Heavy and light quark interaction in instanton liquid model

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Reviews from the works of
D.Diakonov, V.Petrov, P.Pobylitsa,
E.Shuryak, I.Zahed,
M.Musakhanov, others(see ref.s)

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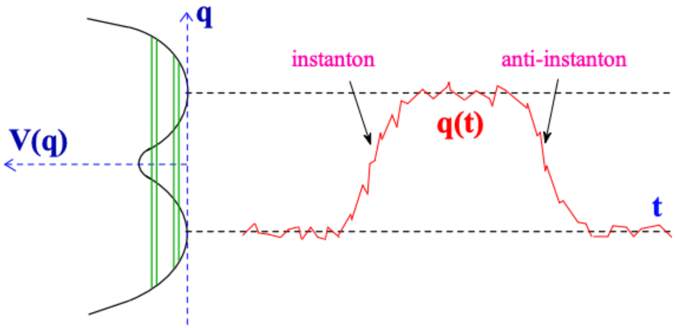
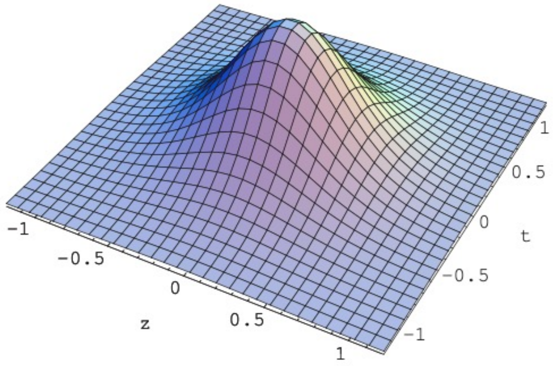
Outline

- Introduction
- Light quarks in ILM
- Heavy quarks in ILM
- Heavy-light quarks interaction in ILM
- Conclusions and Future remarks

What are the instantons?



Tunneling trajectories in a double-well potential



Action density of YM instanton

- Tunneling process in Minkowski space, pseudoparticle in Euclidean space
- Classical solutions, self-dual
- Topologically charged $\frac{1}{32\pi^2}\int_x \tilde{F}F = \pm 1$, strong field inside $\left(F^2\right)^{1/4} \simeq 1.5\ GeV$

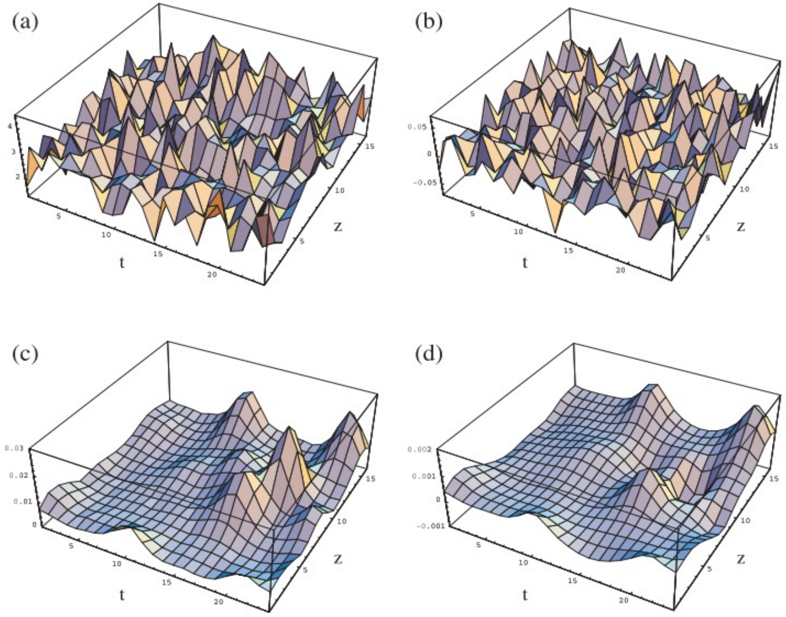
The ensemble of instantons and anti-instantons

$$A_\mu(x) = \sum_{I}^{N_+} A_\mu^I(\xi_I, x) + \sum_{\bar{I}}^{N_-} A_\mu^{\bar{I}}(\xi_{\bar{I}}, x), \qquad \xi = (\rho, z, U)$$

