Supporting evidence for tetraquark mixing model for two light-meson nonets

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Based on EPJC (2022) 82, 1113, PRD (2023) 108, 074016

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Overview

- There are two sets of resonances in $J^P = 0^+$, which we call 'light' and 'heavy' nonets .

I	Light nonet	Mass	Heavy nonet	Mass
1	$a_0(980)$	980	$a_0(1450)$	1474
1/2	$K_0^*(700)$	845	$K_0^*(1430)$	1425
0	$f_0(500)$	400-800	$f_0(1370)$	1200-1500
0	$f_0(980)$	990	$f_0(1500)$	1506

$\Delta M \gtrsim 500$ MeV

- In this talk, we introduce a tetraquark mixing model for them and present various signatures that support this model.

- We also consider a meson molecular model for comparison.

Introduction

- Multiquarks are hadrons composed of four or more constituent quarks,

$qq\bar{q}\bar{q}$	$qqqq\bar{q}$	$qqqqqq$	....
tetraquark	pentaquark	hexaquark	

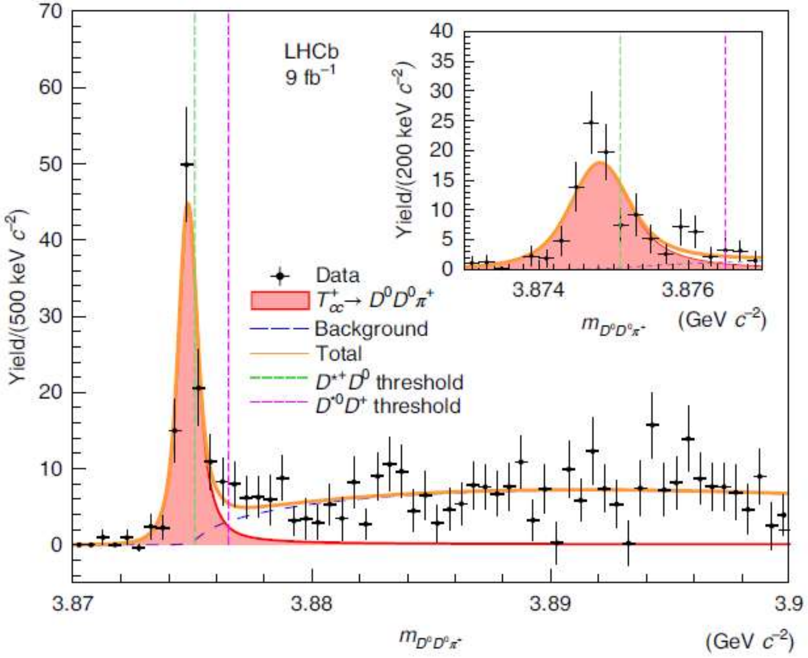
- Multiquarks are different from normal hadrons that are composed of two or three quarks,

$q\bar{q}$	qqq
meson	baryon

- The important issue is that multiquarks have not been confirmed for a long time.
- Since there is no reason for multiquarks not to exist, they have been long anticipated.

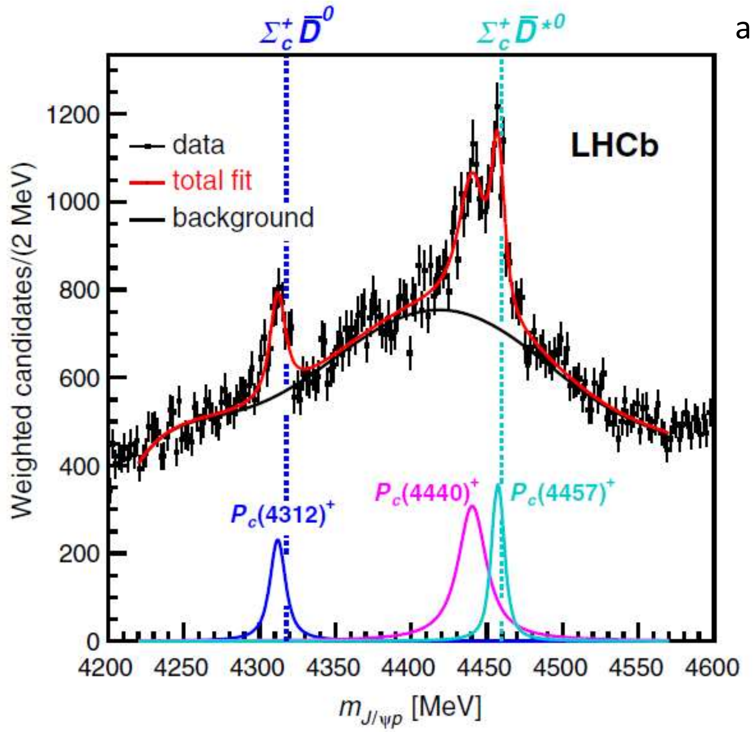
- Recently, promising candidates for multiquarks have been reported in the heavy quark sector, $X(3872)\sim qc\bar{q}\bar{c}$, $T_{cc}(3875)\sim\bar{q}\bar{q}cc$, $P_c\sim qqqc\bar{c}$, etc.

there are various other candidates also.



$T_{cc}^{+}(3875) \Longrightarrow D^0D^0\pi^+$

[LHCb, NP 18, 751 (2022)]



$P_c(4312), P_c(4440), P_c(4457)$

LHCb, PRL 122, 222001 (2019)

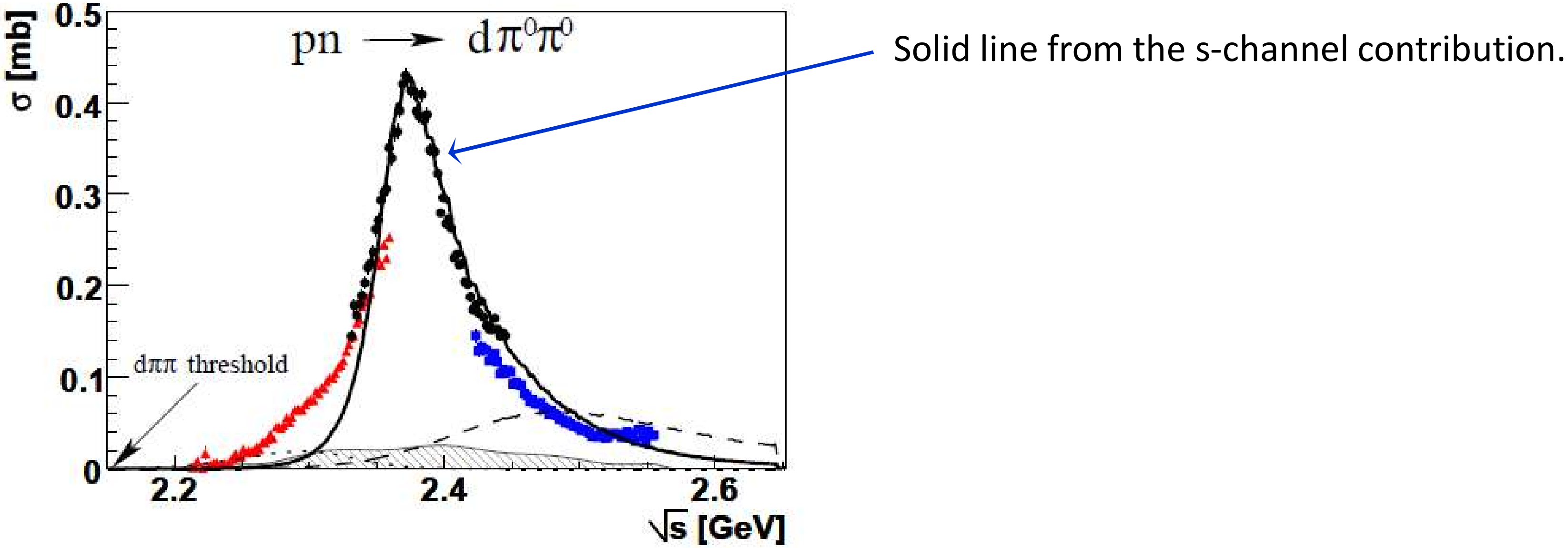
※$P_c(4450)$ in 2015 is splitted into two states, $P_c(4440)$, $P_c(4457)$, in 2019

- They can be interpreted as different states like hadronic molecules also.

