



Search for exotic states in three-meson systems

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Inha University, Incheon, Korea

Outline

Introduction: Exotic states and Fixed Center Approximation (FCA) to Faddeev equations

$f_1(1285) : K^* \bar{K}$ bound state

$f_0(1710) : K^* \bar{K}^*$ bound state

A new $D\bar{D}$ bound state $X(3720)$

Hidden charm $K_{c\bar{c}}(4180)$ state in $D\bar{D}K$ system

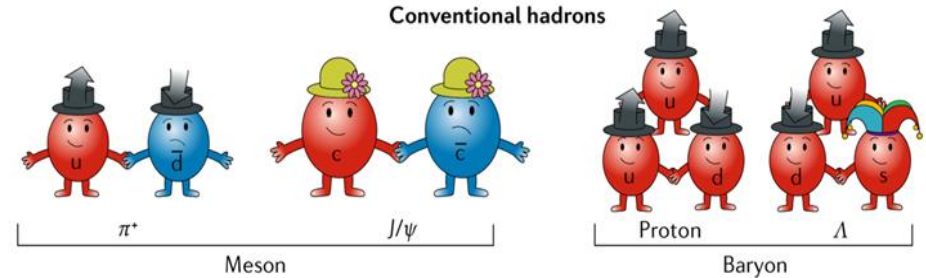
Excited pseudoscalar states in $\pi \bar{K}^* K^*$, $\eta \bar{K}^* K^*$ and $K \bar{K}^* K^*$ systems

Exotic states in $\pi \bar{K} K^*$ and $\eta \bar{K} K^*$ systems

Summary

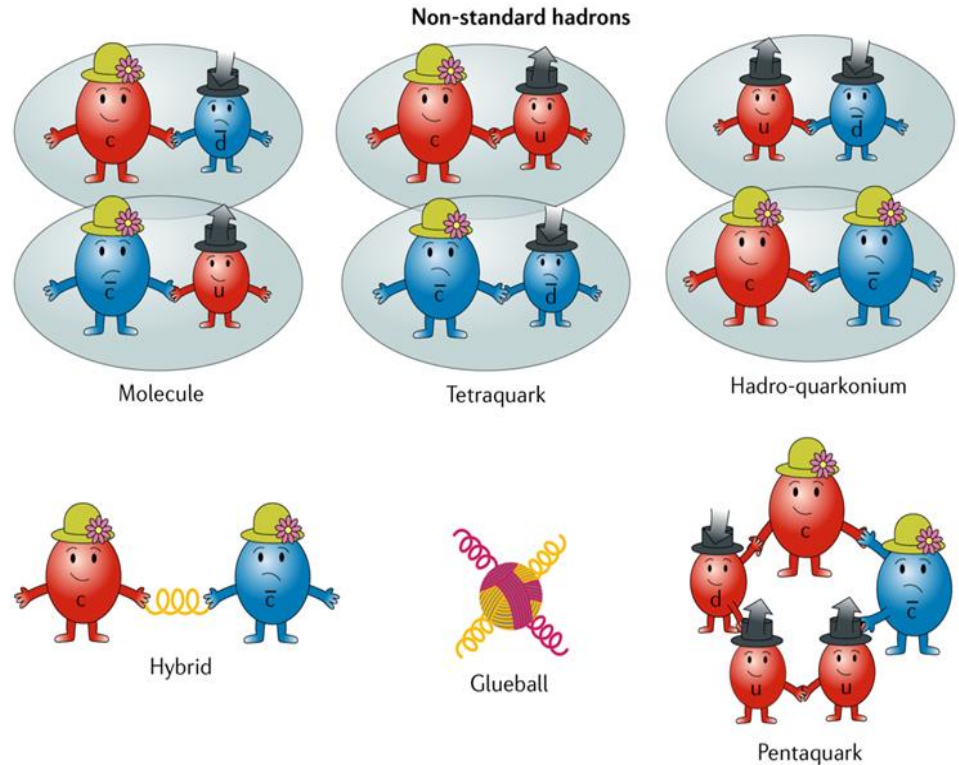
Hadron: In particle physics, a hadron is a composite subatomic particle made of two or more quarks held together by the strong interaction.

Too simple



➤ Exotic states

*Not reached by
 $q\bar{q}$ or qqq ,
need more
quarks or gluons.*



Evidence for a narrow $S = +1$ baryon resonance in photoproduction from the neutron

LEPS Collaboration • T. Nakano (Osaka U., Res. Ctr. Nucl. Phys.) [Show All\(52\)](#)

Jan, 2003

12 pages

Published in: *Phys.Rev.Lett.* 91 (2003) 012002

e-Print: [hep-ex/0301020](#) [hep-ex]

DOI: [10.1103/PhysRevLett.91.012002](#)

Experiments: LEPS

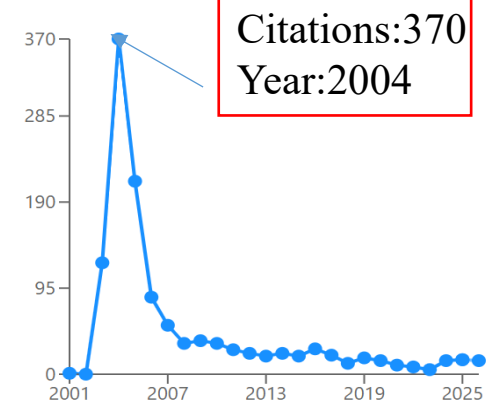
View in: [ADS Abstract Service](#), [Nuclear Science References](#)

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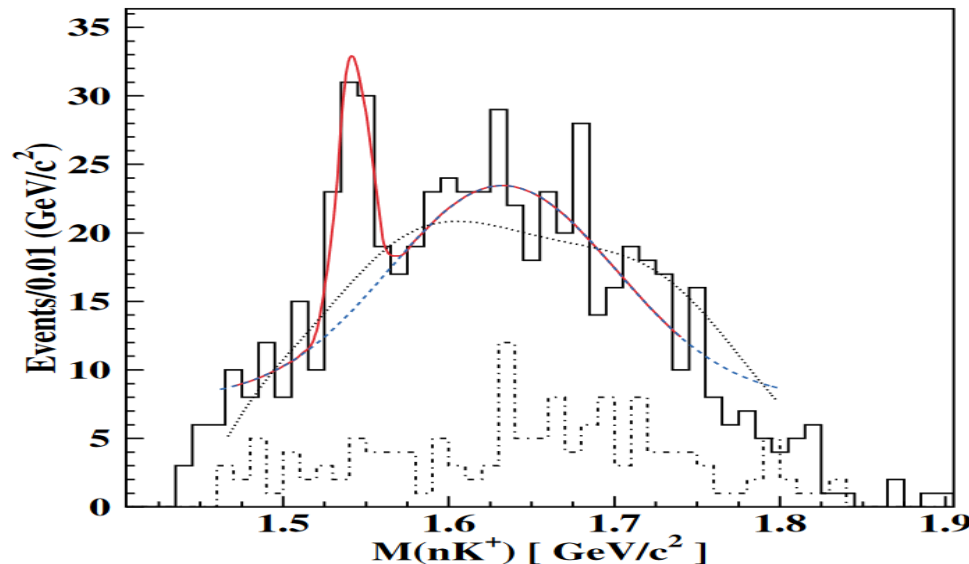
[reference search](#) [↻ 1,227 citations](#)

$uudd\bar{s}$

Citations per year



Citations:370
Year:2004



$$M = 1540 \pm 10 \text{ MeV}$$

$$\Gamma < 25 \text{ MeV}$$

Observation of an exotic $S = +1$ baryon in exclusive photoproduction from the deuteron

CLAS Collaboration • S. Stepanyan (Jefferson Lab) [Show All\(198\)](#)

Jul, 2003

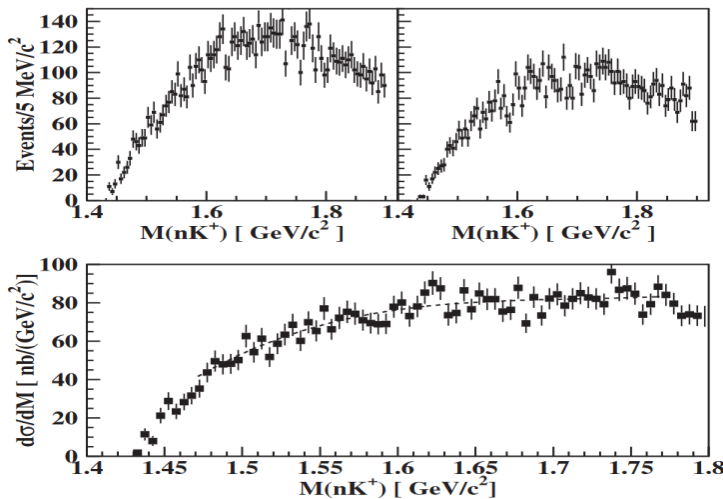
Phys. Rev. Lett. 91 (2003) 012002

F. Huang^a, Z.Y. Zhang^a, Y.W. Yu^a, B.S. Zou^{b,a}

The structure of the pentaquark state $uudd\bar{s}$ is studied in the chiral $SU(3)$ quark model as well as in the extended chiral $SU(3)$ quark model, in which the vector meson exchanges are included. The lowest four configurations of $J^\pi = \frac{1}{2}^-$ and four of $J^\pi = \frac{1}{2}^+$ for the 5 quark cluster are considered. The results show that the isospin $T = 0$ state is always the lowest one for both $J^\pi = \frac{1}{2}^-$ and $J^\pi = \frac{1}{2}^+$ cases in various models. But the theoretical value of the lowest one is still about 200–300 MeV higher than the experimental mass of Θ . It seems that a dynamical calculation should be done for the further study.

PRL 96, 212001 (2006)

PHYSICAL REVIEW LETTERS

week ending
2 JUNE 2006Search for the Θ^+ Pentaquark in the Reaction $\gamma d \rightarrow pK^- K^+ n$ 

A search for the Θ^+ in the reaction $\gamma d \rightarrow pK^- K^+ n$ was completed using the CLAS detector at Jefferson Lab. A study of the same reaction, published earlier, reported the observation of a narrow Θ^+ resonance. The present experiment, with more than 30 times the integrated luminosity of our earlier measurement, does not show any evidence for a narrow pentaquark resonance. The angle-integrated upper limit on Θ^+ production in the mass range of 1.52–1.56 GeV/c² for the $\gamma d \rightarrow pK^- \Theta^+$ reaction is 0.3 nb (95% C.L.). This upper limit depends on assumptions made for the mass and angular distribution of Θ^+ .

