Inha University, Republic of Korea
&
National University of Uzbekistan

Compact Stars in a Meson Mean Field Approach

Ulugbek Yakhshiev

Talk @ 2nd CENuM Workshop in Hadron Physics
December 18-19, 2023, Incheon

Collaborators

Hyun-Chul Kim, Inha University
Namyoung Kim, Inha University
Ghil-Seok Yang, Hoseo University

Content

- Prehistory
- Medium modifications
- Baryons in nuclear matter
- Nuclear matter
- Compact stars

Prehistory: Strategy and Motivation

How to construct a theoretical framework (model of ``nuclear physics")?

Our guiding principles are

- **simplicity (easy to analyse, transparent, etc…) <=> e.g. a small number terms in the Lagrangian;**
- **relation to phenomenology in an attractive way — as much as possible the peculiarities of strong interactions should be taken into account using as less as possible the number of parameters;**
- **universality <=> applicability to**
  - **hadron structure and spectrum studies (from light to heavy sector);**
  - **analysis of NN interactions;**
  - **nuclear many body problems <=> nucleonic systems (finite nuclei) and nuclear matter properties (EOS);**
  - **relation to mesonic atoms;**
  - **hadron structure changes in nuclear environment;**
  - **extreme density phenomena (e.g. neutron stars);**
  - **etc.**

Two possible ways:

- **to construct completely new approach;**
- **a bit fresh look to old ideas (e.g. putting a bit more phenomenological information).**

Prehistory: Studies

The studies were performed and going on in direction of

a single baryon properties

- in separate state considering it as a structure-full system
- nucleon in the community of their partners (EM and EMT form factors)
- nucleon in finite nuclei
- hyperons in nuclear matter
- heavy particles in nuclear matter

as well as on the properties of the whole nucleonic systems

- infinite nuclear matter properties (volume and symmetry energy properties)
- matter under extreme conditions (e.g. neutron stars)
- matter with a strangeness
- neutron, proto-neutron, strange stars
- few/many nucleon systems (symmetric nuclei, mirror nuclei, rare isotopes, halo nuclei,…)
- nucleon knock-out reactions (lepton-nucleus scattering)
- possible changes in in-medium NN interactions
- etc

Prehistory: Possible ways of study

Two important phenomena in low energy region

- Quark confinement
- Chiral symmetry breaking

Two possible ways of development in chiral theories

- Topological approaches
- Non-topological approaches

Prehistory: Baryon

* A baryon can be viewed as a state of Nc quarks bound by mesonic **mean fields** (E. Witten, NPB, 1979 & 1983).

Its mass is proportional to *Nc*, while its width is of order *O(1)*.
→ Mesons are weakly interacting (Quantum fluctuations are suppressed by 1/Nc: *O(1/Nc)*.

Meson mean-field approach (Chiral Quark-Soliton Model)

* Baryons as a state of Nc quarks bound by mesonic mean fields.

$$S_{\text{eff}} = -N_c \text{Tr} \ln \left(-i\partial_\mu + S(\boldsymbol{r}) + P(\boldsymbol{r})i\gamma_5 + V_\mu(\boldsymbol{r})\gamma_\mu + A_\mu(\boldsymbol{r})\gamma_\mu\gamma_5 + T_{\mu\nu}(\boldsymbol{r})\sigma_\mu + i\hat{m}\right)$$

* Key point: Hedgehog Ansatz

$$\pi^a(\boldsymbol{r}) = \begin{cases} n^a F(r),\ n^a = x^a/r, & a = 1, 2, 3 \\ 0, & a = 4, 5, 6, 7, 8. \end{cases}$$

→ It breaks spontaneously $\text{SU}(3)_{\text{flavor}} \otimes \text{O}(3)_{\text{space}} \to \text{SU}(2)_{\text{isospin+space}}$
→ 12 profile functions are only allowed.

Diakonov, Petrov, Vladimirov, PRD 88, 074030 (2013)

*This slide is obtained from H.C.Kim's presentation, which is available in the internet.

Prehistory: Baryon

Collective Hamiltonian

$$
\begin{aligned}
H =& M_{\rm cl} + \frac{1}{2I_1}\sum_{i=1}^{3}\hat{J}_i^2 + \frac{1}{2I_2}\sum_{p=4}^{7}\hat{J}_p^2 \\
&+ (m_{\rm d} - m_{\rm u})\left(\frac{\sqrt{3}}{2}\alpha D_{38}^{(8)}(\mathcal{A}) + \beta\hat{T}_3 + \frac{1}{2}\gamma\sum_{i=1}^{3} D_{3i}^{(8)}(\mathcal{A})\hat{J}_i\right) \\
&+ (m_{\rm s} - \bar{m})\left(\alpha D_{88}^{(8)}(\mathcal{A}) + \beta\hat{Y} + \frac{1}{\sqrt{3}}\gamma\sum_{i=1}^{3} D_{8i}^{(8)}(\mathcal{A})\hat{J}_i\right) + H_{\rm em}
\end{aligned}
$$

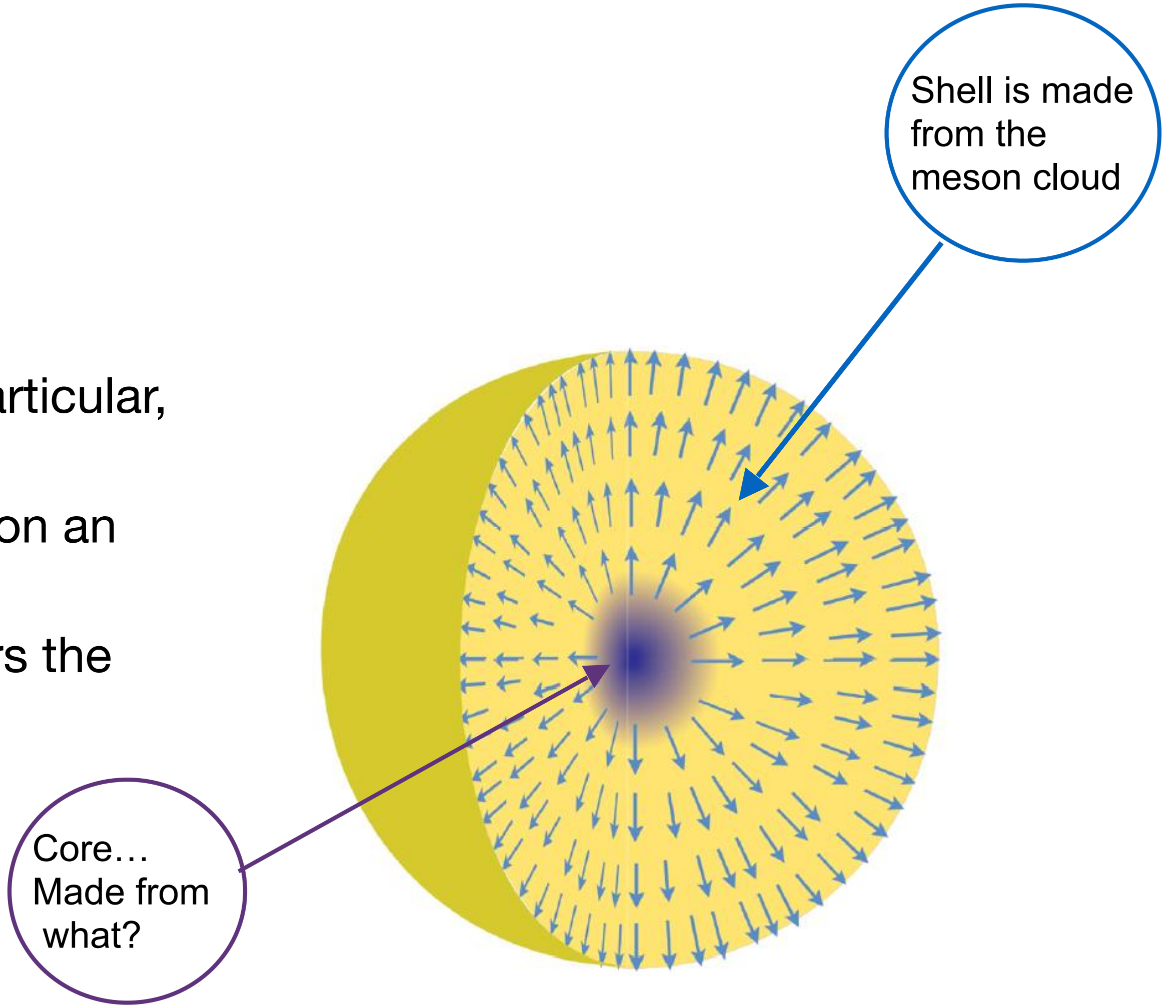
$$
\alpha = -\left(\frac{2}{3}\frac{\Sigma_{\pi N}}{m_{\rm u}+m_{\rm d}} - \frac{K_2}{I_2}\right) \qquad \beta = -\frac{K_2}{I_2} \qquad \gamma = 2\left(\frac{K_1}{I_1} - \frac{K_2}{I_2}\right)
$$

*For more details see the presentations of G.S.Yang in this conference.