- average size $\bar{\rho} \approx 0.3\ fm$, separation $\bar{R} \approx 1\ fm$
- $\bar{\rho}/\ \bar{R} \sim 1/3$ a few percent of 4D space occupied
- Interactions are big enough $\quad \exp|\delta S_{int}| \sim 20 \gg 1$

The ensemble of I's & $\bar{I}$'s $-$ Instanton Liquid Model

[D.Diakonov, V.Petrov 1984; E.Shuryak, T.Schafer 1993]



[Chu et al., 1994]

Motivation

- An analytical approach to non-perturbative QCD
- Lead to the formation of gluon condensate, provide a mechanism of spontaneous symmetry breaking, solution to U(1) problem
- Small packing parameter $\rho^4/R^4 \sim 0.01 - 0.03$ to develop perturbation theory
- Few number of input parameters and obtaining data without fitting

Quark observables in instanton vacuum

- To compute an observable for a given configuration of instantons and anti-instantons, then average over the ensemble

- First, average over the ensemble, and obtain an effective theory written in terms of interacting quarks only. Then, compute observables from this effective theory

Light quarks in ILM

- In the 1-instanton background:
  - Spectrum contain zero & nonzero modes: $S^I(x,y) = -\frac{\phi_0(x)\phi_0^\dagger(y)}{im} + S^{\rm nzm}(x,y)$
  - $p << 1/\rho$:zero modes dominate, $p \to \infty$:free propagator
  - Approximate Green's function: $S^I(x,y) \simeq -\frac{\phi_0(x)\phi_0^\dagger(y)}{im} + S_0(x,y)$
  - Action:
$$\exp(-A^I[\psi^\dagger,\psi]) = \exp\left(\int d^4x\psi^\dagger i\hat{\partial}\psi\right)\left(im - V_I[\psi^\dagger,\psi]\right)$$

- In the background of N_+ I's & N_- $\bar{I}$'s
$$\exp(-A[\psi^\dagger,\psi]) = \prod_f^{N_f}\exp\left(\int d^4x\psi_f^\dagger i\hat{\partial}\psi_f\right)\prod_{I,\bar{I}}\left(im - V_I[\psi_f^\dagger,\psi_f]\right)$$

Light quarks in ILM

- Partition function: $$\mathcal{Z}_{QCD} = \int D\psi D\psi^\dagger \left\langle \exp\left(-A[\psi^\dagger, \psi]\right)\right\rangle$$

  where $\langle \dots \rangle$ means averaging over $I\bar{I}$ ensemble
- Averaging over the ensemble ⇒ averaging over collective coordinates of each I & $\bar{I}$

$$\int DA \to \int \prod_{I,\bar{I}} dz_I dU_I d\rho_I \; d_{\text{eff}}(\rho_I) \to \int \prod_{I,\bar{I}} dz_I dU_I$$

replace all (anti)instantons with average-size one

size dist. func. $d_{\text{eff}}(\rho_I) \xrightarrow{\text{large } N_c} \delta(\rho_I - \bar{\rho})$

- After averaging:

$$\mathcal{Z} = \int d\lambda_+ d\lambda_- \int D\psi_f D\psi_f^\dagger \exp\left(i \int d^4x \sum_{f=1}^{N_f} \psi_f^\dagger i\hat{\partial}\psi_f + i\lambda_+ Y_{N_f}^{(+)} + i\lambda_- Y_{N_f}^{(-)}\right)$$

t'Hooft vertex

- $N_c \to \infty$: saddle point λ_s defines dynamical quark mass $M(k)$.

Instanton induced effective interaction

$$Y_{N_f}^{(\pm)} \sim \int d[k_f]d[l_f]\delta\left(\sum k_f - \sum l_f\right) dU\, [U^\dagger U]^{N_f} \prod_f^{N_f} \sqrt{M(k_f)}\psi_f^\dagger(k_f)\gamma_\pm \sqrt{M(l_f)}\psi_f(l_f)$$

- $N_f = 1$: mass term $\quad Y_1^{(\pm)} \sim M(k)\psi^\dagger(k)\gamma_\pm\psi(k)$
- $N_f = 2$: 4-fermion interaction