Theoretical formalism: two-body hadron-hadron scattering

$$t = v + vgv + vgvgv + \dots = v + vgt \quad (g : \text{two body loop function})$$

$$= [1 - vg]^{-1} v$$

v : from chiral Lagrangian

➤ Two-body loop function

$$G = \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m_1^2 + i\epsilon} \frac{1}{(P - q)^2 - m_2^2 + i\epsilon}$$

➤ **Subtraction constants** in dimensional regularization method

$$G = \frac{1}{16\pi^2} \left(\alpha + \text{Log} \frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s} \text{Log} \frac{m_2^2}{m_1^2} \right. \\ \left. + \frac{p}{\sqrt{s}} \left(\text{Log} \frac{s - m_2^2 + m_1^2 + 2p\sqrt{s}}{-s + m_2^2 - m_1^2 + 2p\sqrt{s}} + \text{Log} \frac{s + m_2^2 - m_1^2 + 2p\sqrt{s}}{-s - m_2^2 + m_1^2 + 2p\sqrt{s}} \right) \right) \quad \alpha'_\mu = \alpha_\mu + \text{Log} \frac{\mu'^2}{\mu^2}$$

➤ or **cutoff values** in cutoff method

$$G = \int_0^{q_{max}} \frac{q^2 dq}{(2\pi)^2} \frac{\omega_1 + \omega_2}{\omega_1 \omega_2 [(P^0)^2 - (\omega_1 + \omega_2)^2 + i\epsilon]}$$

Identify resonances/bound states (Hadronic molecules)

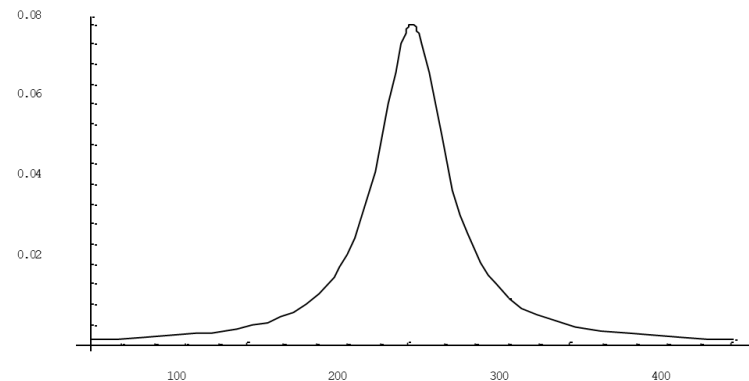
- To identify resonances/bound states, one goes to the complex plane to look for poles.

$$\left. \begin{aligned} T_{ij} &= \frac{g_{ii}g_{jj}}{\sqrt{s} - z_R} \\ z_R &= M_R - i\Gamma_R/2 \end{aligned} \right\} \begin{aligned} M &= \text{Re}\sqrt{s}_{\text{pole}} \quad \text{and} \quad \Gamma = 2 \times \text{Im}\sqrt{s}_{\text{pole}} \end{aligned}$$

- Once poles are found, resonance parameters can also be obtained from amplitudes squared on the real axis.

$$\text{Baryon: } A(s) = \frac{\bar{g}}{\sqrt{s} - M_{BW} + i\frac{\Gamma_{BW}}{2}}$$

$$\text{Meson: } A(s) = \frac{\bar{g}'}{s - M_{BW}^2 + iM_{BW}\Gamma_{BW}}$$



$f_1(1285)$ as a $\bar{K}K^*$ bound state

$$\mathcal{L}_{VVPP} = -\frac{1}{4f^2} \text{Tr} ([V^\mu, \partial^\nu V_\mu][P, \partial_\nu P])$$

$$t = \frac{v}{1 - vG}, \quad \text{Pole position : } (1282, -i0) \text{ MeV}$$

$$\text{Coupling : } g_{K^*\bar{K}}^R = (7555, -i0) \text{ MeV}$$

L. Roca, and E. Oset, “Low lying axial-vector mesons as dynamically generated resonances”, Phys. Rev. D 72, 014002 (2005).

F. Aceti, J.M. Dias and E. Oset, “ $f_1(1285)$ decays into $a_0(980)\pi^0$, $f_0(980)\pi^0$ and isospin breaking,” Eur. Phys. J. A 51, 48 (2015).

R. Molina, M. Döring and E. Oset, “Determination of the compositeness of resonances from decays: the case of the $B_s^0 \rightarrow J/\psi f_1(1285)$,” Phys. Rev. D 93, 114004 (2016).

$f_1(1285)$ in the unitized chiral perturbation theory

TABLE II. Pole positions and couplings in the $(S, I) = (0, 0)$ channel.

	$h_1(1170)$		$h_1(1380)$		$f_1(1285)$	
	LO	NLO	LO	NLO	LO	NLO
$\sqrt{s}$	$918 - i17$	$925 - i29$	$1244 - i7$	$1257 - i0$	$1286 - i0$	$1289 - i0$
$\frac{1}{\sqrt{2}}(\bar{K}^*K + K^*\bar{K})$	...	...	...	...	$7219 + i0$	$7884 + i0$
$\phi\eta$	$-46 + i13$	$69 - i102$	$-3309 + i47$	$-5963 - i38$	...	...
$\omega\eta$	$-24 + i28$	$711 - i427$	$3019 - i22$	$2642 - i47$	...	...
$\rho\pi$	$3452 - i1681$	$3576 - i1909$	$650 - i961$	$134 - i233$	...	...
$\frac{1}{\sqrt{2}}(\bar{K}^*K - K^*\bar{K})$	$-784 + i499$	$-1488 + i757$	$6137 + i183$	$6435 + i35$	...	...

PHYSICAL REVIEW D **90**, 014020 (2014)

Pseudoscalar meson and vector meson interactions and dynamically generated axial-vector mesons

Yu Zhou, Xiu-Lei Ren, Hua-Xing Chen, and Li-Sheng Geng^{*}

$f_1(1285)$

$$I^G(J^{PC}) = 0^+(1^{++})$$

$f_1(1285)$ MASS

$f_1(1285)$ WIDTH

Only experiments giving width error less than 20 MeV are kept for averaging.

VALUE (MeV)	EVTs	DOCUMENT ID	TECN	COMMENT
1281.8 ± 0.5	OUR AVERAGE	Error includes scale factor of 1.7. See the ideogram		

VALUE (MeV)	EVTs	DOCUMENT ID	TECN	COMMENT
23.0 ± 1.1	OUR AVERAGE	Error includes scale factor of 1.6. See the ideogram below.		

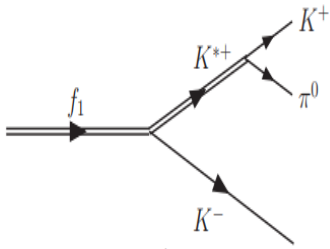
$f_1(1285)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	4π	$(33.5^{+2.0}_{-1.8})\%$	S=1.3
Γ_2	$\pi^0\pi^0\pi^+\pi^-$	$(22.3^{+1.3}_{-1.2})\%$	S=1.3
Γ_3	$2\pi^+2\pi^-$	$(11.2^{+0.7}_{-0.6})\%$	S=1.3
Γ_4	$\rho^0\pi^+\pi^-$	$(11.2^{+0.7}_{-0.6})\%$	S=1.3
Γ_5	$\rho^0\rho^0$	seen	
Γ_6	$4\pi^0$	$< 7 \times 10^{-4}$	CL=90%
Γ_7	$\eta\pi^+\pi^-$	$(35 \pm 15)\%$	
Γ_8	$\eta\pi\pi$	$(52.0^{+1.8}_{-2.1})\%$	S=1.2
Γ_9	$a_0(980)\pi$ [ignoring $a_0(980) \rightarrow K\bar{K}$]	$(38 \pm 4)\%$	
Γ_{10}	$\eta\pi\pi$ [excluding $a_0(980)\pi$]	$(14 \pm 4)\%$	
Γ_{11}	$K\bar{K}\pi$	$(9.1 \pm 0.4)\%$	S=1.1
Γ_{12}	$K\bar{K}^*(892)$	not seen	
Γ_{13}	$\pi^+\pi^-\pi^0$	$(3.0 \pm 0.9) \times 10^{-3}$	
Γ_{14}	$\rho^\pm\pi^\mp$	$< 3.1 \times 10^{-3}$	CL=95%
Γ_{15}	$\gamma\rho^0$	$(5.3 \pm 1.2)\%$	S=2.9
Γ_{16}	$\phi\gamma$	$(7.5 \pm 2.7) \times 10^{-4}$	
Γ_{17}	$\gamma\gamma^*$		

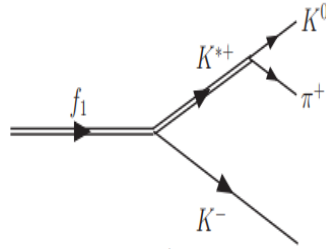
$f_1(1285) \rightarrow \pi K \bar{K}$ decay

$$\frac{1}{\sqrt{2}}(K^* \bar{K} - \bar{K}^* K) = -\frac{1}{2}(K^{*+} K^- + K^{*0} \bar{K}^0 - K^{*-} K^+ - \bar{K}^{*0} K^0) .$$

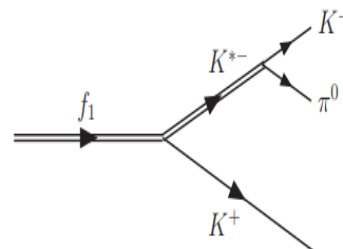
We take the convention $CK^* = -\bar{K}^*$,



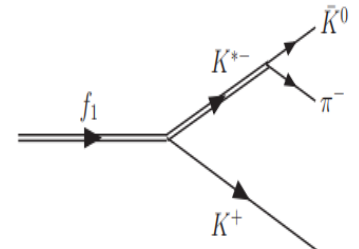
A)



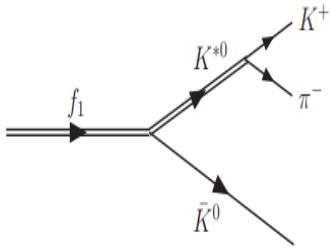
B)



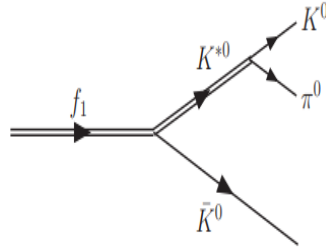
E)