Structure

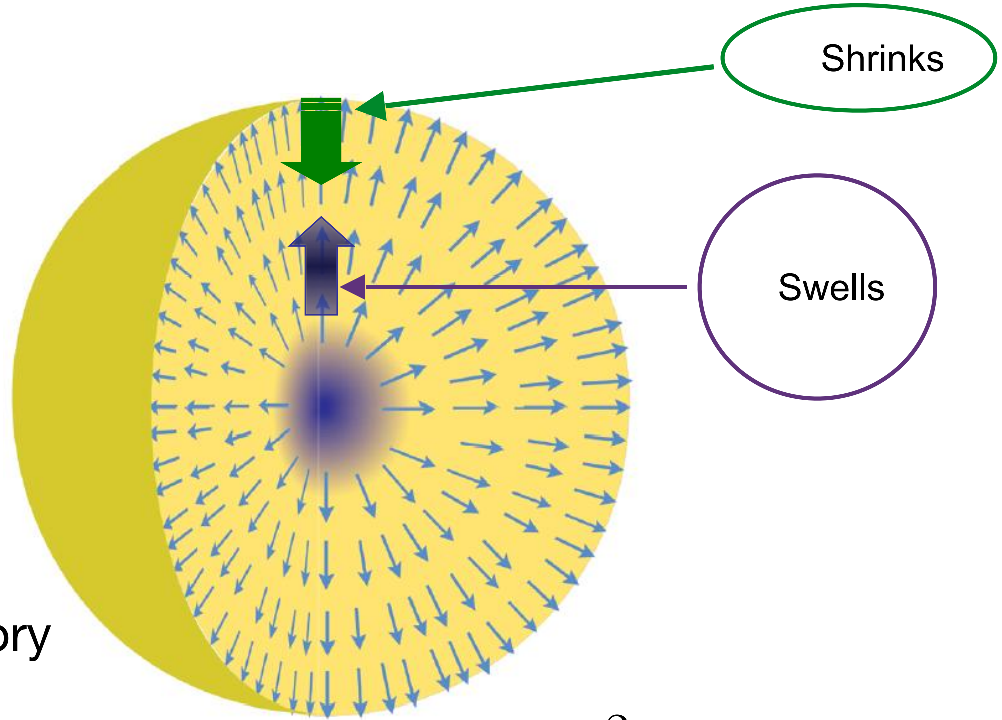
From what made a nucleon and, in particular, its core?

- The structure treatment depends on an energy scale
- At the limit of large number colours the core still has the mesonic content



Stabilization mechanism

- Soliton has the finite size and the finite energy
- One needs at least two counter terms in the effective (mesonic) Lagrangian



Prototype: **Skyrme model**

[T.H.R. Skyrme, Pros.Roy.Soc.Lond. A260 (1961)]

- Nonlinear chiral effective meson (pionic) theory

$$\mathcal{L} = \frac{F_\pi^2}{16}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{1}{16e^2}\,\mathrm{Tr}\,\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U\right]^2$$

Shrinking term — Swelling term

- *Hedgehog* solution (nontrivial mapping)

$$U = \exp\left\{\frac{i\bar{\tau}\,\bar{\pi}}{2F_\pi}\right\} = \exp\{i\bar{\tau}\,\bar{n}F(r)\}$$

What happens in the nuclear medium?

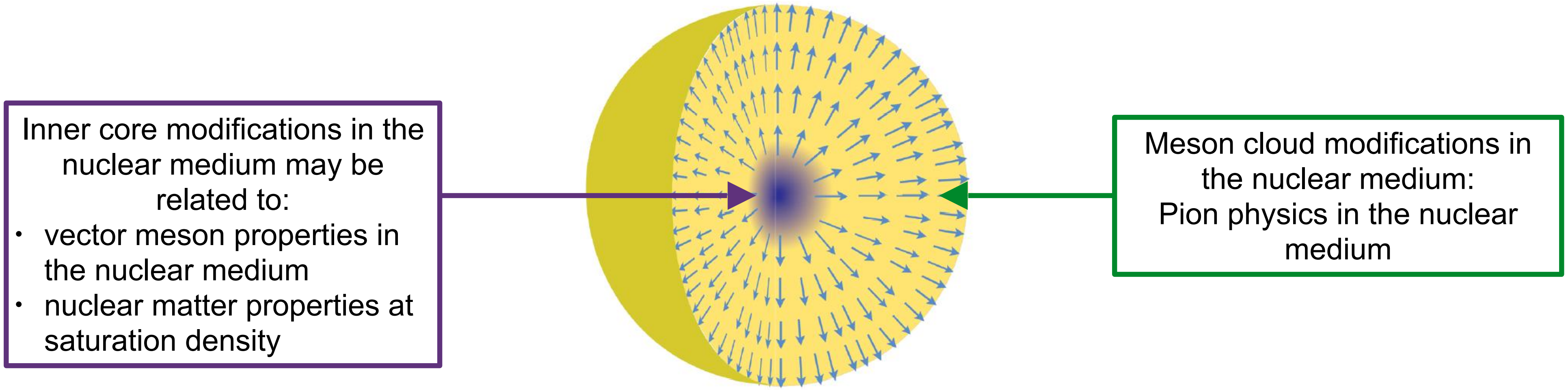
The possible medium effects

- Deformations (swelling or shrinking, multipole deformations) of nucleons
- Characteristic changes in: effective mass, charge distributions, all possible form factors
- NN interactions may change
- etc.

One should be able to describe all those phenomena

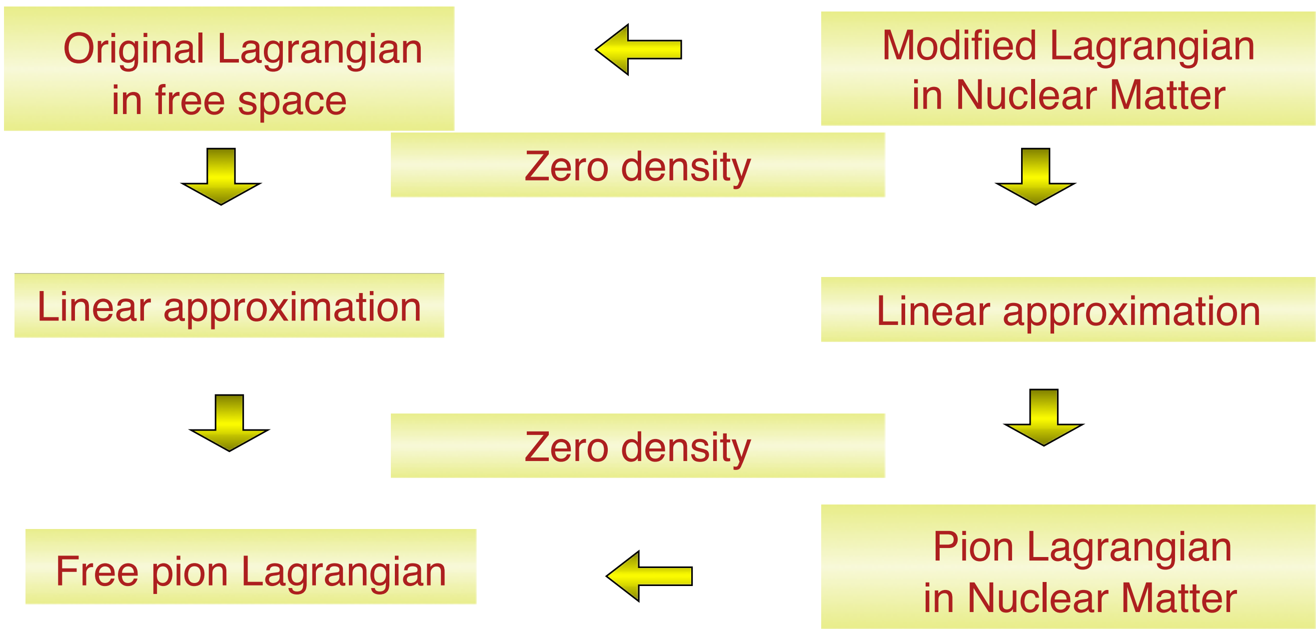
Soliton in the nuclear medium (phenomenological way)

- Outer shell modifications (informations from pionic atoms)
- Inner core modifications, in particular, at large densities (nuclear matter properties)



Medium modifications

- Modifications of the mesonic sector modifies the baryonic sector
- Lagrangian satisfies some limiting conditions



Medium modifications

"Outer shell" modifications

- In free space three types of pions can be treated separately: isospin breaking

$$(\partial^\mu \partial_\mu + m_\pi^2)\vec{\pi}^{(\pm,0)} = 0$$

$$(\partial^\mu \partial_\mu + m_\pi^2 + \hat{\Pi}^{(\pm,0)})\vec{\pi}^{(\pm,0)} = 0$$

- In nuclear matter: three types of polarization operators

$$\hat{\Pi}^0 = 2\omega U_{\rm opt} = \chi_s(\rho, b_0, c_0) + \vec{\nabla} \cdot \chi_p(\rho, b_0, c_0)\vec{\nabla}$$

$$\hat{\Pi}^0 = (\hat{\Pi}^- + \hat{\Pi}^+)/2, \qquad \hat{\Delta}\Pi = (\hat{\Pi}^- - \hat{\Pi}^+)/2$$

- Optic potential approach: parameters from the pion-nucleon scattering (including the isospin dependents)

	π-atom	$T_\pi = 50$ MeV
$b_0\ [m_\pi^{-1}]$	- 0.03	- 0.04
$b_1\ [m_\pi^{-1}]$	- 0.09	- 0.09
$c_0\ [m_\pi^{-3}]$	0.23	0.25
$c_1\ [m_\pi^{-3}]$	0.15	0.16
g'	0.47	0.47

Medium modifications

“Outer shell” modifications in the Lagrangian [U.Meissner *et al.*, EPJ A36 (2008)]

$$\mathcal{L}_2^* = \frac{F_\pi^2}{16}\alpha_\tau \mathrm{Tr}\left(\partial_0 U \partial_0 U^\dagger\right) - \frac{F_\pi^2}{16}\alpha_s \mathrm{Tr}\left(\partial_i U \partial_i U^\dagger\right)$$

$$\mathcal{L}_m^* = -\frac{F_\pi^2 m_\pi^2}{16}\alpha_m \mathrm{Tr}\left(2 - U - U^\dagger\right)$$

$$\hat{\Pi}^0 = 2\omega U_{\mathrm{opt}} = \chi_s(\rho, b_0, c_0) + \vec{\nabla}\cdot\chi_p(\rho, b_0, c_0)\vec{\nabla}$$