$X(3872)\sim D^0\bar{D}^0,\ T_{cc}\approx DD^* \quad P_c(4312)\sim\Sigma_c\bar{D},\ P_c(4440), P_c(4457)\sim\Sigma_c\bar{D}^*$

- Hexaquark candidate : $d^*(2380)$

$qqqqqq \in 1_c$



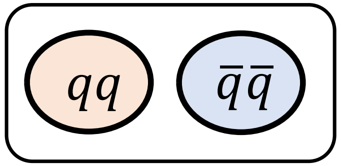
WASA-at-COSY, PRL 106, 242302, (2011)

- It can be interpreted as a hadronic molecule like ΔΔ. $(qqq \in 1_c)(qqq \in 1_c)$

Tetraquark candidates in the light quark system

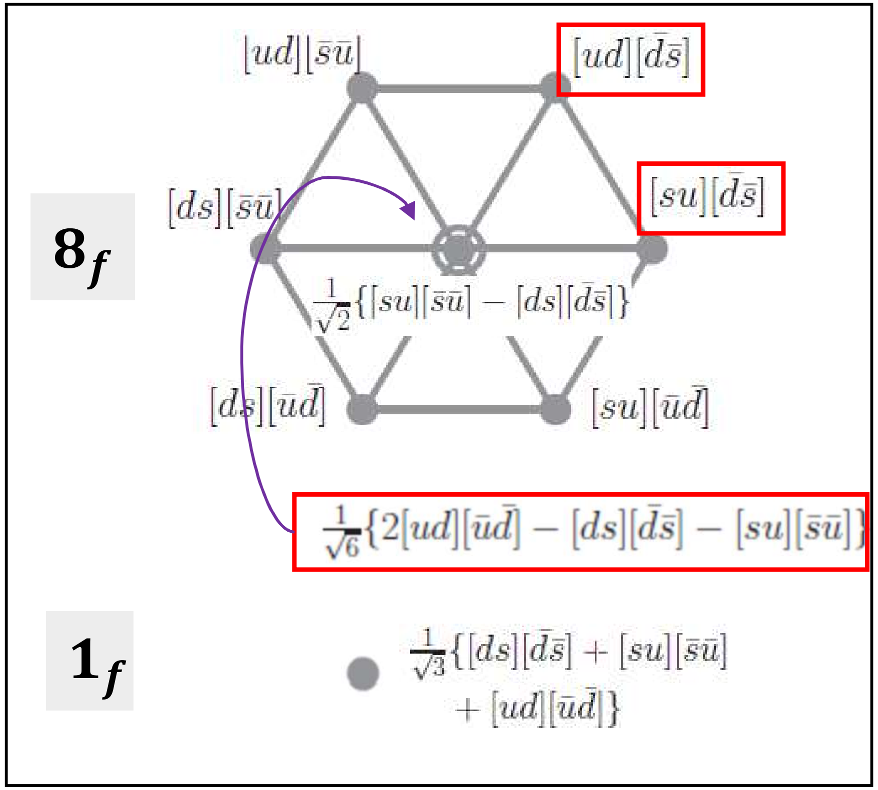
- Long-standing candidates include the light nonet consisting of $a_0(980), K_0^*(700), f_0(500), f_0(980)$ with mass less than 1 GeV. [Jaffe (1977)]

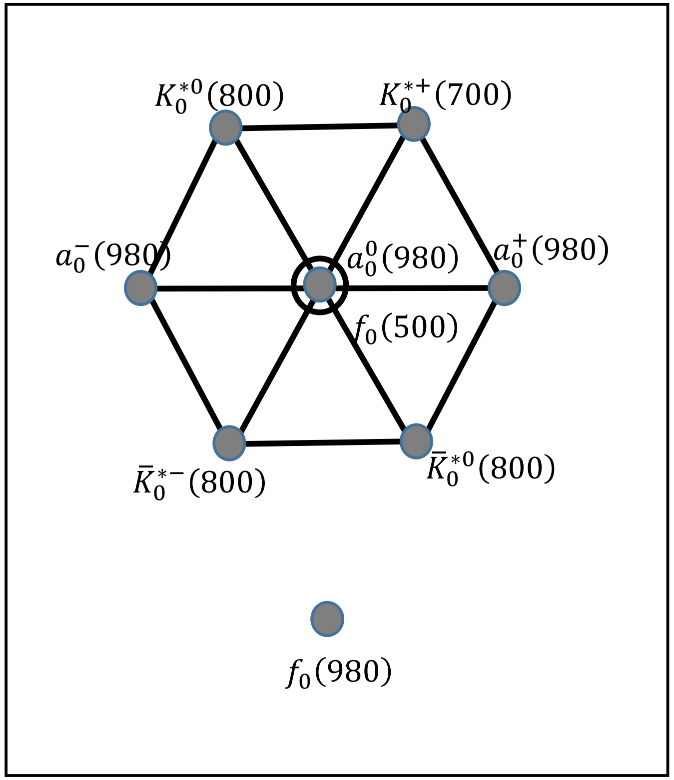
- "Inverted mass ordering" among them, $a_0(980) > K_0^*(700) > f_0(500)$.
- This ordering can be reproduced if the light nonet forms a tetraquark nonet constructed by diquark and antidiquark.



$(q = u, d, s)$

$$\bar{3}_f \otimes 3_f = 8_f \oplus 1_f$$





- Note, this ordering cannot be reproduced if the light nonet is the $q\bar{q}$ $(\ell = 1)$ states.

One problem with this picture is that the light nonet is too light !

- If they are composed of four constituent quarks with $m_{u,d} \approx 330$ MeV, $m_s \approx 500$ MeV, why are their masses less than 1 GeV?

- The hyperfine mass (typically an order of ~ -200 MeV) is not enough to reduce their masses below 1 GeV.

Models different from tetraquarks !

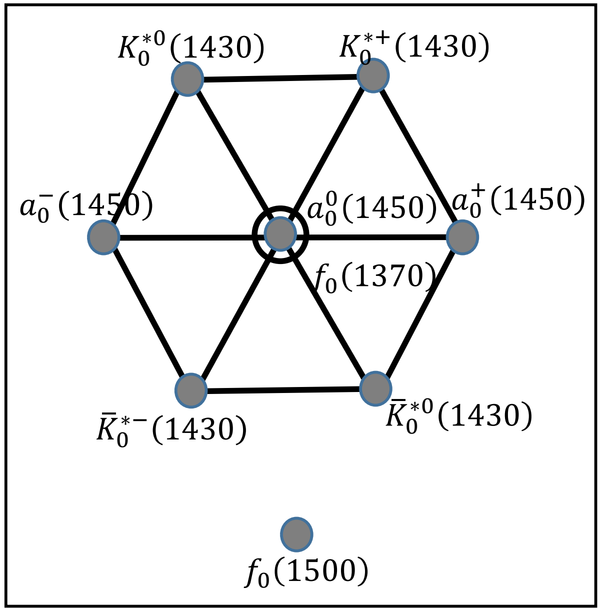
- $a_0(980)$, $f_0(980)$ could be meson molecules like $K\bar{K}$.
  [Weinstein and Isgur, PRD **41**, 2236 (1990)]
- Or they could be the $q\bar{q}$ states with hadronic intermediate states [Törnqvist, ZPC **68**, 647 (1995)], referred as "dynamical generation of the scalar meson".
  [Boglione and Pennington, PRD**65**, 114010 (2002)].

- These models explain only a few members.
  ☞probably cannot explain all members of the light nonet.
- As Boglione and Pennington put this, "it is not possible to reach one common conclusion for all the members of the scalar meson family".

Another tetraquark candidates in the light quark system

- are the heavy nonet composed of $a_0(1450), K_0^*(1430), f_0(1370), f_0(1500)$.
- This heavy nonet is expected to be tetraquarks also forming $\mathbf{9}_f$.
  - ✓ This has the same isospin composition as the light nonet.
  - ✓ This nonet satisfies the inverted mass ordering also,

    $a_0(1450) \gtrsim K_0^*(1430) \gtrsim f_0(1370)$ (marginal)



- Maiani et.al [EPJC50, 609(2007)] also consider this heavy nonet with $f_0(1700)$ as tetraquarks mixed with glueball.

I	Light nonet	Mass	Heavy nonet	Mass
1	$a_0(980)$	980	$a_0(1450)$	1474
1/2	$K_0^*(700)$	845	$K_0^*(1430)$	1425
0	$f_0(500)$	400-800	$f_0(1370)$	1200-1500
0	$f_0(980)$	990	$f_0(1500)$	1506

Models that are not relying on tetraquarks

- Giacosa [PRD **75**(2007)] suggests that $a_0(1450)$, $K_0^*(1430)$ are the $q\bar{q}$ states with $\ell = 1$.
  (but we doubt this because the heavy nonet is too heavy to be the $q\bar{q}$ states.)

- Molina et.al [PRD**78**(2008)] consider $f_0(1370)$ as the $\rho\rho$ molecular state.
- Boglione and Pennington [PRD**65**(2002)] propose that $a_0(1450)$, $K_0^*(1430)$ are the states "dynamical generated" from $q\bar{q}$.

- Again, these models seem to explain only a few members.
  ☞probably cannot explain all members of the heavy nonet

Multiquarks are not established as their candidates can be explained by alternative models like hadronic molecules. Therefore, it is necessary to find a universally accepted approach that can determine multiquarks exclusively.

