The mass-radius relation of the neuron and hyperon stars

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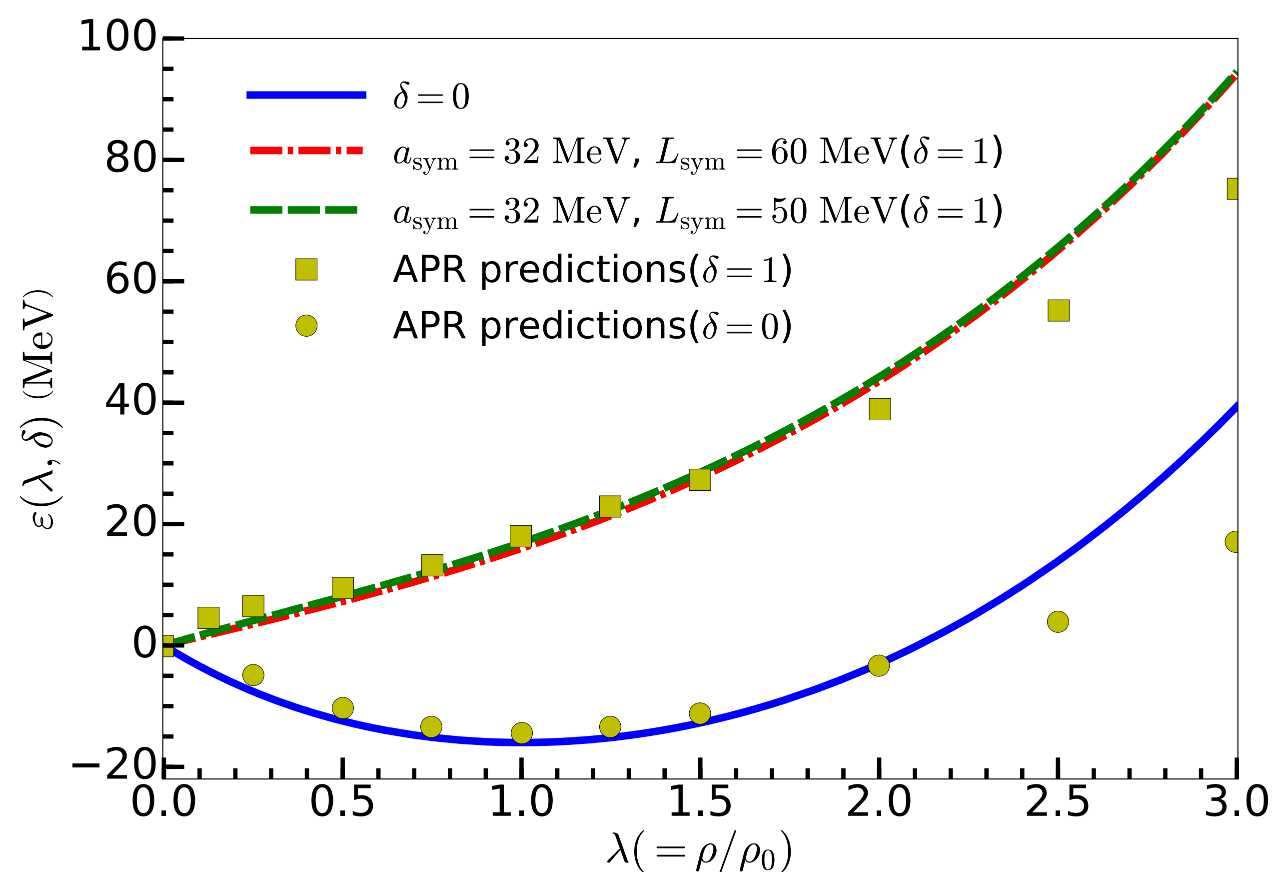
INHA university, Incheon

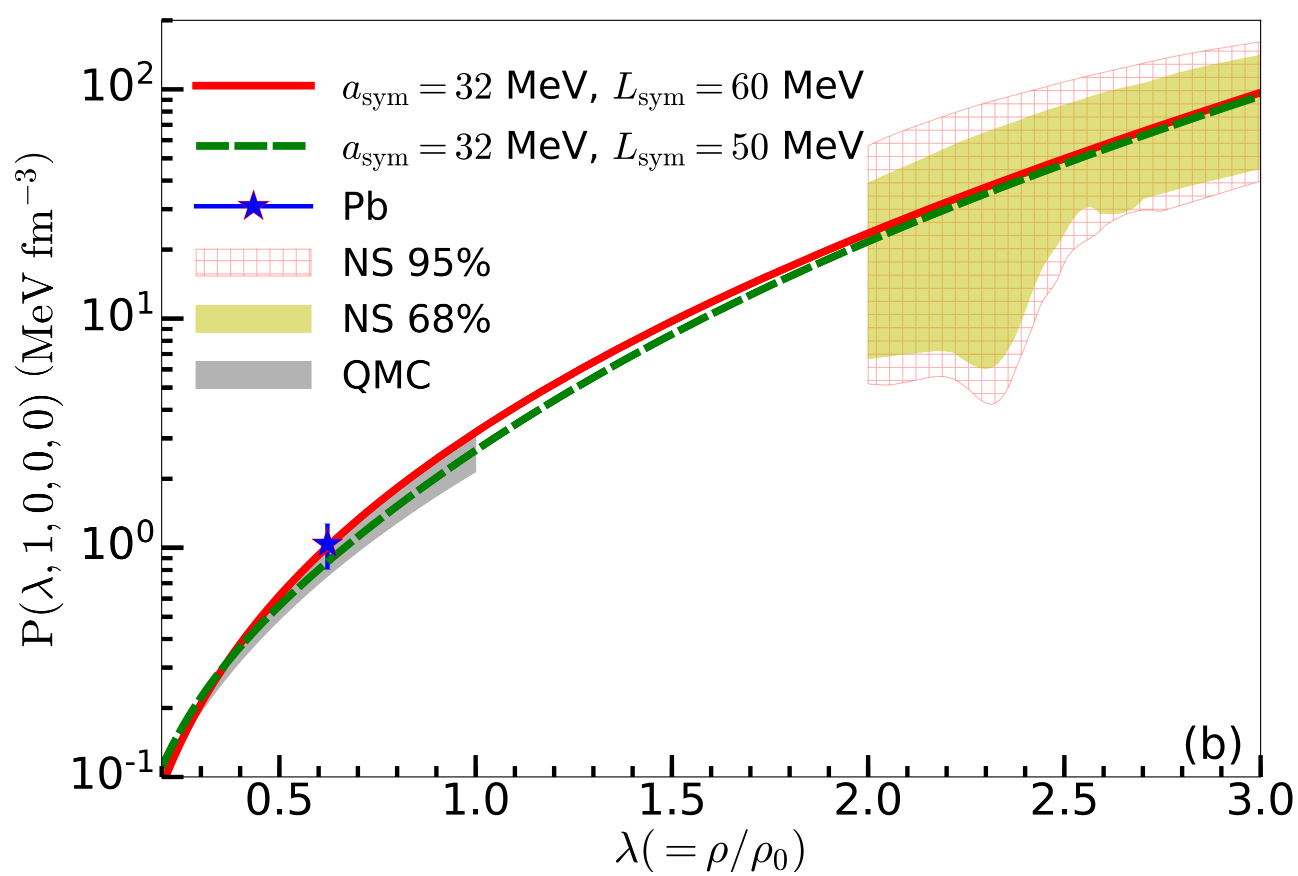
OUTLINE

- Motivation
- Pion mean-field approach
- Medium modification
- Equation of state
- Neutron star
- Result
- Summary

MOTIVATION

- In the previous work[1], the pion mean-field approach successfully explain the nuclear matter properties at low density legion.





[1] N. Y. Ghim, G. S. Yang, H. Ch. Kim, U. Yakhshiev, Phys. Rev. C. **103**, 064306 (2021).