- Due to the nonlocality of optic potential the kinetic term is also modified
- Due to energy and momentum dependence of the optic potential parameters the following parts of the kinetic term are modified in different forms:
  - Temporal part
  - Space part

	π-atom	$T_\pi = 50$ MeV
$b_0\ [m_\pi^{-1}]$	- 0.03	- 0.04
$b_1\ [m_\pi^{-1}]$	- 0.09	- 0.09
$c_0\ [m_\pi^{-3}]$	0.23	0.25
$c_1\ [m_\pi^{-3}]$	0.15	0.16
g'	0.47	0.47

Medium modifications

“Inner core” modifications

[UY & H.Ch. Kim, PRC83 (2011); UY, PRC88 (2013)]

$$\mathcal{L}_4^* = -\frac{1}{16e^2\zeta_\tau}\mathrm{Tr}\left[U^\dagger\partial_0 U, U^\dagger\partial_i U\right]^2 + \frac{1}{32e^2\zeta_s}\mathrm{Tr}\left[U^\dagger\partial_i U, U^\dagger\partial_j U\right]^2$$

may be related to

- Vector meson properties in nuclear matter
- Nuclear matter properties

$$\zeta_{\tau,s} = \zeta_{\tau,s}(\rho, \delta\rho, \text{parameters})$$

Medium modifications

Final Lagrangian
[UY, JKPS62 (2013); UY, PRC88 (2013)]

Separated into two parts

$$\mathcal{L}^* = \mathcal{L}^*_{\rm sym} + \mathcal{L}^*_{\rm asym}$$

- Isoscalar part

$$\mathcal{L}^*_{\rm sym} = \mathcal{L}^*_2 + \mathcal{L}^*_4 + \mathcal{L}^*_m$$

- Isovector part

$$\mathcal{L}^*_{\rm asym} = \mathcal{L}^*_{\delta m} + \mathcal{L}^*_{\delta\rho}$$

- **Nuclear matter stabilization**
- **Asymmetric matter properties**

$$\mathcal{L}^*_2 = \frac{F_\pi^2}{16}\alpha_\tau {\rm Tr}\left(\partial_0 U \partial_0 U^\dagger\right) - \frac{F_\pi^2}{16}\alpha_s {\rm Tr}\left(\partial_i U \partial_i U^\dagger\right)$$

$$\mathcal{L}^*_4 = -\frac{1}{16e^2\zeta_\tau}{\rm Tr}\left[U^\dagger\partial_0 U, U^\dagger \partial_i U\right]^2 + \frac{1}{32e^2\zeta_s}{\rm Tr}\left[U^\dagger\partial_i U, U^\dagger\partial_j U\right]^2$$

$$\mathcal{L}^*_m = -\frac{F_\pi^2 m_\pi^2}{16}\alpha_m {\rm Tr}\left(2 - U - U^+\right)$$

$$\mathcal{L}^*_{\delta m} = -\frac{F_\pi^2}{32}\sum_{a=1}^{2}(m_{\pi^\pm}^2 - m_{\pi^0}^2){\rm Tr}\left(\tau_a U\right){\rm Tr}\left(\tau_a U^\dagger\right)$$

$$\mathcal{L}^*_{\delta\rho} = -\frac{F_\pi^2}{16} m_\pi \alpha_e\ \varepsilon_{ab3}{\rm Tr}\left(\tau_a U\right){\rm Tr}\left(\tau_b \partial_0 U^\dagger\right)$$

Reparametrization
[UY, PRC88 (2013)]

- Five density dependent parameters

Shell modifications

$$F_{\pi,\tau} \to F^*_{\pi,\tau}, \quad F_{\pi,s} \to F^*_{\pi,s},$$

Core modifications

$$e_\tau \to e^*_\tau, \quad e_s \to e^*_s$$

$$m_\pi \to m^*_\pi,$$

- Rearrangment (technical simplification to describe nuclear matter)

$$+C_1 \frac{\rho}{\rho_0} = f_1\left(\frac{\rho}{\rho_0}\right) \equiv \sqrt{\frac{\alpha_p^0}{\gamma_s}}$$

$$+C_2 \frac{\rho}{\rho_0} = f_2\left(\frac{\rho}{\rho_0}\right) \equiv \frac{\alpha_s^{00}}{(\alpha_p^0)^2 \gamma_s}$$

$$+C_3 \frac{\rho}{\rho_0} = f_3\left(\frac{\rho}{\rho_0}\right) \equiv \frac{(\alpha_p^0 \gamma_s)^{3/2}}{\alpha_s^{02}}$$

$$\frac{\alpha_e}{\gamma_s} = f_4\left(\frac{\rho}{\rho_0}\right)\frac{\rho_n - \rho_p}{\rho_0} = \frac{C_4 \frac{\rho}{\rho_0}}{1 + C_5 \frac{\rho}{\rho_0}} \frac{\rho_n - \rho_p}{\rho_0}$$

Nuclear matter

From the Bethe-Weizsacker formula

$$\varepsilon\,(A,Z) = -a_V + a_S \frac{(N-Z)^2}{A^2} + \text{[illegible]}$$

The binding-energy-formula terms in the framework of present model can be obtained considering

We reproduced (Volume term, Asymmetry term)

- Volume term
  - Symmetric infinite nuclear matter
- Asymmetry term
  - Isospin asymmetric environment
- Surface and Coulomb terms
  - Nucleons in a finite volume
- Finite nuclei properties
  - Local density approximation

Nuclear matter

The volume term and Symmetry energy

- At infinite nuclear matter approximation the binding energy per nucleon takes the form

$$\varepsilon(\lambda,\delta) = \varepsilon_V(\lambda) + \varepsilon_S\delta^2 + O(\delta^4) \equiv \varepsilon_V(\lambda) + \varepsilon_A(\lambda,\delta)$$

  - λ is normalised nuclear matter density
  - δ is asymmetry parameter
  - ε_S is symmetry energy

- In our model
  - Symmetric matter
  - Asymmetric matter

$$\varepsilon_V(\lambda) = m^*_{N,s}(\lambda,0) - m^{\text{free}}_N$$

$$\begin{aligned}\varepsilon_A(\lambda,\delta) &= \varepsilon(\lambda,\delta) - \varepsilon_V(\lambda) \\ &= m^*_{N,s}(\lambda,\delta) - m^*_{N,s}(\lambda,0) + m^*_{N,V}(\lambda,\delta)\delta\end{aligned}$$

Nuclear matter properties

- Symmetric matter properties (pressure, compressibility and third derivative)

$$p = \rho_0 \lambda^2 \left. \frac{\partial \varepsilon_V(\lambda)}{\partial \lambda} \right|_{\lambda=1}, \quad K_0 = 9\rho^2 \left. \frac{\partial^2 \varepsilon_V(\lambda)}{\partial \rho^2} \right|_{\rho=\rho_0} \quad Q = 27\lambda^3 \left. \frac{\partial^3 \varepsilon_V(\lambda)}{\partial \lambda^3} \right|_{\lambda=1}$$

- Symmetry energy properties (coefficient, slop and curvature)

$$\varepsilon_s(\lambda) = \varepsilon_s(1) + \frac{L_s}{3}(\lambda - 1) + \frac{K_S}{18}(\lambda - 1)^2 + \cdots$$

Nuclear matter

The binding-energy-formula in a more general case

$$\varepsilon = \frac{E^* - E}{A} = \frac{Z\Delta M_p + N\Delta M_n + \sum_{s=1}^{3} N_s \Delta M_s}{A}$$

$$= \Delta M_N \left(1 - \sum_{s=1}^{3} \delta_s\right) + \frac{1}{2}\delta\Delta M_{np} + \sum_{s=1}^{3} \delta_s \Delta M_s$$

$$M_{np} = M_n - M_p$$

$$\Delta M_N = M_N^* - M_N$$

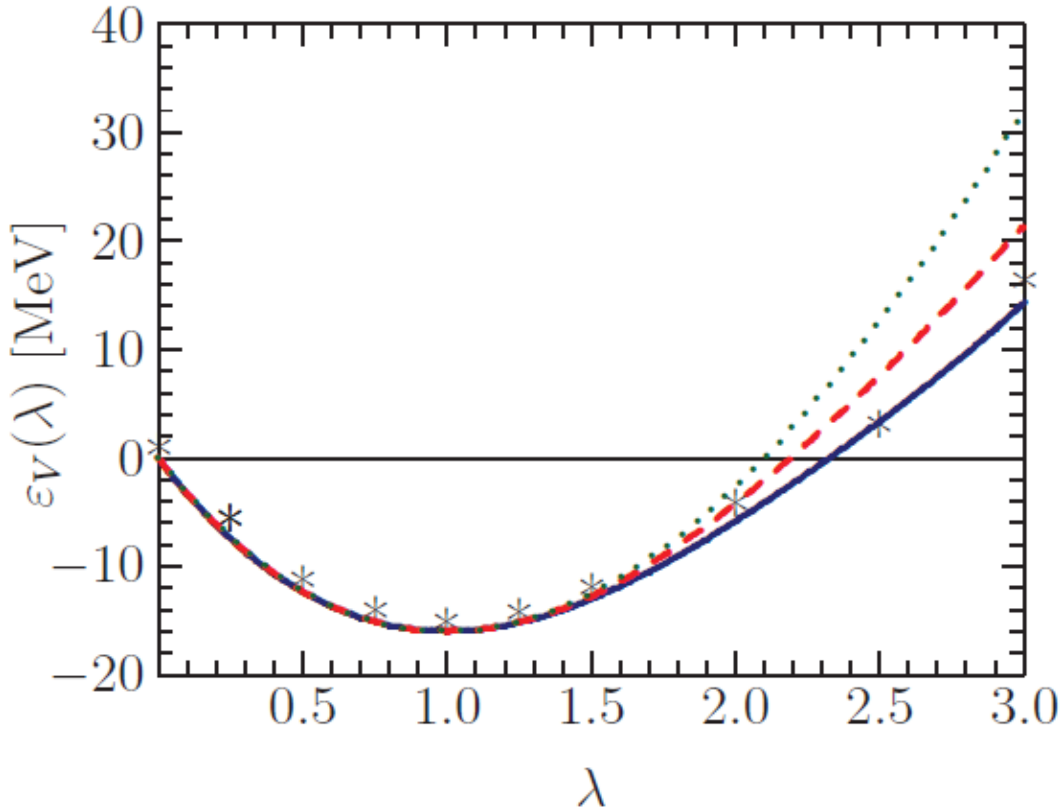
$$\delta = \frac{N - Z}{A}$$

Nuclear matter

Volume energy
[UY, PRC88 (2013)]