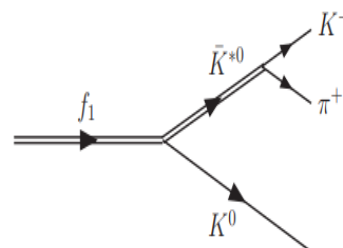
F)



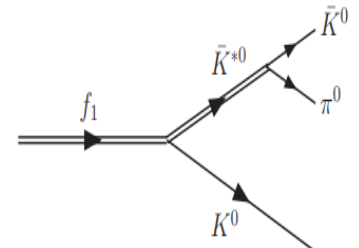
C)



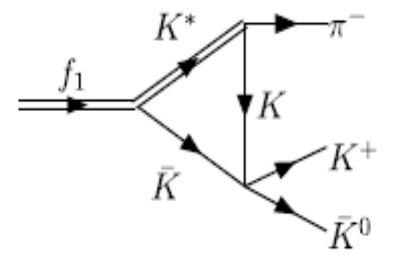
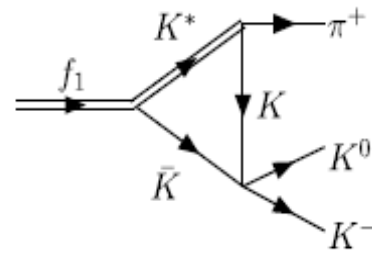
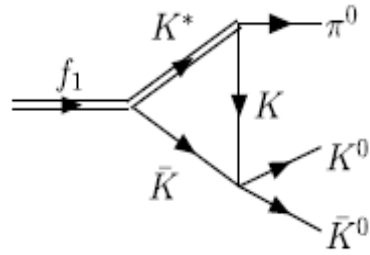
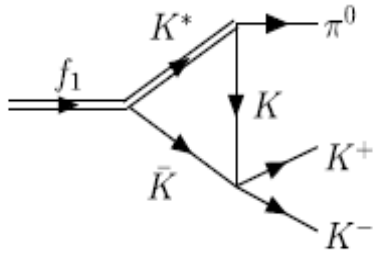
D)



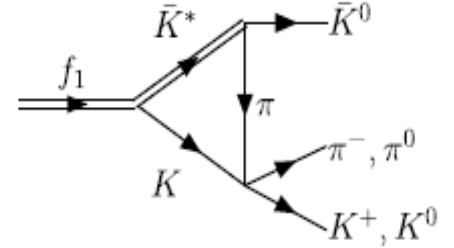
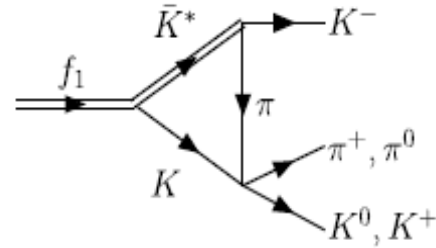
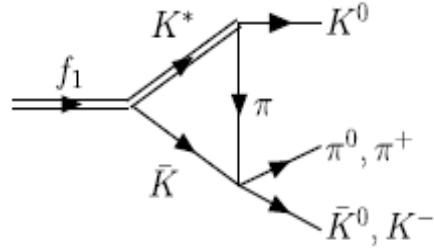
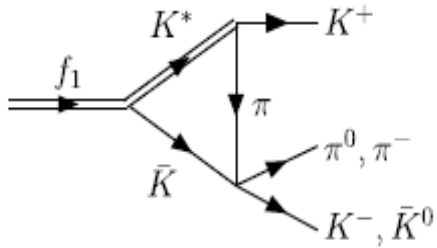
G)



H)



A)



B)

$$\Gamma = 6 \frac{1}{64\pi^3 M_{f_1}} \int \int d\omega_{K^+} d\omega_{K^-} \sum |M|^2 \times \theta(1 - \cos^2 \theta_{K\bar{K}}) \theta(M_{f_1} - \omega_{K^+} - \omega_{K^-} - m_\pi), \quad (24)$$

where M is the full amplitude of the process $f_1(1285) \rightarrow \pi^0 K^+ K^-$ including the FSIs,

$$M = M_{\text{tree}} + M_{\text{FSI}}^{K\bar{K}} + M_{\text{FSI}}^{\pi K}, \quad (25)$$

Theory

$$\text{Br}[f_1(1285) \rightarrow \pi K \bar{K}] = (7.2 \sim 7.8)\%$$

Experiment

$$\text{Br}[f_1(1285) \rightarrow \pi K \bar{K}] = (9.0 \pm 0.4)\%$$

F. Aceti, Ju-Jun Xie and E. Oset, "The $\bar{K}K\pi$ decay of the $f_1(1285)$ and its nature as a $K^* \bar{K}$ molecule," Phys. Lett. B750, 609 (2015).

$f_1(1285) \rightarrow \gamma V$ decay

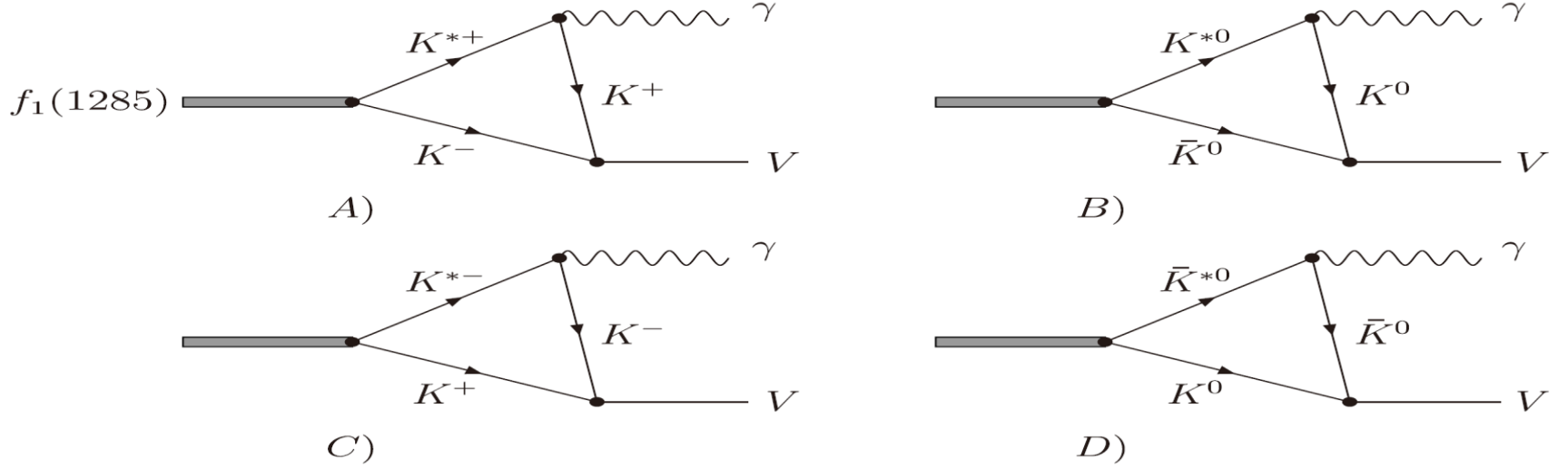
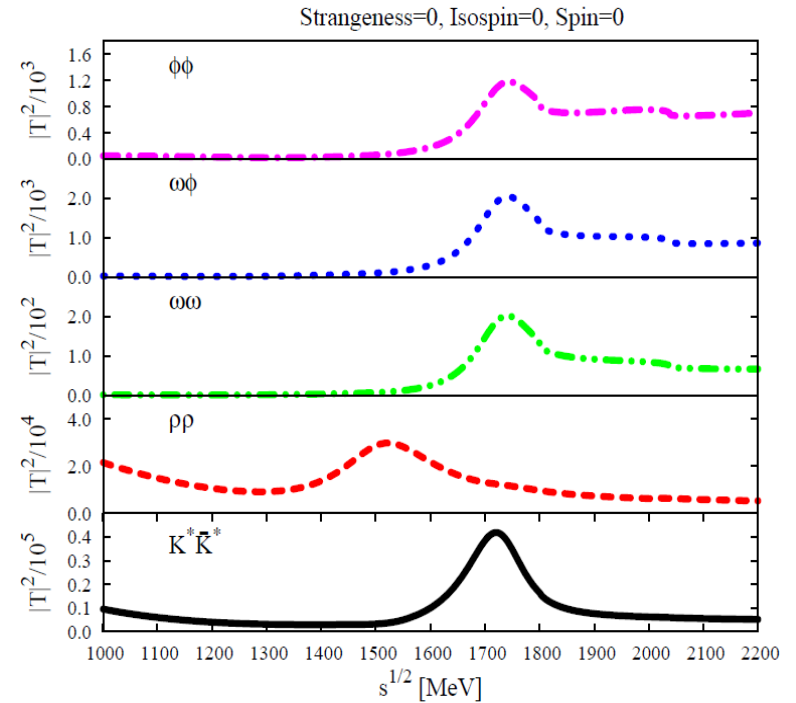
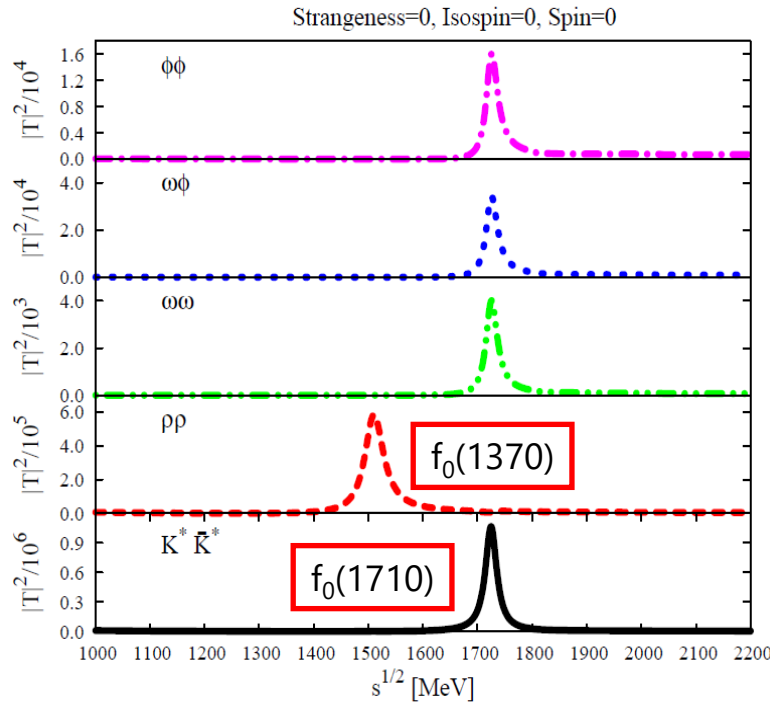


Table 1. Partial decay width for $f_1(1285) \rightarrow \gamma V$. All units are in MeV.