NJL type Lagrangian

$$Y_2^{(+)} + Y_2^{(-)} \sim c_i \left(\sqrt{M(k)}\psi^\dagger(k)\Gamma_i\sqrt{M(l)}\psi(l)\right)^2, \quad \Gamma_i = (\mathbf{1}, \gamma_5, i\tau^a, i\tau^a\gamma_5, \cdots)$$

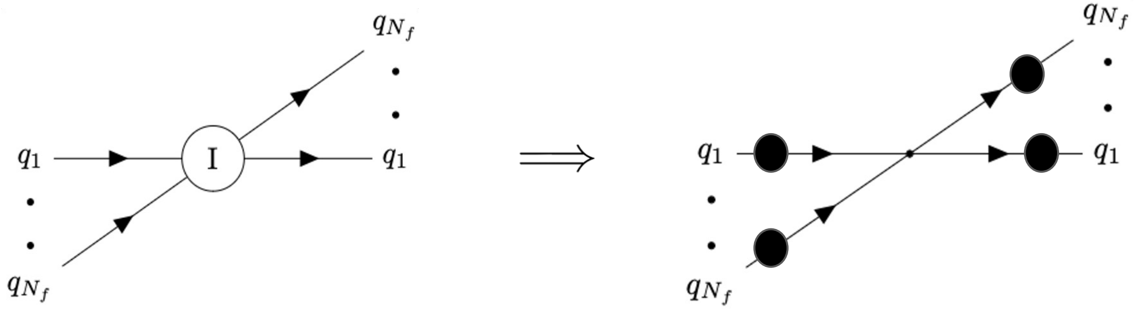
- Any N_f:

$$Y_{N_f}^{(\pm)} \overset{N_c\to\infty}{=} \left(\frac{2V}{N}\right)^{N_f-1} \int d^4x\, \det J^{(\pm)}$$

$$J_{fg}^{(\pm)} = \int \frac{d^4k d^4l}{(2\pi)^8} e^{i(k-l)x} \sqrt{M(k)M(l)}\, \psi_f^\dagger(k)\gamma_\pm\psi^g(l)$$

Instanton induced effective interaction

❑ Diagrammatic interpretation:



form factor function $\sqrt{M(k\bar{\rho})}$ attached each quark leg

[t'Hooft 1976; D.Diakonov, V.Petrov 1986; M.Nowak 1991; R.Rapp, T.Schafer, E.Shuryak, M.Velkovysky 1999 and others]

Low energy QCD

- QCD degrees of freedom with mas $\ll 1/\bar{\rho}$: light quarks and pseudoscalar mesons

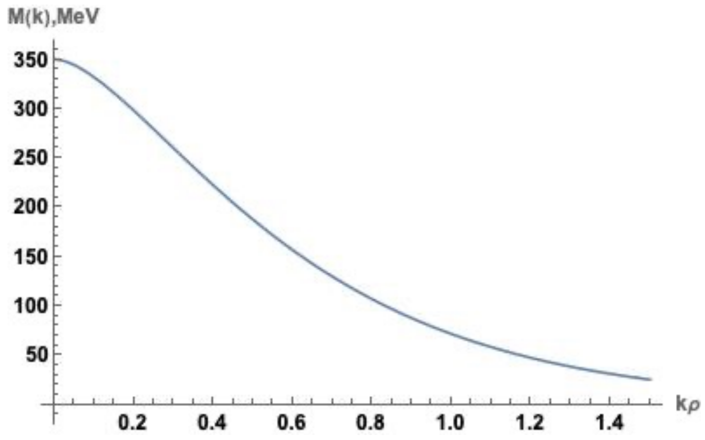
$$Z = \int D\psi D\psi^\dagger \exp\left\{\int_x \psi^\dagger i\hat{\partial}\psi + \lambda\,\mathrm{det}\psi^\dagger\gamma_\pm\psi\right\}$$
$$\to \int D\pi^A D\psi D\psi^\dagger \exp\left\{\int_x \psi^\dagger\left(i\hat{\partial} + iMe^{i\pi^A\tau^A\gamma_5}\right)\psi\right\}$$

Effective interaction between dynamically massive quarks and massless pions

- Cutoff is $1/\bar{\rho}$

- Effective chiral Lagrangian

$$S_{\mathrm{eff}}[\pi] = -N_c \mathrm{Tr}\ln\left(i\partial + iMe^{i\pi^A\tau^A\gamma_5}\right)$$



[D.Diakonov, V.Petrov, P.Pobylitsa 1996-1990]

