2nd CENuM Workshop

Incheon / Inha University
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Nucleon From Instanton Vacuum

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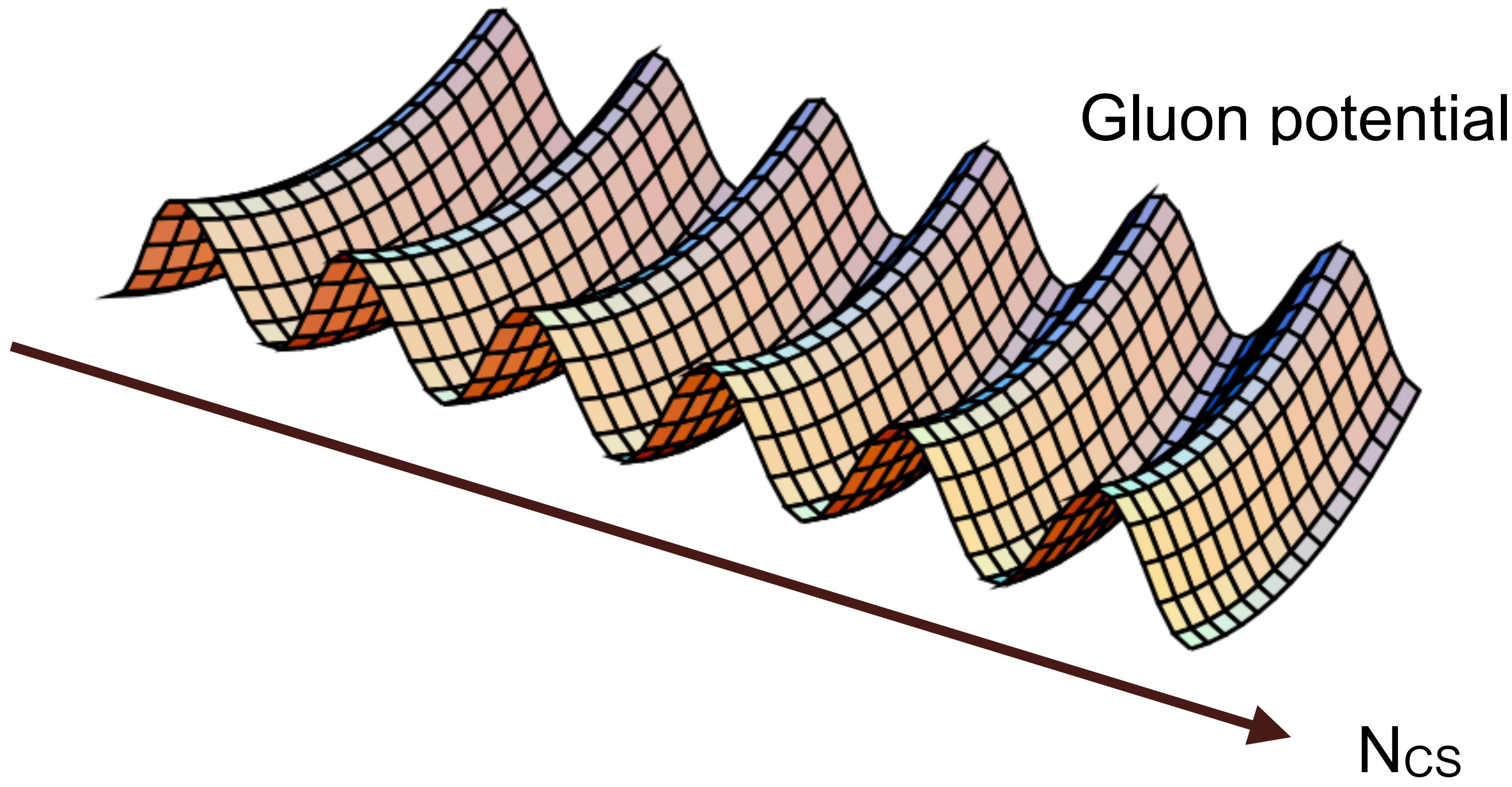
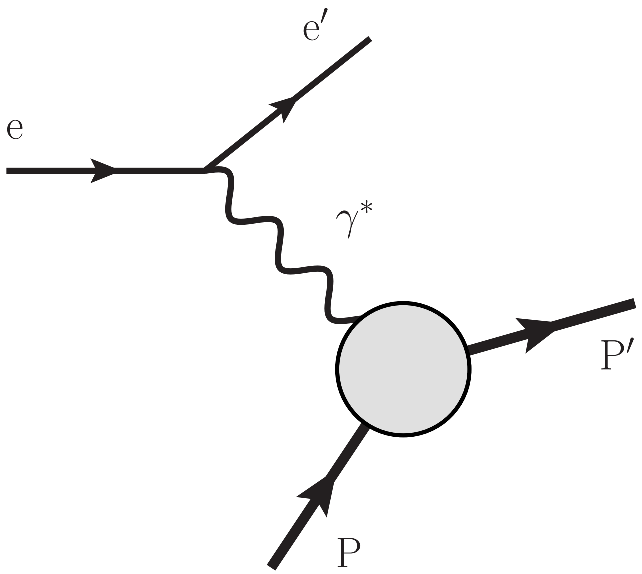
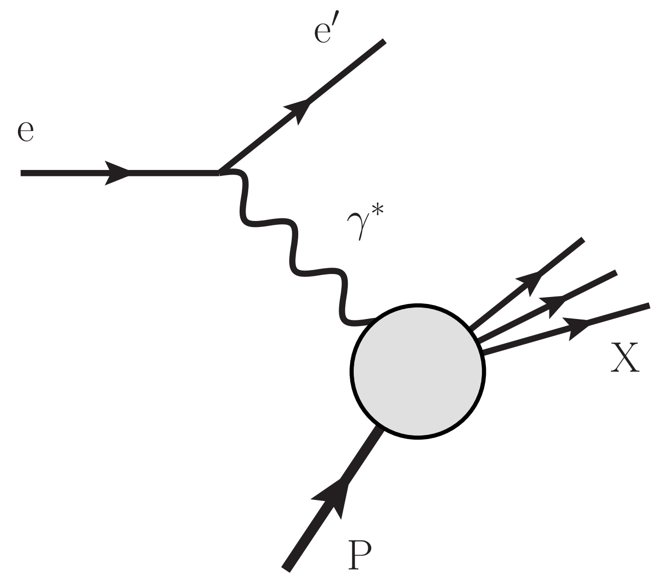