Λ	$f_1 \rightarrow \gamma \rho^0 \Gamma (\bar{\Gamma})$	$f_1 \rightarrow \gamma \omega [\times 10^{-2}]$	$f_1 \rightarrow \gamma \phi [\times 10^{-2}]$	R_1	R_2
800	0.42 (0.34)	1.36	0.71	59	31
1000	0.56 (0.46)	1.87	0.93	60	30
1500	0.88 (0.72)	3.01	1.41	62	29
Ref. [2]	0.311	3.43	—	—	9
Ref. [14] (set I) ¹⁾	0.509	4.8	2.0	25	11
Ref. [14] (set II)	0.565	5.7	0.56	101	10
Exp. [7]	1.2 ± 0.3	—	1.7 ± 0.6	71 ± 30	—
Exp. [1] ^a	0.45 ± 0.18	—	—	—	—

^aThe measured width of $f_1(1285)$ is ~ 6 MeV smaller than the previous world average [7].

$f_0(1710)$ in vector – vector interactions



No decay to two pseudoscalar-pseudoscalar

With decay to two pseudoscalar-pseudoscalar

	pole position (no two p's)	real axis (two p's)
$f_0(1370)$	(1512, 51)	(1523, 257)
$f_0(1710)$	(1726, 28)	(1721, 133)

$f_0(1710)$ confirmed in the chiral quark model

PHYSICAL REVIEW C **84**, 054006 (2011)

Dynamical coupled-channels study of $K^* \bar{K}^*$ and $\omega\phi$ states in a chiral quark model

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²*Theoretical Physics Center for Science Facilities (TPCSF), CAS, Beijing 100049, China*

(Received 6 July 2011; published 21 November 2011)

A dynamical coupled-channels study of a $K^* \bar{K}^*$ state with isospin 0 and $\omega\phi$ state is performed within both the chiral SU(3) quark model and the extended chiral SU(3) quark model by solving a resonating group method (RGM) equation. The model parameters are taken from our previous work, which gave a satisfactory description of the energies of the octet and decuplet baryon ground states, the binding energy of the deuteron, the nucleon-nucleon (NN) scattering phase shifts, and the hyperon-nucleon (YN) cross sections. The results show that the interactions of $K^* \bar{K}^*$ states are attractive, which consequently result in $K^* \bar{K}^*$ bound states with the binding energies of about 10–70 MeV, and contrarily, no $\omega\phi$ bound state is obtained. The channel coupling effect of $K^* \bar{K}^*$ and $\omega\phi$ is found to be considerably large, which makes the binding of $K^* \bar{K}^*$ 5–45 MeV deeper. The plausible interpretation of $f_0(1710)$ and $X(1812)$ being $K^* \bar{K}^*$ dominated states is briefly discussed.

Effects of pseudoscalar-pseudoscalar coupled channels

Further study of $f_0(1710)$ with the coupled-channel approach and the hadron molecular picture

PHYSICAL REVIEW D **104**, 114001 (2021)

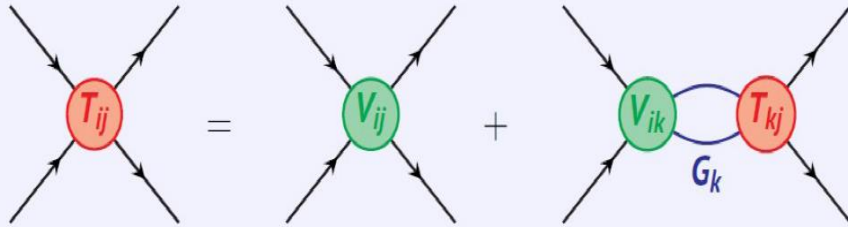
Zheng-Li Wang^{1,2,*} and Bing-Song Zou^{1,2,3,†}

The $f_0(1710)$ was previously proposed to be a dynamically generated state with interactions between vector mesons. We extend the study of $f_0(1710)$ by including its coupling to channels of pseudoscalar mesons within the coupled-channel approach. The channels involved are $K^* \bar{K}^*$, $\rho\rho$, $\omega\omega$, $\phi\phi$, $\omega\phi$, $\pi\pi$, $K\bar{K}$, $\eta\eta$. We show that the pole assigned to $f_0(1710)$ does not change much. Then we calculate the partial decay widths of $f_0(1710) \rightarrow K^* \bar{K}^* \rightarrow \pi\pi, K\bar{K}, \eta\eta$ as the coupled channel dynamically generated state as well as assuming it to be a pure $K^* \bar{K}^*$ molecule. In both cases the ratios of partial decay widths agree fairly with that in PDG.

$D\bar{D}$ and $D_s^+ D_s^-$ interactions in chiral unitary approach

9. $C = 0, S = 0$

States	Amplitude
$D_s^+ D_s^- \rightarrow D_s^+ D_s^-$	$-\frac{1}{3f^2}(\psi_3(s+t-2u) + 2m_D^2 + 2m_K^2 - 2m_\pi^2)$
$\rightarrow D^+ D^-$	$-\frac{1}{6f^2}(\psi_5(t-u) + s-u + 2m_D^2 + m_K^2 - m_\pi^2)$
$\rightarrow D^0 \bar{D}^0$	$-\frac{1}{6f^2}(\psi_5(t-u) + s-u + 2m_D^2 + m_K^2 - m_\pi^2)$
$D^+ D^- \rightarrow D^+ D^-$	$-\frac{1}{3f^2}(\psi_3(s+t-2u) + 2m_D^2)$
$\rightarrow D^0 \bar{D}^0$	$-\frac{1}{6f^2}(\psi_5(t-u) + s-u + 2m_D^2)$
$D^0 D^0 \rightarrow D^0 D^0$	$-\frac{1}{3f^2}(\psi_3(s+t-2u) + 2m_D^2)$



Bethe-Salpeter equation: $T = V + VGT \implies T = (\mathbb{I} - VG)^{-1} V$

TABLE IV. Pole positions for the phenomenological model.

Resonance ID	C	S	I	RE($\sqrt{s}$) (MeV)	IM($\sqrt{s}$) (MeV)
f_0	0	0	0	918.45	-18.76
σ	0	0	0	616.19	-143.77
(?)	0	0	0	3718.93	-0.06
a_0	0	0	1	987.68	-38.29
κ	0	1	$\frac{1}{2}$	831.58	-147.24
$D_{s0}^*(2317)$	1	1	0	2317.25	0
$D_0^*(2400)$	1	0	$\frac{1}{2}$	2129.26	-157.00
(?)	1	0	$\frac{1}{2}$	2694.69	-441.89
(?)	1	1	1	2704.31	-459.50
(?)	1	-1	0	2709.39	-445.73

*X(3720) : corresponding
mostly to a $D\bar{D}$ state*

Bound $D\bar{D}$ state in other calculations

PHYSICAL REVIEW D **74**, 014013 (2006)

Possible S-wave bound-states of two pseudoscalar mesons

Yin-Jie Zhang,¹ Huan-Ching Chiang,^{2,1} Peng-Nian Shen,^{3,1} and Bing-Song Zou^{1,4,*}

POSSIBLE S-WAVE BOUND-STATES OF TWO ...

TABLE I. Dependence of binding energies on the cut-off parameter Λ for various isoscalar two-pseudoscalar-meson systems

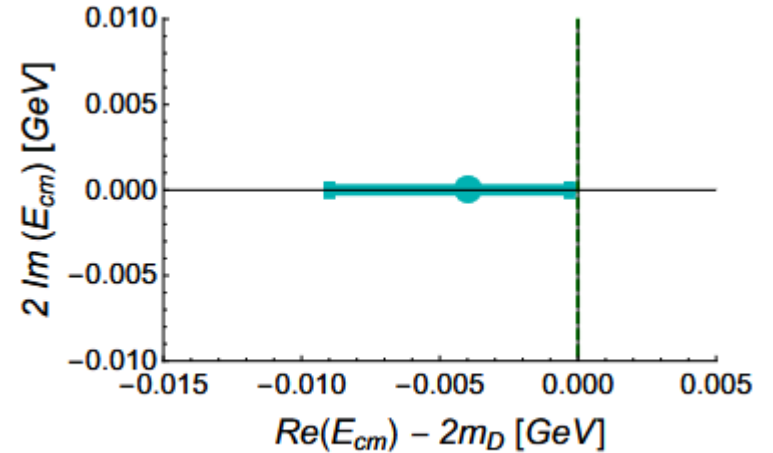
E(MeV) Λ (GeV)	DB	$D\bar{D}$	$B\bar{B}$	$B\bar{K}$	$B\bar{D}$	$D\bar{K}$
1.4	-1.2	—	—	—	—	—
1.5	-5.7	—	—	—	—	—
1.6	-13.4	—	—	—	—	—
1.7	-24.3	—	—	—	—	—
1.8	-38.1	-9.3	-0.8	—	—	—
1.9	-54.6	-17.7	-4.3	—	—	—
2.0	-73.6	-28.7	-10.9	—	—	—
2.1	-95.0	-42.4	-21.0	—	—	—
2.2	-118.6	-58.8	-35.1	-1.4	—	—
2.3	-144.2	-77.9	-53.8	-3.1	—	—
2.4	-171.8	-100.0	-78.2	-5.4	—	—
2.5	-201.3	-125.1	-110.1	-8.2	-0.3	-5.8
2.6	-232.7	-153.7	-152.4	-11.6	-2.5	-55.2
2.7	-265.8	-186.0	-210.3	-15.5	-6.9	-302.7
2.8	-300.6	-222.8	-295.9	-19.9	-13.9	—
2.9	-337.2	-264.8	-450.9	-24.7	-24.3	—
3.0	-375.6	-313.3	—	-30.0	-39.0	—

$$M_{D\bar{D}} \sim 3735 \text{ MeV}$$

Charmonium-like resonances with $J^{PC} = 0^{++}, 2^{++}$ in coupled $D\bar{D}$, $D_s\bar{D}_s$ scattering on the lattice