Heavy Quark Effective Theory

- Static HQET Lagrangian : $\mathcal{L}_{\mathrm{HQET}_0} = Q^\dagger i D_0 Q + \mathcal{O}\left(m_Q^{-1}\right)$
- HQ symmetry: 1. Conservation of HQ spin $\frac{1+\gamma_0}{2} Q = Q$

  2. In leading order on m_Q^{-1}, the results are the same for all HQ flavors

- Infinitely HQ interacts 0^{th} component of gauge fields (in Minkowski space)
- HQ propagator: $w_A = (i\partial_0 + A_0)^{-1}$

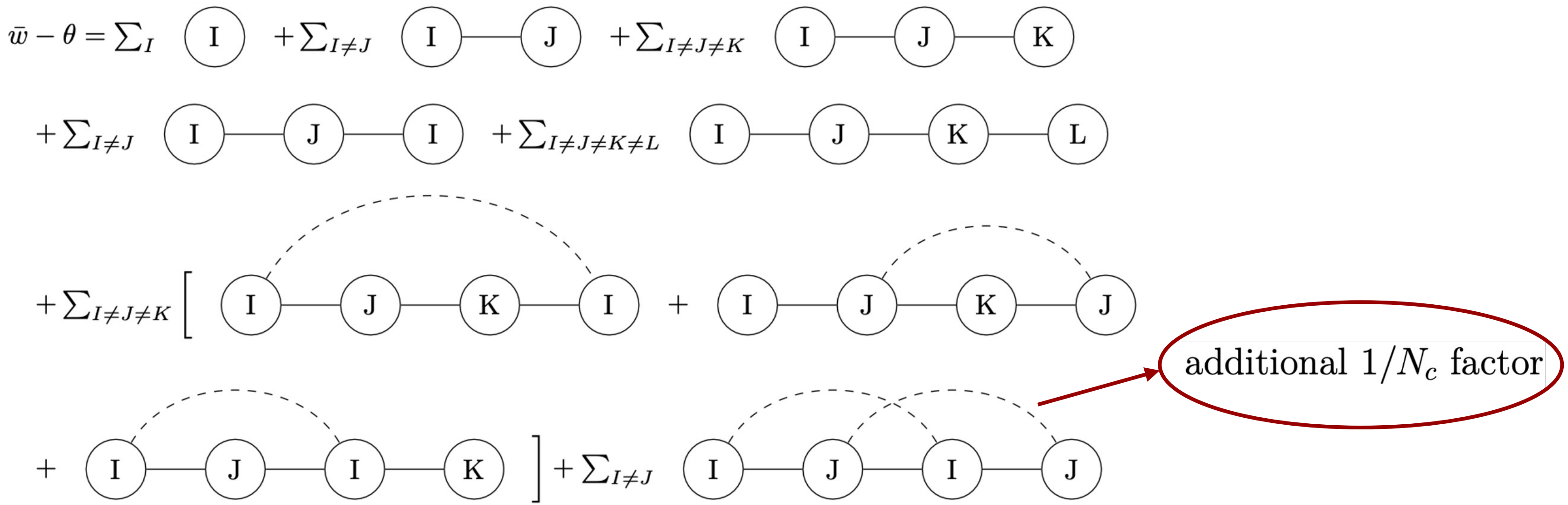
$$w_A(\vec{x}, x_0; \vec{x}', x_0') = -i\theta(x_0 - x_0')\delta^{(3)}(\vec{x} - \vec{x}')P\exp\left(-ig\int_{x_0'}^{x_0} d\tau A_a^0(\vec{x}, \tau)\lambda^a\right)$$

[papers by H.Georgi, M.Wise, N.Isgur and others]

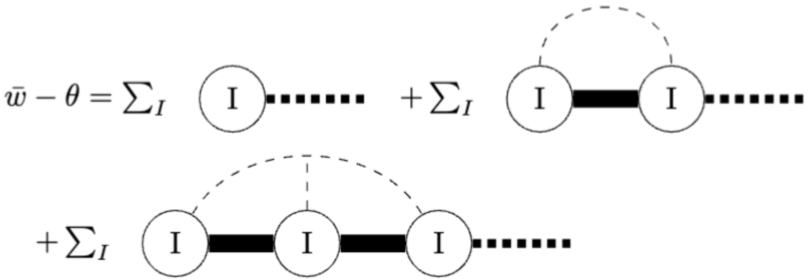
Heavy quark propagator in an instanton ensemble