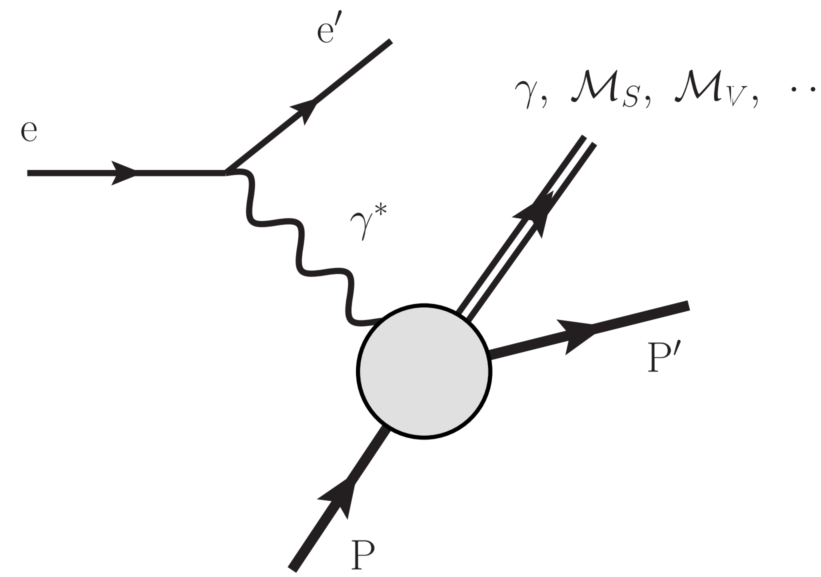
Hadron Structure



Elastic scattering



Inclusive / Inelastic



Exclusive / Inelastic

Form Factors (FFs)

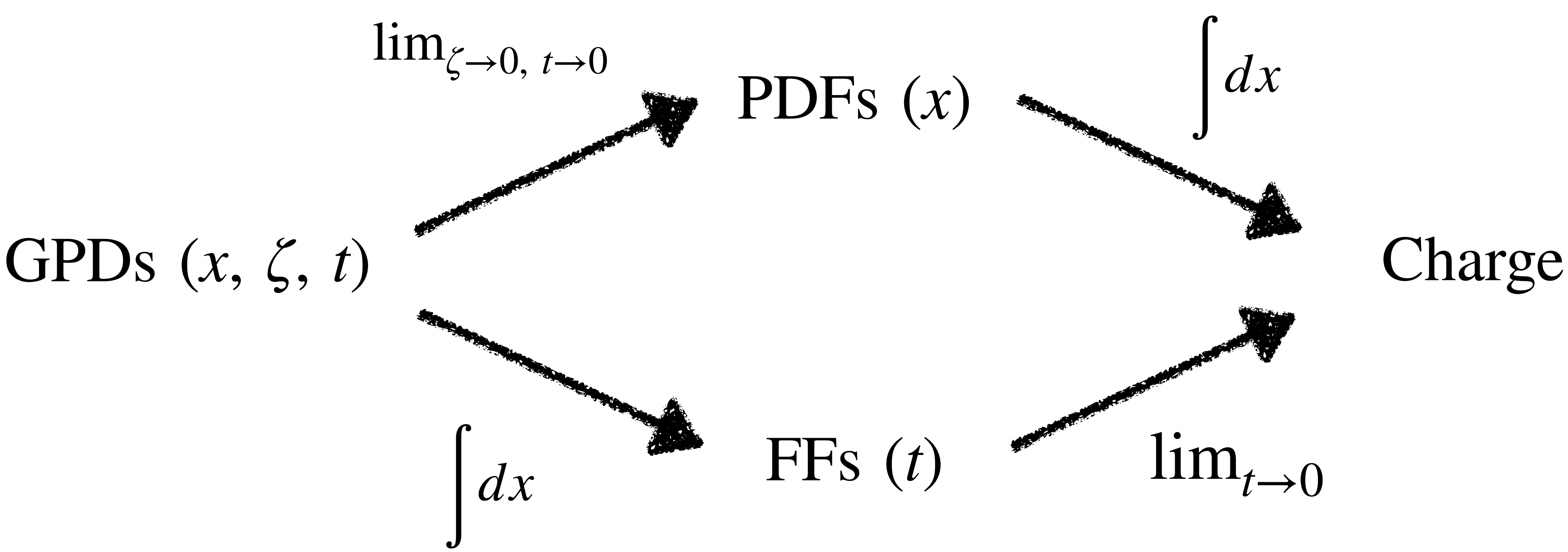
Density

Parton Distribution Functions (PDFs)

Momentum

Generalized Parton Distributions (GPDs)

Angular momentum

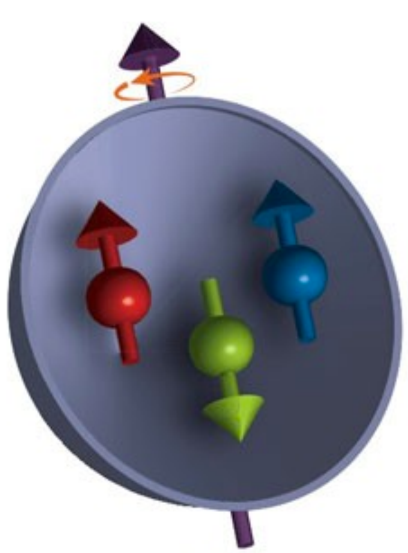


Why Effective Models?

Parton Distribution Functions (PDFs)
- Valence quarks PDF
- Sea quarks PDF
- Gluon PDF

Form Factors (FFs)
- 1st Melin moment - Electromagnetic, …
- 2nd Melin moment - Energy-Momentum Tensor, …
  - Valence quark
  - Sea quark
  - Gluon

1960s



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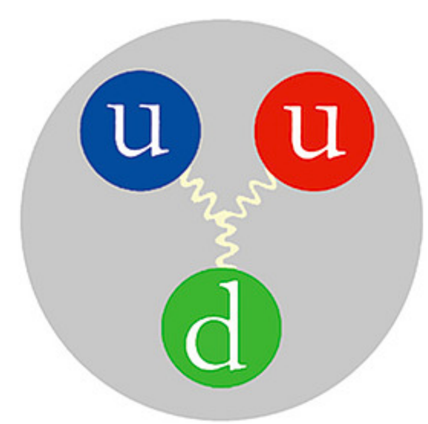
Now