Pion mean-field approach

- Collective Hamiltonian [2,3]

$$
\begin{aligned}
H =& M_{\rm cl} + \frac{1}{2I_1}\sum_{i=1}^{3}\hat{J}_i^2 + \frac{1}{2I_2}\sum_{p=4}^{7}\hat{J}_p^2 \\
&+ (m_{\rm d} - m_{\rm u})\left(\frac{\sqrt{3}}{2}\alpha D_{38}^{(8)}(\mathcal{A}) + \beta\hat{T}_3 + \frac{1}{2}\gamma\sum_{i=1}^{3}D_{3i}^{(8)}(\mathcal{A})\hat{J}_i\right) \\
&+ (m_{\rm s} - \bar{m})\left(\alpha D_{88}^{(8)}(\mathcal{A}) + \beta\hat{Y} + \frac{1}{\sqrt{3}}\gamma\sum_{i=1}^{3}D_{8i}^{(8)}(\mathcal{A})\hat{J}_i\right) + H_{\rm em}
\end{aligned}
$$

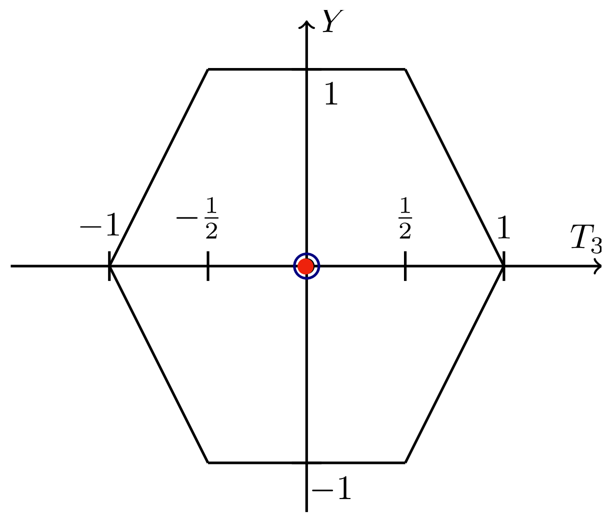
$$
\alpha = -\left(\frac{2}{3}\frac{\Sigma_{\pi\rm N}}{m_{\rm u}+m_{\rm d}} - Y\frac{K_2}{I_2}\right) \qquad \beta = -\frac{K_2}{I_2} \qquad \gamma = 2\left(\frac{K_1}{I_1} - \frac{K_2}{I_2}\right)
$$

[2] G. S. Yang and H.-Ch. Kim, Phys. Rev. D. **94**, 07152 (2016).
[3] G. S. Yang, H. Ch. Kim, Phys. Lett. B. **808**, 135619 (2020).

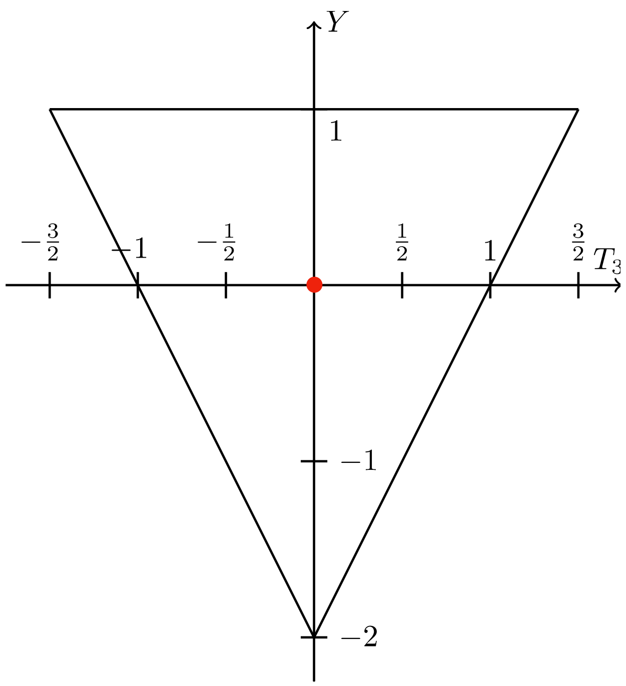
Pion mean-field approach

- Collective Hamiltonian [2,3]

$$H = \mathrm{M}_{\mathrm{cl}} + \boxed{\frac{1}{2}\left(\frac{1}{I_1} - \frac{1}{I_2}\right)\sum_{i=1}^{3} \hat{J}_i \hat{J}_i + \frac{1}{2I_2}\sum_{a=1}^{8} \hat{J}_a \hat{J}_a - \frac{1}{2I_2}\hat{J}_8^2}$$



Octet



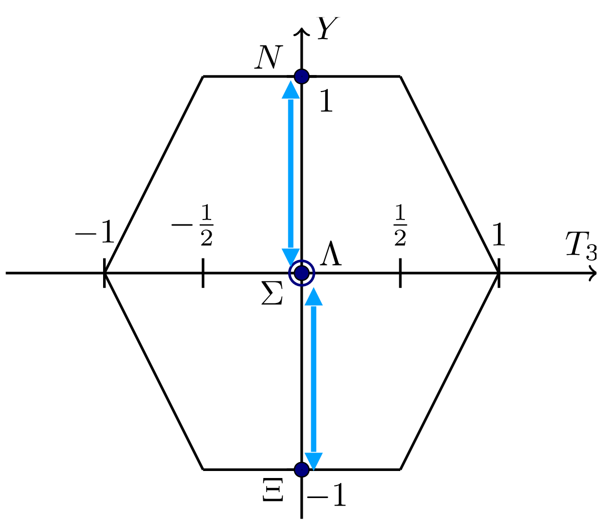
Decuplet

[2] G. S. Yang and H.-Ch. Kim, Phys. Rev. D. **94**, 07152 (2016).
[3] G. S. Yang, H. Ch. Kim, Phys. Lett. B. **808**, 135619 (2020).

Pion mean-field approach

- Collective Hamiltonian [2,3]

$$(m_s - \hat{m})\left(\alpha D^{(8)}_{88}(R) + \beta \hat{Y} + \frac{1}{\sqrt{3}}\gamma \sum_{i=1}^{3} D^{(8)}_{8i}(R)\hat{J}_i\right)$$



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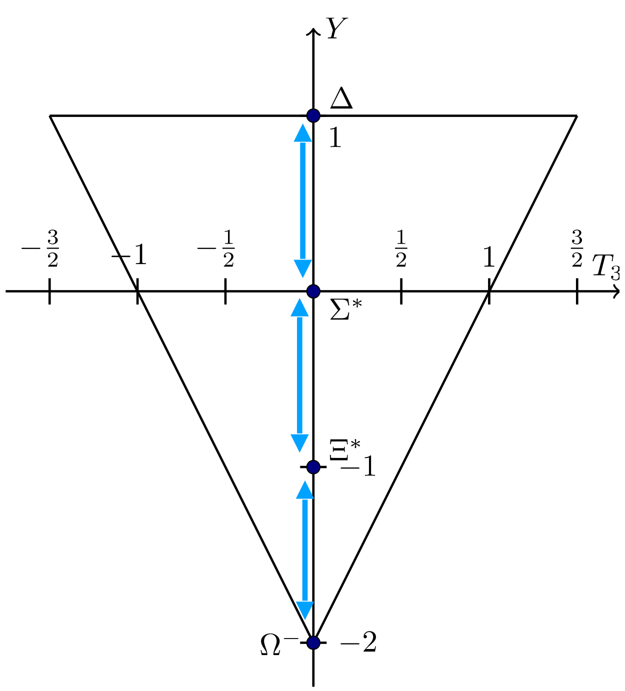

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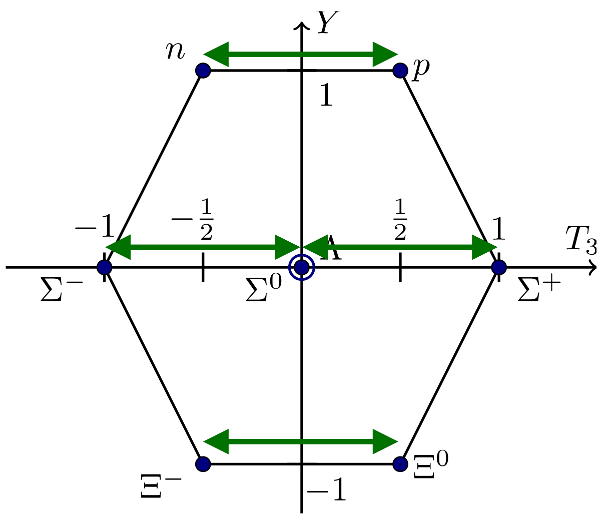
Pion mean-field approach

- Collective Hamiltonian [2,3]

$$(m_d - m_u)\left(\frac{\sqrt{3}}{2}\alpha D^{(8)}_{38}(R) + \beta\hat{T}_3 + \frac{1}{2}\gamma\sum_{i=1}^{3} D^{(8)}_{3i}(R)\hat{J}_i\right)$$