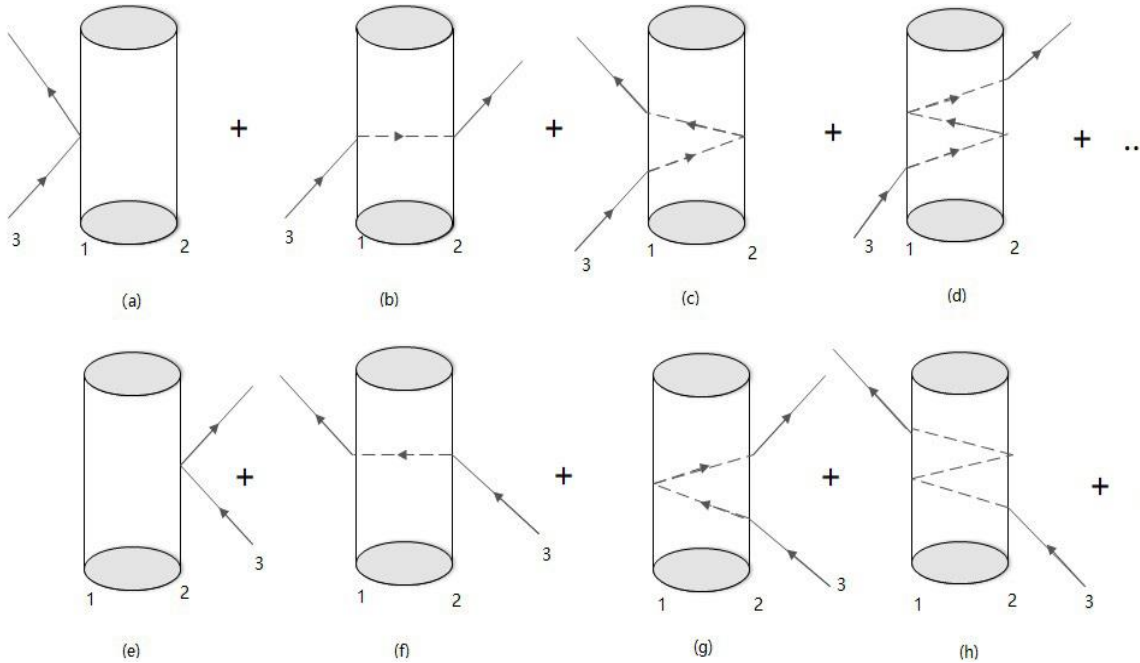
Sasa Prelovsek,^{a,b,c} Sara Collins,^a Daniel Mohler,^{d,e,f} M. Padmanath^{d,e} and Stefano Piemonte^a

JHEP 06 (2021) 035



ABSTRACT: We present the first lattice investigation of coupled-channel $D\bar{D}$ and $D_s\bar{D}_s$ scattering in the $J^{PC} = 0^{++}$ and 2^{++} channels. The scattering matrix for partial waves $l = 0, 2$ and isospin zero is determined using multiple volumes and inertial frames via Lüscher's formalism. Lattice QCD ensembles from the CLS consortium with $m_\pi \simeq 280$ MeV, $a \simeq 0.09$ fm and $L/a = 24, 32$ are utilized. The resulting scattering matrix suggests the existence of three charmonium-like states with $J^{PC} = 0^{++}$ in the energy region ranging from slightly below $2m_D$ up to 4.13 GeV. We find a so far unobserved $D\bar{D}$ bound state just below threshold and a $D\bar{D}$ resonance likely related to $\chi_{c0}(3860)$, which is believed to be $\chi_{c0}(2P)$. In addition, there is an indication for a narrow 0^{++} resonance just below the $D_s\bar{D}_s$ threshold with a large coupling to $D_s\bar{D}_s$ and a very small coupling to $D\bar{D}$. This

Fixed Center Approximation to Faddeev equations (FCA)



$$T_1 = \tilde{t}_1 + \tilde{t}_1 G_0 T_2$$

$$T_2 = \tilde{t}_2 + \tilde{t}_2 G_0 T_1$$

$$T = T_1 + T_2$$



$$T = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\tilde{t}_1\tilde{t}_2 G_0}{1 - \tilde{t}_1\tilde{t}_2 G_0^2}$$

$$G_0 = \frac{1}{2m_{cls}} \int \frac{d^3 \vec{q}}{(2\pi)^3} F_{cls}(q) \frac{1}{q^0{}^2 - \vec{q}^2 - m^2 + i\varepsilon}$$

$$F_{cls}(q) = \frac{1}{N} \int_{\substack{|\vec{p}| < \Lambda \\ |\vec{p} - \vec{q}| < \Lambda}} d^3 \vec{p} \frac{1}{m_{cls} - w_1(\vec{p}) - w_2(\vec{p})} \frac{1}{m_{cls} - w_1(\vec{p} - \vec{q}) - w_2(\vec{p} - \vec{q})} \\ \frac{1}{2w_1(\vec{p})} \frac{1}{2w_2(\vec{p})} \frac{1}{2w_1(\vec{p} - \vec{q})} \frac{1}{2w_2(\vec{p} - \vec{q})}$$

$$\tilde{t}_1 = \frac{m_{cls}}{m_1} t_1$$

$$\tilde{t}_2 = \frac{m_{cls}}{m_2} t_2$$

$$N = \int_{|\vec{p}| < \Lambda} d^3 \vec{p} \left(\frac{1}{2w_1(\vec{p})} \frac{1}{2w_2(\vec{p})} \frac{1}{m_{cls} - w_1(\vec{p}) - w_2(\vec{p})} \right)^2$$

$D\bar{D}K$ system in FCA

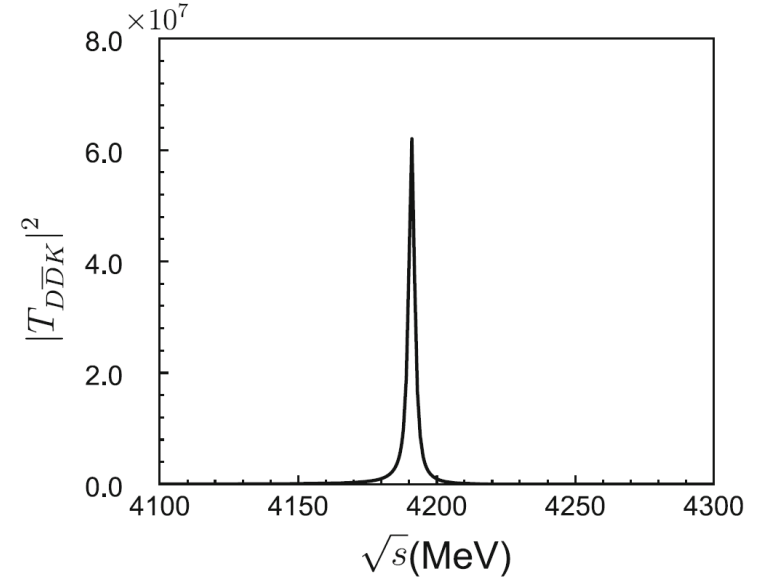
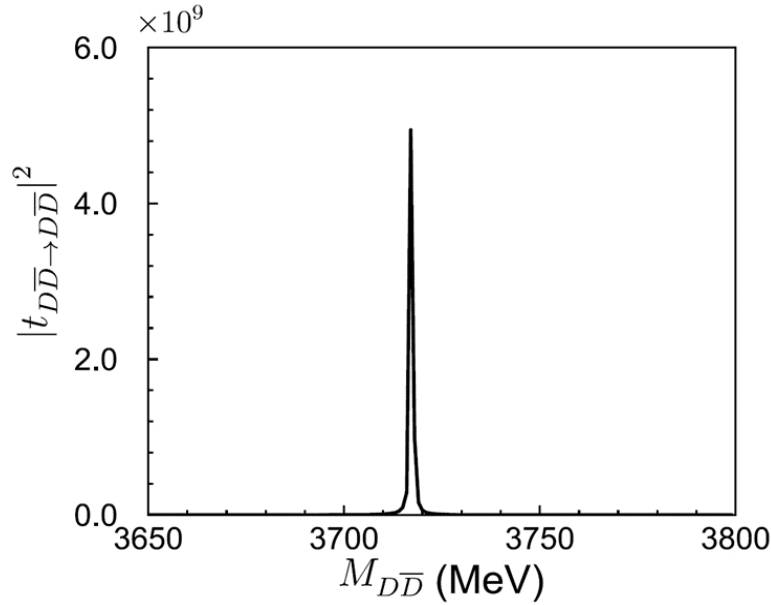


Fig. 5 Modulus squared of the total scattering amplitude of $D\bar{D}K$ three body system in isospin $I = 1/2$

Table 2 Obtained masses (in MeV) for $X(3720)$ and $K_{c\bar{c}}(4180)$ states with different Λ

Λ (MeV)	700	750	800	850	900	950	1000
$X(3720)$	3736	3732	3727	3721	3713	3703	3692
$K_{c\bar{c}}(4180)$	4194	4194	4193	4192	4189	4185	4181

Xiang Wei, Q.H. Shen, Ju-Jun Xie, Faddeev fixed-center approximation to the $\bar{D}DK$ system and the hidden charm $K_{c\bar{c}}(4180)$ state, Eur. Phys. J. C82, 718, (2022).