- Recently, we proposed a tetraquark mixing model that can describe the light and heavy nonets in one mixing framework.

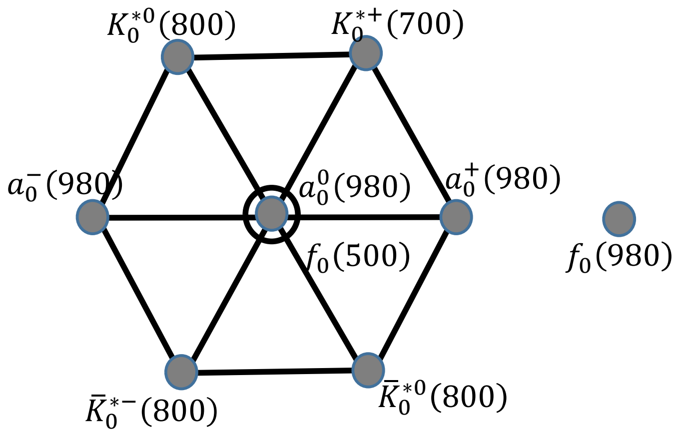
Light nonet: $a_0(980), K_0^*(700), f_0(500), f_0(980)$
Heavy nonet: $a_0(1450), K_0^*(1430), f_0(1370), f_0(1500)$

- In this model, the two nonets are represented by a linear combination of two types of tetraquarks.
- This mixing model has successful features that cannot be reproduced in models other than tetraquarks.

Tetraquark mixing model for the two nonets in $J^P = 0^+$

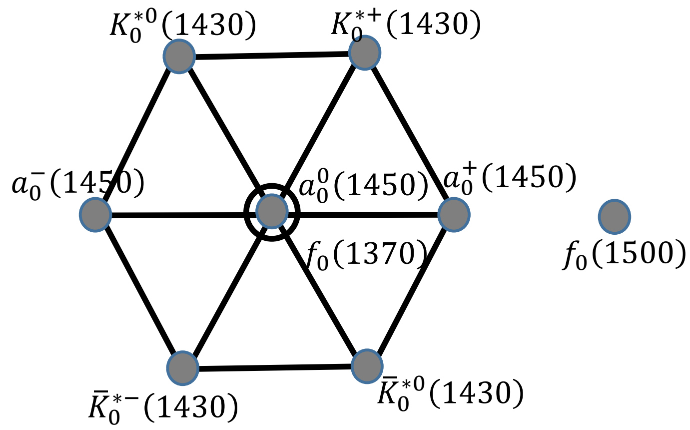
Light nonet

$a_0(980), K_0^*(800), f_0(500), f_0(980)$

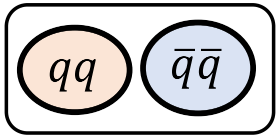


Heavy nonet

$a_0(1450), K_0^*(1430), f_0(1370), f_0(1500)$



- This mixing model utilizes two types of tetraquarks in the diquark-antidiquark form, which we denoted as $|\text{Type1}\rangle, |\text{Type2}\rangle$



$(q = u, d, s)$

$|\text{Type1}\rangle$: $[qq \in (J = 0, \bar{3}_c, \bar{3}_f)] \otimes [\bar{q}\bar{q} \in (J = 0, 3_c, 3_f)] \Longrightarrow qq\bar{q}\bar{q} \in (J = 0, 1_c, 9_f)$

- Well-known tetraquark type, originally constructed for the light nonet (Jaffe).
- $qq \in (J = 0, \bar{3}_c, \bar{3}_f)$: most compact among all possible diquarks.

$|\text{Type2}\rangle$: $[qq \in (J = 1, 6_c, \bar{3}_f)] \otimes [\bar{q}\bar{q} \in (J = 1, \bar{6}_c, 3_f)] \Longrightarrow qq\bar{q}\bar{q} \in (J = 0, 1_c, 9_f)$

- Another type proposed in 2017 [EPJC77, 3 (2017)].
- $qq \in (J = 1, 6_c, \bar{3}_f)$: second most compact

- Note, $|\text{Type1}\rangle, |\text{Type2}\rangle$ have the same flavor structure, $\mathbf{9}_f$.
  ☞ both generate "inverted mass ordering", the ordering satisfied by the two nonets.
  $a_0(980) > K_0^*(700) > f_0(500)$, $a_0(1450) \gtrsim K_0^*(1430) \gtrsim f_0(1370)$

- But $|\text{Type1}\rangle, |\text{Type2}\rangle$ have different color and spin configurations.

- An important observation to make is that $|\text{Type1}\rangle$, $|\text{Type2}\rangle$ are mixed through V_{CS},

  $\langle \text{Type2}|V_{CS}|\text{Type1}\rangle \neq 0$

  $\Rightarrow$ V_{CS} forms a 2x2 matrix for each flavor member.

$$V_{CS} \propto -\sum_{i<j} \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i\, m_j}$$

Color-spin interaction

- In fact, this mixing is very large !

  ex) For the $I = 1$ channel,

$\langle V_{CS}\rangle$	$\|\text{Type1}\rangle$	$\|\text{Type2}\rangle$
$\|\text{Type1}\rangle$	-173.9	-222.3
$\|\text{Type2}\rangle$	-222.3	-331.5

strong mixing

- This mixing implies that $|\text{Type1}\rangle$, $|\text{Type2}\rangle$ are not eigenstates of V_{CS}.
  ☞ $|\text{Type1}\rangle$, $|\text{Type2}\rangle$ cannot represent physical states.

- Physical states can be generated by combinations of the two types that diagonalize the 2x2 matrix of V_{CS}.
- The diagonalization process produces two sets of "physical" states,

$$
\begin{aligned}
|\text{Heavy nonet}\rangle &= -\alpha|\text{Type1}\rangle + \beta|\text{Type2}\rangle \Rightarrow a_0(1450), K_0^*(1430), f_0(1370), f_0(1500) \\
|\text{Light nonet}\rangle &= \beta|\text{Type1}\rangle + \alpha|\text{Type2}\rangle \Rightarrow a_0(980), K_0^*(800), f_0(500), f_0(980)
\end{aligned}
$$

and the mixing parameters, α, β, for each flavor member,

I	α	β
1	0.8167	0.5770
1/2	0.8130	0.5822
0	0.8136	0.5814
0	0.8157	0.5784

- In fact, α and β are almost independent of the isospin states.
  $\Rightarrow$ $|\text{Light nonet}\rangle$, $|\text{Heavy nonet}\rangle$ also form $\mathbf{9}_f$ just as $|\text{Type1}\rangle$, $|\text{Type2}\rangle$.

This is the tetraquark mixing model for the two nonets !

EPJC77, 3 (2017); EPJC77, 435 (2017); PRD97, 094005 (2018); PRD 99, 014005 (2019)
PRD100, 034021(2019); EPJC82, 1113 (2022); PRD108, 074016 (2023).

Supporting evidence for the tetraquark mixing model from $\langle V_{CS} \rangle$

1. $\langle V_{CS} \rangle \approx -500$ MeV for the light nonet
   $\Rightarrow$ explain qualitatively why $\mathrm{M(light\ nonet)} \lesssim 1$ GeV.

2. $\langle V_{CS} \rangle \approx (-29) \sim (-17)$ MeV for the heavy nonet
   $\Rightarrow$ explain qualitatively why $\mathrm{M(heavy\ nonet)} \approx 4m_q$.

3. $\Delta M \approx \Delta \langle V_{CS} \rangle$!

The same mass splitting formula works for the mass difference between vector meson –pseudoscalar meson, baryon octet- baryon decuplet.

Light nonet	$\langle V_{CS} \rangle$	Heavy nonet	$\langle V_{CS} \rangle$	ΔM	$\Delta \langle V_{CS} \rangle$
$a_0(980)$	-488.5	$a_0(1450)$	-16.8	494	472
$K_0^*(700)$	-592.7	$K_0^*(1430)$	-26.9	580	566
$f_0(500)$	-667.5	$f_0(1370)$	-29.2	750	612
$f_0(980)$	-535.1	$f_0(1500)$	-20.1	516	542

SU(3) (bracketing the $f_0(500)/f_0(1370)$ and $f_0(980)/f_0(1500)$ rows)