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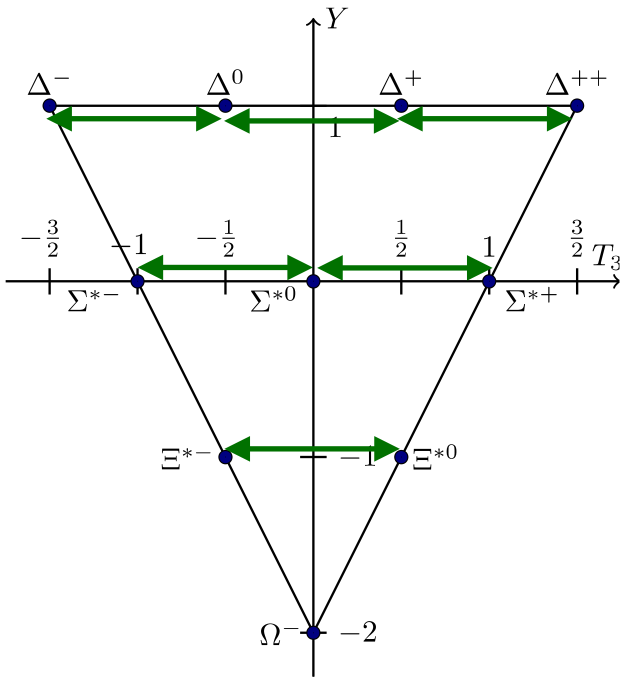

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Medium modification

- Binding energy per baryon[1]

$$\varepsilon = \frac{E^* - E}{A} = \frac{Z\Delta M_p + N\Delta M_n + \sum_{s=1}^{3} N_s \Delta M_s}{A}$$

$$= \Delta M_N \left(1 - \sum_{s=1}^{3} \delta_s\right) + \frac{1}{2}\delta \Delta M_{np} + \sum_{s=1}^{3} \delta_s \Delta M_s$$

$$M_{np} = M_n - M_p$$

$$\Delta M_N = M_N^* - M_N$$

$$\delta = \frac{N - Z}{A}$$

$$\delta_s = \frac{N_s}{A}$$

[1] N. Y. Ghim, G. S. Yang, H. Ch. Kim, U. Yakhshiev, Phys. Rev. C. **103**, 064306 (2021).

Medium modification

- The properties of nuclear matter[1]

  - Volume energy:

$$\varepsilon(\lambda, 0, 0, 0, 0) = \varepsilon_V(\lambda) = \Delta M_N(\lambda)$$

  - Pressure:

$$P(\lambda) = \rho_0 \lambda^2 \frac{\partial \varepsilon_V(\lambda)}{\partial \lambda}$$

  - Compressibility:

$$K(\lambda) = 9\lambda^2 \frac{\partial^2 \varepsilon_V(\lambda)}{\partial \lambda^2}$$

[1] N. Y. Ghim, G. S. Yang, H. Ch. Kim, U. Yakhshiev, Phys. Rev. C. **103**, 064306 (2021).

Medium modification

- The properties of nuclear matter[1]

- Symmetry energy: $\varepsilon_{\text{sym}}(\lambda) = \frac{1}{2!}\frac{\partial^2\varepsilon(\lambda,\delta,0,0,0)}{\partial\delta^2}\Big|_{\delta=0}$

- The slope parameter: $L_{sym} = 3\frac{\partial\varepsilon_{\text{sym}}(\lambda)}{\partial\lambda}\Big|_{\lambda=1}$

[1] N. Y. Ghim, G. S. Yang, H. Ch. Kim, U. Yakhshiev, Phys. Rev. C. **103**, 064306 (2021).

Medium modification

- Medium functions[1]

$$M^*_{\mathrm{cl}} = M_{\mathrm{cl}}\left(1 + C_{\mathrm{cl}}\lambda\right)$$

$$I^*_1 = I_1\left(1 + C_1\lambda\right)$$

$$I^*_2 = I_2\left(1 + C_2\lambda\right)$$

$$\frac{K^{I*}_{1,2}}{I^*_{1,2}} = \frac{K^I_{1,2}}{I_{1,2}}\left(1 + \frac{C_{\mathrm{num}}\lambda\delta}{1 + C_{\mathrm{den}}\lambda}\right)$$

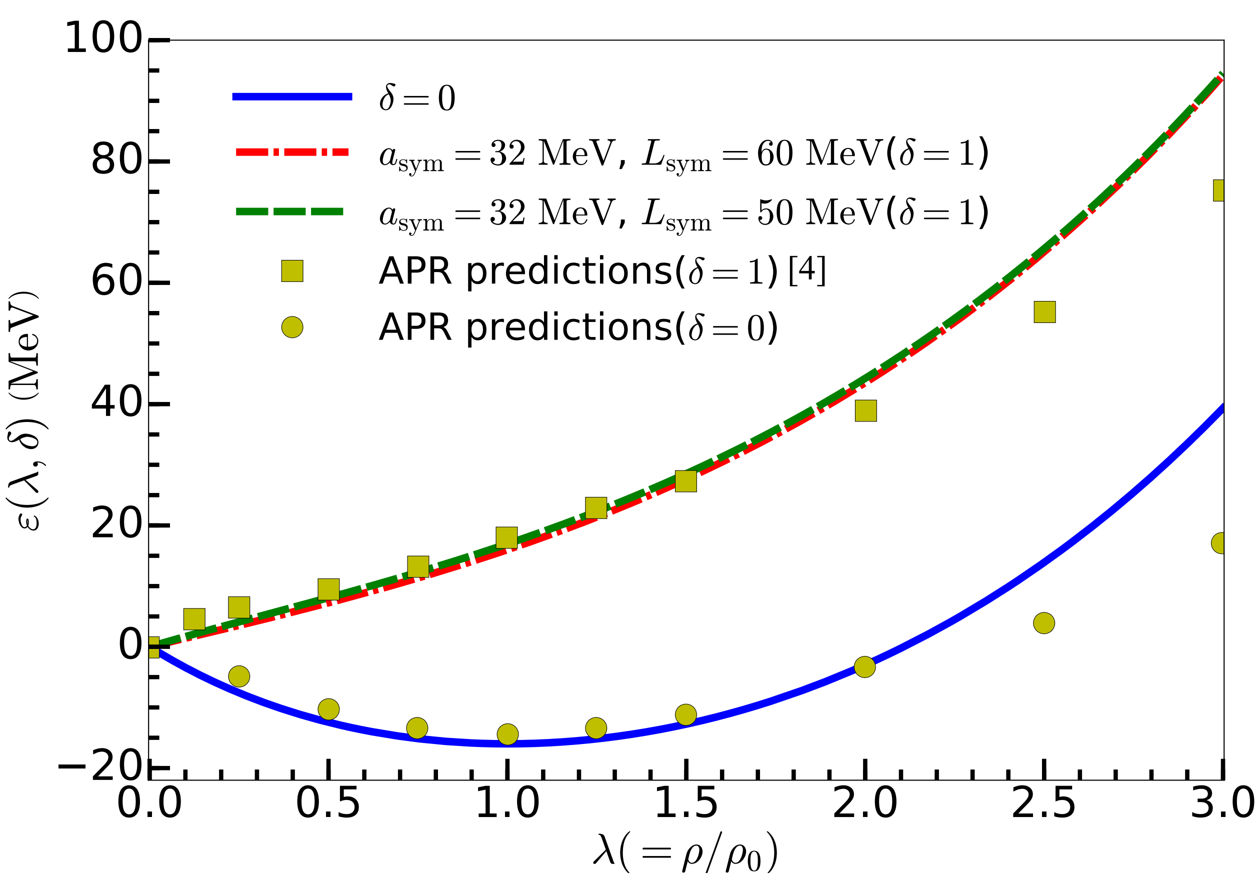
- Density-dependent parameters[1]