Quark Model



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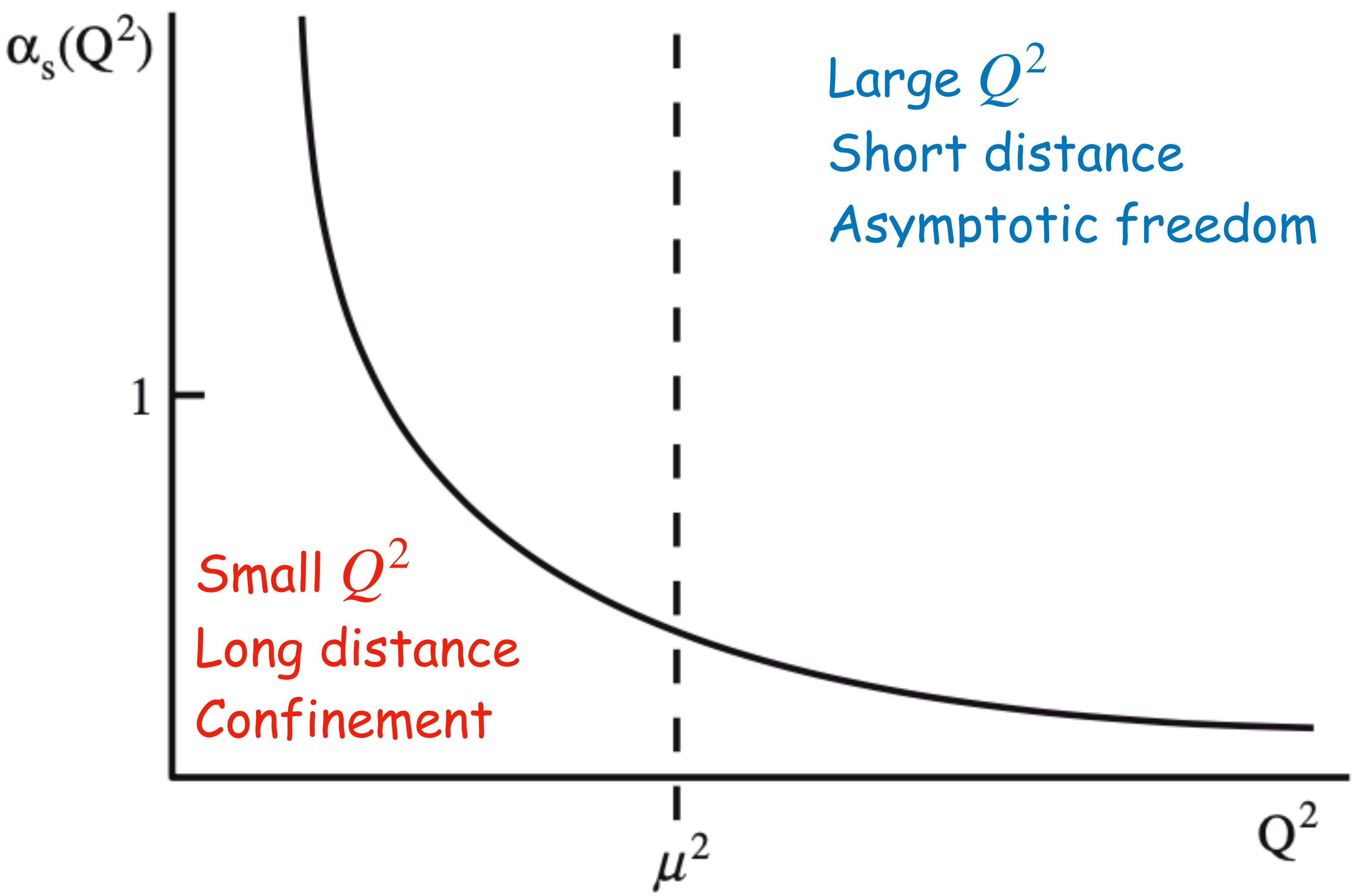
Why Effective Models?

At low energy, cannot use QCD directly

because of non-perturbative nature with QCD running coupling.



- Quark model
- Bag model
- Nambu–Jona-Lasinio model
- Chiral quark soliton model
- Skyrme model
- Lattice QCD

⋮

Instanton Vacuum

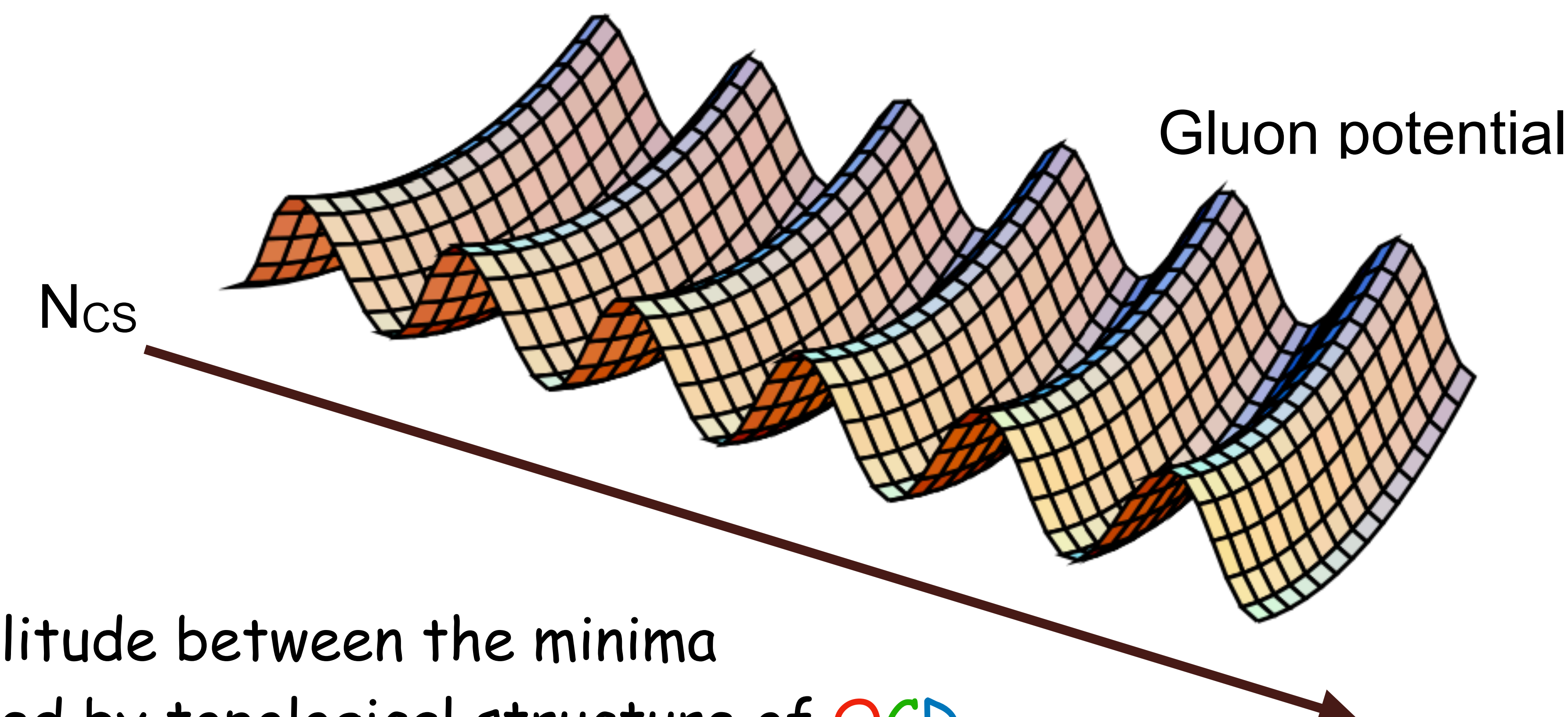
Classical ground state solution of QCD in Euclidean space

Non-perturbative part of gluons is replaced by instantons :

QCD DOF : quarks & gluons => quarks & instantons

Provide spontaneous chiral symmetry breaking

& non-local interaction between quarks !