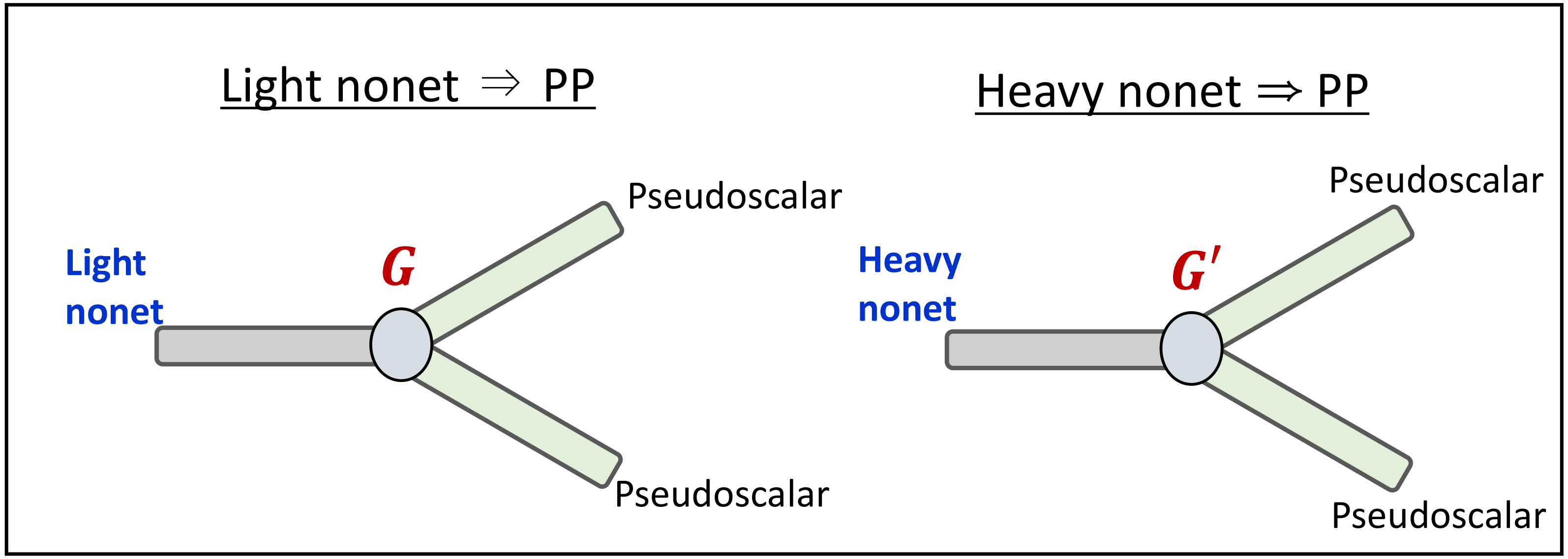
$\langle V_{CS} \rangle$ in the physical basis

- We emphasize that these successful aspects originate from the mixing of two types of tetraquarks.

Another supporting evidence from coupling strengths

- The tetraquark mixing model predicts that,

$$|G|(\text{light nonet}) \gg |G'|(\text{heavy nonet})$$



- This prediction is indeed supported by the experimental partial decay widths,

$$\Gamma_{exp}(\text{light nonet}) \gtrsim \Gamma_{exp}(\text{heavy nonet})$$

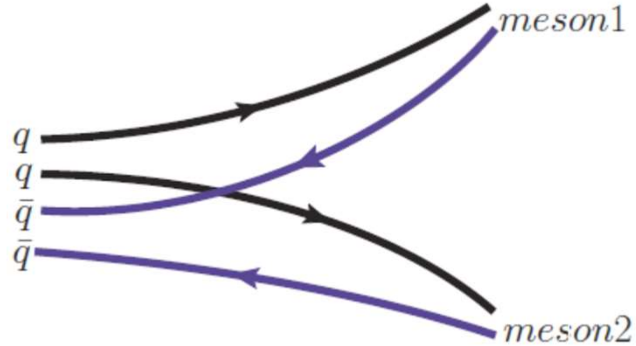
Why $|G|$(light nonet) $\gg$ $|G'|$ (heavy nonet) ?

Tetraquarks, either in Type1 or Type2, can decay into two pseudoscalar mesons through the component of $(q\bar{q})_{\mathbf{1}_c}(q\bar{q})_{\mathbf{1}_c}$.

$$[q_1 q_2 \bar{q}_3 \bar{q}_4]_{1_c} \Longrightarrow \underbrace{(q_1\bar{q}_3)_{\mathbf{1}_c} \otimes (q_2\bar{q}_4)_{\mathbf{1}_c}}_{\text{two-meson comp.}} \oplus \underbrace{[(q_1\bar{q}_3)_{\mathbf{8}_c} \otimes (q_2\bar{q}_4)_{\mathbf{8}_c}]_{1_c}}_{\text{hidden-color comp.}}$$

(24)

(13)



$qq\bar{q}\bar{q}$ fall-apart decay

$$|\text{Heavy nonet}\rangle = -\alpha|\text{Type1}\rangle + \beta|\text{Type2}\rangle$$
$$|\text{Light nonet}\rangle = \beta|\text{Type1}\rangle + \alpha|\text{Type2}\rangle$$

Due to the difference in relative signs,
☞ the coupling strength of the PP mode is enhanced in the light nonet but suppressed in the heavy nonet.

Coupling strength(G) to PP mesons

$G = \langle PP|\text{Light nonet}\rangle$
$G' = \langle PP|\text{Heavy nonet}\rangle$

Light nonet mode	$G = \langle PP\|\text{Light nonet}\rangle$	Heavy nonet mode	$G' = \langle PP\|\text{Heavy nonet}\rangle$
$a_0^+(980) \to \pi^+\eta$	0.6076	$a_0^+(1450) \to \pi^+\eta$	0.1406
$a_0^+(980) \to K^+\bar{K}^0$	0.7441	$a_0^+(1450) \to K^+\bar{K}^0$	0.1722
$K_0^{*+}(700) \to \pi^0 K^+$	0.5253	$K_0^{*+}(1430) \to \pi^0 K^+$	0.1251
$f_0(500) \to \pi^0\pi^0$	−0.3310	$f_0(1370) \to \pi^0\pi^0$	−0.0785
$f_0(980) \to \pi^0\pi^0$	−0.1690	$f_0(1500) \to \pi^0\pi^0$	−0.0394
$f_0(980) \to K^0\bar{K}^0$	−0.4685	$f_0(1500) \to K^0\bar{K}^0$	−0.1093

- So, the tetraquark mixing model predicts that

  $|G|$(light nonet) $\gg$ $|G'|$ (heavy nonet)

  $$\Rightarrow \frac{G(\text{light nonet})}{G'(\text{heavy nonet})} \approx 4!$$

- This prediction from the tetraquark mixing model is unique and it is very unlikely to reproduce this from models other than tetraquarks.

- This prediction can be experimentally verified by examining partial decay widths.

$$\Gamma_{partial} = G^2 \Gamma_{kin}$$

So, we examine [EPJC82, 1113 (2022) by H.Kim and K.S.Kim]

coupling strength cont.

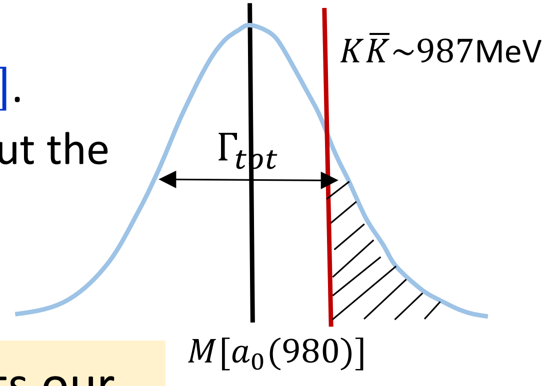
Experimental partial decay widths estimated from PDG (2022)

Light nonet		Heavy nonet	
Decay mode	Γ_{exp}(MeV)	Decay mode	Γ_{exp}(MeV)
$a_0(980) \to \pi\eta$	60	$a_0(1450) \to \pi\eta$	15.4–20.5
$K_0^*(700) \to \pi K$	468	$K_0^*(1430) \to \pi K$	251.1
$f_0(980) \to \pi\pi$	50	$f_0(1500) \to \pi\pi$	38.1
$f_0(980) \to K\bar{K}$	9.5–46.2	$f_0(1500) \to K\bar{K}$	9.5
$f_0(500) \to \pi\pi$	NC	$f_0(1370) \to \pi\pi$	NC
$a_0(980) \to K\bar{K}$	10.6	$a_0(1450) \to K\bar{K}$	13.5–18.0

Note, some partial widths appear in PDG recently.