	Values
C_{cl}	-0.0561
C_1	0.6434
C_2	-0.1218
C_{num}	65.60
C_{den}	0.60

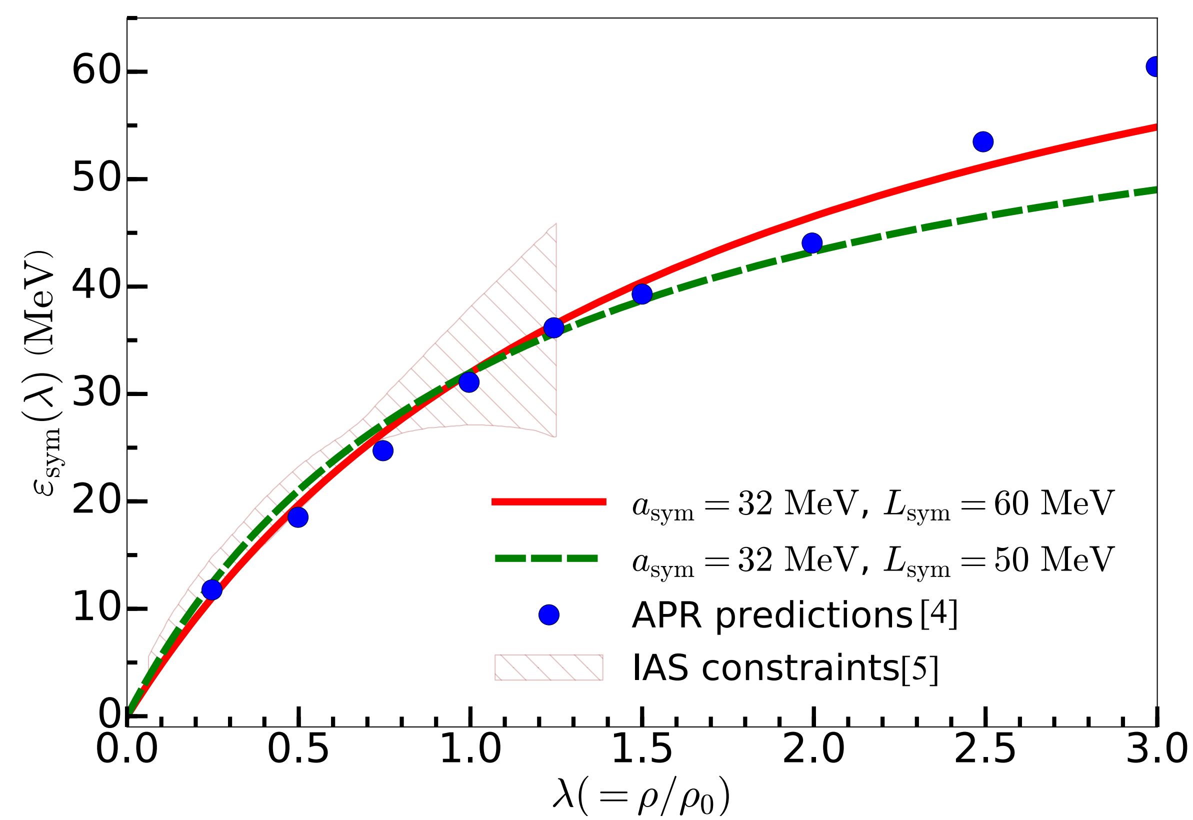
[1] N. Y. Ghim, G. S. Yang, H. Ch. Kim, U. Yakhshiev, Phys. Rev. C. **103**, 064306 (2021).

Equation of state

- Equation of state[1]



Binding energy of nuclear matter

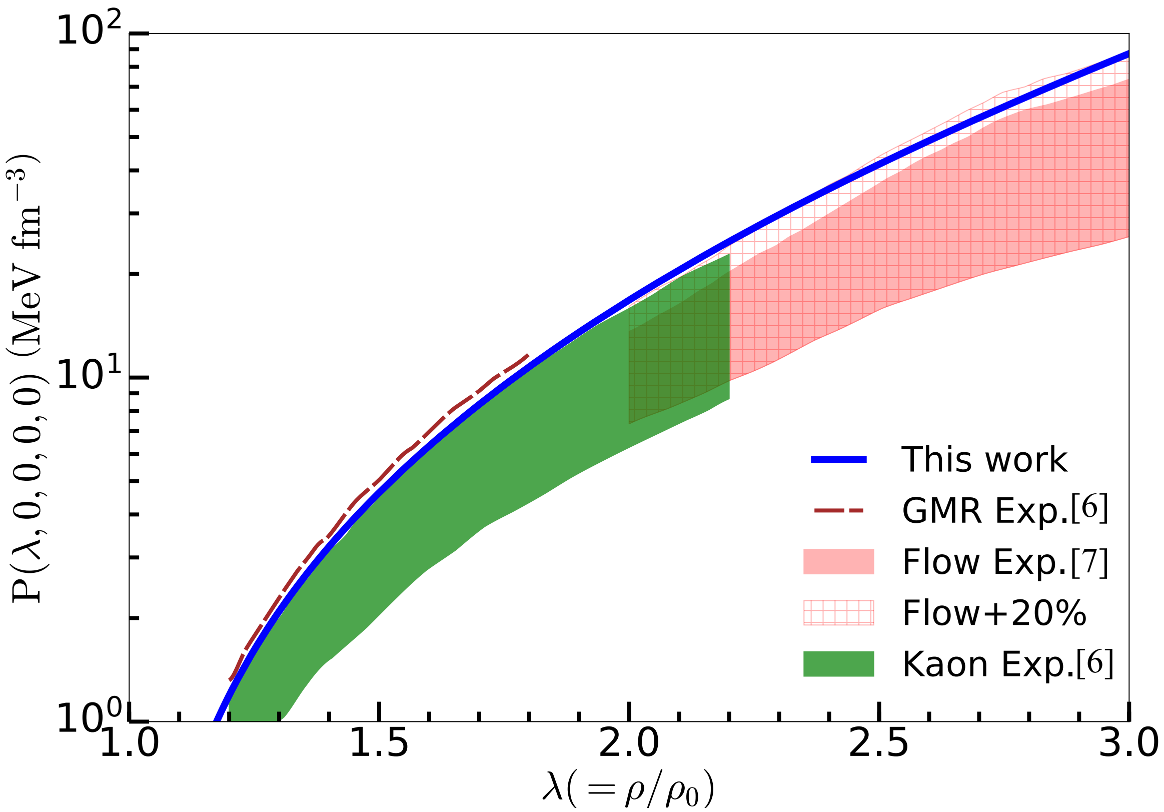


Symmetry energy of nuclear matter

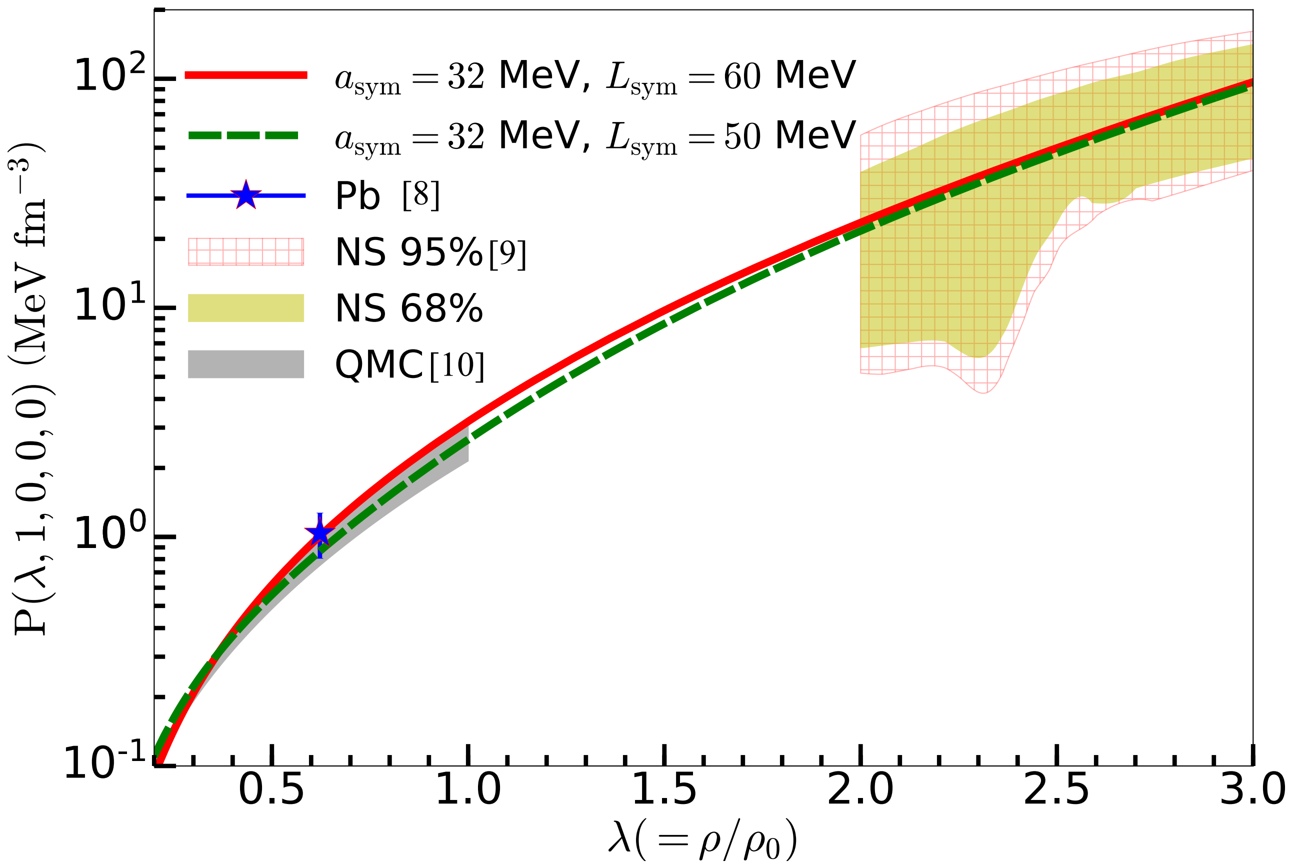
[4] A. Akmal, V. R. Pandharipande and D. G. Ravenhall, Phys. Rev. C 58, 1804 (1998).
[5] P. Danielewicz and J. Lee, Nucl. Phys. A 922, 1 (2014).