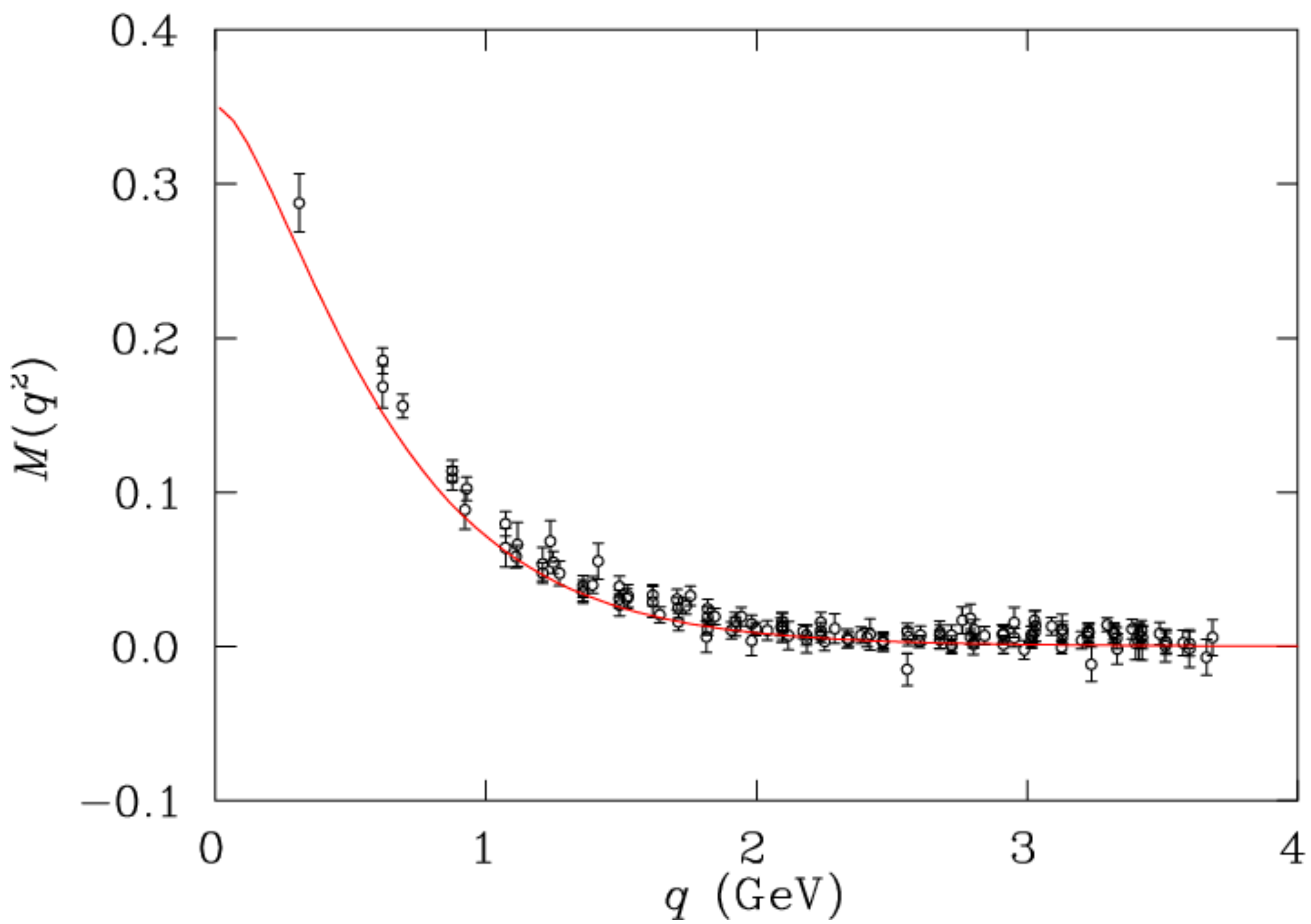


Tunneling amplitude between the minima
is characterized by topological structure of QCD

Momentum Dependent Dynamical Quark Mass

Fourier transform of fermion zero mode solution :

$$F(k^2) = F(\omega^2 + |\mathbf{k}|^2) = -z\,\frac{d}{dz}\Big(I_0(z)K_0(z) - I_1(z)K_1(z)\Big)\Big|_{z=k\bar{\rho}}$$



$$M(k) = M(0)\;F^2(k)$$

Lattice data from

P.O. Bowman et al., Nucl. Phys. Proc. Suppl. 128, 23 (2004)

In the model, it plays roles of natural UV **regulator**.

Non-Local Effective Action

Instanton induced non-local action :

$$\mathcal{S}_{eff} = \int d^4x \left[\psi_f^\dagger(x)\Big(i\partial + im\Big)\psi^f(x) + \boxed{\frac{1}{2G}\Big(\sigma^2(x) + \pi^2(x)\Big)} \right.$$

$$\left. +i\int \frac{d^4k}{(2\pi)^4}\int \frac{d^4l}{(2\pi)^4}\, e^{i(k-l)\cdot x}\, \psi^\dagger(k)\boxed{F(k)}\Big(U(x)\frac{1+\gamma_5}{2} + U^\dagger(x)\frac{1-\gamma_5}{2}\Big)\boxed{F(l)}\psi(l)\, \right]$$

with chiral fields,

$$U(x)\frac{1+\gamma_5}{2} + U^\dagger(x)\frac{1-\gamma_5}{2} = \exp\Big(i\pi^a(\mathbf{x})\tau^a\gamma_5\Big) = \sigma(x) + i\gamma^5\,\vec{\tau}\cdot\vec{\pi}(\mathbf{x}) \equiv M_0 U^{\gamma_5}$$

momentum dependent quark mass : $M(k) = M(0)\; F^2(k)$

$F(k) = 1 \quad \rightarrow \quad$ Chiral Quark Soliton Model (local model)

Fixing model parameters

Model parameters : $\bar{\rho}, \;\; M_0, \;\; G, \;\; m_0$

Instanton vacuum : $\bar{\rho} \simeq 0.3 \;\, \mathrm{fm},$

$$M(0) = M_0 \simeq 350 \;\, [\mathrm{MeV}]$$

Gap equation : $$\frac{1}{G} = \int \frac{d^4k}{(2\pi)^4} \frac{F^2}{\left(k^2 + F^4 M_0^2\right)^2}$$

Gell-Mann-Oakes-Renner relation : $m_\pi^2 \; f_\pi^2 = -\, m_0 \langle \bar{\psi}\psi \rangle$

No free parameter !