NC: Not conclusive

- For most decay modes, we see an unnatural trend
  Γ_{exp}(light nonet) $\gtrsim$ Γ_{exp}(heavy nonet).
- For $f_0(500) \longrightarrow \pi\pi$, $f_0(1370) \longrightarrow \pi\pi$, expt. data are not conclusive but we expct $\Gamma_{exp}[f_0(500) \longrightarrow \pi\pi]$ to be larger because $f_0(500)$ is famous for its broad width.
- $\exists$ exceptional case, $\Gamma_{exp}[a_0(980) \to K\bar{K}] < \Gamma_{exp}[a_0(1450) \to K\bar{K}]$. $\Gamma_{exp}[a_0(980) \to K\bar{K}]$ is small due to the kinematical cutoff. Without the cutoff, the above trend is expected to continue in this case also.
  [See EPJC77, 435 (2017) by K.S.Kim and H.Kim]



- Qualitatively, Γ_{exp}(light nonet) $\gtrsim$ Γ_{exp}(heavy nonet) supports our prediction, $|G|$(light nonet) $\gg$ $|G'|$ (heavy nonet) ! (Next slide)

- Partial decay width can be written as

  $$\Gamma_{partial} = G^2\,\Gamma_{kin}$$

- The kinematical widths satisfy
  Γ_{kin}(light nonet) $\ll$ Γ_{kin}(heavy nonet).

  ex) $\frac{\Gamma_{kin}[a_0(1450)\to\pi\eta]}{\Gamma_{kin}[a_0(980)\to\pi\eta]} \approx 1.68$

- If we multiply by G^2 (G'^2), the inequality needs to be reversed,

  $$G^2\,\Gamma_{kin}(\text{light nonet}) \gtrsim G'^2\,\Gamma_{kin}(\text{heavy nonet}),$$

  in order to reproduce the unnatural trend of the experimental partial widths,
  Γ_{exp}(light nonet) $\gtrsim$ Γ_{exp}(heavy nonet),

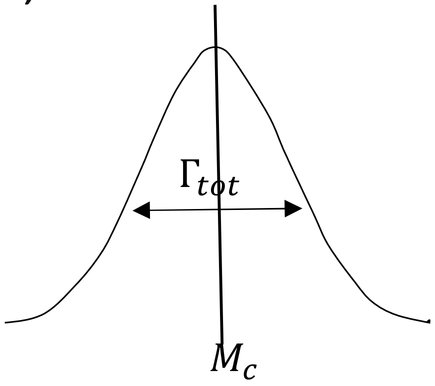
- The only way to reproduce the expt. partial widths is to have
  $|G|$(light nonet) $\gg$ $|G'|$(heavy nonet)(!)
- Thus, the prediction from the tetraquark mixing model is qualitatively supported by expt. partial decay widths.

To do a quantitative comparison

- We also calculate the theoretical partial decay width, Γ_{theory}, that includes the mass distribution caused by the total width,

$$\Gamma_{theory}(M_c, \Gamma_{tot}) = \frac{\int_{m_1+m_2}^{\infty} \Gamma(M \longrightarrow m_1, m_2) f(M) dM}{\int_{m_1+m_2}^{\infty} f(M) dM}$$

with G fixed by the tetraquark mixing model.



$$f(M) \sim e^{-(M-M_c)^2[4\ln 2/\Gamma_{tot}^2]}$$

- And compare Γ_{theory} with Γ_{exp} (through some ratios).
- Our analysis shows that the quantitative comparison is not precise enough.

- Nevetheless(!) this result does not undermine the qualitative conclusion that, $|G|$(light nonet) $\gg$ $|G'|$(heavy nonet)!
- The tetraquark mixing model is clearly supported by the experimental partial decay widths.

We now consider a different approach like

Meson molecular model for the two nonets

- It was suggested that $a_0(980)$, $f_0(980)$ in the light nonet could be meson molecules like $K\bar{K}$.
- If the light nonet really forms $\mathbf{9}_f$, other members need to be meson molecules also [by SU(3)].
- Similarly, one can look for a meson molecular model for the heavy nonet also.
- Here, we test whether the two nonets can be described by meson molecules composed of two pseudoscalar (PS) mesons.

Multiplets in meson molecules

- In this approach, we construct two nonets by combining two PS nonets.

Note, $P_j^j = 0$,
and $(P_j^i, P_l^k) = \delta^{ik}\delta_{jl} - \frac{1}{3}\delta_j^i\delta_l^k$

Pseudoscalar (PS) nonet

$$\mathbf{1}_f = \eta_1$$

$$\mathbf{8}_f:\ P_j^i = \begin{bmatrix} P_1^1 & P_1^2 & P_1^3 \\ P_2^1 & P_2^2 & P_2^3 \\ P_3^1 & P_3^2 & P_3^3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 \end{bmatrix}$$

- Two-meson molecules can form the following multiplets,

PS ⊗ PS	two−meson multiplets

$$\mathbf{1}_f \otimes \mathbf{1}_f = \boxed{\mathbf{1}'}$$

$$\mathbf{8}_f \otimes \mathbf{1}_f = \boxed{\mathbf{8}''}$$

$$\mathbf{8}_f \otimes \mathbf{8}_f = \mathbf{27} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \boxed{\mathbf{8}} \oplus \boxed{\mathbf{8}'} \oplus \boxed{\mathbf{1}}$$

- Possible nonets can be obtained from those marked as $\boxed{}$.

Trivial nonet of two-meson molecules is obtained from

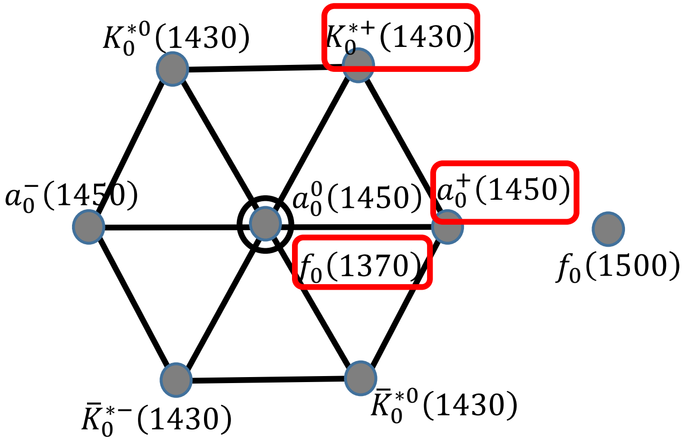
PS ⊗ PS

$$\mathbf{1}_f \otimes \mathbf{1}_f = \mathbf{1}'$$

two-meson rep.

$$\mathbf{1}' = \eta_1\eta_1$$

$$\mathbf{8}_f \otimes \mathbf{1}_f = \mathbf{8}''$$



$$(\mathbf{8}'')^3_1 = \eta_1 K^+ \qquad (\mathbf{8}'')^3_2 = \eta_1 K^0$$

$$(\mathbf{8}'')^2_1 = \eta_1\pi^+ \quad \frac{1}{\sqrt{2}}[(\mathbf{8}'')^1_1 - (\mathbf{8}'')^2_2] = \eta_1\pi^0 \quad (\mathbf{8}'')^1_2 = \eta_1\pi^-$$

$$(\mathbf{8}'')^3_3 = \eta_1\eta_8$$

$$(\mathbf{8}'')^2_3 = \eta_1\bar{K}^0 \qquad (\mathbf{8}'')^1_3 = \eta_1 K^-$$

This trivial nonet cannot represent either of the two nonets in PDG.

- This trivial nonet is too heavy to represent the light nonet with mass $\lesssim$ 1 GeV.
- This nonet has the mass ordering opposite to that of the heavy nonet.
  - ✓ The mass ordering, $M(\eta_1\pi^-) < M(\eta_1 K^0) < M(\eta_1\eta_8)$, is opposite to $a_0(1450) \gtrsim K_0^*(1430) \gtrsim f_0(1370)$.
- The decay modes do not match those of the two nonets.

Another nonet is possible from $\mathbf{8}_f \otimes \mathbf{8}_f = \mathbf{27} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{8} \oplus \mathbf{8}' \oplus \mathbf{1}$

The multiplets, $\mathbf{1}, \mathbf{8}, \mathbf{8}'$ can be obtained from the general two-meson states, $P^i_j P^k_l$.

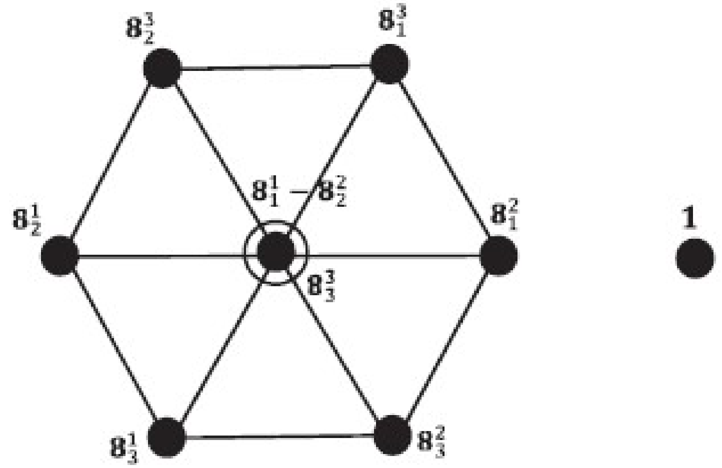
- We found that

$$\mathbf{1} = \mathrm{Tr}(PP)$$

$$\mathbf{8}^k_j = (PP)^k_j - \frac{1}{3}\delta^k_j \mathrm{Tr}(PP) \qquad (\mathbf{8}^i_j, \mathbf{8}^k_l) = \frac{7}{3}\left(\delta^{ik}\delta_{jl} - \frac{1}{3}\delta^i_j \delta^k_l\right)$$

$$(\mathbf{8}')^k_j = \mathbf{8}^k_j$$

$$P^i_j = \begin{bmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 \end{bmatrix}$$



