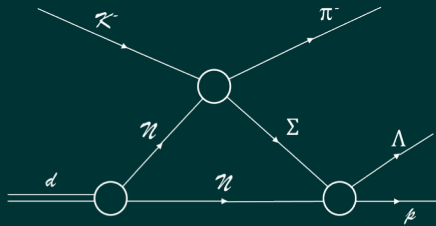


Femtoscscopy and Cusp: New Methods to Study the Hadron-Hadron Interaction

YUDAI ICHIKAWA (TOHOKU UNIVERSITY)

NEW PERSPECTIVES ON EXOTIC AND HEAVY HADRONS, 2026/08/25-28

1. Reaction mechanism



2. Theoretical formula

$$\Gamma(\bar{K}d \rightarrow \pi\Sigma) \sim \Gamma(\bar{K}n \rightarrow \pi\Sigma) \mathcal{F}_d(q, k_\Sigma) \Gamma(\Sigma n \rightarrow \Lambda n)$$

$$\mathcal{F}_d(q, k_\Sigma) = \int d^3r \{ \sin(qr) | \langle qn \rangle \} \{ \exp(ik_\Sigma r) | r \} \psi^2 dr$$

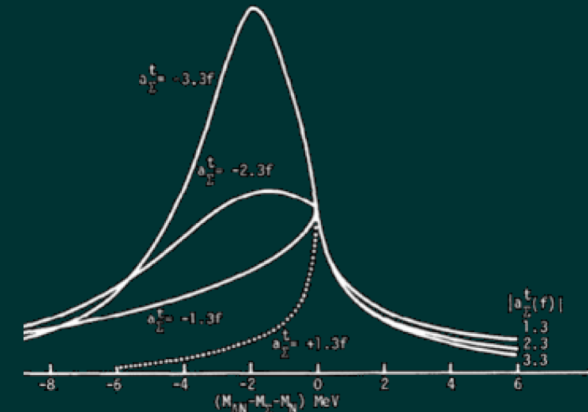
$$\Gamma(\Sigma n \rightarrow \Lambda n) = b_\Sigma / (1 - ik_\Sigma \mathcal{A}_\Sigma), \quad \mathcal{A}_\Sigma = a - ib$$

$$\text{Above threshold: } \mathcal{R}_i = 4\pi b \{ (1 + k_\Sigma a)^2 + (k_\Sigma b)^2 \},$$

$$\text{Below threshold: } \mathcal{R}_i = 4\pi b \{ (1 + |k_\Sigma| a)^2 + k_\Sigma^2 b^2 \}$$