Saddle-Point Approx. & Hedgehog Ansatz

"Hedgehog Ansatz"

(the specific configuration of chiral field)

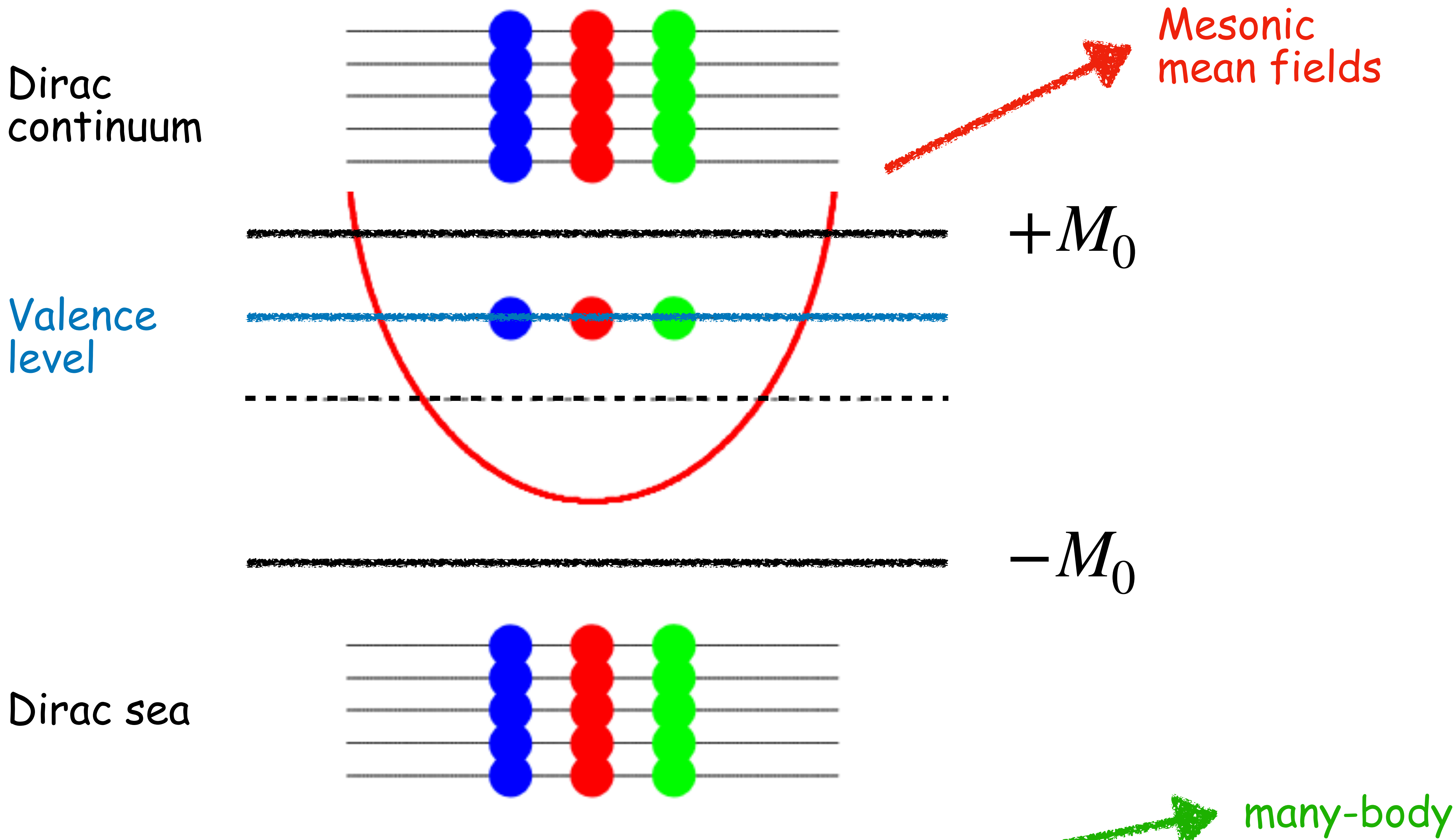
$$\sigma(\vec{r}) = \sigma(r), \qquad \vec{\pi}(\vec{r}) = \pi(r)\ \hat{r}$$

To solve non-linear differential equation,

we use Saddle-Point Approximation as

$$\left.\frac{\delta S_{eff}(\sigma,\, \vec{\pi})}{\delta \sigma}\right|_{\sigma=\sigma_c,\ \pi=\pi_c} = 0\ , \qquad \left.\frac{\delta S_{eff}(\sigma,\, \vec{\pi})}{\delta \vec{\pi}}\right|_{\sigma=\sigma_c,\ \pi=\pi_c} = 0$$

Baryon Picture in CQSM

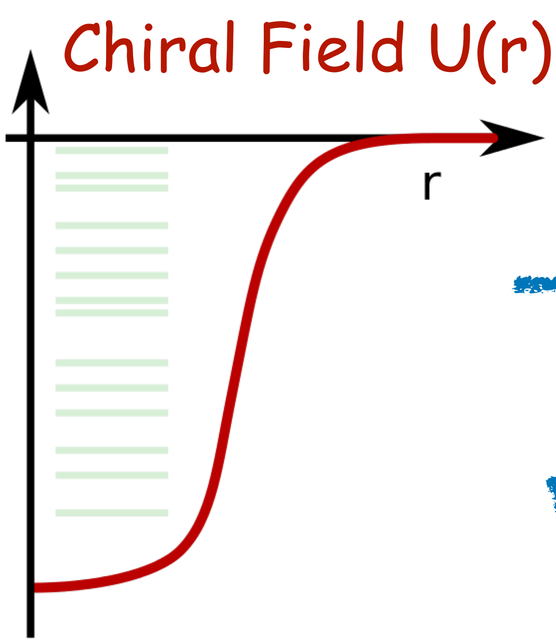


A **baryon** can be viewed as a state of **Nc quarks** bound by **mesonic mean fields** (E. Witten, NPB, 1979 & 1983).

Mean Field Approx. & Self-Consistent Calc.

Interactions between quarks are average meson fields.