- In this approach, there is only one molecular nonet that can match either the light nonet or the heavy nonet.
- Thus, this molecular approach cannot explain the existence of two nonets.

two-meson rep. of the nontrivial nonet

short-hand notations

$$\vec{\pi}\cdot\vec{\pi} = \pi^+\pi^- + \pi^-\pi^+ + \pi^0\pi^0,$$
$$\bar{K}K = K^-K^+ + \bar{K}^0K^0,$$
$$(\bar{K}K)^\dagger = K^+K^- + K^0\bar{K}^0.$$

$I=0$ ▪ $\mathbf{1} = \mathrm{Tr}(PP) = \vec{\pi}\cdot\vec{\pi} + \bar{K}K + (\bar{K}K)^\dagger + \eta_8\eta_8$

▪ $\mathbf{8}^k_j = (PP)^k_j - \frac{1}{3}\delta^k_j \mathrm{Tr}(PP)$

$I=1/2$

$$\mathbf{8}^3_1 = \frac{1}{\sqrt{2}}\pi^0K^+ + \frac{1}{\sqrt{6}}\eta_8K^+ + \pi^+K^0 - \sqrt{\frac{2}{3}}K^+\eta_8$$
$$\mathbf{8}^3_2 = \pi^-K^+ - \frac{1}{\sqrt{2}}\pi^0K^0 + \frac{1}{\sqrt{6}}\eta_8K^0 - \sqrt{\frac{2}{3}}K^0\eta_8$$

$I=1$

$$\mathbf{8}^2_1 = \frac{1}{\sqrt{2}}(\pi^0\pi^+ - \pi^+\pi^0) + \frac{1}{\sqrt{6}}(\eta_8\pi^+ + \pi^+\eta_8) + K^+\bar{K}^0$$
$$\mathbf{8}^1_1 - \mathbf{8}^2_2 = \frac{1}{\sqrt{3}}\left[\pi^0\eta_8 + \eta_8\pi^0 + \sqrt{3}(\pi^+\pi^- - \pi^-\pi^+ + K^+K^- - K^0\bar{K}^0)\right]$$
$$\mathbf{8}^1_2 = \frac{1}{\sqrt{2}}(\pi^-\pi^0 - \pi^0\pi^-) + \frac{1}{\sqrt{6}}(\eta_8\pi^- + \pi^-\eta_8) + K^0K^-$$

$I=0$

$$\mathbf{8}^3_3 = \frac{1}{\sqrt{3}}[-\vec{\pi}\cdot\vec{\pi} - (\bar{K}K)^\dagger + 2\bar{K}K + \eta_8\eta_8]$$

$I=1/2$

$$\mathbf{8}^2_3 = K^-\pi^+ - \frac{1}{\sqrt{2}}\bar{K}^0\pi^0 + \frac{1}{\sqrt{6}}\bar{K}^0\eta_8 - \sqrt{\frac{2}{3}}\eta_8\bar{K}^0$$
$$\mathbf{8}^1_3 = \frac{1}{\sqrt{2}}K^-\pi^0 + \bar{K}^0\pi^- + \frac{1}{\sqrt{6}}K^-\eta_8 - \sqrt{\frac{2}{3}}\eta_8K^-$$

- To justify these, we need to compare these with two-meson modes from the tetraquark mixing model.
- Then one can determine the realistic model from experimental decay modes.

Two-meson modes from the tetraquark mixing model

$$|\text{Heavy nonet}\rangle = -\alpha|\text{Type1}\rangle + \beta|\text{Type2}\rangle$$
$$|\text{Light nonet}\rangle = \beta|\text{Type1}\rangle + \alpha|\text{Type2}\rangle$$

$$[q_1 q_2 \bar{q}_3 \bar{q}_4]_{1_c} \Longrightarrow \underbrace{(q_1\bar{q}_3)_{\mathbf{1}_c} \otimes (q_2\bar{q}_4)_{\mathbf{1}_c}}_{\text{two-meson modes.}} \oplus \underbrace{[(q_1\bar{q}_3)_{\mathbf{8}_c} \otimes (q_2\bar{q}_4)_{\mathbf{8}_c}]_{1_c}}_{\text{hidden-color comp.}}$$

(24) exchange indicated between q_2 and $\bar{q}_3$; (13) exchange indicated between q_1 and $\bar{q}_3$.

For example, two-meson modes for $a_0^+(980)$ can be calculated as

$$\underbrace{\left(\frac{\beta}{\sqrt{12}}+\frac{\alpha}{\sqrt{2}}\right)}\underbrace{\frac{1}{2}\left\{\bar{K}^0K^+ + K^+\bar{K}^0 + \sqrt{\frac{2}{3}}(\eta_8\pi^+ + \pi^+\eta_8) - \frac{1}{\sqrt{3}}(\eta_1\pi^+ + \pi^+\eta_1)\right\}}$$

color-spin recombination factor

from flavor recombination

Two-meson modes of the light nonet in the tetraquark mixing model

$f_0(980)$

$$\left(\frac{\beta}{\sqrt{12}}+\frac{\alpha}{\sqrt{2}}\right)\left\{\frac{1}{3}\left[(\sqrt{2}a-b)\eta_1\eta_1-\left(\frac{a}{2}+\frac{b}{\sqrt{2}}\right)\eta_1\eta_8-\left(\frac{a}{2}+\frac{b}{\sqrt{2}}\right)\eta_8\eta_1-\left(\sqrt{2}a+\frac{b}{2}\right)\eta_8\eta_8\right]\right.$$
$$\left.+\frac{b}{2}\vec{\pi}\cdot\vec{\pi}-\frac{a}{2\sqrt{2}}[\bar{K}K+(\bar{K}K)^\dagger]\right\}$$

$K_0^{*+}(700)$

$$\left(\frac{\beta}{\sqrt{12}}+\frac{\alpha}{\sqrt{2}}\right)\frac{1}{2}\left\{\pi^+K^0+K^0\pi^++\frac{1}{\sqrt{2}}(K^+\pi^0+\pi^0K^+)-\frac{1}{\sqrt{6}}(K^+\eta_8+\eta_8K^+)-\frac{1}{\sqrt{3}}(K^+\eta_1+\eta_1K^+)\right\}$$

$a_0^+(980)$

$$\left(\frac{\beta}{\sqrt{12}}+\frac{\alpha}{\sqrt{2}}\right)\frac{1}{2}\left\{\bar{K}^0K^++K^+\bar{K}^0+\sqrt{\frac{2}{3}}(\eta_8\pi^++\pi^+\eta_8)-\frac{1}{\sqrt{3}}(\eta_1\pi^++\pi^+\eta_1)\right\}$$

$f_0(500)$

$$\left(\frac{\beta}{\sqrt{12}}+\frac{\alpha}{\sqrt{2}}\right)\left\{\frac{1}{3}\left[(a+\sqrt{2}b)\eta_1\eta_1+\left(\frac{a}{\sqrt{2}}-\frac{b}{2}\right)\eta_1\eta_8+\left(\frac{a}{\sqrt{2}}-\frac{b}{2}\right)\eta_8\eta_1+\left(\frac{a}{2}-\sqrt{2}b\right)\eta_8\eta_8\right]\right.$$
$$\left.-\frac{a}{2}\vec{\pi}\cdot\vec{\pi}-\frac{b}{2\sqrt{2}}[\bar{K}K+(\bar{K}K)^\dagger]\right\}$$

Two-meson modes of the heavy nonet in the tetraquark mixing model

$f_0(1500)$

$$\left(-\frac{\alpha}{\sqrt{12}}+\frac{\beta}{\sqrt{2}}\right)\left\{\frac{1}{3}\left[(\sqrt{2}a-b)\eta_1\eta_1-\left(\frac{a}{2}+\frac{b}{\sqrt{2}}\right)\eta_1\eta_8-\left(\frac{a}{2}+\frac{b}{\sqrt{2}}\right)\eta_8\eta_1-\left(\sqrt{2}a+\frac{b}{2}\right)\eta_8\eta_8\right]\right.$$
$$\left.+\frac{b}{2}\vec{\pi}\cdot\vec{\pi}-\frac{a}{2\sqrt{2}}[\bar{K}K+(\bar{K}K)^\dagger]\right\}$$

$K_0^{*+}(1430)$

$$\left(-\frac{\alpha}{\sqrt{12}}+\frac{\beta}{\sqrt{2}}\right)\frac{1}{2}\left\{\pi^+K^0+K^0\pi^++\frac{1}{\sqrt{2}}(K^+\pi^0+\pi^0K^+)-\frac{1}{\sqrt{6}}(K^+\eta_8+\eta_8K^+)-\frac{1}{\sqrt{3}}(K^+\eta_1+\eta_1K^+)\right\}$$