Equation of state

▸Equation of state[1]



Pressure of symmetric nuclear matter

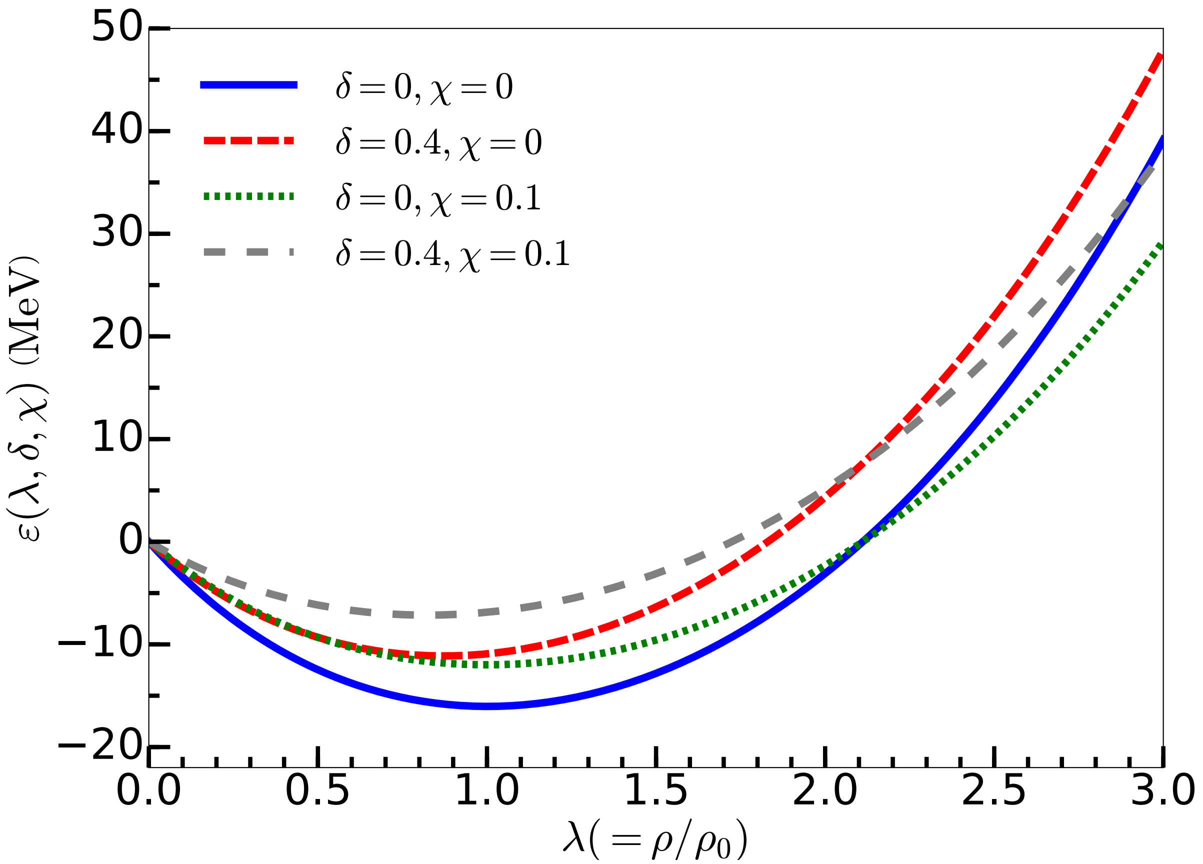


Pressure of neutron matter

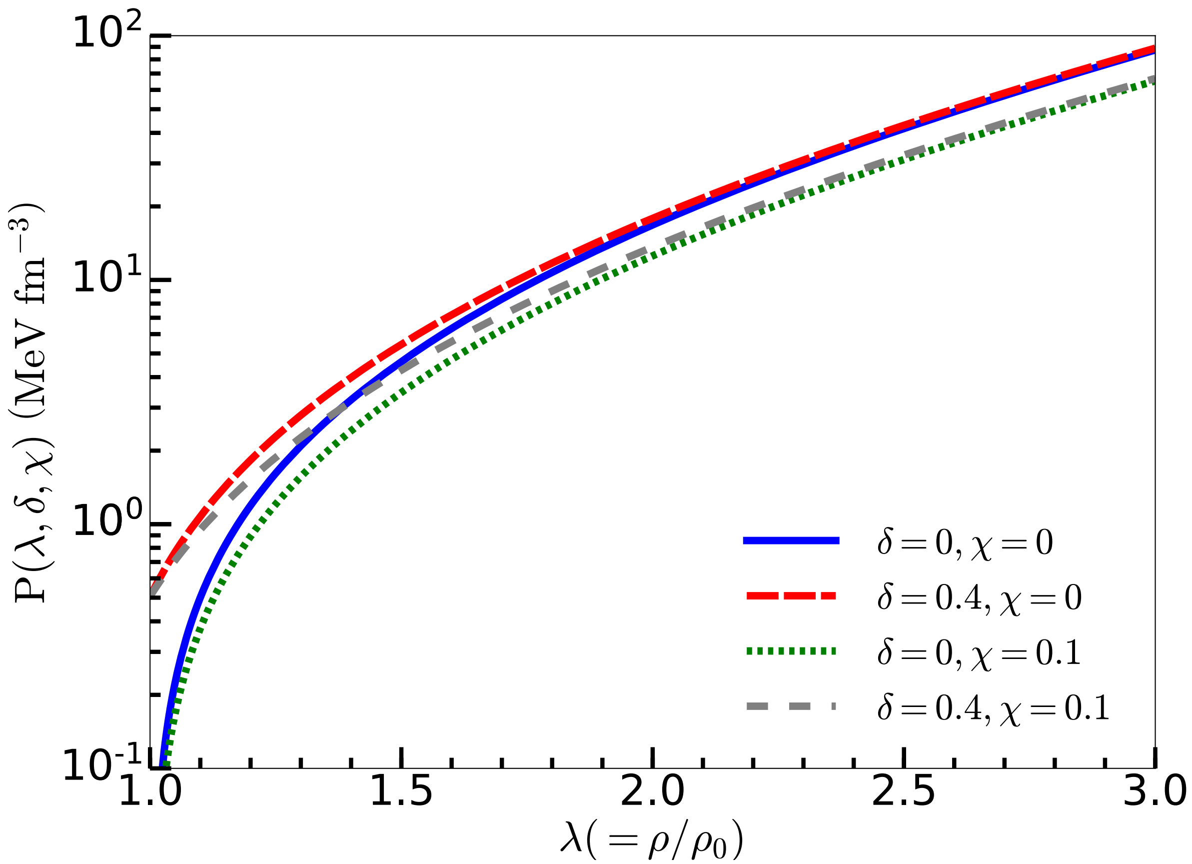
[6] D. H. Youngblood, H. L. Clark and Y.-W. Lui, Phys. Rev. Lett. 82, 691 (1999)
[7] P. Danielewicz, R. Lacey and W. G. Lynch, Science 298, 1592 (2002)
[8] M. B. Tsang et al., Phys. Rev. C 86, 015803 (2012)
[9] A. W. Steiner, J. M. Lattimer and E. F. Brown, Astro- phys. J. Lett. 765, L5 (2013)
[10] S. Gandolfi, J. Carlson and S. Reddy, Phys. Rev. C 85, 032801 (2012)

Equation of state

▶ Equation of state[1]



Binding energy of baryonic matter



Pressure of baryonic matter

[1] N. Y. Ghim, G. S. Yang, H. Ch. Kim, U. Yakhshiev, Phys. Rev. C. **103**, 064306 (2021).

