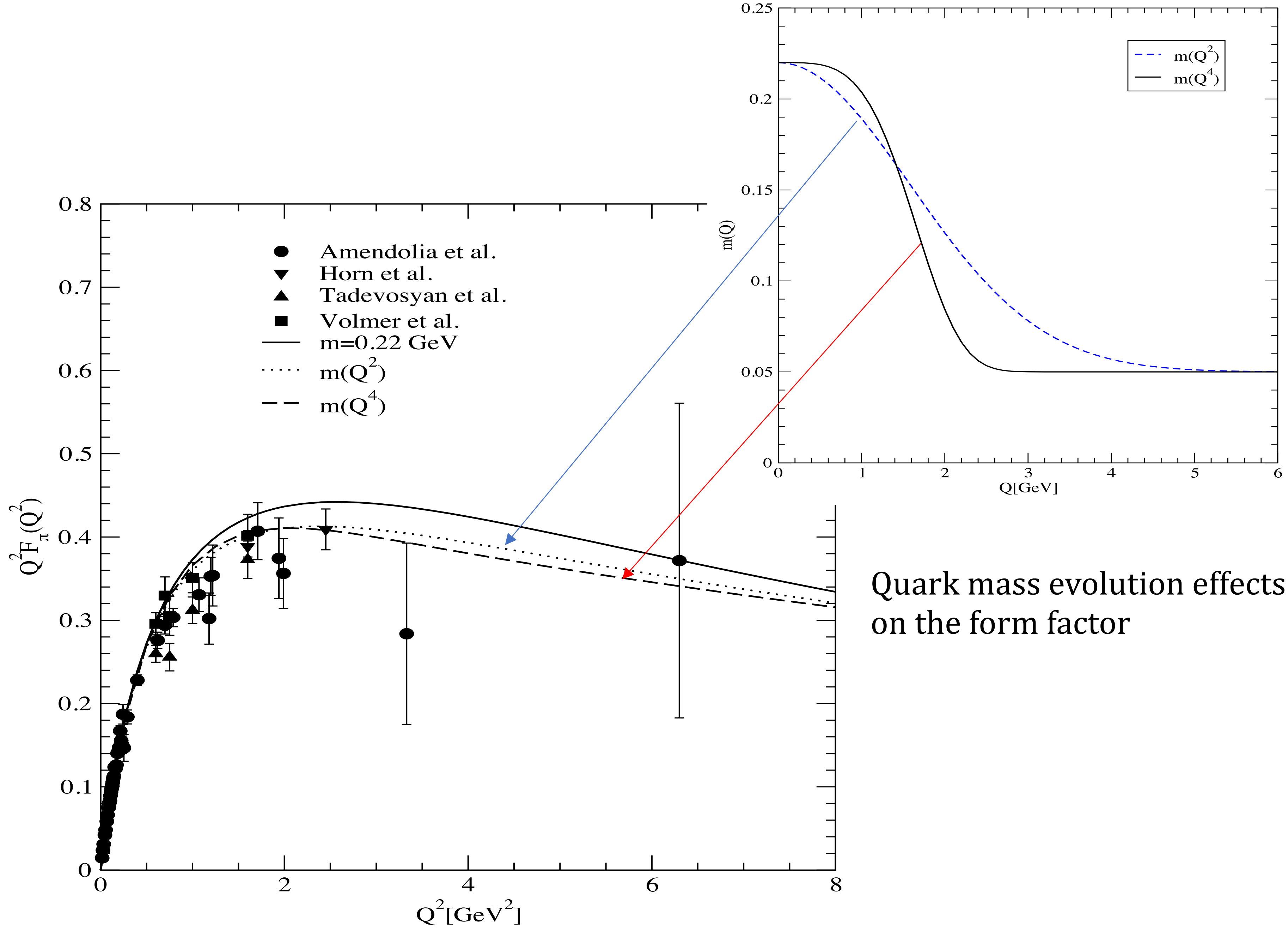
$f_\pi^{LFQM} = 130$ MeV

(Exp.=131 MeV)