$a_0^+(1450)$

$$\left(-\frac{\alpha}{\sqrt{12}}+\frac{\beta}{\sqrt{2}}\right)\frac{1}{2}\left\{\bar{K}^0K^++K^+\bar{K}^0+\sqrt{\frac{2}{3}}(\eta_8\pi^++\pi^+\eta_8)-\frac{1}{\sqrt{3}}(\eta_1\pi^++\pi^+\eta_1)\right\}$$

$f_0(1370)$

$$\left(-\frac{\alpha}{\sqrt{12}}+\frac{\beta}{\sqrt{2}}\right)\left\{\frac{1}{3}\left[(a+\sqrt{2}b)\eta_1\eta_1+\left(\frac{a}{\sqrt{2}}-\frac{b}{2}\right)\eta_1\eta_8+\left(\frac{a}{\sqrt{2}}-\frac{b}{2}\right)\eta_8\eta_1+\left(\frac{a}{2}-\sqrt{2}b\right)\eta_8\eta_8\right]\right.$$
$$\left.-\frac{a}{2}\vec{\pi}\cdot\vec{\pi}-\frac{b}{2\sqrt{2}}[\bar{K}K+(\bar{K}K)^\dagger]\right\}$$

Note, the heavy nonet and the light nonet have the same two-meson modes. They differ by the (overall) color-spin factor.

Comparison of two-meson modes in two models

$K_0^{*+}(700), K_0^{*+}(1430)$ meson molecules

$$\mathbf{8}_1^3 = \frac{1}{\sqrt{2}}\pi^0 K^+ + \frac{1}{\sqrt{6}}\eta_8 K^+ + \pi^+ K^0 - \sqrt{\frac{2}{3}} K^+ \eta_8$$

tetraquark mixing

$$\propto \frac{1}{2}\left\{\pi^+ K^0 + K^0 \pi^+ + \frac{1}{\sqrt{2}}(K^+\pi^0 + \pi^0 K^+) - \frac{1}{\sqrt{6}}(K^+\eta_8 + \eta_8 K^+) - \frac{1}{\sqrt{3}}(K^+\eta_1 + \eta_1 K^+)\right\}$$

- They differ, for example, by branching fractions, the presence (or absence) of η_1 in some modes.

$f_0(500), f_0(1430)$

$$\mathbf{8}_3^3 = \frac{1}{\sqrt{3}}[-\vec{\pi}\cdot\vec{\pi} - (\bar{K}K)^\dagger + 2\bar{K}K + \eta_8\eta_8]$$

$$\propto \left\{\frac{1}{3}\left[(a+\sqrt{2}b)\eta_1\eta_1 + \left(\frac{a}{\sqrt{2}} - \frac{b}{2}\right)\eta_1\eta_8 + \left(\frac{a}{\sqrt{2}} - \frac{b}{2}\right)\eta_8\eta_1 + \left(\frac{a}{2} - \sqrt{2}b\right)\eta_8\eta_8\right] - \frac{a}{2}\vec{\pi}\cdot\vec{\pi} - \frac{b}{2\sqrt{2}}[\bar{K}K + (\bar{K}K)^\dagger]\right\}$$

$f_0(980), f_0(1500)$

$$\mathbf{1} = \vec{\pi}\cdot\vec{\pi} + \bar{K}K + (\bar{K}K)^\dagger + \eta_8\eta_8$$

$$\propto \left\{\frac{1}{3}\left[(\sqrt{2}a - b)\eta_1\eta_1 - \left(\frac{a}{2} + \frac{b}{\sqrt{2}}\right)\eta_1\eta_8 - \left(\frac{a}{2} + \frac{b}{\sqrt{2}}\right)\eta_8\eta_1 - \left(\sqrt{2}a + \frac{b}{2}\right)\eta_8\eta_8\right] + \frac{b}{2}\vec{\pi}\cdot\vec{\pi} - \frac{a}{2\sqrt{2}}[\bar{K}K + (\bar{K}K)^\dagger]\right\}$$

- However, most of the differences are difficult to verify experimentally due to the ambiguity from the $\eta - \eta'$ mixing and the flavor mixing parameters, a, b, etc.

- But(!), in this comparison, one can see a clear distinction in the isovector channel.

$a_0^+(980)$, $a_0^+(1450)$

meson molecules

$$\mathbf{8}_1^2 = \frac{1}{\sqrt{2}}(\pi^0\pi^+ - \pi^+\pi^0) + \frac{1}{\sqrt{6}}(\eta_8\pi^+ + \pi^+\eta_8) + K^+\bar{K}^0$$

tetraquark mixing

$$\propto \frac{1}{2}\left\{\bar{K}^0K^+ + K^+\bar{K}^0 + \sqrt{\frac{2}{3}}(\eta_8\pi^+ + \pi^+\eta_8) - \frac{1}{\sqrt{3}}(\eta_1\pi^+ + \pi^+\eta_1)\right\}$$

- The two-pion mode appears in the meson molecular model but is absent in the tetraquark mixing model.

- Experimentally, $a_0(980)$, $a_0(1450)$ do not have the two-pion decay mode.
  ☞ This clearly supports the tetraquark mixing model.

- Limitations of the meson molecular model for the two nonets.

1. This molecular approach cannot explain the existence of two nonets.
2. It is impossible to reproduce the coupling strengths as in the tetraquark mixing model.
3. It is also not clear whether this model can generate the inverted mass ordering.
4. This model predicts two-pion modes from the isovector resonances but there are no experimental supports for them.

- Tetraquark mixing model seems better than meson molecules in explaining the two nonets. [PRD (2023) 108, 074016, H.Kim and K.S.Kim]

Summary

- In this talk, we demonstrate that the tetraquark mixing model,

$$|\text{Heavy nonet}\rangle = -\alpha|\text{Type1}\rangle + \beta|\text{Type2}\rangle$$
$$|\text{Light nonet}\rangle = \beta|\text{Type1}\rangle + \alpha|\text{Type2}\rangle,$$

can explain reasonably well the two nonets in $J^P = 0^+$:

Heavy nonet: $a_0(1450), K_0^*(1430), f_0(1370), f_0(1500)$
Light nonet : $a_0(980), K_0^*(800), f_0(500), f_0(980)$

- This model can describe qualitatively the masses of the two nonets and the difference in mass between them.
- One striking prediction of this model, $|G|$(light nonet) $\gg$ $|G'|$ (heavy nonet), is qualitatively supported by Γ_{exp}(light nonet) $\gtrsim$ Γ_{exp}(heavy nonet).
- This could be strong evidence in support of the tetraquark mixing model.
- All successful aspects of the tetraquark mixing model come from the fact that two tetraquark types are mixed to generate the physical states.
- These aspects may be difficult to reproduce in models other than tetraquarks, such as meson molecules or $q\bar{q}$ states with hadronic intermediate states(because they do not rely on mixing to generate the two nonets).
- In particular, we demonstrate that the meson molecular model is not successful in explaining the two nonets.

- We believe that this type of study can establish tetraquarks in the light quark system as well as those in the heavy quark system.

Thank you for your attention !

Additional slides

to answer some questions that may arise in the talk.

additional nonets ?

digression

- The spin-1 diquark scenario requires additional nonets to be found in $J^P = 1^{+-}, 2^{++}$ corresponding to the configurations in Type 2.

$$|111\rangle_{6_c,\bar{6}_c} \qquad |211\rangle_{6_c,\bar{6}_c}$$

※ One can prove that C-parity is negative for $J = 1$, positive for $J = 2$.

Are there such nonets in PDG ? My answer is 'Maybe'.

- There are lots of resonances to choose but the candidate selection is not definite.

Name	I	J^{PC}	Mass(MeV)	Γ(MeV)
$h_1(1170)$	0	1+-	1170.0	360
$b_1(1235)$	1	1+-	1229.5	142
$h_1(1380)$	?	1+-	1386.0	91
$h_1(1595)$	0	1+-	1594.0	384
$K_1(1270)$	1/2	1+	1272.0	90
$K_1(1400)$	1/2	1+	1403.0	172
$K_1(1650)$	1/2	1+	1650.0	150

$J^{P(C)} = 1^{+(-)}$ resonances

- Highlighted members can be selected but with some ambiguity,
  - unknown isospin of $h_1(1380)$,
  - the mass ordering, slightly violated, $M[b_1(1235)] < M[K_1(1270)]$

Name	I	J^{PC}	Mass(MeV)	Γ(MeV)
$f_2(1270)$	0	2++	1275.1	185.1
$a_2(1320)$	1	2++	1318.3	105
$f_2(1430)$	0	2++	1430.0	?
$f'_2(1525)$	0	2++	1525.0	73
$f_2(1565)$	0	2++	1562.0	134
$f_2(1640)$	0	2++	1639.0	99
$a_2(1700)$	1	2++	1732.0	194
$f_2(1810)$	0	2++	1815.0	197
$f_2(1910)$	0	2++	1903.0	196
$f_2(1950)$	0	2++	1944.0	472
$f_2(2010)$	0	2++	2011.0	202
$f_2(2150)$	0	2++	2157.0	152
$f_2(2300)$	0	2++	2300.0	149
$f_2(2340)$	0	2++	2345.0	322
$K_2^*(1430)$	1/2	2+	1425.0	98.5
$K_2^*(1980)$	1/2	2+	1973.0	373

$J^{P(C)} = 2^{+(+)}$ resonances

digression

The selection is ambiguous

- maybe due to further mixings with additional tetraquarks constructed by other diquarks, and possible contamination from two-quark component with $\ell = 1$.
- This ambiguity does not mean that $|111\rangle$, $|211\rangle$ do not exist.
  $\Rightarrow$ It simply says that the candidates do not stand out in a well-separated entity.
  $\Rightarrow$ It does not rule out our mixing framework in the 0^+ channel.

