

Low-energy $N\phi$ scattering from a pole-enhanced triangle diagram

Mao-Jun Yan

Southwest University, Chongqing, China

In collaboration with C.S. An and C.R. Deng

August 26th, 2026

Motivation

Theor. predictions

- The non-relativistic Yukawa potential $V_{(Q\bar{Q})A} = -\alpha e^{-\mu r}/r$ is matched to the Pomeron interaction. S. J. Brodsky et al, PRL64(1990); H. Gao et al, PRC63(2001).
- $\{N\phi, \Lambda K^*\}$ coupled channel scattering, F. Huang et al, PRC73(2006); J.J Xie et al PLB774(2017); C.S. An et al, PRC98(2018); J. He et al, PRD98(2018).
- Analysis in correlation function: attractive $N\phi$ scattering E. Chizzali et al, PLB848(2024); L. M. Abreu et al, PLB860(2025).

Latt. prediction

- $J = 3/2$ $N\phi$ scattering length $(-1.7, -0.9)$ fm fitted to two-pion exchange with $m_\pi = 146.4$ MeV by HAL QCD, Yan Lyu et al, PRD106(2022)

Exp. prediction

- Re. $N\phi$ scattering length around -1 fm w/o $\Lambda K^*, \Sigma K^*$ coupled channel scattering by Alice, S. Acharya et al, PRL127(2021)

$N\phi$ interaction to one loop

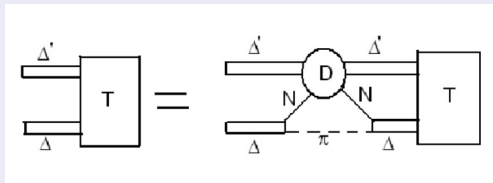
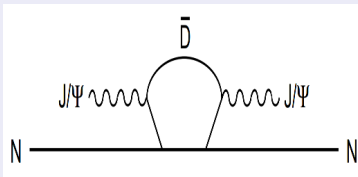
OBE

- η -exchange is perturbative.
- $\omega - \phi$ mixing is tiny.
- σ coupling to the strange quark is not clear.

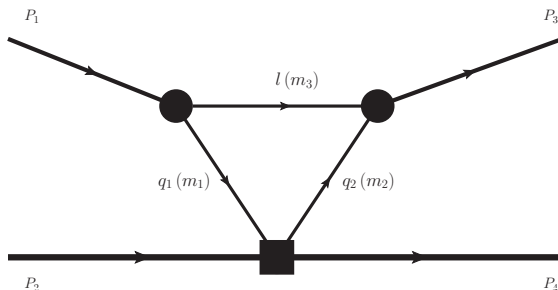
Two-pion exchange in bubbles

- Two-pion exchange in bubbles is negligible with a vanishing WT in $\phi\pi(K)$ scattering.

Dynamics in triangle diagrams S.J. Brodsky et al, PLB 412 (1997); A. Gal et al, PRL111(2013).



Dynamics in the triangle diagram



- $\mathcal{L}_{\phi_{K\bar{K}}} = ig\phi^\mu (\bar{K}\partial_\mu \cdot K - K\partial_\mu \cdot \bar{K})$
- $L_1 = \langle \bar{B} i\gamma^\mu \frac{1}{4F_\pi^2} [(\Phi\partial_\mu\Phi - \partial_\mu\Phi\Phi)B - B(\Phi\partial_\mu\Phi - \partial_\mu\Phi\Phi)] \rangle$
- Poles driven by a WT in $NK(\bar{K})$ scattering
- Spin-independent interaction

Dynamics in the triangle diagram

The S-wave projected loop integral w/o couplings

$$I(l) = \int \frac{d^4 l}{(2\pi)^4} \frac{-g_{\text{eff}} [4\vec{l}^2 + \vec{P}_1 \cdot \vec{P}_3] u(P_2) \bar{u}(P_4)}{(l^2 - m_1^2) [(P_1 - l)^2 - m_2^2] [(P_3 - l)^2 - m_3^2] [q_\Lambda^2 - m_\Lambda^2]},$$

- For the pole $z_A : l^0 = \omega_1 - i\epsilon$

$$2\pi i \text{Res } I(z_A) = \frac{\int \frac{d^3 l}{(2\pi)^3} \frac{4\vec{l}^2 * g_{\text{eff}} / (8 m_K^3)}{P_1^0 - \omega_1 - \omega_2} \frac{u(P_2) \bar{u}(P_4)}{P_3^0 - \omega_1 - \omega_3}}{1 \over 2m_\Lambda (P_1^0 + P_2^0 - \omega_1 - \omega_\Lambda)}$$