$r_\pi^{LFQM} = 0.654$ fm

(Exp.=0.659(4)fm)

$$F_\pi^{(+)}(Q^2) = F_\pi^{(\perp)}(Q^2) = F_\pi^{(-)}(Q^2)$$



Quark mass evolution effects on the form factor

4. Unpolarized TMDs of pion

C. Lorcé, B. Pasquini, and P. Schweitzer,
EPJC76,415(2016)

$$\int \frac{[dz]}{2(2\pi)^3} e^{ip\cdot z} \langle P|\bar{\psi}(0)\gamma^+\psi(z)|P\rangle|_{z^+=0} = f_1^q(x, p_T),$$

$$\int \frac{[dz]}{2(2\pi)^3} e^{ip\cdot z} \langle P|\bar{\psi}(0)\gamma_T^j\psi(z)|P\rangle|_{z^+=0} = \frac{p_T^j}{P^+} f_3^q(x, p_T),$$

$$\int \frac{[dz]}{2(2\pi)^3} e^{ip\cdot z} \langle P|\bar{\psi}(0)\gamma^-\psi(z)|P\rangle|_{z^+=0} = \left(\frac{m_\pi}{P^+}\right)^2 f_4^q(x, p_T),$$

which are related with the forward matrix elements $\langle P|\bar{q}\ \gamma^\mu\ q|\mathrm{P}\rangle$ as

$$2P^+ \int dx\, f_1^q(x) = \langle P|\bar{\psi}(0)\gamma^+\psi(0)|P\rangle,$$

$$2p_T \int dx\, f_3^q(x) = \langle P|\bar{\psi}(0)\gamma^\perp\psi(0)|P\rangle,$$

$$4P^- \int dx\, f_4^q(x) = \langle P|\bar{\psi}(0)\gamma^-\psi(0)|P\rangle,$$

4. Unpolarized TMDs of pion

C. Lorcé, B. Pasquini, and P. Schweitzer,
EPJC76,415(2016)

$$\int \frac{[dz]}{2(2\pi)^3} e^{ip\cdot z}\langle P|\bar{\psi}(0)\gamma^+\psi(z)|P\rangle|_{z^+=0} = f_1^q(x,p_T),$$

$$\int \frac{[dz]}{2(2\pi)^3} e^{ip\cdot z}\langle P|\bar{\psi}(0)\gamma_T^j\psi(z)|P\rangle|_{z^+=0} = \frac{p_T^j}{P^+} f_3^q(x,p_T),$$

$$\int \frac{[dz]}{2(2\pi)^3} e^{ip\cdot z}\langle P|\bar{\psi}(0)\gamma^-\psi(z)|P\rangle|_{z^+=0} = \left(\frac{m_\pi}{P^+}\right)^2 f_4^q(x,p_T),$$

which are related with the forward matrix elements $\langle P|\bar{q}\ \gamma^\mu\ q|\mathrm{P}\rangle$ as

$$2P^+\int dx\, f_1^q(x) = \langle P|\bar{\psi}(0)\gamma^+\psi(0)|P\rangle,$$

$$2p_T\int dx\, f_3^q(x) = \langle P|\bar{\psi}(0)\gamma^\perp\psi(0)|P\rangle,$$

$$4P^-\int dx\, f_4^q(x) = \langle P|\bar{\psi}(0)\gamma^-\psi(0)|P\rangle,$$

PDF TMD

$$f(x) = \int d^2p_T\, f(x,p_T).$$

4. Unpolarized TMDs of pion

C. Lorcé, B. Pasquini, and P. Schweitzer,
EPJC76,415(2016)

Sum rules :

$$\int dx\ f_1^q(x) = N_q$$

$$\sum_q \int dx\ x\, f_1^q(x) = 1$$

$$2\int dx\ f_4^q(x) = N_q$$

$(e.g. N_u = N_{\bar{d}} = 1 \text{ in } \pi^+)$

Positivity inequalities:

$$f_1^q\,(x, p_T) \geq 0$$

$$f_4^q\,(x, p_T) \geq 0$$

4. Unpolarized TMDs of pion

C. Lorcé, B. Pasquini, and P. Schweitzer,
EPJC76,415(2016)

Sum rules :

$$\int dx\ f_1^q(x) = N_q$$

$$\sum_q \int dx\ x\, f_1^q(x) = 1$$

$$2\int dx\ f_4^q(x) = N_q$$

$(e.g. N_u = N_{\bar{d}} = 1 \text{ in } \pi^+)$

Positivity inequalities:

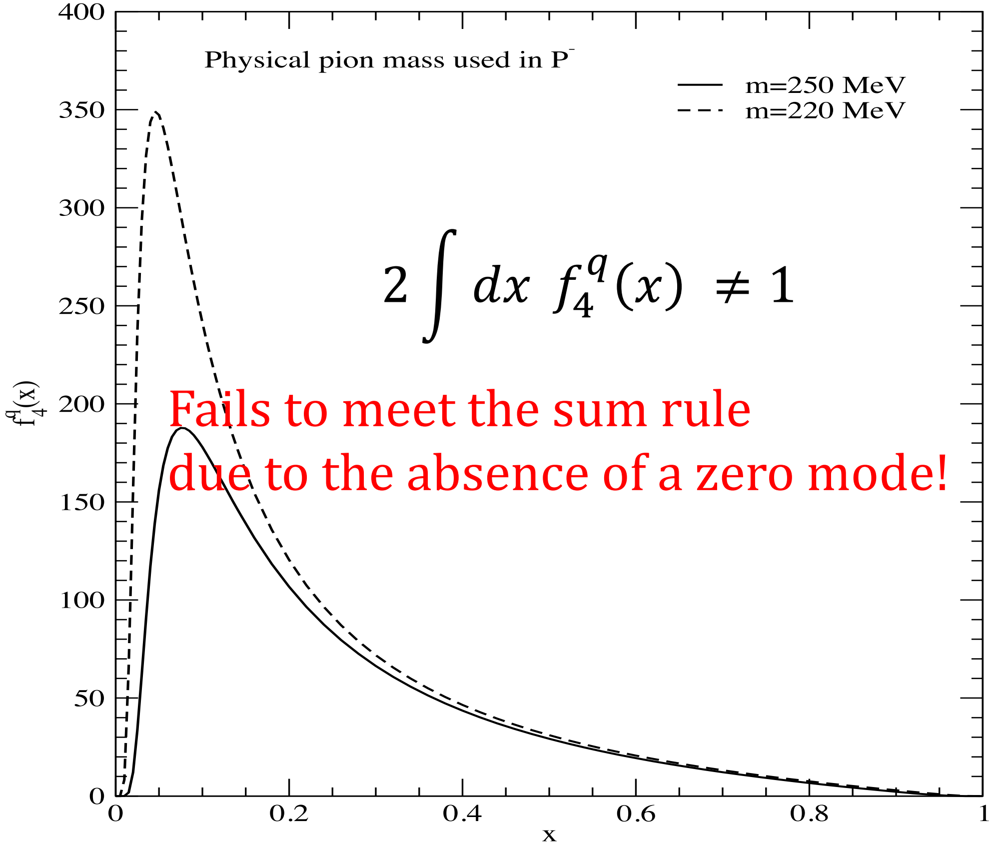
$$f_1^q\,(x, p_T) \geq 0$$

$$f_4^q\,(x, p_T) \geq 0$$

In free quark model:

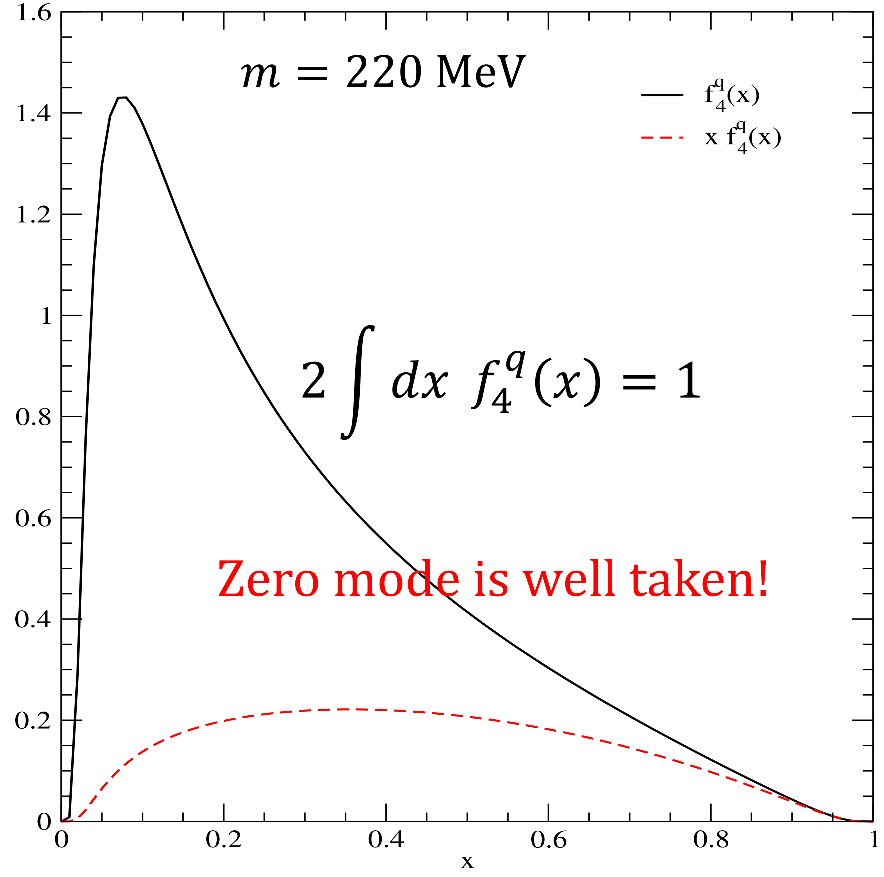
$$x f_3^q(x, p_T) = f_1^q(x, p_T)$$

LF Zero-Mode for twist-4 PDF and its Resolution



C. Lorcé, B. Pasquini, and P. Schweitzer,
EPJC76,415(2016)

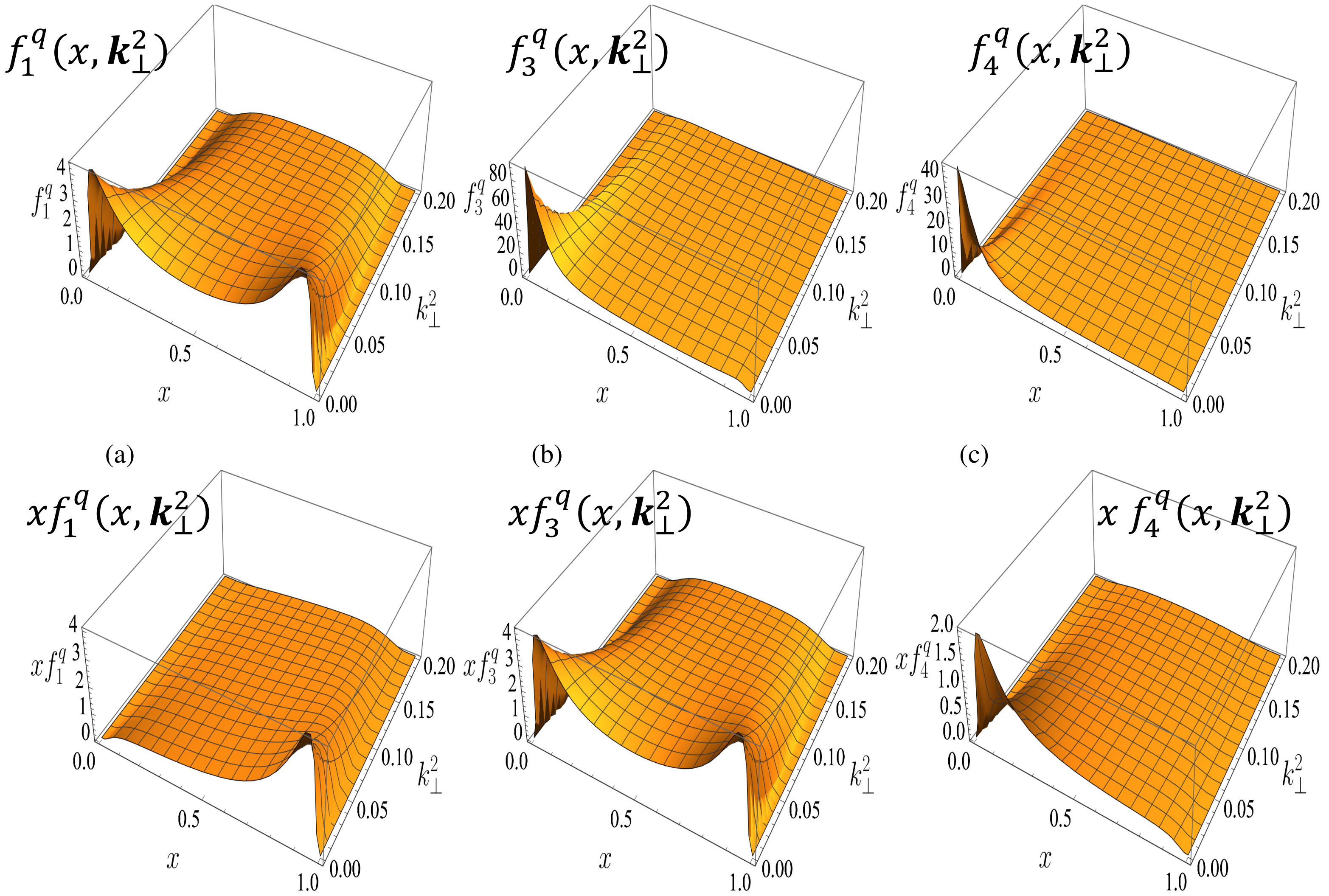
Preliminary



This work!