Why two diquark types ?

- The two diquark types, $qq \in (J = 0, \bar{3}_c, \bar{3}_f)$, $qq \in (J = 1, 6_c, \bar{3}_f)$, are adopted in constructing $|\text{Type1}\rangle$, $|\text{Type2}\rangle$ respectively, as their hyperfine potential is negative, $\langle V_{CS}\rangle < 0$.

〈Possible qq structure〉

$Spin$	Color	Flavor	$\langle V_{CS}\rangle$	Type
0	$\bar{3}_c$	$\bar{3}_f$	-2	Attractive
1	6_c	$\bar{3}_f$	$-1/3$	Attractive
1	$\bar{3}_c$	6_f	$2/3$	Repulsive
0	6_c	6_f	1	Repulsive

$$V_{CS} \propto -\sum_{i<j} \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i\, m_j}$$

Hyperfine color-spin interaction

λ_i: Gell-Mann matrix for color

J_i: spin

m_i: constituent quark mass $\sim$330 MeV

Other models to explain the two nonets

with some limitations.

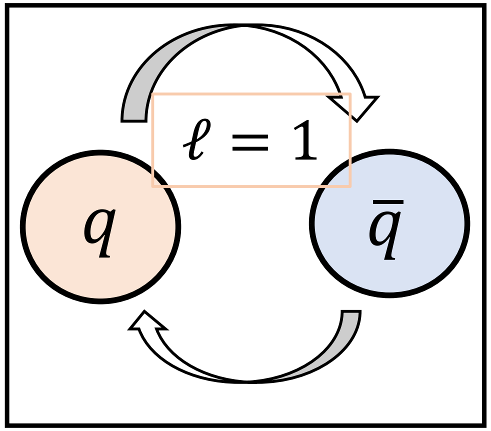
Two-quark picture($q\bar{q}$) with $\ell = 1$

- can make nonets also with $J^P = 0^+$.
- Does this picture explain the two nonets in PDG ? My answer is `No'

$$q\bar{q}: (S = 0,1)\otimes(\ell = 1) \Longrightarrow J = 0,1,2$$

Total J	Configuration	# of confs.
$J = 0$	$(S = 1, \ell = 1)$	one
$J = 1$	$(S = 0, \ell = 1), (S = 1, \ell = 1)$	two
$J = 2$	$(S = 1, \ell = 1)$	one

- This picture has only one configuration in $J^P = 0^+$, not enough to explain the two nonets in $J^P = 0^+$.

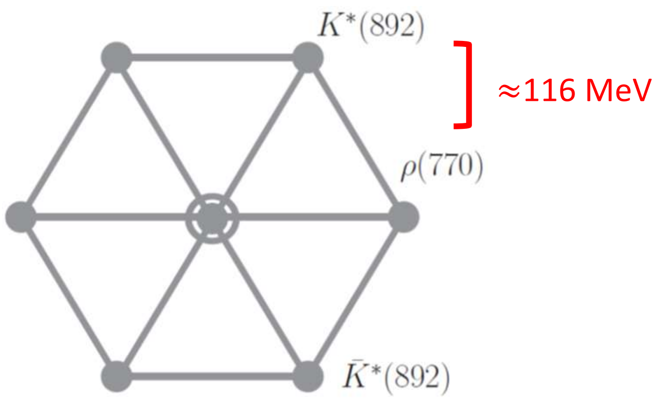


Alternatively, one may treat that

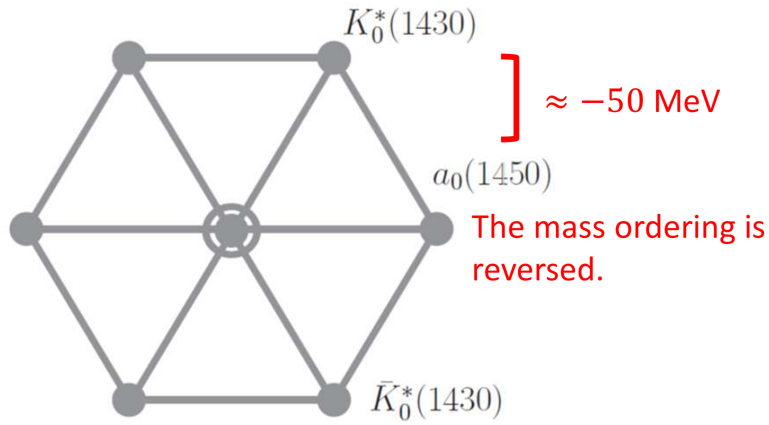
- the heavy nonet can be represented by $q\bar{q}$ ($\ell = 1$) and the light nonet by $qq\bar{q}\bar{q}$.

- The heavy nonet must have the configuration
  $(S = 1$, vector nonet$) \otimes (\ell = 1) \Longrightarrow J = 0$
  $\Rightarrow$ orbital excitations of the vector mesons, ρ, ω, K^*, ϕ.

- In this picture, the spin-orbit (SO) coupling can make the heavy nonet 'heavier' than the vector nonet.



$$\underset{(\ell = 1)}{\Longrightarrow}$$



- To reproduce the reversed gap (≈ -50 MeV), SO must have strong isospin dependence, strong enough to flip the mass ordering established by the quark masses. ☞ not realistic !

Mixing model

Mixture of $q\bar{q}$, $qq\bar{q}\bar{q}$

One may view the two nonets as a mixture of $q\bar{q}$ $(\ell = 1)$ and $qq\bar{q}\bar{q}$?
Black et.al, PRD 59(1999)

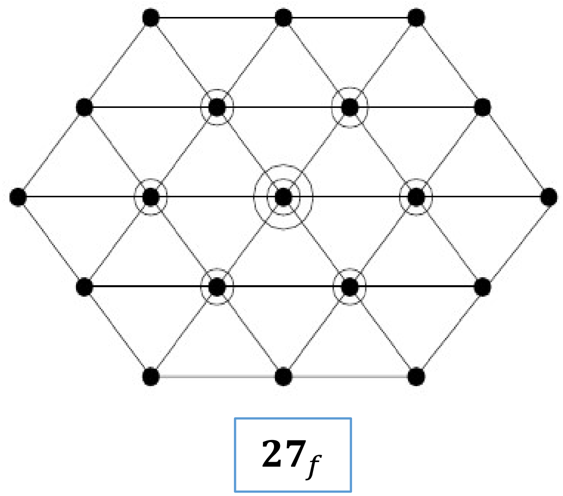
- Black et.al introduce the effective fields corresponding to $q\bar{q}$ and $qq\bar{q}\bar{q}$ nonets and make SU(3) invariant Lagrangian among them.
- As pointed by Maiani et.al. EPJC50(2007), the required mixing seems too large given the fact that very different configurations are involved.

- $q\bar{q}(\ell = 1)$, $qq\bar{q}\bar{q}$ do not mix under the color-spin interaction !
  $\langle q\bar{q}|qq\bar{q}\bar{q}\rangle = 0$, $\langle q\bar{q}|V_{CS}|qq\bar{q}\bar{q}\rangle = 0$.
- It is hard to establish such a mixing from well-known quark-quark interactions.

One may view the two nonets as meson-meson bound states.

- Since mesons are colorless, this model suggests shallow bound states.
  ex) $f_0(980) \sim K\bar{K}$ since $M[f_0(980)] \sim 2M_K$.
  But it is hard to view $f_0(500)$ as a shallow bound state of $\pi\pi$.

- Why meson-meson states form nonet only (?)
  ⇒Since the lowest-lying mesons form a nonet in flavor, the meson-meson states can form various multiplets including the 27-plet
  $$8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8 \oplus 8 \oplus 1$$
  ⇒ PDG does not support this picture. (ex. no 0^+ resonances with $I = 2$.)



$I = 1$

$I = \frac{3}{2}, \frac{1}{2}$

$I = 2,1,0$