- HQ propagator in instanton ensemble: $\bar{w} = \left\langle \left(\theta^{-1} - \sum_I a_I \right)^{-1} \right\rangle$

- In terms of single instanton propagators: $\bar{w} = \theta + \sum_I \langle w_I - \theta \rangle + \sum_{I \neq J} \langle w_I - \theta \rangle \theta^{-1} \langle w_J - \theta \rangle + \cdots$



Heavy quark in an instanton ensemble



- In the ensemble of N/2 instantons and N/2 anti-instantons:

$$\bar{w}^{-1} - \theta^{-1} = \frac{N}{2}\left\langle(\bar{w} - a_I^{-1})^{-1}\right\rangle + \frac{N}{2}\left\langle(\bar{w} - a_{\bar{I}}^{-1})^{-1}\right\rangle$$

- After averaging: $\bar{w}^{-1} - \theta^{-1} = \frac{N}{2VN_c}\mathrm{tr_c}\left(\int d^4 z_I (\bar{w} - a_I^{-1})^{-1} + \int d^4 z_{\bar{I}} (\bar{w} - a_{\bar{I}}^{-1})^{-1}\right)$
- Perturbative parameter: $\frac{\bar{\rho}^4 N}{VN_c} = \frac{\bar{\rho}^4}{\bar{R}^4 N_c} \simeq 0.004$
- HQ propagator in the instanton ensemble:

$$\bar{w}^{-1} = \theta^{-1} - \frac{N}{2VN_c}\mathrm{tr_c}\left(\int d^4 z_I \theta^{-1}(w_I - \theta)\theta^{-1} + (I \to \bar{I})\right) + \mathcal{O}\left(\left(\frac{N}{VN_c}\right)^2\right)$$

[D.Diakonov, V.Petrov, P.Pobylitsa 1989]

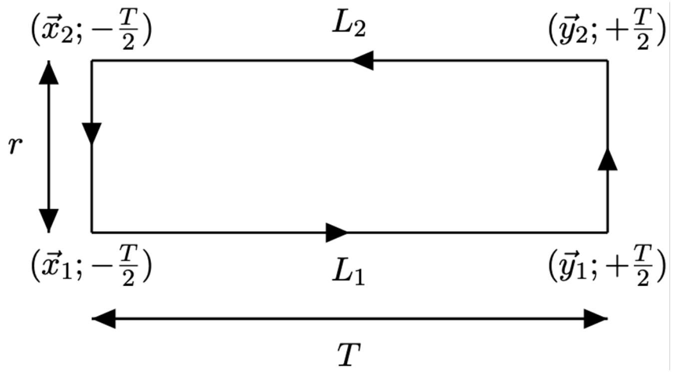
Application of Pobylitsa equation

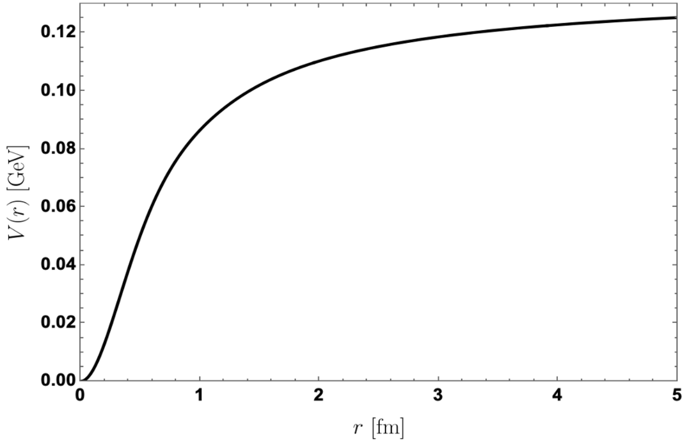
❑ Direct instanton contribution to HQ mass:

$$\Delta M_Q^{\rm dir} = \frac{N}{2VN_c}\sum_{\pm}\int d^3z_\pm\, \mathrm{tr}_c\left(1 - \mathrm{P}\exp\left(i\int d\tau A_{4,\pm}(\tau)\right)\Big|_{z_{4,\pm}=0}\right) \simeq 70\ \mathrm{MeV}$$