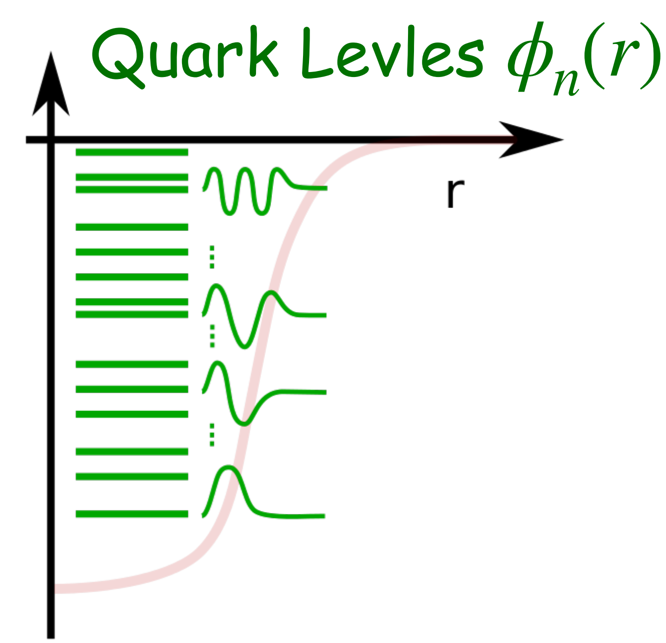
$$H = \sum_i T_i + \sum_{i<j} V_{ij} \approx \sum_i \left[T_i + \boxed{U(r_i)} \right] = \sum_i h_i$$



Diagonalize

Hamiltonian

$$\hat{H}\phi_n = e_n \phi_n$$



Min. E_{Soliton}?

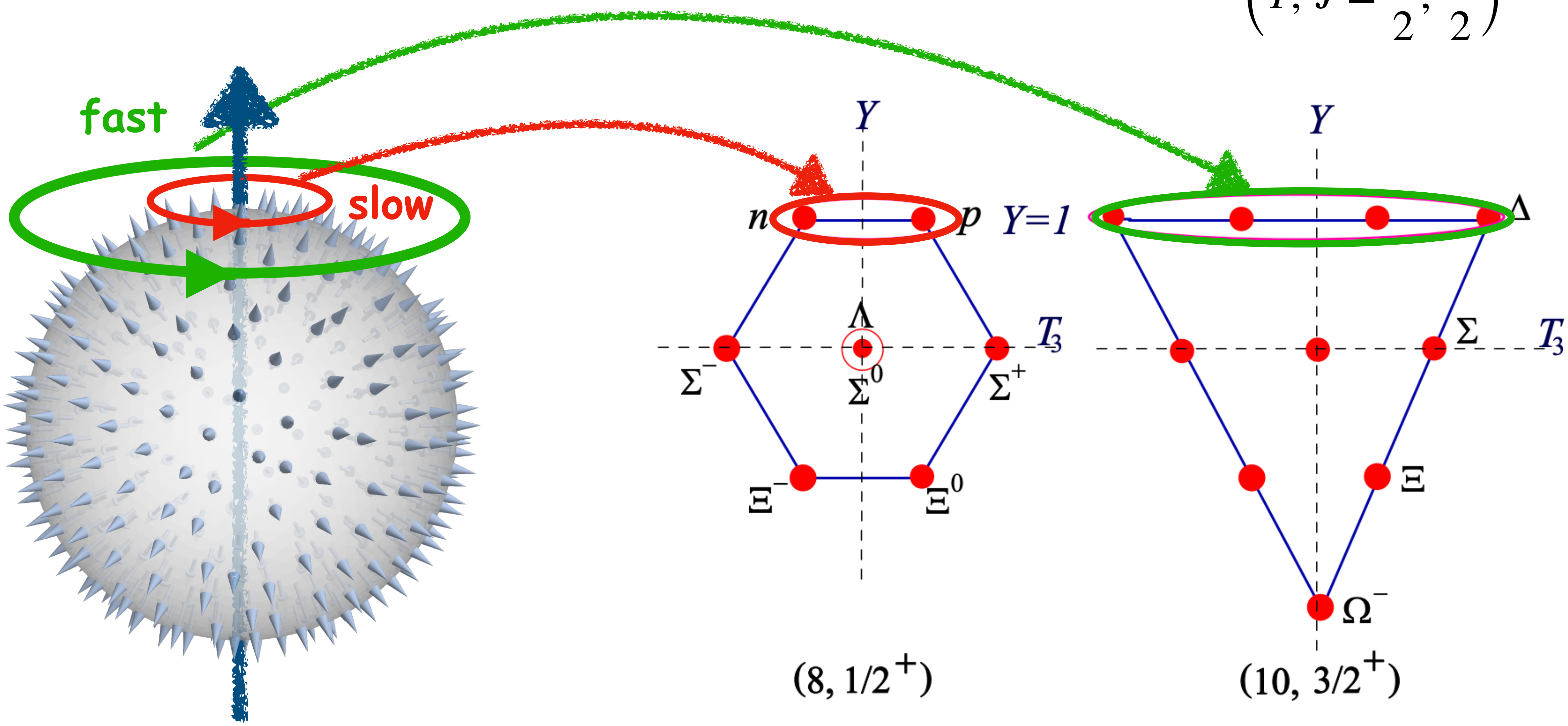
Rotating Soliton with Hedgehog Ansatz

Soliton rotates addiabatically to assign quantum numbers,

It break isospin, spatial symmetry simultaneously!

$$\mathscr{R}\left(\sigma(r)+i\gamma_5(\hat{r}\cdot\vec{\tau})\pi(r)\right)\mathscr{R}^\dagger$$

$$\left(T,\,J=\frac{1}{2},\,\frac{3}{2}\right)$$



Current Conservation

Current is not conserved in non-local model,

To fix it, we introduce gauge connection,

$$L(x,y) = \mathscr{P} \exp\left(i \int_x^y ds^\mu \ A_\mu(s) \right)$$

For example, baryon current is

Local

$$\hat{B} = \frac{\delta \mathcal{S}'_{eff}}{\delta A(\tau)}\bigg|_{A=0} = \frac{1}{N_c} \int d^3x \, \boxed{\psi^\dagger(x)\psi(x)}$$

Non-local

$$+ \frac{i}{N_c} \int d^3x \left(\boxed{\psi^\dagger(x) F_0 \Phi(x) F \psi(x) + \psi^\dagger(x) F \Phi(x) F_0 \psi(x)} \right),$$