- Set I – solid
- Set II – dashed
- Set III – dotted

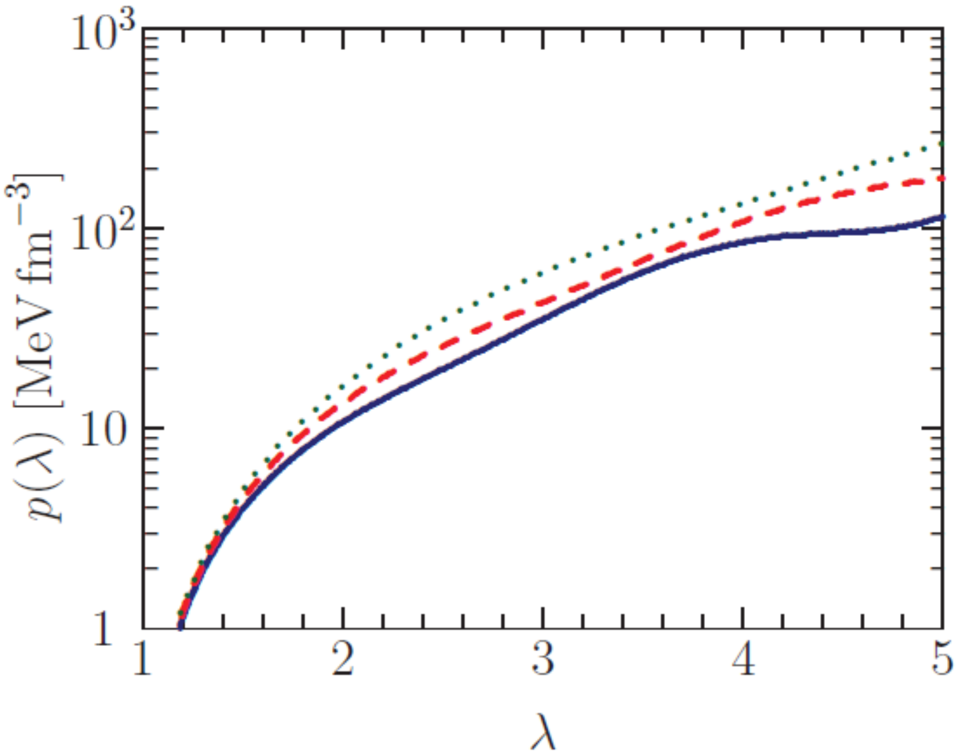
For comparison: Akmal-Pandharipande-Ravenhall (APR) predictions
[PRC 58, 1804 (1998)]
are given by stars.
(From Arigonna 2 body interactions + 3 body interactions)

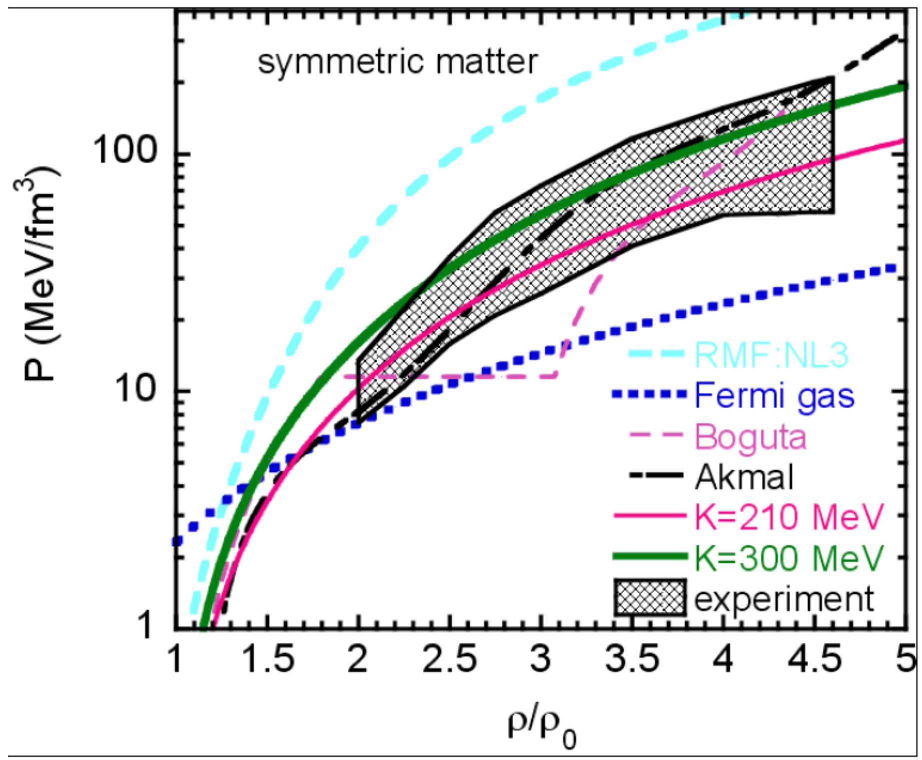


Set	C_1	C_2	C_3	$\varepsilon_V(\rho_0)$ (MeV)	K_0 (MeV)	Q (MeV)
I	−0.279	0.737	1.782	−16	240	−410
II	−0.273	0.643	1.858	−16	250	−279
III	−0.277	0.486	2.124	−16	260	−178

Pressure

[UY, PRC88 (2013)]





For comparison: Right figure from

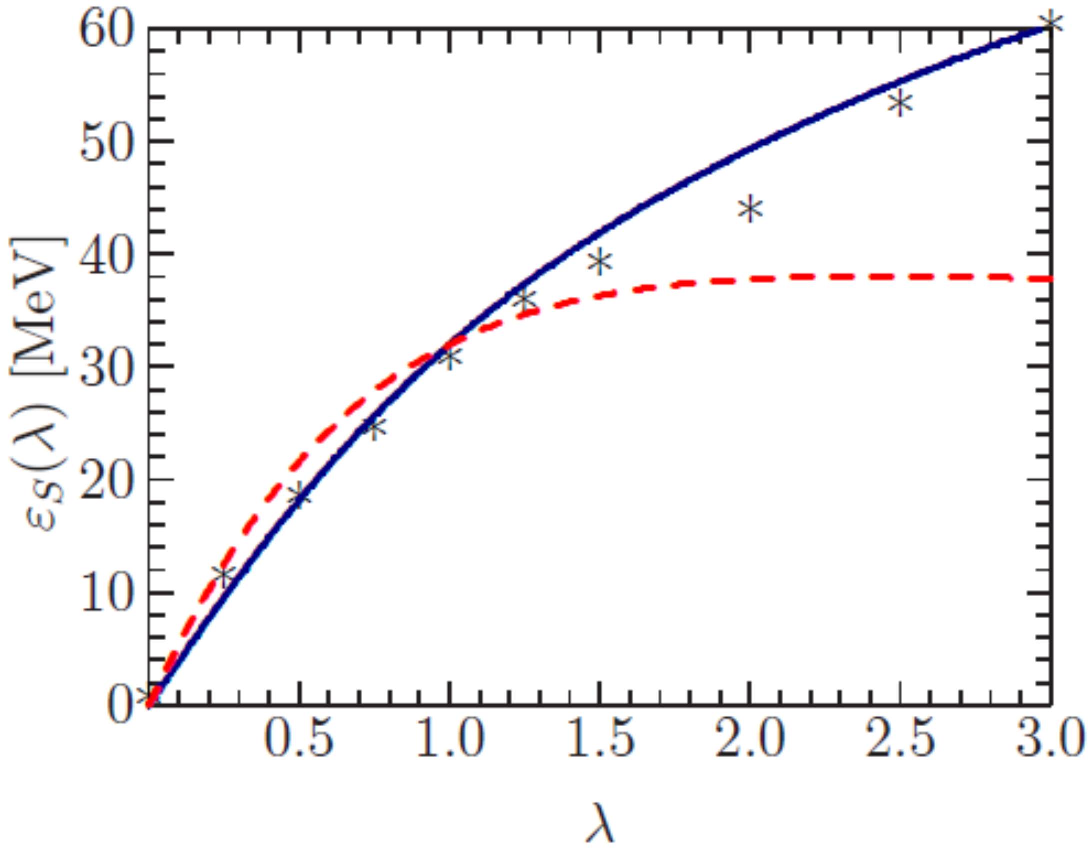
Danielewicz- Lacey-Lynch, Science 298, 1592 (2002).