❑ Instanton induced quark – anti-quark potential:

$$V^{\rm dir}(r) = \frac{N}{2VN_c}\sum_{\pm}\int d^4z_\pm\, \mathrm{tr_c}\left[1 - P\exp\left(i\int_{L_1} d\tau A_{4,\pm}(\tau)\right)P\exp\left(-i\int_{L_2} d\tau A_{4,\pm}(\tau)\right)\right]_{z_{4,\pm}=0}$$





[D.Diakonov, V.Petrov, P.Pobylitsa 1989]

Heavy quark propagator in the presence of light quarks

- QCD partition function:

$$\mathcal{Z}_{\rm QCD} \sim \int D\psi^\dagger D\psi DQ_\pm^\dagger DQ_\pm \left\langle \exp\left(\int d^4x \left[\sum_f^{N_f} \psi_f^\dagger i\hat{\partial}\psi_f + Q_\pm^\dagger (\theta^{-1} - \sum_{I,\bar{I}} a_I) Q_\pm \right] \right) \prod_f^{N_f} \prod_{I,\bar{I}} (i m_f - V_I[\psi_f^\dagger, \psi_f]) \right\rangle$$

- Heavy quark propagator:

$$\bar{w} \sim \int D\psi^\dagger D\psi \, e^{-S_q} \left\langle \prod_f^{N_f} \prod_{I,\bar{I}} (-V_I[\psi_f^\dagger, \psi_f]) \left(\theta^{-1} - \sum_I a_I \right)^{-1} \right\rangle$$

$$\left\langle \prod_f^{N_f} \prod_{I,\bar{I}} (-V_I[\psi_f^\dagger, \psi_f]) \right\rangle \left\langle \prod_f^{N_f} \prod_{I,\bar{I}} (-V_I[\psi_f^\dagger, \psi_f]) \right\rangle^{-1}$$

develop Pobylitsa type eq.

$$w[\psi, \psi^\dagger]$$

"heavy quark functional"

$$\int D\psi D\psi^\dagger \exp\left[\int d^4x \sum_f^{N_f} \psi_f^\dagger i\hat{\partial}\psi + iY_{N_f}^{(+)} + iY_{N_f}^{(-)} \right]$$

- Pobylitsa eq. for HQ functional: $\bar{w}^{-1}[\psi^\dagger, \psi] = \theta^{-1} - \frac{N}{2} \sum_\pm \left\langle \prod_f^{N_f} (-) V_\pm[\psi_f^\dagger, \psi_f] \right\rangle^{-1} \Delta_{H,\pm}[\psi^\dagger, \psi] + \mathcal{O}\left(\frac{N^2}{V^2} \right)$

where $\Delta_{H,\pm}[\psi^\dagger, \psi] = \int d\zeta_\pm \prod_f^{N_f} (-) V_\pm[\psi_f^\dagger, \psi_f] \theta^{-1} (w_\pm - \theta) \theta^{-1}$

Heavy and light quarks interaction

- QCD partition function can be rewritten as

$$\mathcal{Z}_{QCD} \sim \int D[\text{fermions}] \exp\left(-S_q - S_Q + \int d^4x\, Q^\dagger i\lambda \sum_{\pm} \Delta_{H,\pm}[\psi^\dagger, \psi] Q + [\text{sources}]\right)$$

$$S_{Qq} \sim \sum_{\pm} \int d[\zeta_\pm, k, l, x, y] \prod_f^{N_f} \sqrt{M(k_f)M(l_f)}\psi_f^\dagger(k_f)\psi_f(l_f)\; Q^\dagger(x)\langle x|\theta^{-1}(w_\pm - \theta)\theta^{-1}|y\rangle Q(y)$$