$N\phi$ potential w/ $2m_K - m_\phi = \delta$ in HAL QCD

- $\frac{V_{LO}^s}{C} = \arctan\left(\frac{\Lambda}{a_0}\right) \left[\frac{a_0}{2(b^2 - a_0^2)} + \frac{a_0(a_0^2 - 2b^2)}{(a_0^2 - b^2)^2} \right] + \frac{a_0}{2(b^2 - a_0^2)} \frac{a_0 \Lambda}{a_0^2 + \Lambda^2} + \arctan\left(\frac{\Lambda}{b}\right) \frac{b^3}{(b^2 - a_0^2)^2}$, w/ $C = -2m_K^2 g_{\text{eff}} / \pi^2$, $a_0 = \sqrt{-m_K \delta}$
 $b^2 = -2m_\Lambda \Delta E$, $\delta = m_\phi - 2m_K$ and $\Delta E = m_\phi + m_p - \omega_1 - m_\Lambda$

Dynamics in the triangle diagram

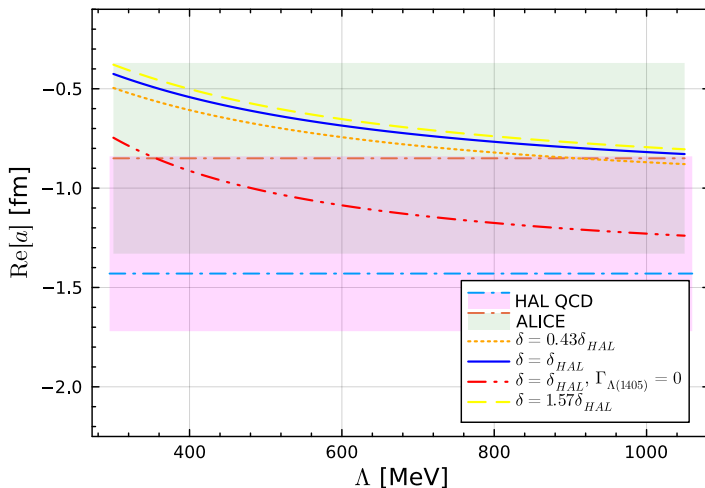
Physical $N\phi$ potential, w/ $k_0 = \sqrt{2m_K\delta}$

$$\frac{V_{LO}^s}{C} = \frac{k_0}{4(b^2+k_0^2)} \left(\log \frac{\Lambda+k_0}{\Lambda-k_0} - i\pi - \frac{2k_0\Lambda}{\Lambda^2-k_0^2} \right) + \frac{b^3}{(b^2+k_0^2)^2} \arctan\left(\frac{\Lambda}{b}\right) \\ + \frac{k_0(k_0^2+2b^2)}{2(b^2+k_0^2)^2} \left(\log \frac{\Lambda-k_0}{\Lambda+k_0} + i\pi \right),$$

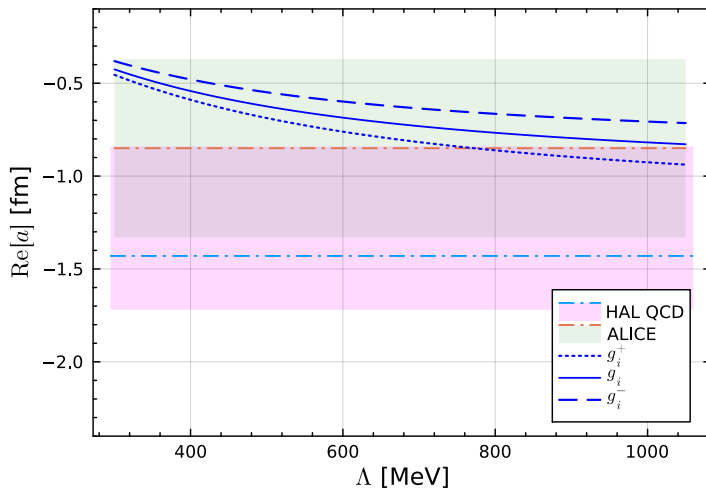
$N\phi$ potential and ERE

- $\frac{8\pi th.}{T} = V^{-1} - \textcolor{violet}{G} = -\frac{1}{a} - ik, -\frac{1}{a_{N\phi}} = \frac{1}{V_{LO}^s}. \Lambda_G = 200 - 500 \text{ MeV}.$
- $t = -\lim_{s \rightarrow s_R} \frac{g_i^2}{s-s_R} \text{ w/ } \sqrt{s_R} = 1417_{-4}^{+4} - i24_{-4}^{+7} \text{ MeV and coupling } g_i = 7.7_{-0.6}^{+1.2} \text{ GeV, } \textcolor{brown}{D. Jido et al, NPA725(2003); Z.H. Guo and J. A. Oller, PRC87(2013)}$
- $\frac{g_i^{\text{unphy. } 2}}{g_i^2} \simeq \left(\frac{m_N^{\text{unphy.}}}{m_N} \right)^2 \left(\frac{m_K^{\text{unphy.}}}{m_K} \right)^2 \approx 1.19.$