Excited K meson, $K_c(4180)$, with hidden charm as a $D\bar{D}K$ bound state

Tian-Wei Wu,¹ Ming-Zhu Liu,² and Li-Sheng Geng^{1,3,4,5,*}

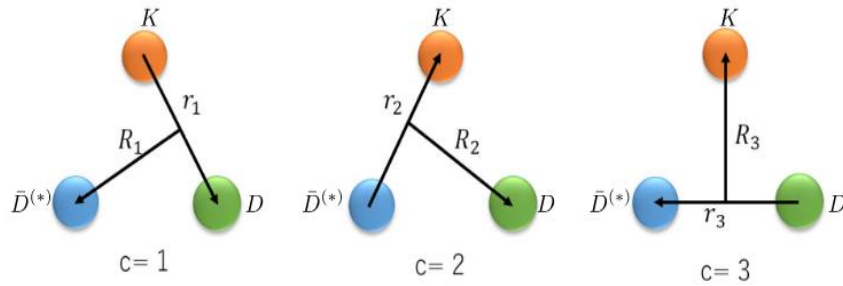


FIG. 1. Three permutations of the Jacobi coordinates for the $D\bar{D}^{(*)}K$ system.

$$\Psi = \sum_{c=1}^3 \Psi(\mathbf{r}_c, \mathbf{R}_c)$$

$$\Psi(\mathbf{r}_c, \mathbf{R}_c) = C_{c,\alpha} H_{t,T}^c \Phi_{lL,\lambda}(\mathbf{r}_c, \mathbf{R}_c)$$

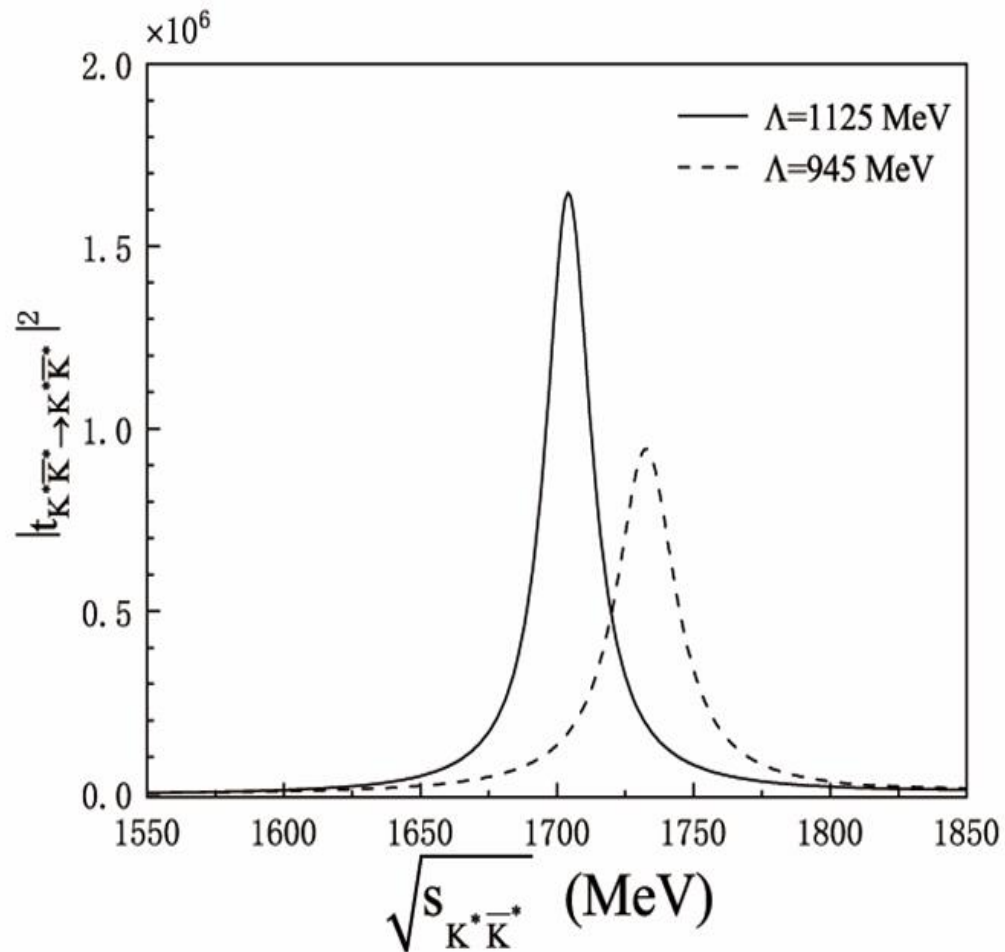
The $D\bar{D}K$ system is found to bind with quantum numbers $I(J^P) = \frac{1}{2}(0^-)$ and a three-body binding energy

$$B_3(D\bar{D}K) \simeq 48.9_{-2.4}^{+1.4} \text{ MeV}.$$

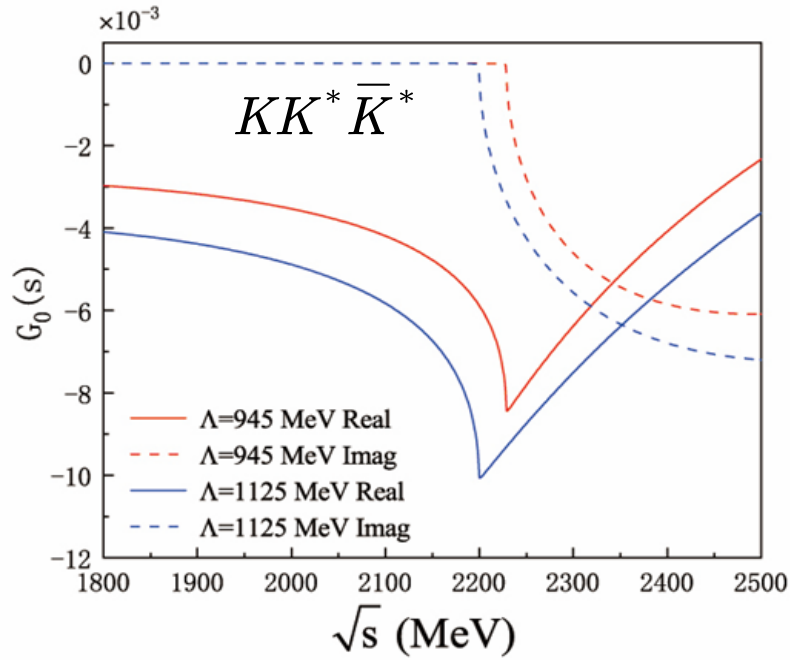
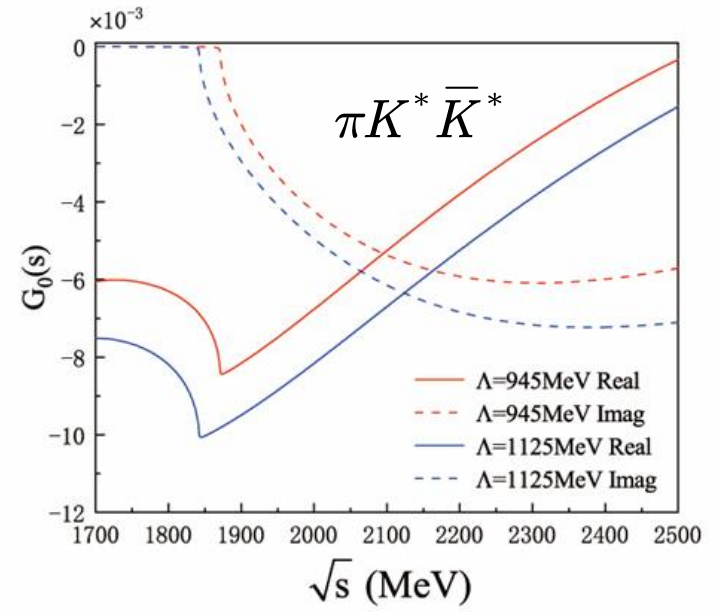
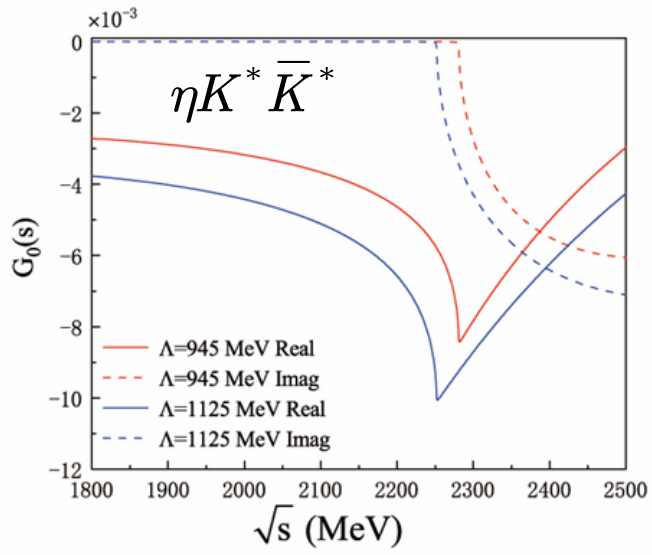
$$M_{K_{c\bar{c}}} = 4181.2_{-1.4}^{+2.4} \text{ MeV}$$

*Excited pseudoscalar states in $\pi\bar{K}^*K^*$, $\eta\bar{K}^*K^*$ and $K\bar{K}^*K^*$ systems*

$$\bar{K}^*K^* \rightarrow f_0(1710)$$

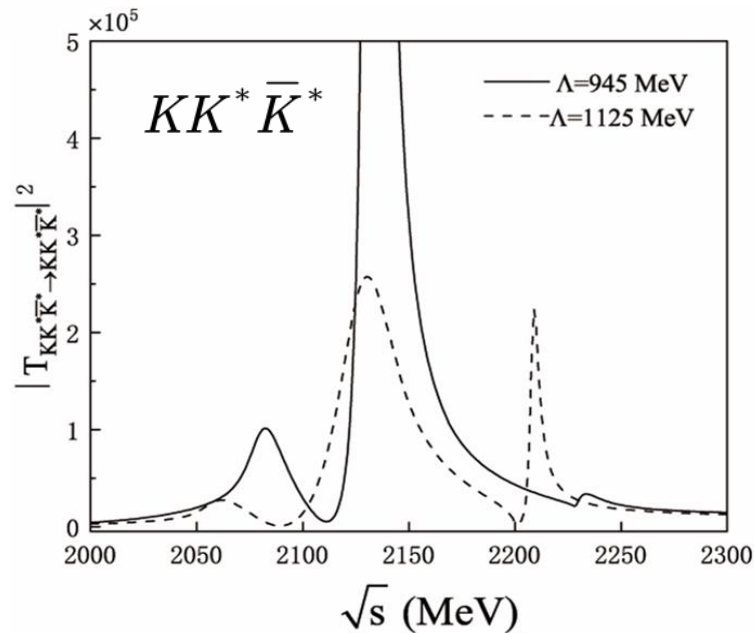
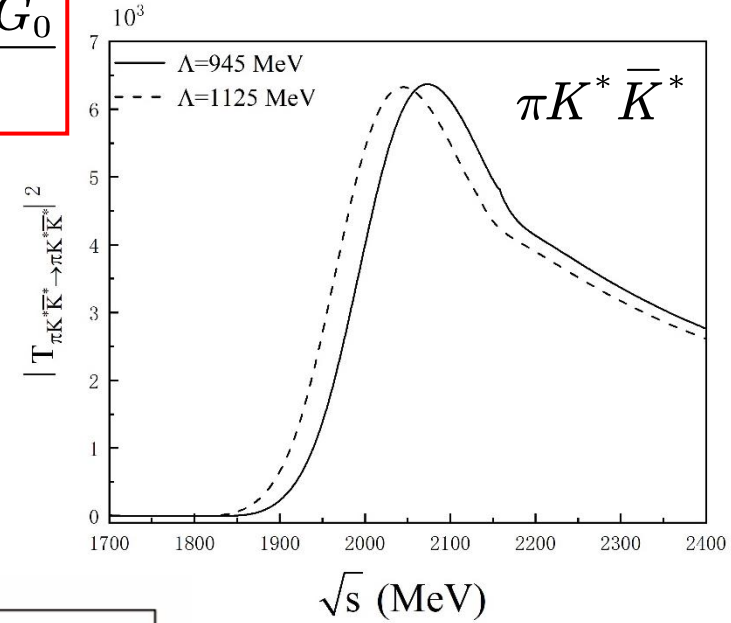
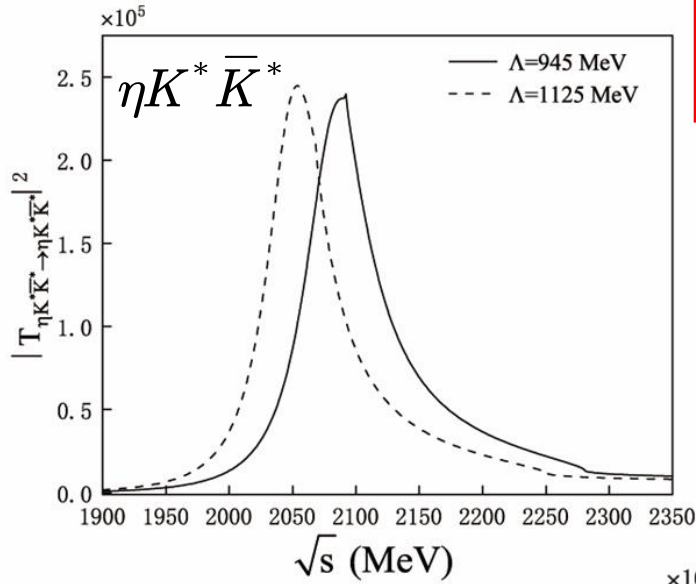