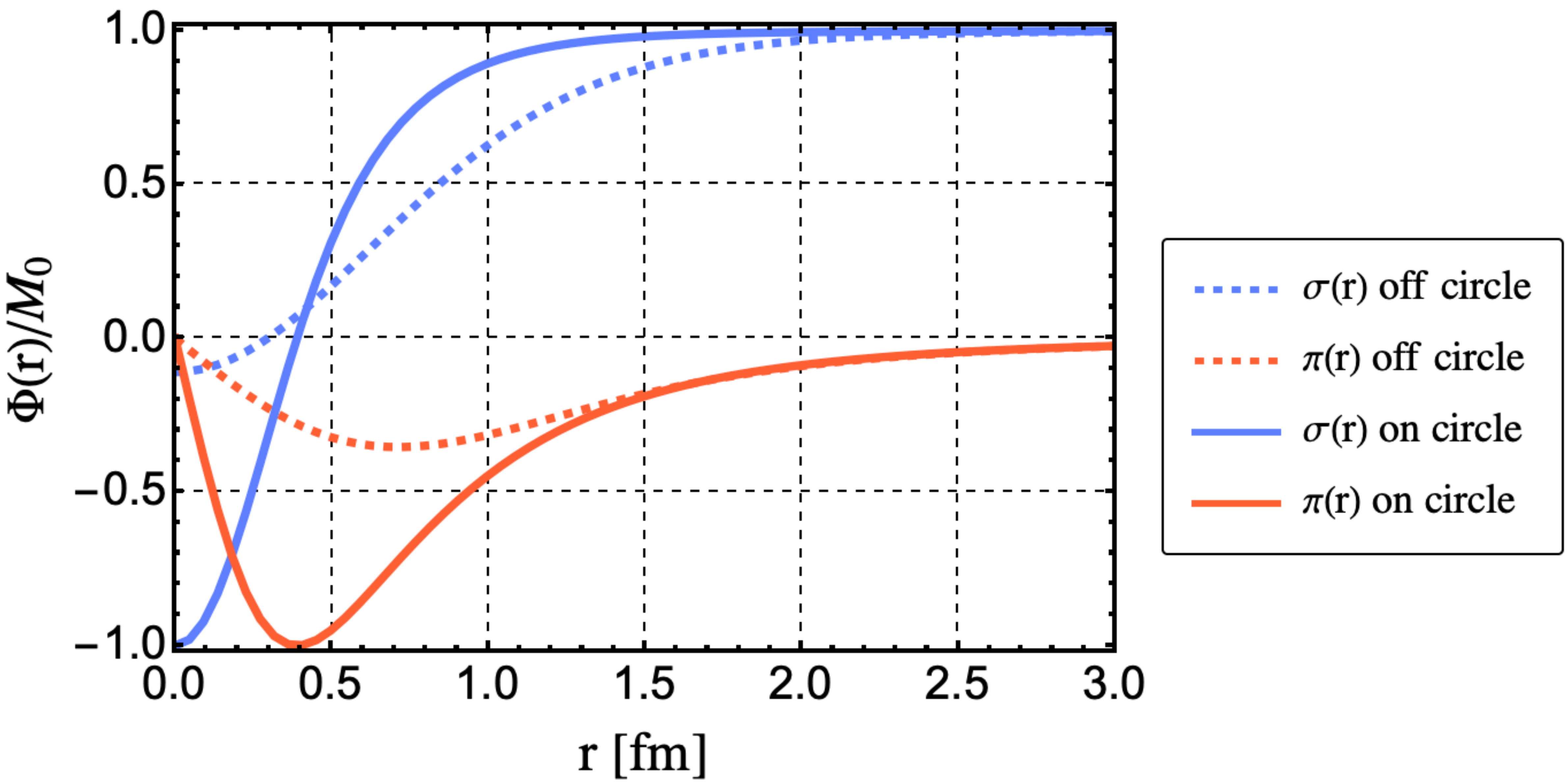
where $\hat{F}_0 = \partial F(k)/\partial k^0$

Chiral Fields on & off Chiral-Circle

In local model, stable soliton only if chiral fields impose chiral-circle,

but, **in non-local model, can get stable soliton in any case**.

$$\sigma^2(r) + \pi^2(r) = M_0^2 \ (?!)$$



Moment of Inertia

In **rotational quantization**, second variation is

$$\frac{\delta^2 \mathcal{S}'_{eff}}{\delta\Omega_a \delta\Omega_b} \sim I_{total} = I_{sea} + I_{val} \sim M_{N-\Delta}$$

$$I_{sea} = N_c \int \frac{d\omega}{2\pi} \sum_{n_\omega, m_\omega} \frac{\langle n_\omega | w^a | m_\omega \rangle}{\omega - ie_n(\omega^2)} \frac{\langle m_\omega | w^b | n_\omega \rangle}{\omega - ie_m(\omega^2)} - N_c \int \frac{d\omega}{2\pi i} \sum_{n_\omega} \frac{\langle n_\omega | W^{ab} | n_\omega \rangle}{\omega - ie_n(\omega^2)},$$

$$I_{val} = -2N_c \sum_{m_\omega} \frac{z_{val}}{e_m(\omega^2) - e_v} \langle val | w^a | m_\omega \rangle \langle m_\omega | w^b | val \rangle \Big|_{\omega = ie_v} - \frac{N_c}{2} z_{val} \langle val | W^{ab} | val \rangle \Big|_{\omega = ie_v},$$

where $$w^{a,b} = \mathcal{O}(\Omega) = \boxed{i\frac{\tau^{a,b}}{2}} + F_0 \frac{\tau^{a,b}}{2} \gamma^0 \Phi_c F + F\gamma^0 \Phi_c \frac{\tau^{a,b}}{2} F_0$$

$$W^{ab} = \mathcal{O}(\Omega^2) = \frac{\delta^{ab}}{4} F_{00} \gamma^0 \Phi_c F + F_0 \frac{\tau^a}{2} \gamma^0 \Phi_c \frac{\tau^b}{2} F_0 + F_0 \frac{\tau^b}{2} \gamma^0 \Phi_c \frac{\tau^a}{2} F_0 + F\gamma^0 \Phi_c \frac{\delta^{ab}}{4} F_{00}$$

Moment of Inertia - Results

$$M_\Delta - M_N = \frac{3}{2I} =_{\text{Exp.}} 294 \text{ [MeV]} \ \rightarrow \ I =_{\text{Exp.}} 1.00679 \text{ [fm]}$$

Without any free parameters,

Model (Chiral-circle)	$I_{\text{sea}}\ [\mathcal{O}(F_0)]$	$I_{\text{sea}}\ [\mathcal{O}(F_{00})]$	$I_{\text{val}}\ [\mathcal{O}(F_0)]$	$I_{\text{val}}\ [\mathcal{O}(F_{00})]$	I_{total} [fm]
Local (O)	~ 0.5		~ 1.2		~ 1.7
Non-local (X)	0.00654	-0.00471	2.73067	0.04391	2.77641
Non-local (O)	-0.1721	0.01594	1.32559	0.07249	1.24192

Slightly free from a restriction, $M_0 = 350$ [MeV] $\rightarrow$ $350 \sim 400$ [MeV]

Can reach an almost exact agreement !