$N\phi$ scattering length with unphysical Kaon mass

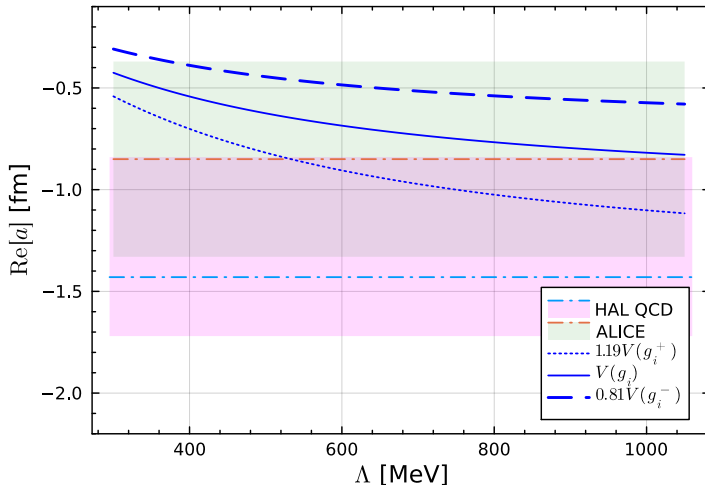


$N\phi$ scattering length with unphysical Kaon mass



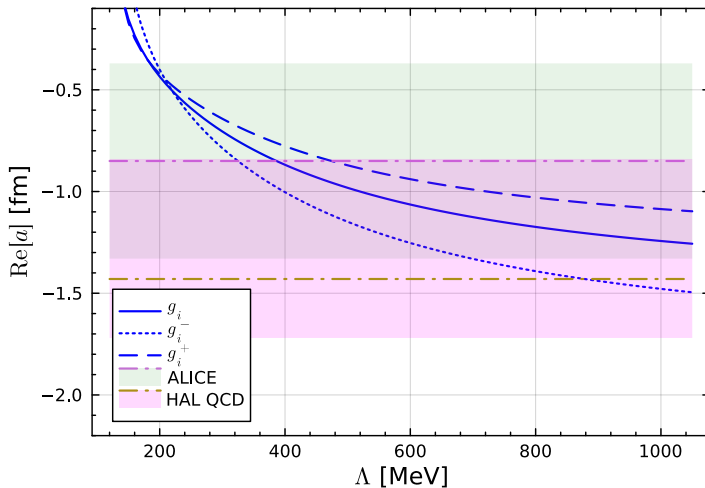
Uncertainty from the pole of $\Lambda(1405)$.

$N\phi$ scattering length with unphysical Kaon mass



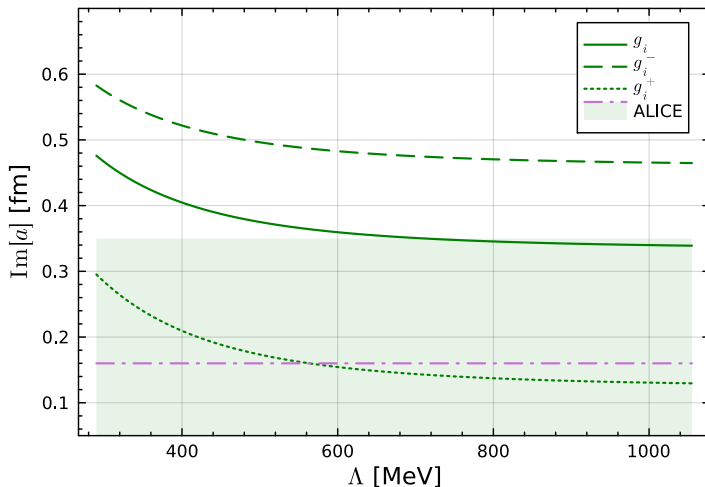
$$\text{Err.} : \left(\frac{m_N^{\text{unphy.}}}{m_N} \right)^2 \left(\frac{m_K^{\text{unphy.}}}{m_K} \right)^2.$$