Neutron star

- Spherically symmetric approximation.

$$\mathcal{M}(r) = 4\pi \int_0^r \mathrm{d}r \ r^2 \mathcal{E}(r)$$

- TOV Equation

$$-\frac{d\ P(r)}{dr} = \frac{G\mathcal{E}(r)\mathcal{M}(r)}{r^2}\left(1 - \frac{2G\mathcal{M}(r)}{r}\right)^{-1}\left(1 + \frac{P(r)}{\mathcal{E}(r)}\right)\left(1 + \frac{4\pi r^3 P(r)}{\mathcal{M}(r)}\right)$$

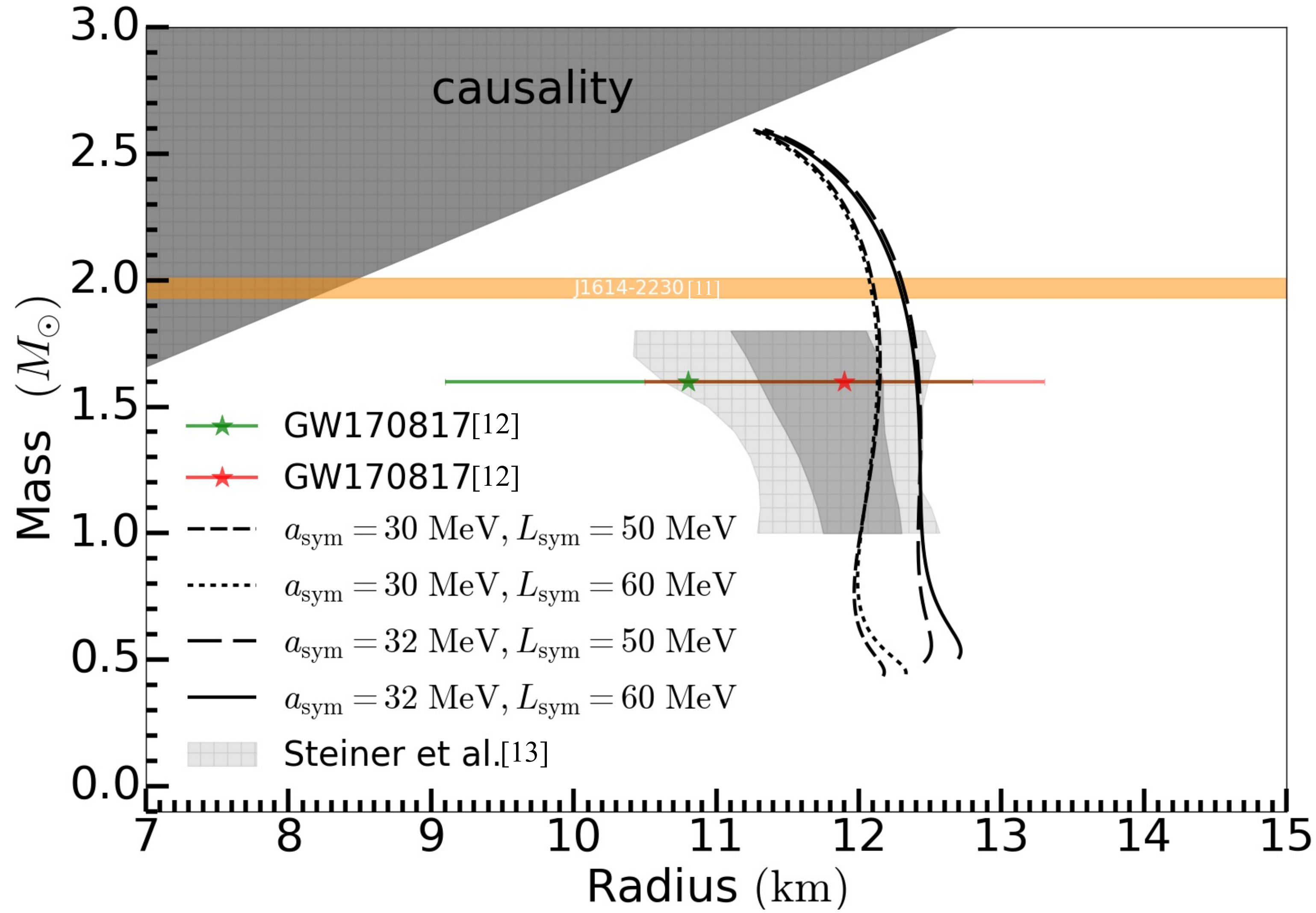
- Boundary condition

$$\mathcal{M}(0) = 0, \quad \mathcal{E}(0) = \mathcal{E}_{\mathrm{cent}}, \quad P(r = R) = 0$$

Pressure : $P(\lambda) = \rho_0 \lambda^2 \dfrac{\partial \varepsilon(\lambda, 1)}{\partial \lambda}$ Density: $\mathcal{E}(\lambda) = \left[\varepsilon(\lambda, 1) + m_{\mathrm{N}}\right] \lambda \rho_0$

RESULT

The mass-radius relation

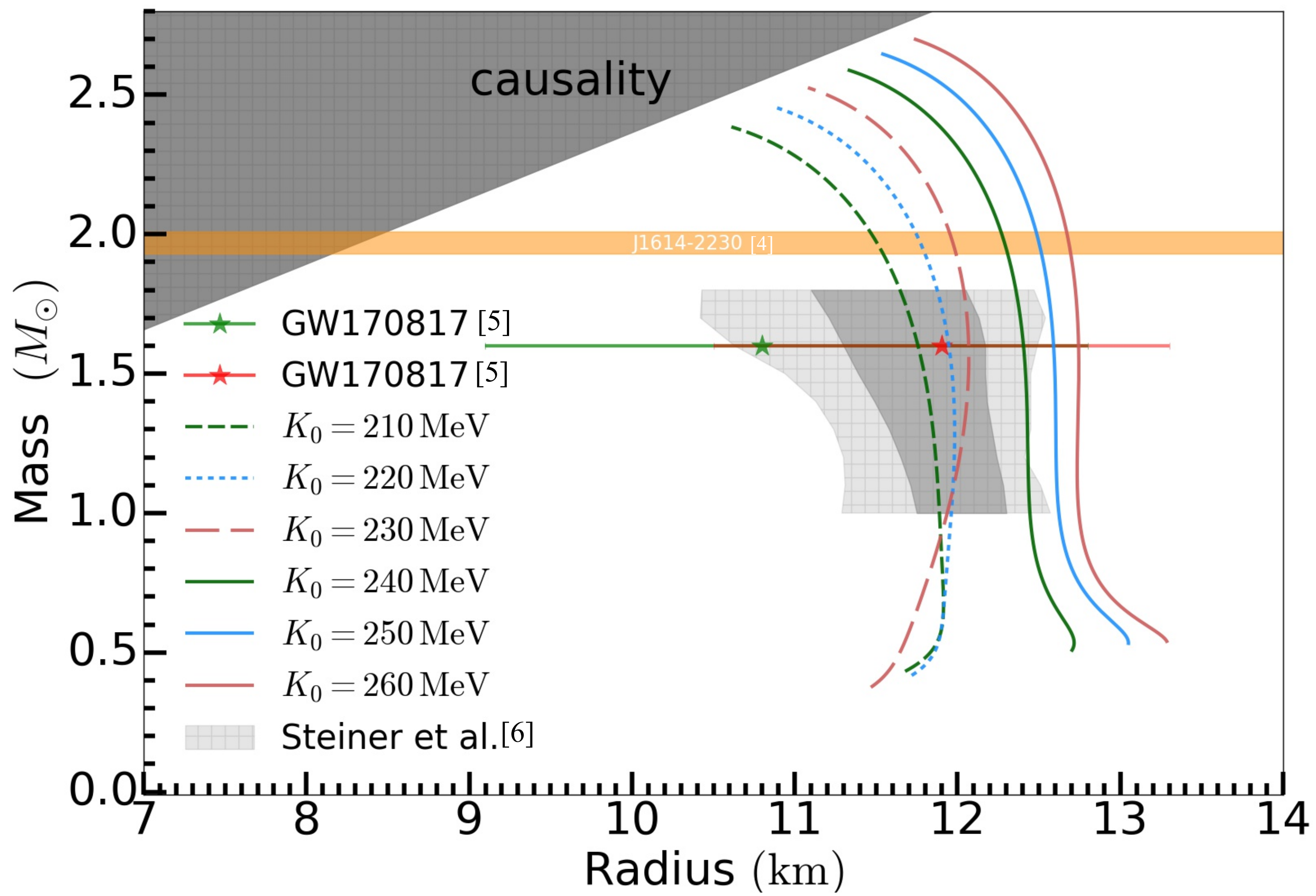


Pure neutron matter ($K_0 = 240$ MeV)

[11] Demorest, P., Pennucci, T., Ransom, S. *et al.* A two-solar-mass neutron star measured using Shapiro delay. *Nature* **467,** 1081–1083 (2010)
[12] B. P. Abbott *et al.* (The LIGO Scientific Collaboration and the Virgo Collaboration) Phys. Rev. Lett. **121**, 161101
[13] Andrew W. Steiner *et al.* 2010 *ApJ* **722.** 33

RESULT

The mass-radius relation



Pure neutron matter (a_{sym} = 32 MeV, L_{sym} = 60 MeV)