(Deduced from experimental flow data and simulations studies)

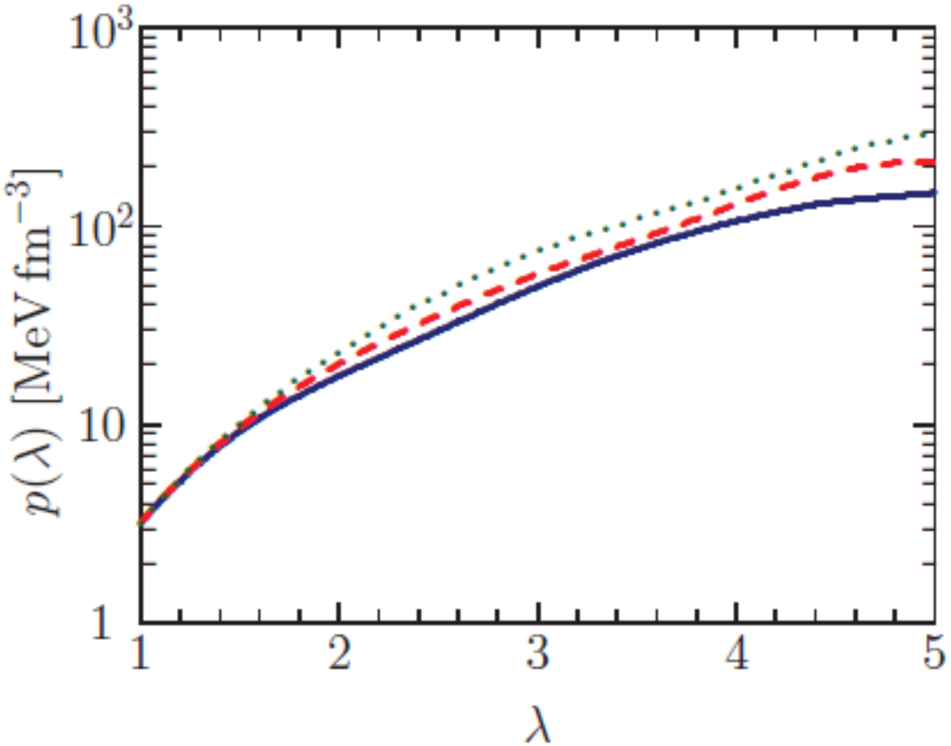
Symmetry energy

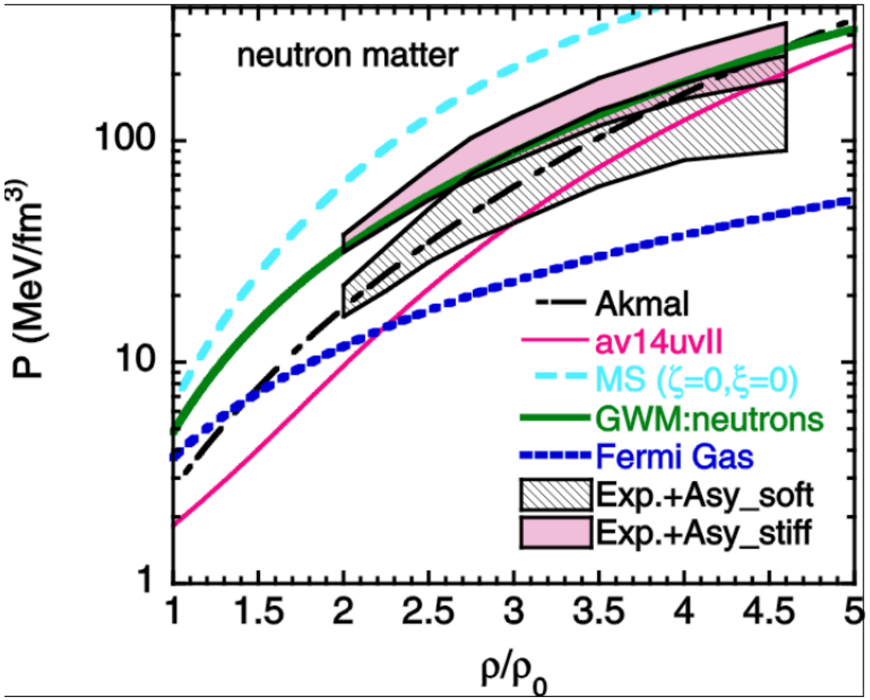
- Solid $L_s = 70\,\mathrm{MeV}$
- Dashed $L_s = 40\,\mathrm{MeV}$

For comparison: Akmal-Pandharipande-Ravenhall (APR) predictions [PRC 58, 1804 (1998)] are given by stars.
(From arigonna 2 body interactions + 3 body interactions)



Pressure in neutron matter
[UY, PRC88 (2013)]





For comparison: Right figure from

Danielewicz- Lacey-Lynch, Science 298, 1592 (2002).

(Deduced from experimental flow data and simulations studies)

Low density behaviour of symmetry energy

For comparison:
Trippa-Colo-Vigezzi
[PRC 77, 061304 (2008)];
From analysis of GDR
(208Pb).

$$23.3 < \varepsilon_s(\rho = 0.1\mathrm{fm}^{-3}) < 24.9\ \mathrm{MeV}$$

Consequently one can predict in this model:

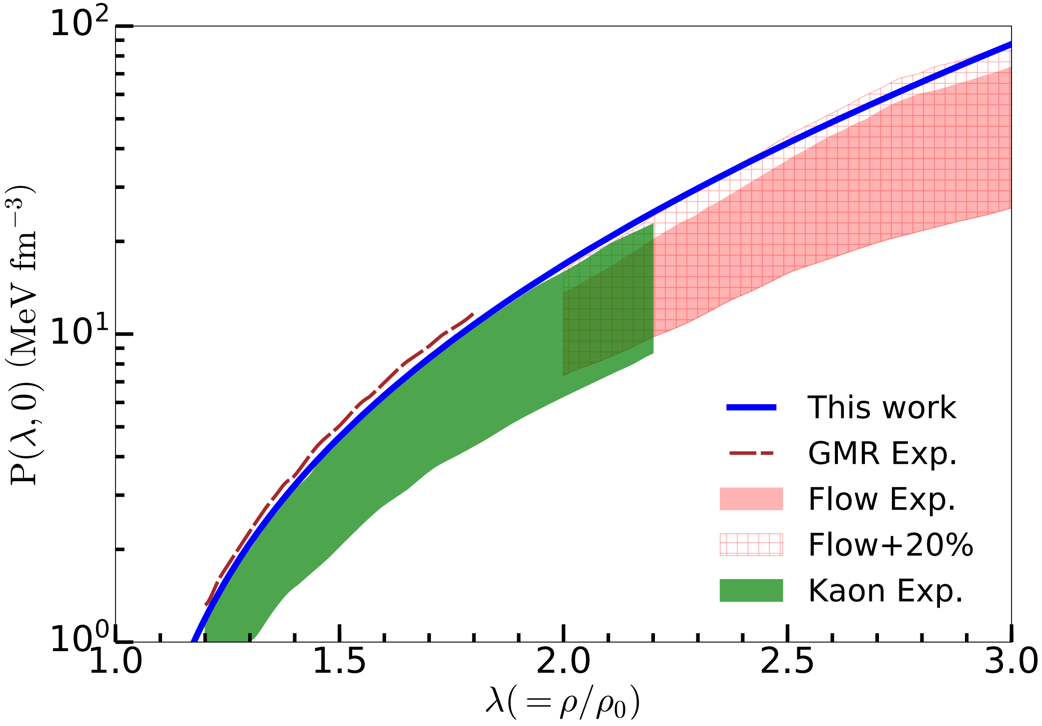
$$K_\tau = K_s - 6L_s$$

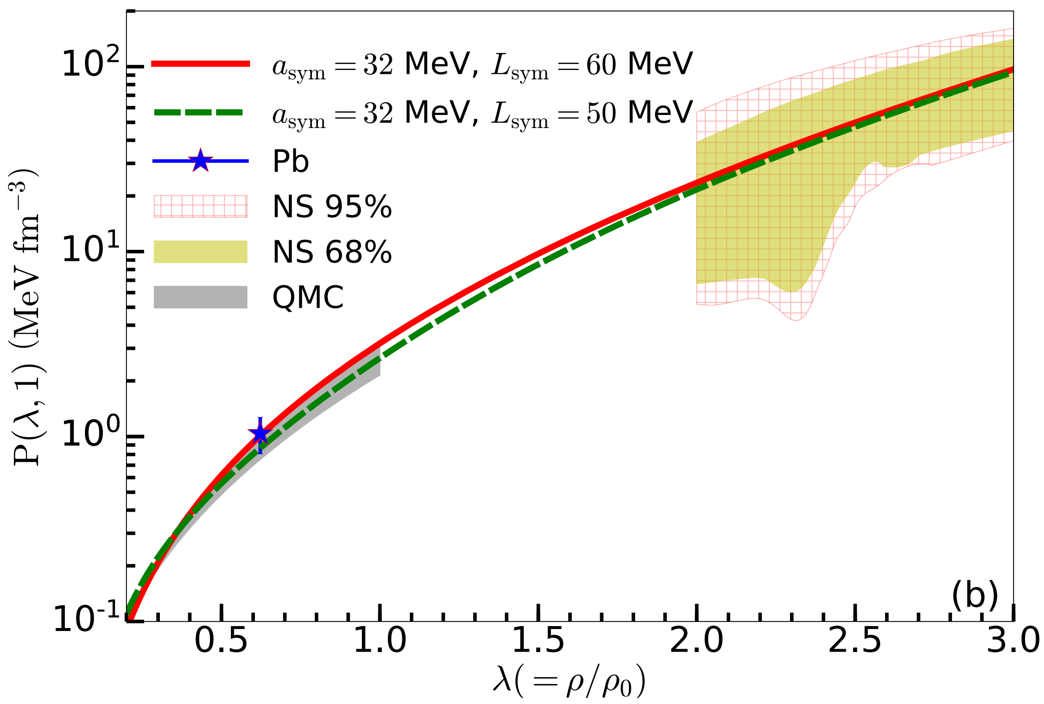
$$K_{0,2} = K_\tau - \frac{Q}{K_0} L_S$$

$\varepsilon_S(\rho_0)$ [MeV]	L_S [MeV]	K_S [MeV]	K_τ [MeV]	$K_{0,2}$ [MeV]	$\varepsilon_S(0.1\mathrm{fm}^{-3})$ [MeV]
32	40	−181	−301	−257	25.15
32	50	−160	−310	−254	24.15
32	60	−126	−306	−239	23.22
32	70	−80	−290	−211	22.37
32	80	−21	−261	−172	21.57
32	90	50	−220	−119	20.82
32	100	134	−166	−55	20.13