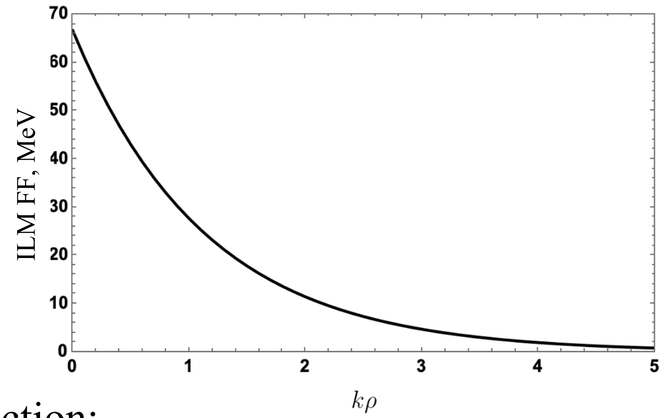
- Effective HQ & N_f light quarks interaction vertex

$$\int d[k_f]d[l_f]d[p]\delta^{(4)}\left(\sum k_f - \sum l_f + p\right) dU\; [U^\dagger U]_q^{N_f}\; [U^\dagger U]_Q$$
$$\times \sqrt{M(k_f)}\psi_f^\dagger(k_f)\gamma_\pm \sqrt{M(l_f)}\psi_f(l_f) \cdot Q_+^\dagger(p_1)\langle p_1|\theta^{-1}(w_\pm - \theta)\theta^{-1}|p_2\rangle Q_+(p_2)$$

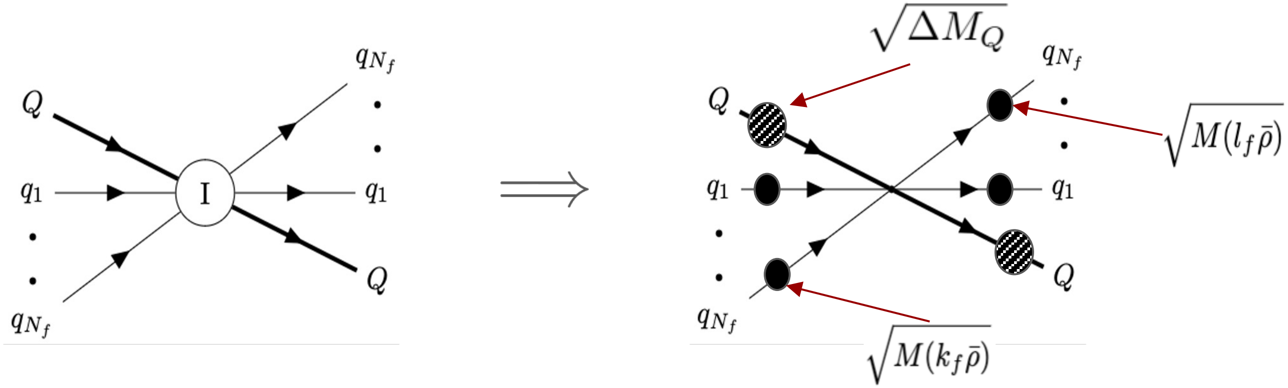
After color integration leads to momentum dependent ‘non-slashed’ term

Heavy and light quarks interaction

$$\lim_{|\vec{p}|\to 0} \frac{N}{2VN_c} \mathrm{tr}_c \langle 0|\theta^{-1}(w_\pm - \theta)\theta^{-1}|p\rangle = \Delta M_Q$$



- Diagrammatic representation of the effective interaction:



Heavy-light quark interaction

- $N_f = 1$ case:

$$
\begin{aligned}
S_{Qq} \quad \propto \quad & \int \frac{d^4k_1 d^4k_2}{(2\pi)^8} \frac{d^4p_1 d^4p_2}{(2\pi)^8} (2\pi)^4 \delta^{(4)}(k_1 - k_2 + p_1 - p_2) \sqrt{M(k_1)M(k_2)} \frac{\Delta M_Q}{N/V} \\
& \times \left[\frac{N_c^2}{N_c^2 - 1} \left(1 - \frac{1}{2N_c}\right) \left(\psi^\dagger(k_1)\psi(k_2)\right) \left(Q^\dagger(p_1)Q(p_2)\right) \right. \\
& \left. + \frac{N_c^2}{8(N_c^2 - 1)} \left(1 - \frac{2}{N_c}\right) \sum_i \left(\psi^\dagger(k_1)\Gamma_i Q(p_1)\right) \left(Q^\dagger(p_2)\Gamma_i \psi(k_2)\right) \right]
\end{aligned}
$$

where $\Gamma_i = \left(\mathbf{1},\ \gamma_5,\ \gamma_\mu,\ i\gamma_\mu\gamma_5,\ \sigma_{\mu\nu}/\sqrt{2}\right)$