Unpolarized TMDs for Pion

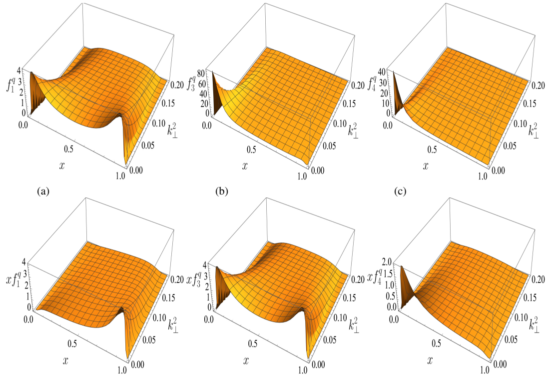
Preliminary



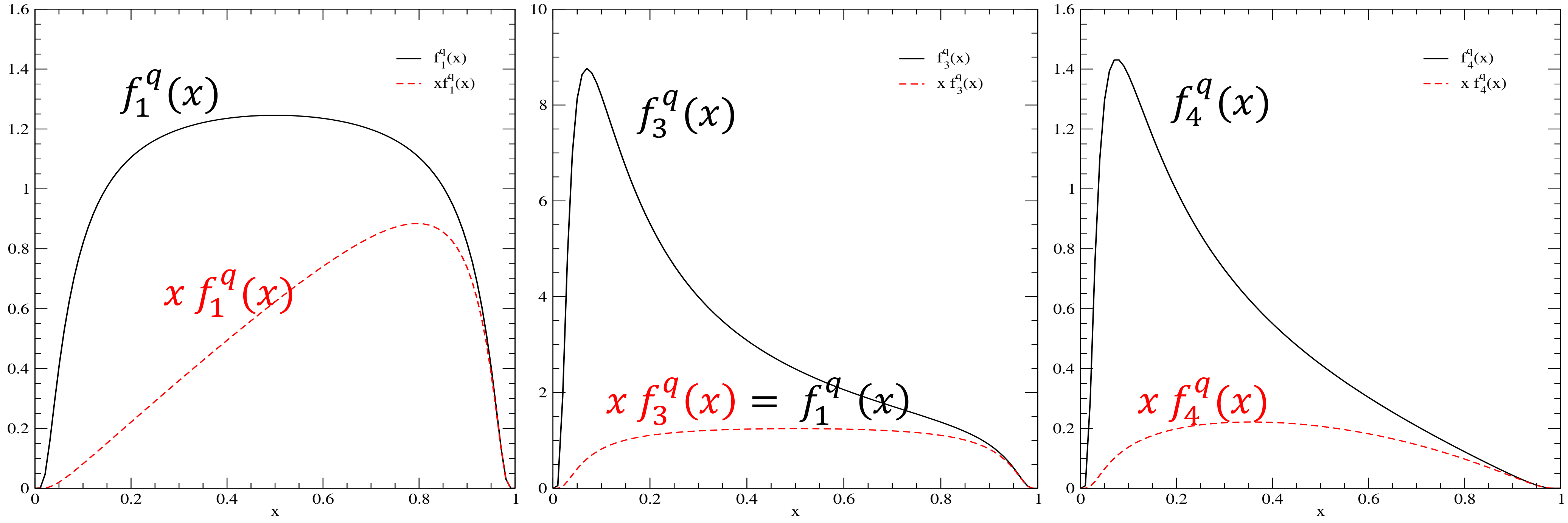
$$xf_3^q(x, \boldsymbol{k}_\perp^2) = f_1^q(x, \boldsymbol{k}_\perp^2)$$