Nuclear matter
(SU(3) model independent approach with hyperons)

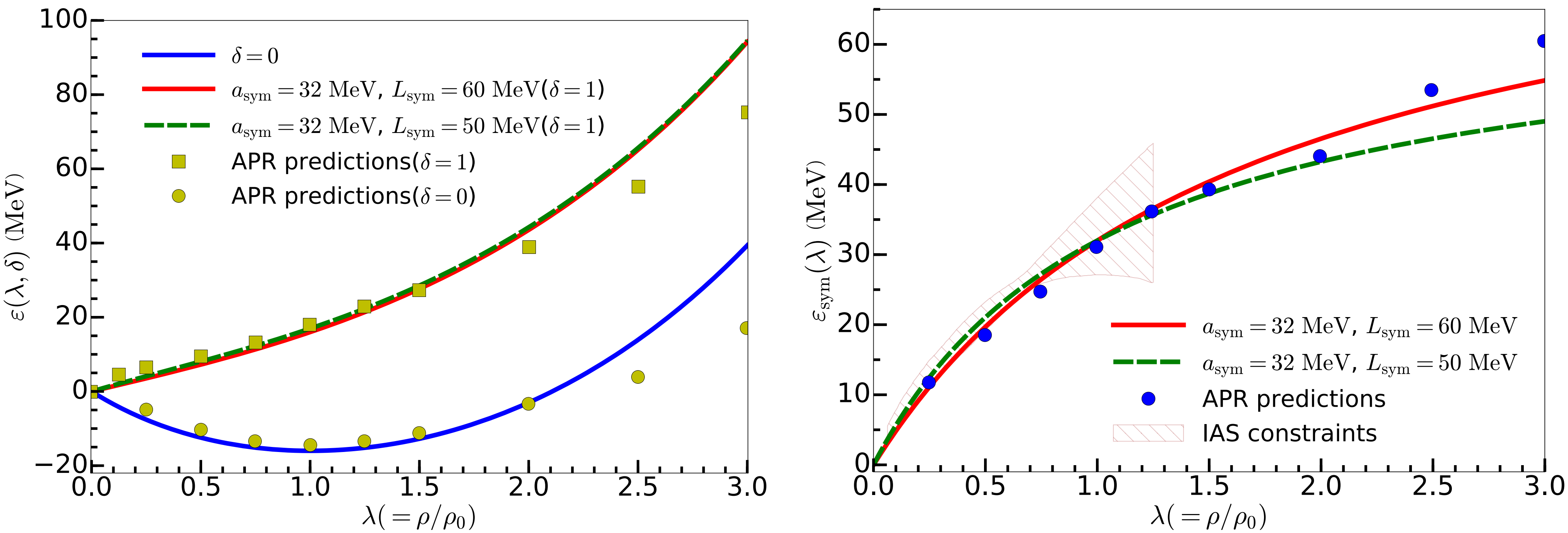
Pressure [N.Y.Ghim, G.S.Yang, H.Ch.Kim, UY, PRC103 (2021)]





Nuclear matter
(SU(3) model independent approach with hyperons)

Volume and symmetry energy [N.Y.Ghim, G.S.Yang, H.Ch.Kim, UY, PRC103 (2021)]



Neutron star properties

- TOV equations

$$-\frac{dP(r)}{dr} = \frac{G\mathcal{E}(r)\mathcal{M}(r)}{r^2}\left(1-\frac{2G\mathcal{M}(r)}{r}\right)^{-1}\left(1+\frac{P(r)}{\mathcal{E}(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{\mathcal{M}(r)}\right)$$

- Energy-pressure relation

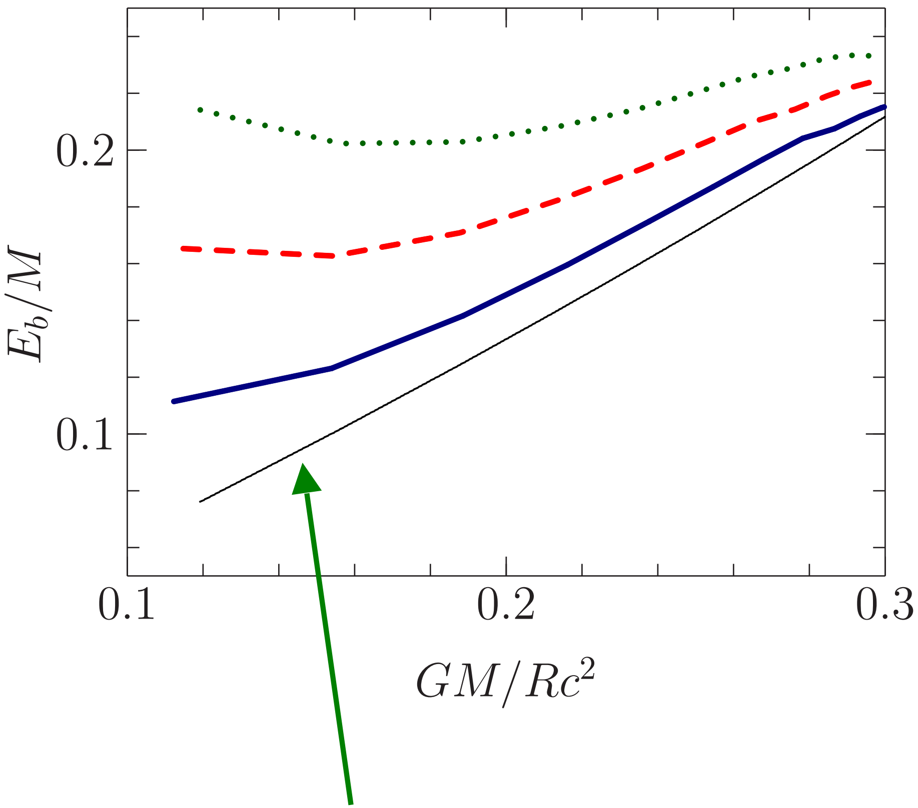
$$P = P(\mathcal{E})$$

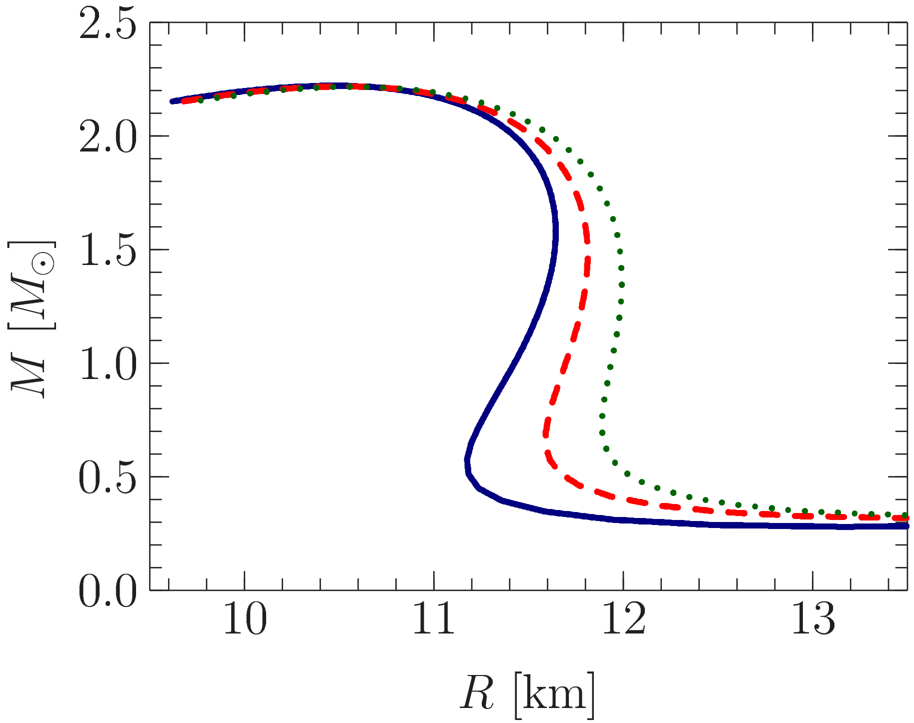
$$\begin{aligned} P(\lambda) &= \rho_0\lambda^2\frac{\partial\varepsilon(\lambda,1)}{\partial\lambda}, \\ \mathcal{E}(\lambda) &= [\varepsilon(\lambda,1)+m_N]\lambda\rho_0. \end{aligned}$$

- Neutron star's mass

$$\mathcal{M}(r) = 4\pi\int_0^r \mathrm{d}r\, r^2\mathcal{E}(r)\,.$$

Neutron star properties [UY, PLB749 (2015)]





From Ref. [J.M. Lattimer & M. Prakash, Astrophys. J. 550 (2001)].