- $N_f = 2$ case, interaction terms:

$$
\begin{aligned}
S_{Qq} \quad \sim \quad & C_{SSS}(u^\dagger u)(d^\dagger d)(Q^\dagger Q) + C_{\Gamma\Gamma S}(u^\dagger \Gamma_i d)(d^\dagger \Gamma_i u)(Q^\dagger Q) \\
& + C_{\Gamma S \Gamma}(u^\dagger \Gamma_i Q)(d^\dagger d)(Q^\dagger \Gamma_i u) + C_{S\Gamma\Gamma}(u^\dagger u)(d^\dagger \Gamma_i Q)(Q^\dagger \Gamma_i d) + \cdots
\end{aligned}
$$

- Applications you will see in Mr. KiHoon Hong’s talk (this workshop)

Heavy-light quark interaction

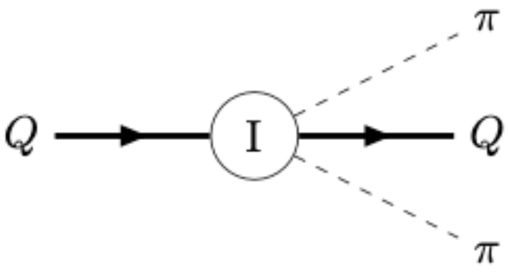
- In $N_f \geq 2$ case many-fermion vertices can be linearized introducing integration over boson fileds

$$\mathcal{Z} \sim \int DU \int D\psi^\dagger D\psi \int DQ^\dagger DQ \exp\left\{ \int \psi^\dagger i\hat{\partial}\psi + Q^\dagger \theta^{-1} Q + iM\psi^\dagger U \psi \Delta M_Q Q^\dagger Q \right\}$$

- Integrating out light quark fields leads to effective HQ & light mesons interaction

$$S_{Q\pi} \sim F_{\pi Q}^2 \operatorname{Tr}\left[\partial_\mu U \partial_\mu U^\dagger\right] \Delta M_Q Q^\dagger Q$$

$$F_{\pi Q}^2 = 2N_c \int \frac{d^4p}{(2\pi)^4} \frac{p^2 M^2(p)}{\left[p^2 + M^2(p)\right]^3}$$



Heavy quarkonium light quark interaction

- Repeat the same procedure which was done for HQ propagator to obtain heavy $Q\bar{Q}$ functional
- Use perturbation theory over $\lambda = \rho^4/R^4$ (Pobylitsa type equation)
- Write corresponding partition function and extract interaction action using saddle point method

$$S_{Q\bar{Q}q} \sim \int d[k]d[l]Q^\dagger(k_2)\bar{Q}^\dagger(l_2)\, V(|\vec{k}_2 - \vec{l}_2|, |\vec{k}_1 - \vec{l}_1|, \omega_k, \omega_l)\, Q(k_1)\bar{Q}(l_1)\cdots$$

$$\mathrm{FT}\left\{V(|\vec{k}_2 - \vec{l}_2|, |\vec{k}_1 - \vec{l}_1|, 0, 0)\right\} \to V^{\mathrm{dir}}(r)$$

- $N_f \geq 2$ case do bosonization procedure to linearize multi-fermion vertex. Integrating out light quark fields leads to heavy quarkonium and meson interaction

$$S_{Q\bar{Q}\pi} \sim F_{\pi Q}^2 \mathrm{Tr}\left[\partial_\mu U \partial_\mu U^\dagger\right] Q^\dagger \bar{Q}^\dagger V Q\bar{Q}$$

Conclusions and Future remarks

- Instanton-induced multi light quark interaction is effective interaction of light quarks that a form factor $\sqrt{M(k)}$ attached each quark leg. In case of instanton induced heavy –light quark effective interaction heavy quarks also get a form factor ($\sqrt{\Delta M_Q}$ -instanton generated dynamical contribution to the mass).
- Bosonization (integrating out light quark degrees of freedom) leads to heavy quark and light mesons effective interaction.
- Instantons generate heavy quarkonium light mesons effective interaction which is obtained through bosonization of heavy quark – anti-quark and light quarks interaction
- Develop the calculations to real world case ($N_f = 3$).
- Develop to HQ and light diquark interaction.
- Add flavor number to HQ (N_f^q light and N_f^Q heavy quarks interaction).

Thanks for the attention