Unpolarized PDFs for Pion

Preliminary



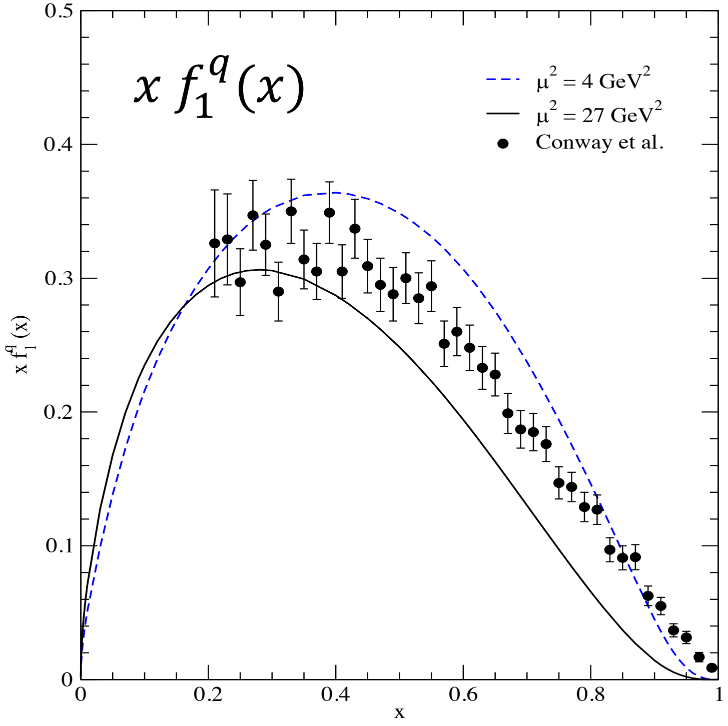
$\int d^2\boldsymbol{k}_\perp$ at the initial scale $\mu_0 = 1$ GeV