Q.H. Shen, and Ju-Jun Xie, Phys. Rev. D 107, 034019 (2023), arXiv:2211.04911 [hep-ph]



Three-body scattering amplitudes

$$T = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\tilde{t}_1\tilde{t}_2G_0}{1 - \tilde{t}_1\tilde{t}_2G_0^2}$$



Numerical results

TABLE: Peak positions of squared of three-body scattering amplitude corresponding to different value of cutoff parameter Λ . Unit in MeV.

Λ	855	945	1035	1125	1215	PDG
$M_{f_0(1710)}$	1747	1733	1718	1704	1690	1704 ± 12 (eva.) 1733^{+8}_{-7} (ave.)
$M_{\eta K^* \bar{K}^*}$	2104	2092	2070	2054	2038	2050^{+30+75}_{-24-26} 2103 ± 50 $\eta(2100)$
$M_{\pi K^* \bar{K}^*}$	2089	2073	2059	2045	2031	2070 ± 35 $\pi(2070)$
$M_{KK^* \bar{K}^*}$	State 1	2092	2083	2072	2062	2053 $2075 \pm 12 \pm 5$ $X(2075)$
	State 2	2140	2134	2130	2130	2134 ?
	State 3	■■■	2233	2219	2209	2211 ?

Qing-Hua Shen, and Ju-Jun Xie, Phys. Rev. D 107, 034019 (2023), arXiv:2211.04911 [hep-ph]

Exotic (quantum numbers) state $\pi_1(1600)$

$\pi_1(1600)$

$$I^G(J^{PC}) = 1^-(1^-+)$$

$\pi_1(1600)$ T-Matrix Pole $\sqrt{s}$

Note that $\Gamma = -2 \operatorname{Im}(\sqrt{s})$.

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
(1480–1680) – i (150–300) OUR ESTIMATE			
$(1623 \pm 47^{+24}_{-75}) - i (228 \pm 44^{+72}_{-88})$	¹ KOPF	21	RVUE 0.9 $p\bar{p} \rightarrow \pi^0\pi^0\eta$, $\pi^0\eta\eta$, $\pi^0K^+K^-$ and 191 $\pi^-p \rightarrow$ $\pi^-\pi^-\pi^+p$
$(1564 \pm 24 \pm 86) - i (246 \pm 27 \pm 51)$	² RODAS	19	RVUE 191 $\pi^-p \rightarrow \eta^{(\prime)}\pi^-p$

$\pi_1(1600)$ DECAY MODES

Mode	Fraction (Γ_i/Γ)
$\Gamma_1 \quad \pi\pi\pi$	seen
$\Gamma_2 \quad \rho^0\pi^-$	seen
$\Gamma_3 \quad f_2(1270)\pi^-$	not seen
$\Gamma_4 \quad b_1(1235)\pi$	seen
$\Gamma_5 \quad \eta'(958)\pi^-$	seen
$\Gamma_6 \quad \eta\pi$	seen
$\Gamma_7 \quad f_1(1285)\pi$	seen

Breit-Wigner mass and width of $\pi_1(1600)$

	Mass (MeV)	Width (MeV)
$\eta\pi$ mode	1354 ± 25	330 ± 35
non- $\eta\pi$ mode	1645^{+40}_{-17}	370^{+50}_{-60}

Mesons with quantum numbers $J^{PC} = 1^{-+}$

cannot be formed with a $q\bar{q}$ pair.

Hybrid?

Tetraquark?

$$\bar{K}K^* \implies J^{PC} = 1^{++} \quad f_1(1285)$$

$$\pi f_1(1285) \implies J^{PC} = 1^{-+} \quad ?$$

$\pi_1(1600) \longleftrightarrow$

On the other hand, from Fig. 5 we see that there is no any bump structure around $\sqrt{s} \sim 1400$ MeV, which can be assigned as the $\pi_1(1400)$ state. This may indicate that the $\pi_1(1400)$ can not be dynamically generated from the $\pi f_1(1285)$ interaction.



Citation: S. Navas *et al.* (Particle Data Group), Phys. Rev. D 110, 030001 (2024)

$\pi_1(1600)$

$$I^G(J^{PC}) = 1^-(1^{-+})$$

Coupled channel analyses favor the existence of only one broad 1^{-+} isovector state consistent with $\pi_1(1600)$ in the 1400–1600 MeV region. Accordingly, the $\pi_1(1400)$ entries of the previous Reviews have been moved into this section. See the review on "Spectroscopy of Light Meson Resonances."

$\pi\bar{K}K^*$ system

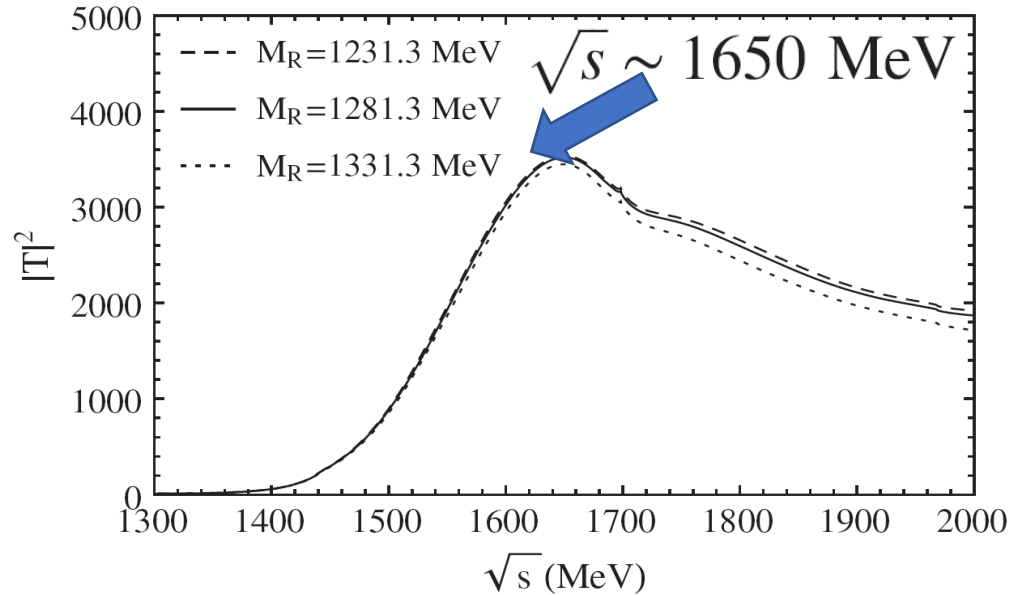


FIG. 5. Modulus squared of the $\pi\bar{K}K^*$ three-body scattering amplitude.

Xu Zhang, Ju-Jun Xie, and X.R. Chen, Phys. Rev. D 95, 056014 (2017).

$$\bar{K}K^* \Rightarrow J^{PC} = 1^{++} \quad f_1(1285)$$

$$\eta f_1(1285) \Rightarrow I^G J^{PC} = 0^+ 1^{-+} \quad ?$$

$$\eta K^* \Rightarrow I(J^P) = \frac{1}{2}(1^+) \quad K_1(1270)$$

$$\bar{K}K_1(1270) \Rightarrow I(J^P) = 0(1^-) \quad ?$$



Chinese Physics C Vol. 44, No. 5 (2020) 054104

Prediction of possible exotic states in the $\eta\bar{K}K^*$ system*

Xu Zhang(张旭)^{1,2;1)} Ju-Jun Xie(谢聚军)^{1,2,3;2)}

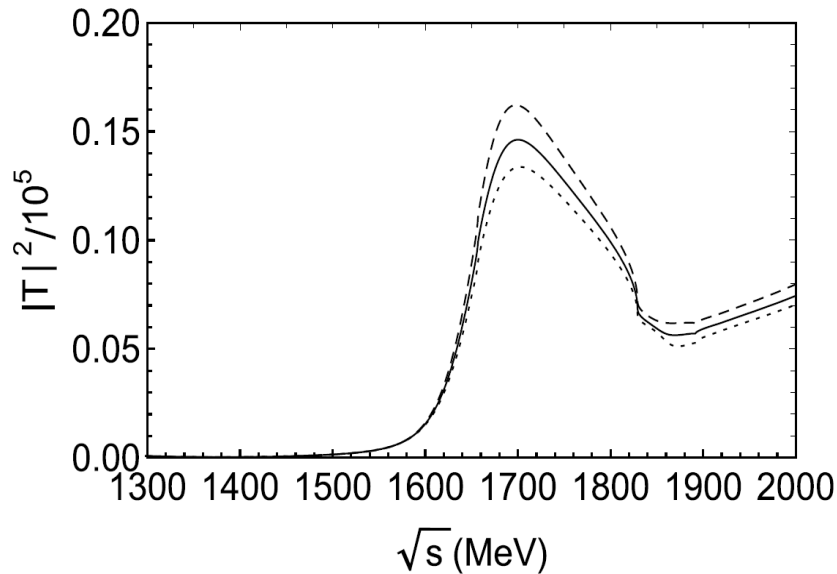


Fig. 7. Modulus squared of the total amplitude T for the η - $(\bar{K}K^*)_{f_1(1285)}$ system. The dashed, solid and dotted curves are for $\Lambda = 890, 990$, and 1090 MeV, respectively.