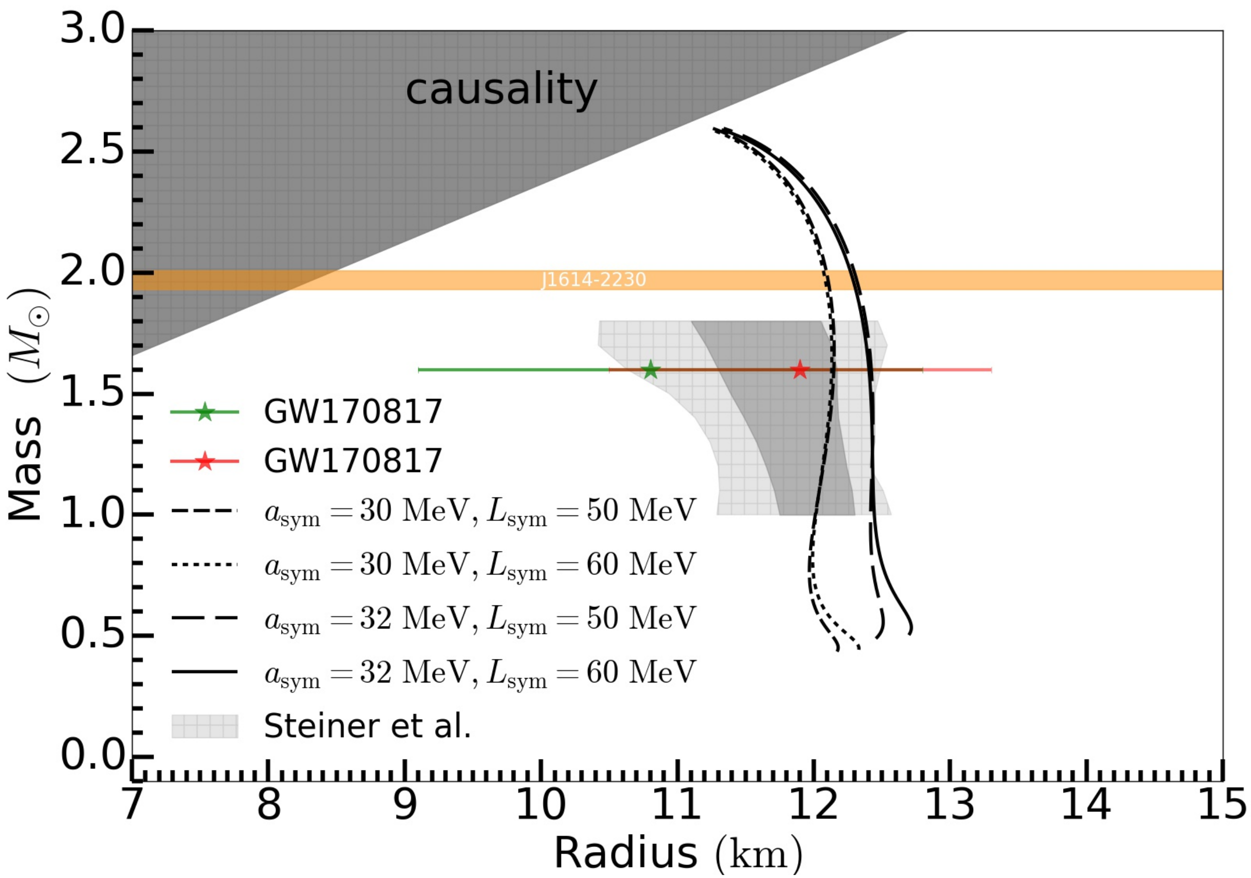
Neutron star properties [UY, PLB749 (2015)]

TABLE III: Properties of the neutron stars from the different sets of parameters (see Tables I and II for the values of parameters): n_c is central number density, ρ_c is central energy-mass density, R is radius of the neutron star, $M_{\max}$ is possible maximal mass, A is number of baryons in the star, E_b is binding energy of the star. In the left panel we represent the neutron star properties corresponding to the maximal mass $M_{\max}$ and in right panel approximately 1.4 solar mass neutron star properties. The last two lines are results from the Ref. [21].

Set	n_c [fm^{-3}]	ρ_c [10^{15}g/cm^3]	R [km]	$M_{\max}$ [$M_\odot$]	A [10^{57}]	E_b [10^{53}erg]	n_c [fm^{-3}]	ρ_c [10^{15}g/cm^3]	R [km]	M [$M_\odot$]	A [10^{57}]	E_b [10^{53}erg]
III-a	1.046	2.445	10.498	2.226	3.227	8.721	0.479	0.861	11.587	1.402	1.898	3.503
III-b	1.045	2.444	10.547	2.223	3.216	8.557	0.471	0.861	11.772	1.402	1.895	3.453
III-c	1.037	2.424	10.616	2.221	3.200	8.397	0.460	0.832	11.953	1.402	1.887	3.339
III-d	1.047	2.452	10.494	2.221	3.213	8.598	0.481	0.867	11.619	1.402	1.893	3.422
III-e	1.044	2.440	10.554	2.218	3.203	8.495	0.473	0.858	11.809	1.403	1.890	3.384
III-f	1.040	2.433	10.609	2.216	3.189	8.311	0.464	0.842	11.992	1.403	1.887	3.334
SLy230a [21]	1.15	2.69	10.25	2.10	2.99	7.07	0.508	0.925	11.8	1.4	1.85	2.60
SLy230b [21]	1.21	2.85	9.99	2.05	2.91	6.79	0.538	0.985	11.7	1.4	1.85	2.61

Compact stars

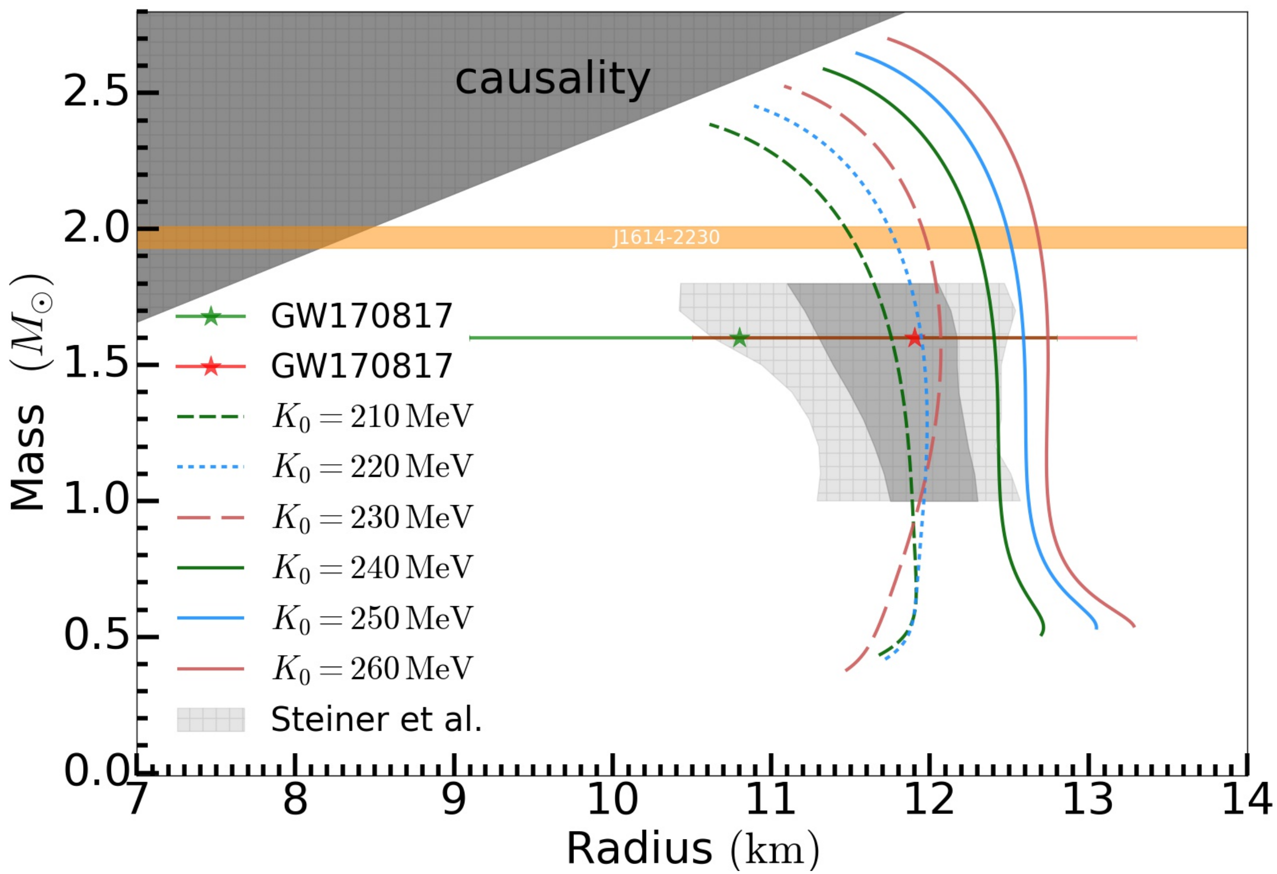
- Pure neutron matter ($K_0 = 240$ MeV).



*N.Y.Ghim, G.S.Yang, H.Ch.Kim, UY, In preparation.