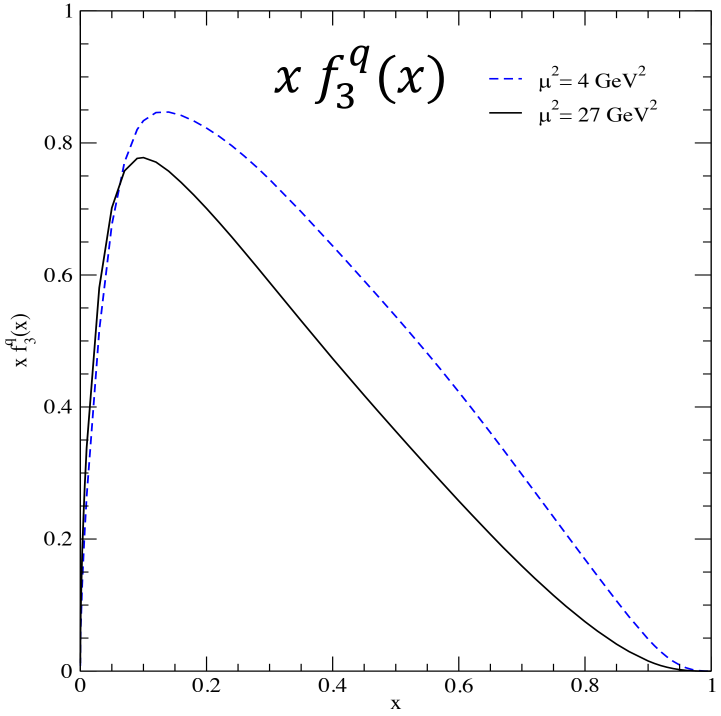


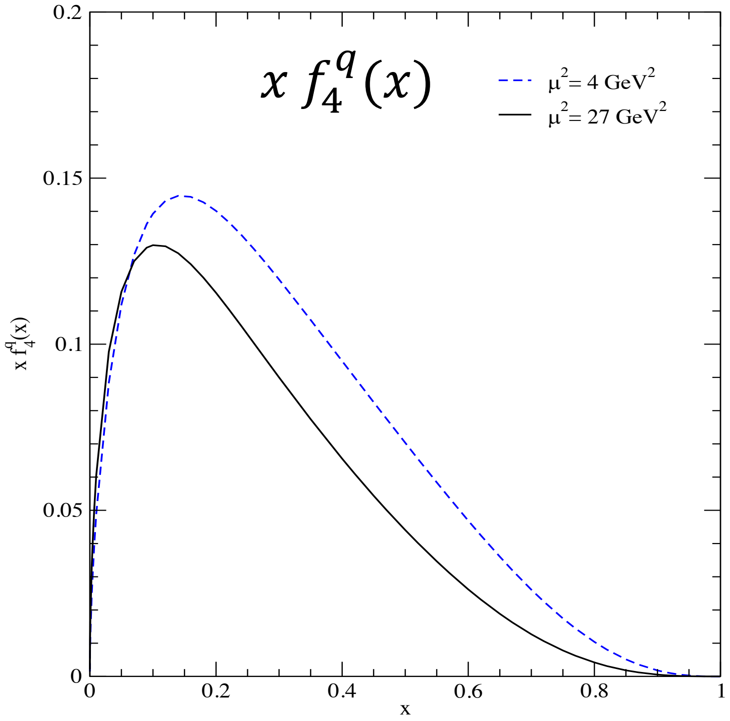
4. QCD Evolution of Pion PDFs

Preliminary

Evolved from $\mu_0^2 = 1\ \text{GeV}^2$ to $\mu^2 = 4$ and $27\ \text{GeV}^2$







We use the **Higher Order Perturbative Parton Evolution toolkit (HOPPET)** to solve the NNLO DGLAP equation.

Mellin moments: $\langle x^n \rangle = \int_0^1 dx\ x^n f(x)$

Preliminary

Twist-2 PDF

$\mu^2 = 4\ \text{GeV}^2$	$\langle x \rangle^u_{t2}$	$\langle x^2 \rangle^u_{t2}$	$\langle x^3 \rangle^u_{t2}$	$\langle x^4 \rangle^u_{t2}$
This work	0.236	0.101	0.055	0.033
[64]	0.2541(26)	0.094(12)	0.057(4)	0.015(12)
[65]	0.2075(106)	0.163(33)	–	–
[39]	0.24(2)	0.098(10)	0.049(7)	–
[40]	0.24(2)	0.094(13)	0.047(8)	–

Twist-2 PDF

$\mu^2 = 27\ \text{GeV}^2$	$\langle x \rangle^u_{t2}$	$\langle x^2 \rangle^u_{t2}$	$\langle x^3 \rangle^u_{t2}$	$\langle x^4 \rangle^u_{t2}$
This work	0.182	0.069	0.034	0.019
[66]	0.18(3)	0.064(10)	0.030(5)	–
[40]	0.20(2)	0.074(10)	0.035(6)	–
[48]	0.184	0.068	0.033	0.018
[53]	0.217(11)	0.087(5)	0.045(3)	–

Twist-3 PDF

	$\langle x \rangle^u_{t3}$	$\langle x^2 \rangle^u_{t3}$	$\langle x^3 \rangle^u_{t3}$	$\langle x^4 \rangle^u_{t3}$
$\mu^2 = 4\ \text{GeV}^2$	0.471	0.164	0.079	0.045
$\mu^2 = 27\ \text{GeV}^2$	0.365	0.111	0.049	0.026

Twist-4 PDF

	$\langle x \rangle^u_{t4}$	$\langle x^2 \rangle^u_{t4}$	$\langle x^3 \rangle^u_{t4}$	$\langle x^4 \rangle^u_{t4}$
$\mu^2 = 4\ \text{GeV}^2$	0.069	0.021	0.009	0.005
$\mu^2 = 27\ \text{GeV}^2$	0.053	0.014	0.006	0.003

5. Conclusions

We developed a new method for the covariant analysis of LFQM

Our LFQM: Noninteracting Q & $\bar{Q}$ representation consistent with the Bakamjian-Thomas(BT) constuction!

$P^- = p_q^- + p_{\bar{q}}^-$, i.e. $M^2 \rightarrow M_0^2$

$\langle 0|\bar{q}\,\Gamma^\mu q|\mathrm{P}\rangle = \mathfrak{F}\,\wp^\mu$

$\mathfrak{F}$: physical observables ($\mathfrak{F} = f_P, F \cdots$)

$\wp^\mu$: Lorentz factors ($\wp = P^\mu \cdots$)

$$\mathfrak{F} = \left\langle 0 \middle| \bar{q}\,\frac{\Gamma^\mu}{\wp^\mu}\,q \middle| \mathrm{P} \right\rangle = \iint dx\, d^2\mathbf{k}_\perp \cdots \left(\frac{\Gamma^\mu}{\wp^\mu}\right) \cdots$$

Constrained by BT construction!

⇒ We obtain physical observables independent of μ, paving the way for the development of the self-consistency in the LFQM.