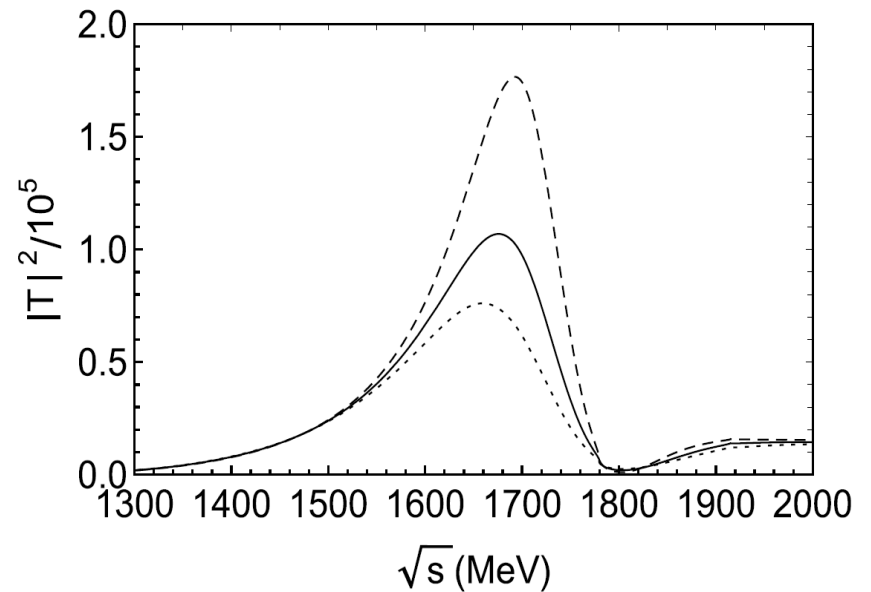


Fig. 8. Modulus squared of the total amplitude T for the $\bar{K}$ - $(\eta K^*)_{K_1(1270)}$ system. The dashed, solid and dotted curves are for $\Lambda = 900, 1000$, and 1100 MeV, respectively.

Three-body systems	Quantum numbers	Mass (MeV)	Width (MeV)
$\eta - (\bar{K}K^*)_{f_1(1285)}$	$I^G(J^{PC}) = 0^+(1^{-+})$	~ 1700	~ 180
$\bar{K} - (\eta K^*)_{K_1(1270)}$	$I(J^P) = 0(1^-)$	~ 1680	~ 160

Exotic state $\eta_1(1855)$

Observation of an Isoscalar Resonance with Exotic $J^{PC} = 1^{-+}$
Quantum Numbers in $J/\psi \rightarrow \gamma\eta\eta'$

(BESIII Collaboration)

PHYSICAL REVIEW LETTERS **129**, 192002 (2022)

Partial wave analysis of $J/\psi \rightarrow \gamma\eta\eta'$

M. Ablikim *et al.* (BESIII Collaboration)

Phys. Rev. D **106**, 072012 – Published 31 October 2022

$$M_{\eta_1(1855)} = 1855 \pm 9_{-1}^{+6} \text{ MeV}$$

$$\Gamma_{\eta_1(1855)} = 188 \pm 18_{-8}^{+3} \text{ MeV}$$

SCIENCE CHINA

Physics, Mechanics & Astronomy



June 2022 Vol. 65 No. 6: 261011

<https://doi.org/10.1007/s11433-022-1887-5>

Interpretation of the $\eta_1(1855)$ as a $K\bar{K}_1(1400) + \text{c.c.}$ molecule

Xiang-Kun Dong^{1,2}, Yong-Hui Lin³, and Bing-Song Zou^{1,2,4*}

Chinese Physics C Vol. 46, No. 5 (2022) 051001

Editors' Suggestion

Towards the establishment of the light $J^{P(C)} = 1^{-(+)}$ hybrid nonet*

Lin Qiu(邱林)^{1,2†} Qiang Zhao(赵强)^{1,2,3†}



universe

Article

On the $\eta_1(1855)$, $\pi_1(1400)$ and $\pi_1(1600)$ as Dynamically Generated States and Their SU(3) Partners

Mao-Jun Yan^{1,*}, Jorgivan M. Dias¹, Adolfo Guevara¹, Feng-Kun Guo^{1,2,3,*} and Bing-Song Zou^{1,2,4}

Chin. Phys. Lett. **39**, 051201 (2022)

QCD Axial Anomaly Enhances the $\eta\eta'$ Decay of the Hybrid Candidate $\eta_1(1855)$

Hua-Xing Chen(陈华星)^{1*}, Niu Su(苏妞)^{1*}, and Shi-Lin Zhu(朱世琳)^{2*}

Possible structure of the newly found exotic state $\eta_1(1855)$ Bing-Dong Wan^{1,2,†} Sheng-Qi Zhang^{2,‡} and Cong-Feng Qiao^{2,*}**TABLE I.** The continuum thresholds, Borel parameters, and predicted masses of tetraquark states.

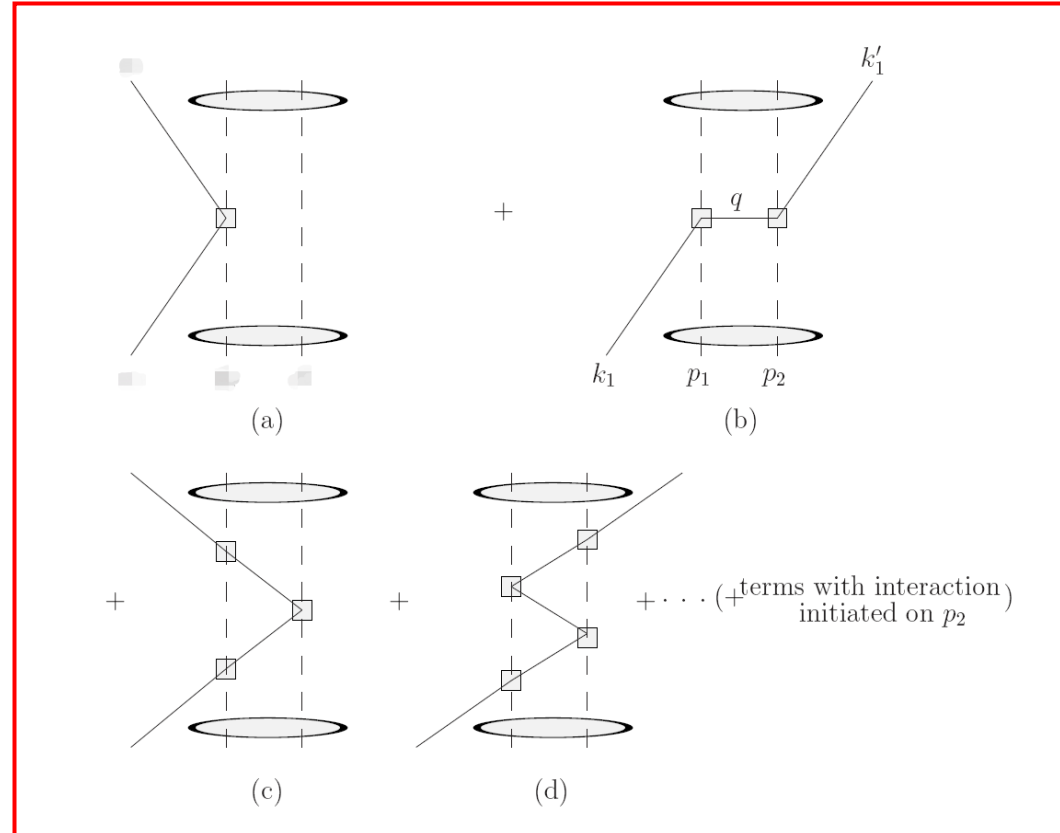
Configuration	Current	$\sqrt{s_0}$ (GeV)	M_B^2 (GeV ²)	M^X (GeV)
$[1_c]_{\bar{s}s} \otimes [1_c]_{\bar{q}q}$	<i>A</i>	2.2 ± 0.1	1.1–1.6	1.87 ± 0.08
	<i>B</i>	2.1 ± 0.1	1.1–1.6	1.75 ± 0.08
$[1_c]_{\bar{s}q} \otimes [1_c]_{\bar{q}s}$	<i>C</i>	2.4 ± 0.1	1.3–1.8	2.05 ± 0.07
	<i>D</i>	2.1 ± 0.1	1.2–1.7	1.63 ± 0.12
$[1_c]_{\bar{s}s} \otimes [1_c]_{\bar{s}s}$	<i>A</i>	2.5 ± 0.1	1.3–1.9	2.14 ± 0.07
	<i>D</i>	2.2 ± 0.1	1.3–1.8	1.71 ± 0.11

Radiative Decay Width of $J/\psi \rightarrow \gamma \eta_{(2)}$ from $N_f=2$ Lattice QCDXiangyu Jiang^{1,2,*} Feiyu Chen^{1,2} Ying Chen^{1,2,†} Ming Gong^{1,2} Ning Li³ Zhaofeng Liu^{1,2,4}
Wei Sun¹ and Renqiang Zhang^{1,2} **1^-+ hybrid meson in J/ψ radiative decays from lattice QCD**Feiyu Chen^{1,2,*} Xiangyu Jiang^{1,2,†} Ying Chen^{1,2,‡} Ming Gong^{1,2}
Zhaofeng Liu^{1,2,3} Chunjiang Shi^{1,2} and Wei Sun¹

Summary

We conclude that

The FCA is an effective and technically straightforward tool for studying three-body systems!



And,

exotic hadron states are made of *hadrons*.

Thank you very much for your attention!