Matrix Element for Local Current

With local gauge transformation, local current is

$$\left.\frac{\delta \mathcal{S}'_{\rm eff}}{\delta A_\mu(x)}\right|_{A=0} = J^\mu(x)$$

The matrix element is

Local

$$\langle N' | J^\mu(x) | N \rangle = \langle N' | \bar\psi(x)\gamma^\mu\psi(x) | N \rangle$$

Non-local

$$+\langle N' | \bar\psi(x)\hat{F}_\mu\gamma^\mu\gamma^0 U^{\gamma_5}(x)\hat{F}\psi(x) + \bar\psi(x)\hat{F}\gamma^\mu\gamma^0 U^{\gamma_5}(x)\hat{F}_\mu\psi(x) | N \rangle$$

where $\hat{F}_\mu = \partial F(k)/\partial k^\mu$

Also, need to consider **rotational quantization**,

$$\langle N' | J^\mu(x) | N \rangle = \langle N' | J^\mu(x) | N \rangle_{\mathcal{O}(\Omega^0)} + \langle N' | J^\mu(x) | N \rangle_{\mathcal{O}(\Omega^1)} + \langle N' | J^\mu(x) | N \rangle_{\mathcal{O}(\Omega^2)} + \cdots$$

Effective Gluon Operator

Effective gluon operator (including higher twist) is equivalent to **four-fermion operator** in instanton vacuum.

$$\mathcal{O}_{\alpha_1\cdots\alpha_r\beta 1\cdots\beta s}[\psi^\dagger,\psi;\pi](x) = -\,i\psi^\dagger(x)\frac{\lambda^a}{2}\Gamma_{\alpha_1\cdots\alpha_r}\psi(x)$$

J. Balla, M.V. Polyakov, C. Weiss
Nucl. Phys. B 510 (1997) 327

$$\times\left[\,(Y_{\mathcal{F}+})^a_{\beta_1\cdots\beta_s}(x)_{\rm bosonized} + (Y_{\mathcal{F}-})^a_{\beta_1\cdots\beta_s}(x)_{\rm bosonized}\,\right]$$

where

$$(Y_{\mathcal{F}+})^a_{\beta_1\cdots\beta_s}(x)_{\rm bosonized} = \frac{iM}{N_c}\int d^4z\,\mathcal{F}_{\pm,\mu\nu\beta_1\cdots\beta_s}(x-z)\sum_{f,g=1}^{N_f} J^a_{\pm\mu\nu}(z)_{fg}\,U_\pm(z)_{fg}$$

Chiral fields ($U_\pm(z)_{fg}$)

Two quarks with regulators ! ($J^a_{\pm\mu\nu}(z)_{fg}$)

To obtain gluon contents and higher twist contributions, it is essential to develop non-local model.

Summary

I. We developed **non-local chiral quark soliton model**

with instanton induced interaction.

II. We subtlely applied procedures such as Hedgehog ansatz,

saddl-point approximation, and mean-field approach to the model.

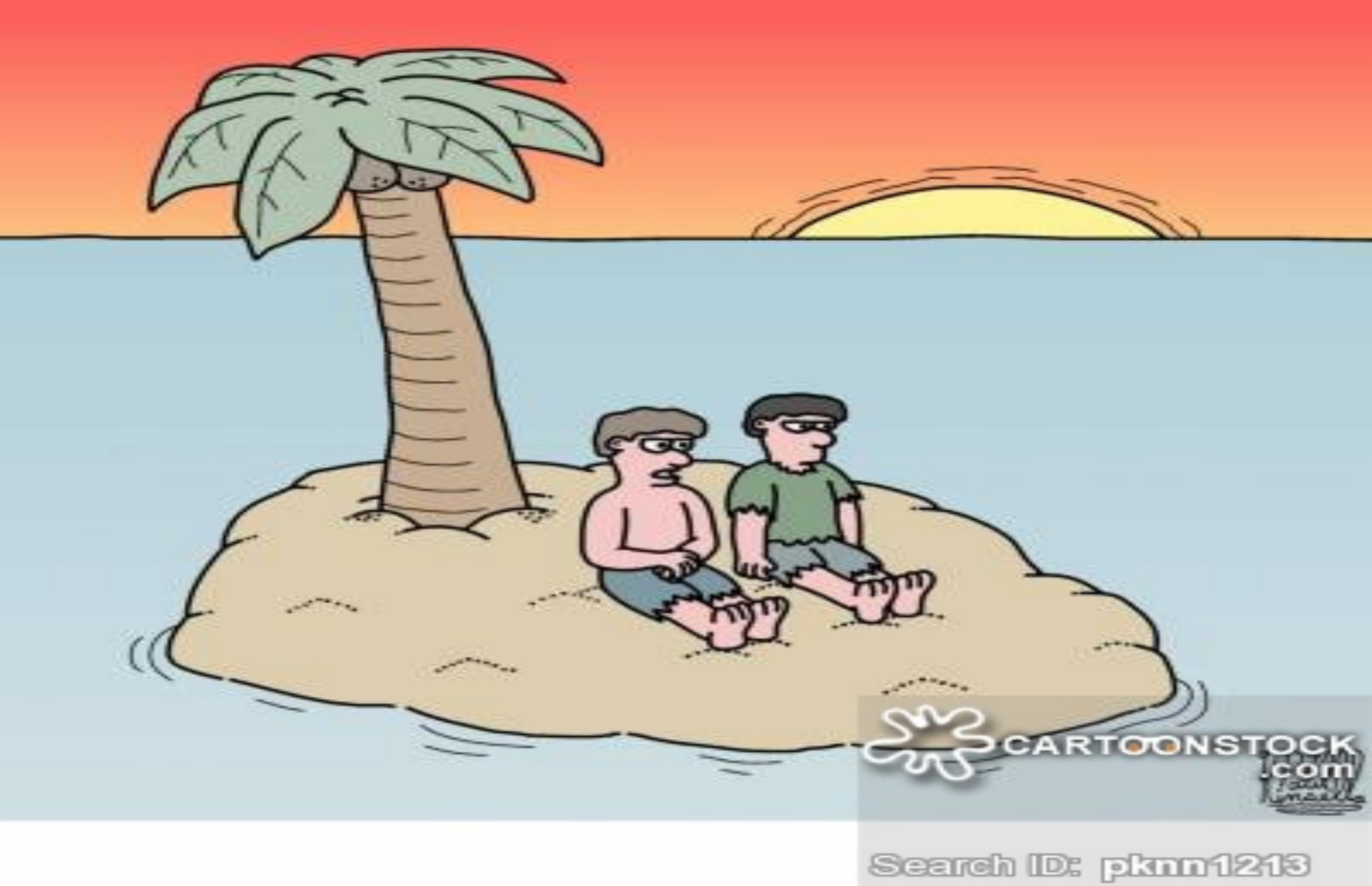
III. Since current is not conserved in non-local model,

consider extra terms to obtain physical quantities.

IV. **Without fitting, free parameter,**

obtain **moment of inertia, ~1.2 [fm]** which is comparable to Exp. ~1.0.

V. Using this model, will study hadron structure farther and farther away.



"Thank you for listening."