$N\phi$ scattering length with physical Kaon mass



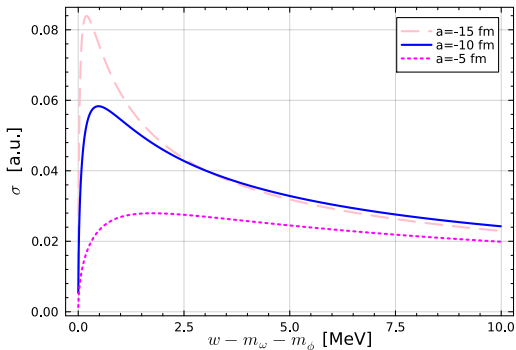
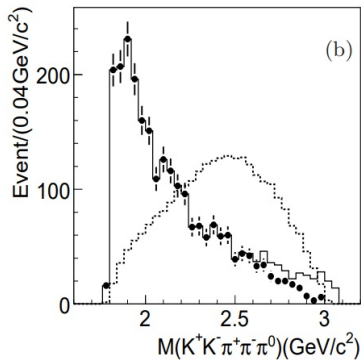
The momentum carried by the outgoing K is 118.17 MeV in $\phi \rightarrow K\bar{K}$.

$N\phi$ scattering length with physical Kaon mass



The momentum carried by the outgoing K is 118.17 MeV in $\phi \rightarrow K\bar{K}$.

$\omega\phi$ threshold enhancement



M. Ablikim et al. (BESIII), PRD87, 032008 (2013)

A pole from $\omega\phi$ scattering around the threshold in addition to $f_0(1710)$?

Summary

- 1 The $N\phi$ scattering length evaluated in the triangle diagram with including $\Lambda(1405)$ matches the one from HAL QCD, where the open channel effect in $\Sigma\pi$, $N\bar{K}$ scattering is ignored.
- 2 The scattering length is a power law of a_0 and b and differs from two-pion exchange and the Van der Waals force.
- 3 When the 3-B pole closes to the 2-B threshold, the 3B pole contributes to the scattering length.
- 4 This dynamic driven from the triangle diagram is adaptable to $\omega\phi$ threshold enhancement.
- 5 $P_c(4312)\pi$ couple to $I = 3/2$ sector.
- 6 $w_{c1}\pi$, $X(3872)\pi$ couple to $Z_c(4020)$.

Summary

- 1 The $N\phi$ scattering length evaluated in the triangle diagram with including $\Lambda(1405)$ matches the one from HAL QCD, where the open channel effect in $\Sigma\pi$, $N\bar{K}$ scattering is ignored.
- 2 The scattering length is a power law of a_0 and b and differs from two-pion exchange and the Van der Waals force.
- 3 When the 3-B pole closes to the 2-B threshold, the 3B pole contributes to the scattering length.
- 4 This dynamic driven from the triangle diagram is adaptable to $\omega\phi$ threshold enhancement.
- 5 $P_c(4312)\pi$ couple to $I = 3/2$ sector.
- 6 $w_{c1}\pi$, $X(3872)\pi$ couple to $Z_c(4020)$.
- 7 To be tested in $N\phi\phi$ and $NN\phi$ scattering [Wen-Liang Zhen et al, PRD 111\(2025\)](#)

Summary

- 1 The $N\phi$ scattering length evaluated in the triangle diagram with including $\Lambda(1405)$ matches the one from HAL QCD, where the open channel effect in $\Sigma\pi$, $N\bar{K}$ scattering is ignored.
- 2 The scattering length is a power law of a_0 and b and differs from two-pion exchange and the Van der Waals force.
- 3 When the 3-B pole closes to the 2-B threshold, the 3B pole contributes to the scattering length.
- 4 This dynamic driven from the triangle diagram is adaptable to $\omega\phi$ threshold enhancement.
- 5 $P_c(4312)\pi$ couple to $I = 3/2$ sector.
- 6 $w_{c1}\pi$, $X(3872)\pi$ couple to $Z_c(4020)$.
- 7 To be tested in $N\phi\phi$ and $NN\phi$ scattering [Wen-Liang Zhen et al, PRD 111\(2025\)](#)

Thanks !

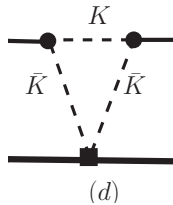
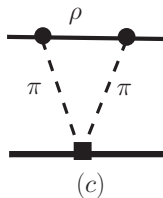
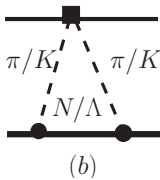
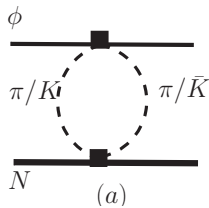
The SU(3) matrices for the mesons and the baryons are the following

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix},$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}.$$

Hadron	Lattice [MeV]	Expt. [MeV]
K	524.7	495.6
ϕ	1048.0	1019.5
N	954.0	938.9

Backup



w/: WT in $\phi\pi \rightarrow \phi\pi$ and $\phi K \rightarrow \phi K$ vanish; $g_{\phi K \bar{K}} \gg g_{\phi \rho \pi}$.