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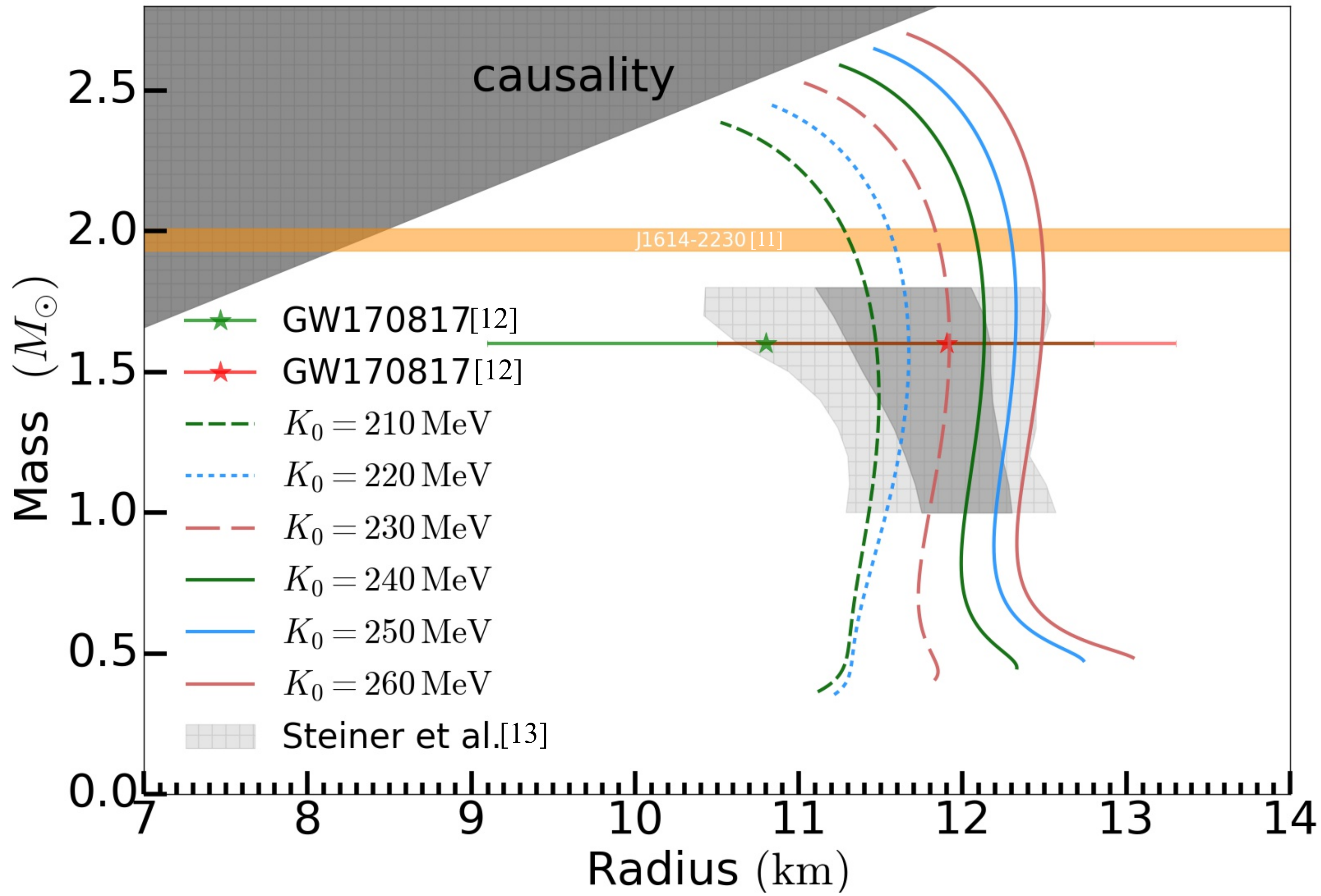
RESULT

The mass-radius relation



Pure neutron matter (a_{sym} = 32 MeV, L_{sym} = 50 MeV)

[11] Demorest, P., Pennucci, T., Ransom, S. *et al.* A two-solar-mass neutron star measured using Shapiro delay. *Nature* **467,** 1081–1083 (2010)
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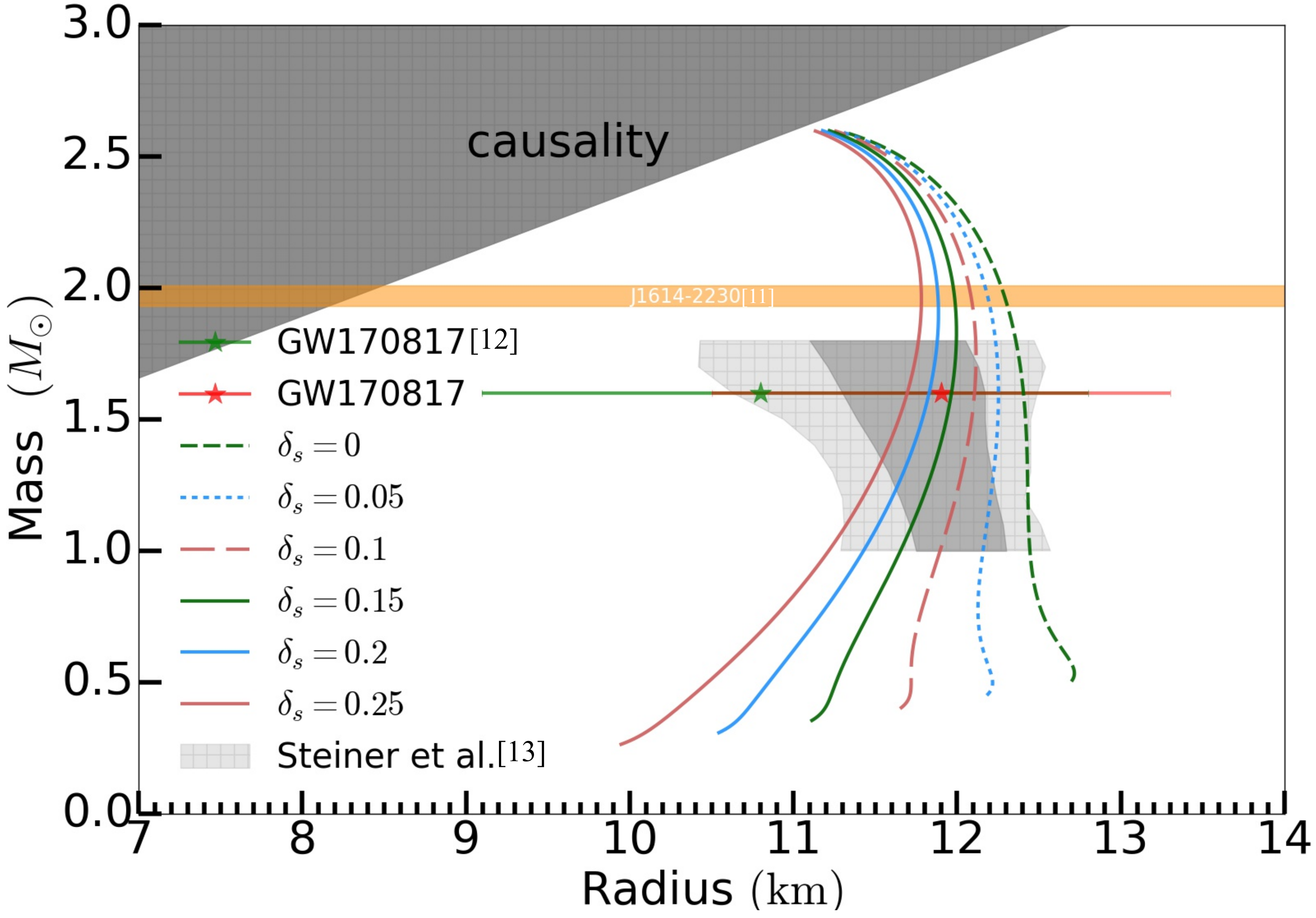
RESULT

The mass-radius relation



Hyperon mixed neutron matter ($K_0 = 240$ MeV, $a_{\rm sym} = 32$ MeV, $L_{\rm sym} = 50$ MeV)