Compact stars

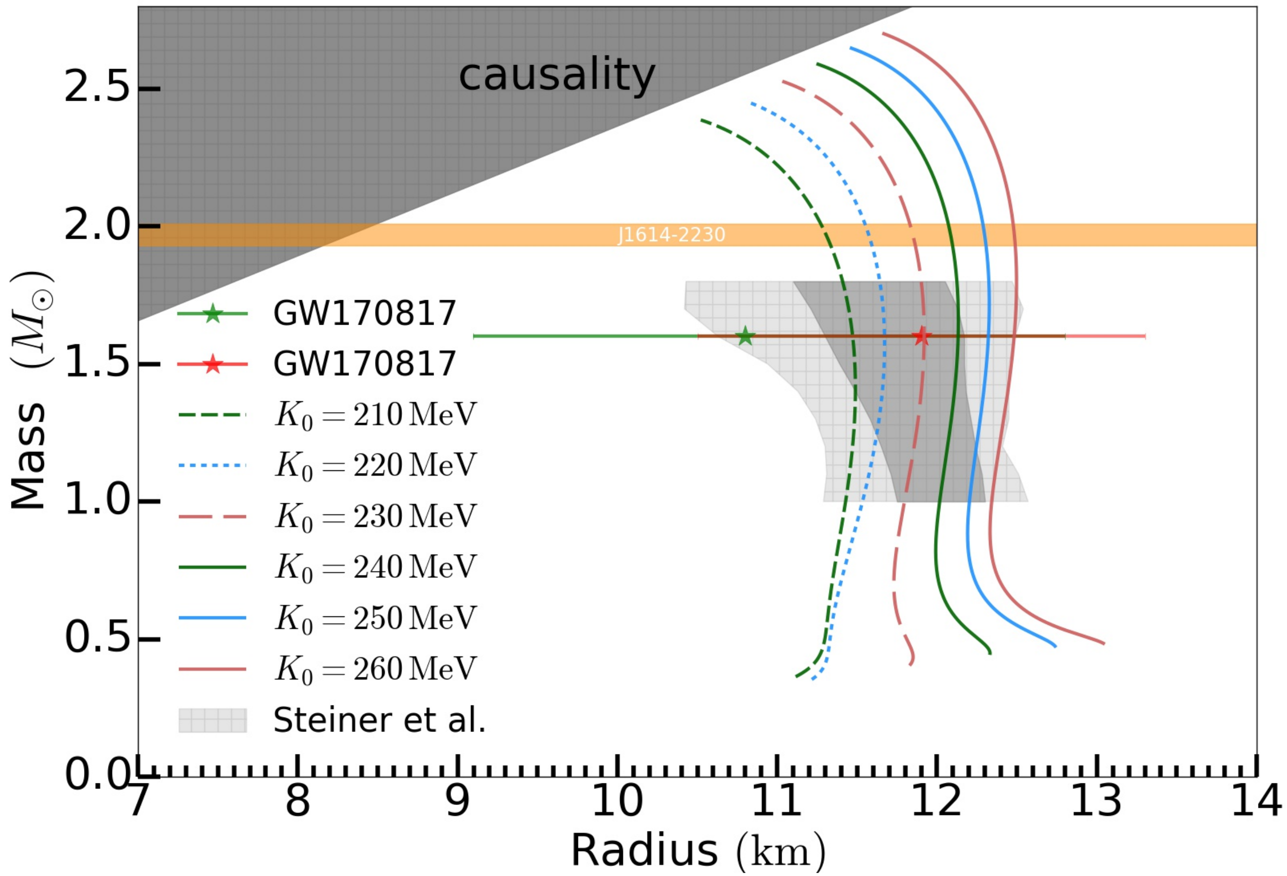
- Pure neutron matter ($a_{\rm sym} = 32$ MeV, $L_{\rm sym} = 60$ MeV)



*N.Y.Ghim, G.S.Yang, H.Ch.Kim, UY, In preparation.

Compact stars

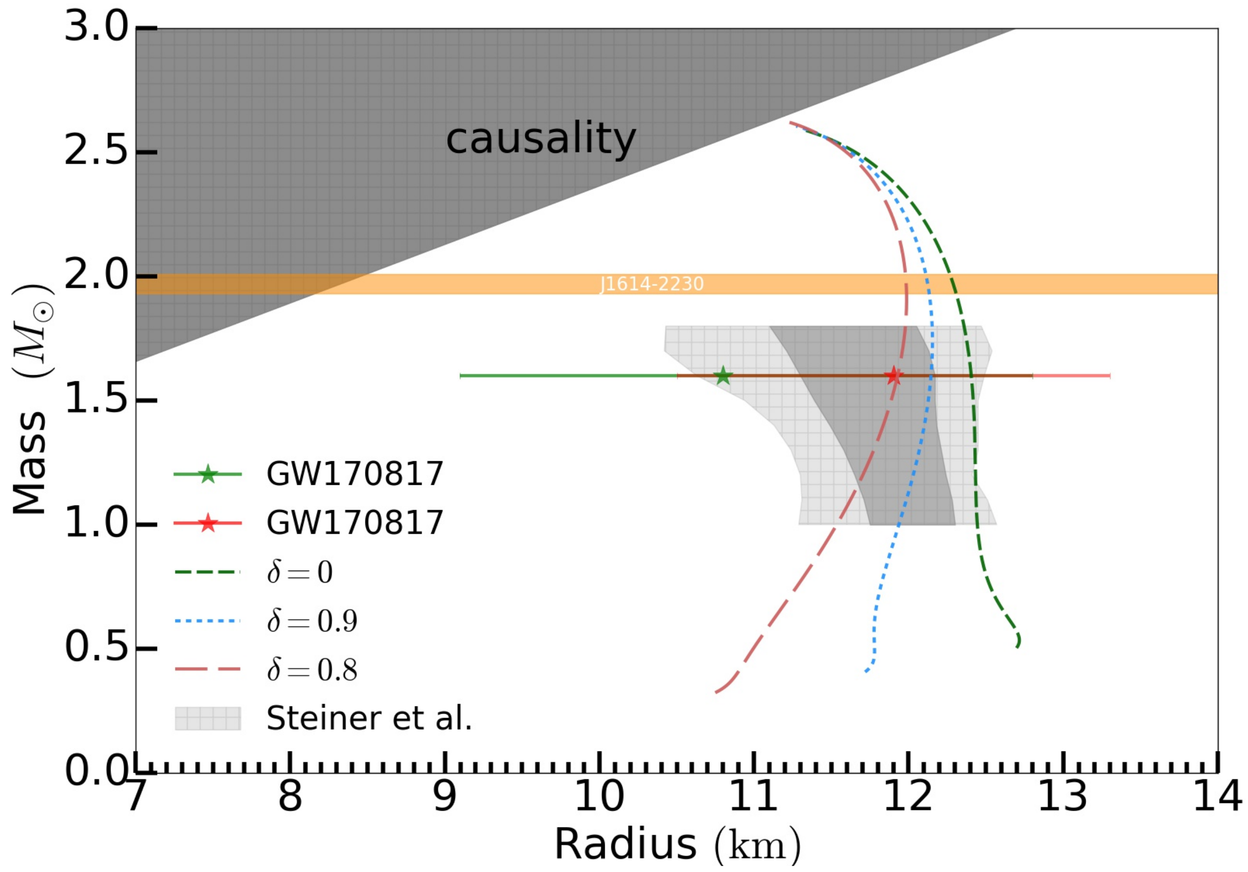
- Pure neutron matter ($a_{\rm sym} = 32$ MeV, $L_{\rm sym} = 50$ MeV)



*N.Y.Ghim, G.S.Yang, H.Ch.Kim, UY, In preparation.

Compact stars

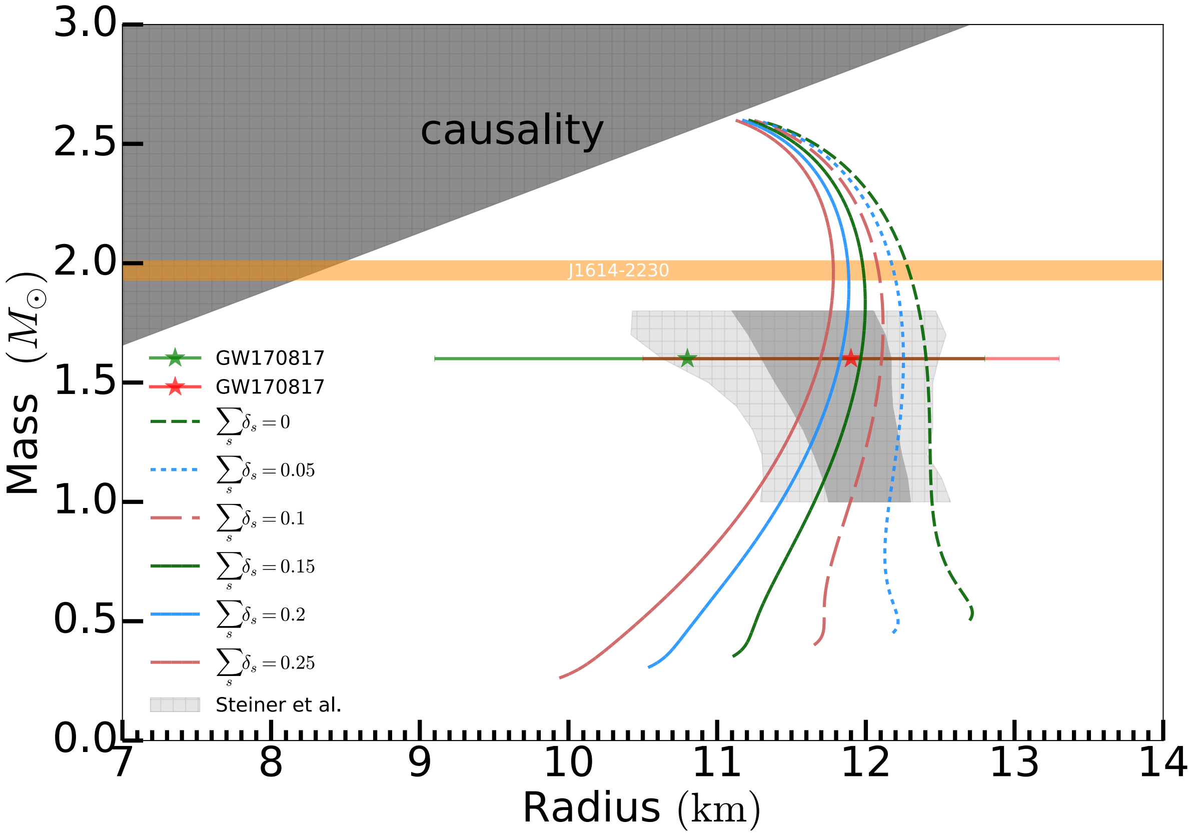
- Proto-neutron star ($K_0 = 240$ MeV, $a_{\rm sym} = 32$ MeV, $L_{\rm sym} = 60$ MeV)



*N.Y.Ghim, G.S.Yang, H.Ch.Kim, UY, In preparation.

Compact stars

- Hyperon mixed neutron matter ($K_0 = 240$ MeV, $a_{sym} = 32$ MeV, $L_{sym} = 50$ MeV)



*N.Y.Ghim, G.S.Yang, H.Ch.Kim, UY, In preparation.

Thank you very much for your attention!