[11] Demorest, P., Pennucci, T., Ransom, S. *et al.* A two-solar-mass neutron star measured using Shapiro delay. *Nature* **467,** 1081–1083 (2010)
[12] B. P. Abbott *et al.* (The LIGO Scientific Collaboration and the Virgo Collaboration) Phys. Rev. Lett. **121**, 161101
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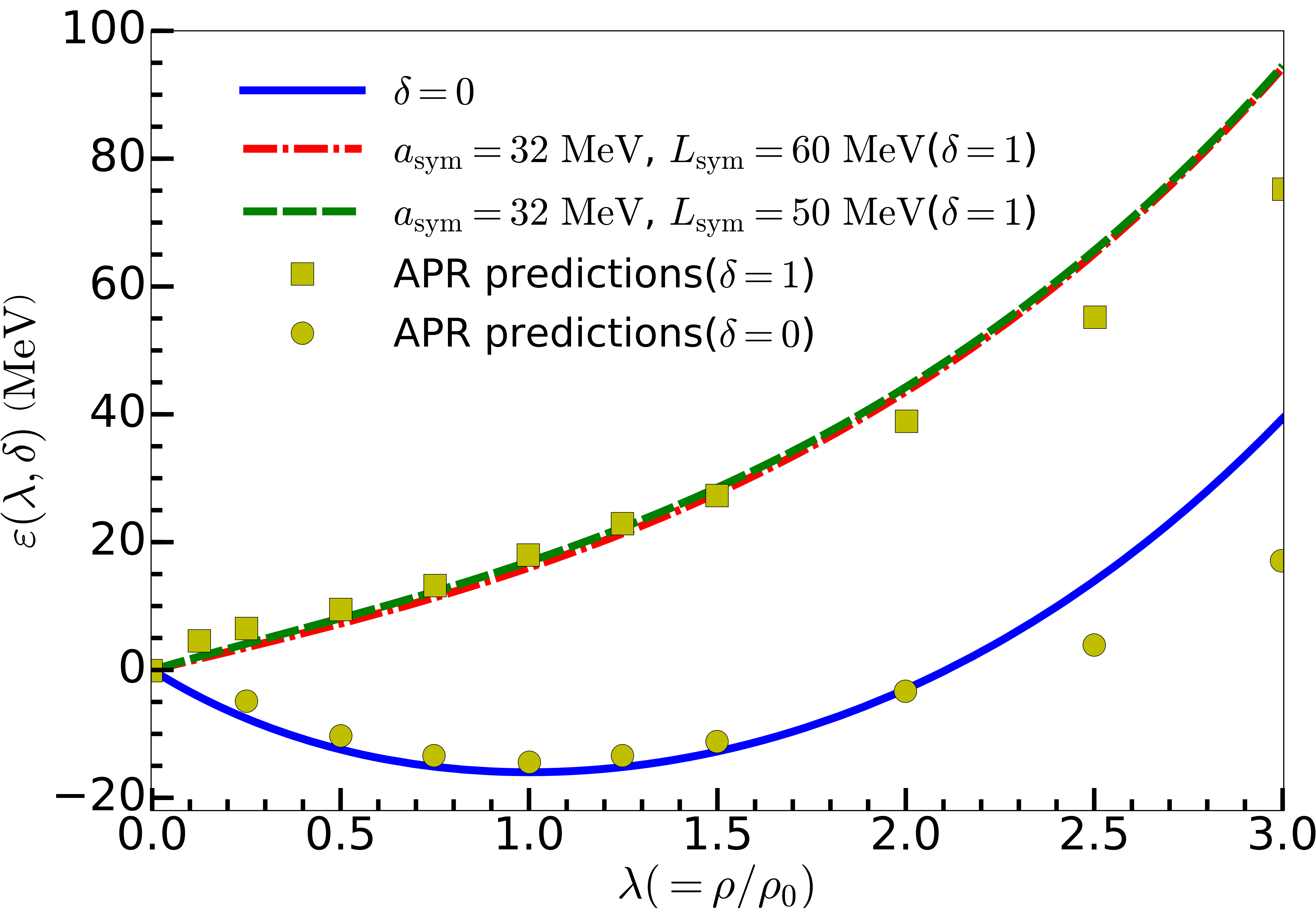
Summary

- We investigated the masses and radii of neutron stars based on a pion mean-field approach and linear-response approximation.

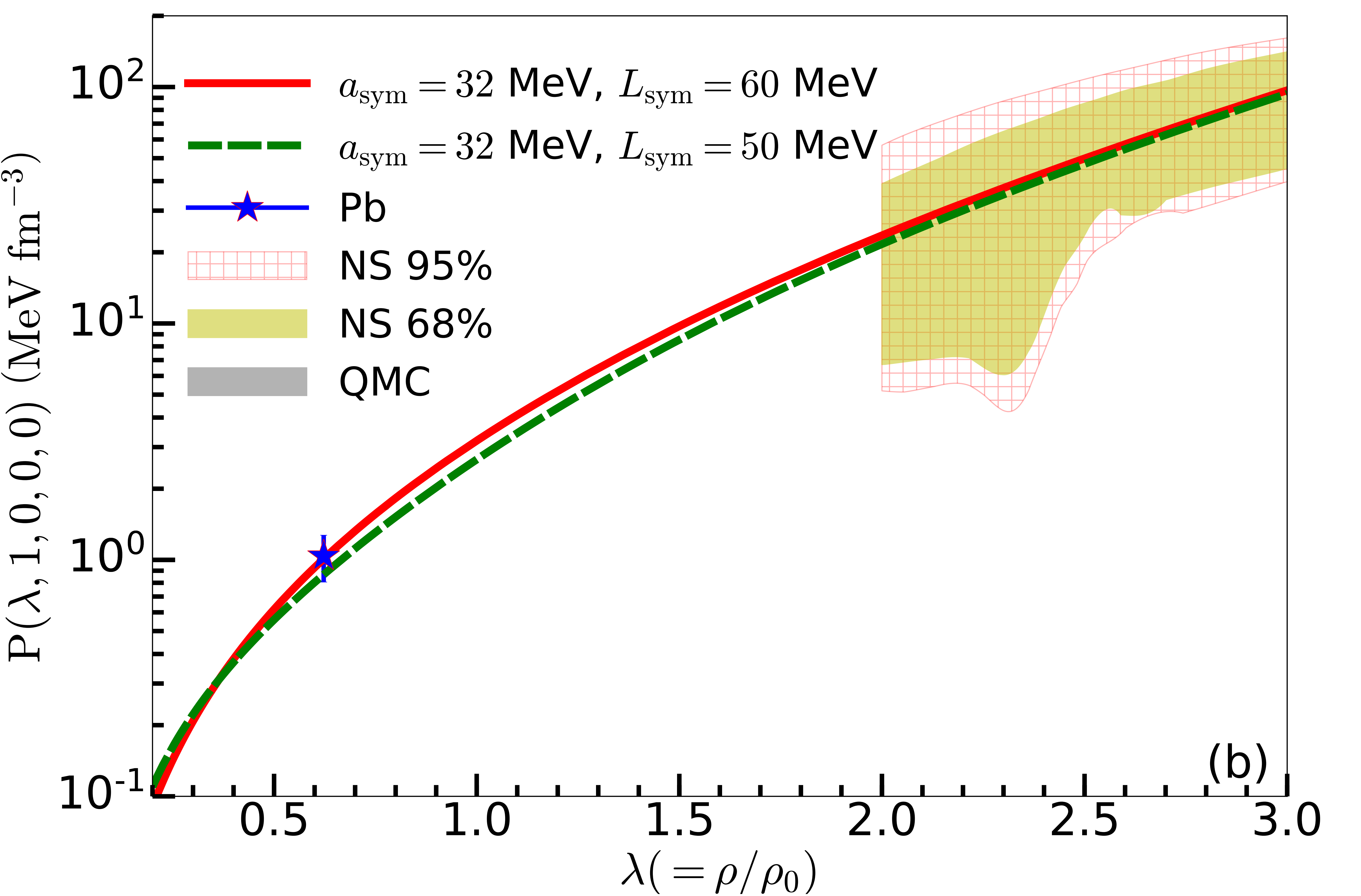
- The density-dependent functions and parameters are determined using empirical data at normal nuclear density.

- The mass-radius relations of neutron star are in good agreement with the GW170817 measurements and Shapiro delay measurements.

Thank you very much



100
80
60
40
20
0
−20
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\delta = 0$
$a_{\rm sym} = 32$ MeV, $L_{\rm sym} = 60$ MeV($\delta = 1$)
$a_{\rm sym} = 32$ MeV, $L_{\rm sym} = 50$ MeV($\delta = 1$)
APR predictions($\delta = 1$)
APR predictions($\delta = 0$)
$\varepsilon(\lambda, \delta)$ (MeV)
$\lambda(= \rho/\rho_0)$



$a_{\rm sym} = 32$ MeV, $L_{\rm sym} = 60$ MeV
$a_{\rm sym} = 32$ MeV, $L_{\rm sym} = 50$ MeV
Pb
NS 95%
NS 68%
QMC
(b)
10^2
10^1
10^0
10^{-1}
0.5
1.0
1.5
2.0
2.5
3.0
$\mathrm{P}(\lambda, 1, 0, 0, 0)$ $(\mathrm{MeV\ fm^{-3}})$
$\lambda (= \rho/\rho_0)$