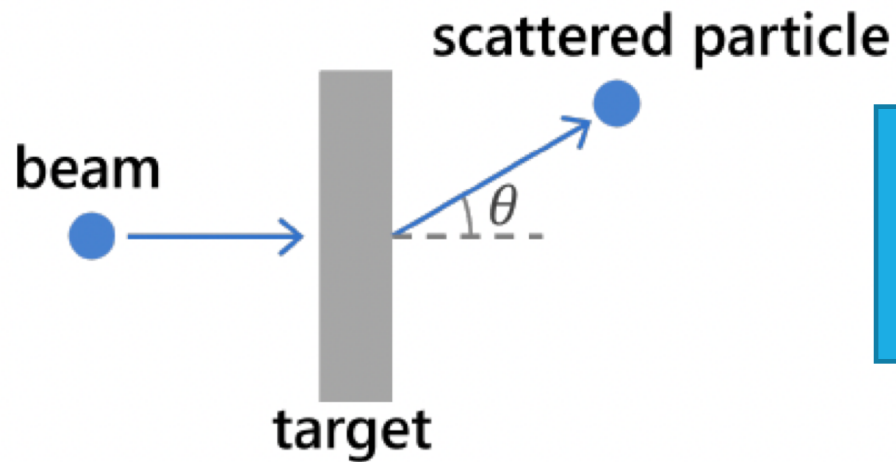
$$(k_\Sigma = i|k_\Sigma|)$$

3. Calculated spectrum



SCATTERING EXPERIMENT: CONVENTIONAL TOOL TO STUDY THE HADRON INTERACTION

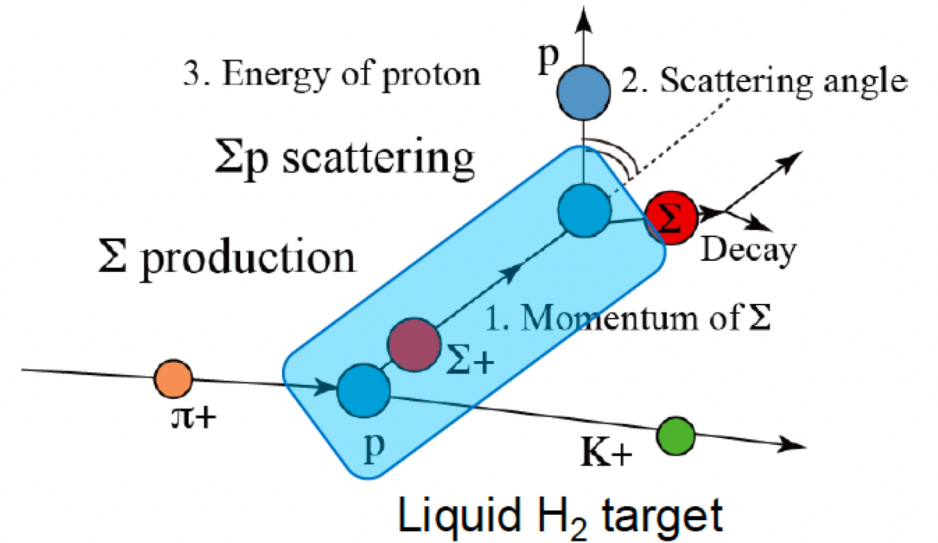
Ordinary, hadron interaction is studied by the scattering experiment.
 pp , pn , πp , Kp (Limited channels)



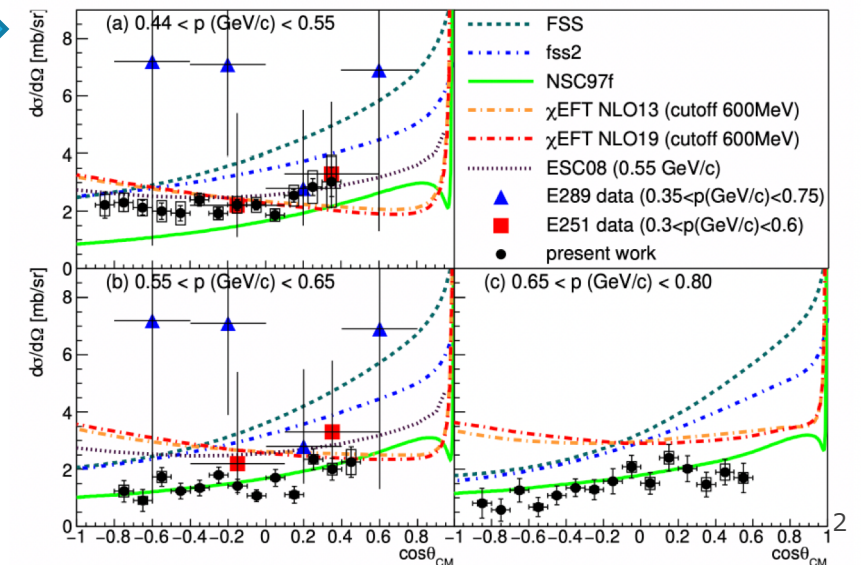
$$\frac{d\sigma}{d\Omega} = \frac{1}{4k^2} \left| \sum_{l=0}^{\infty} (2l+1)(S_l - 1)P_l(\cos\theta) \right|^2$$

$$= \frac{1}{k^2} \left| \sum_{l=0}^{\infty} (2l+1)e^{i\delta_l} \sin \delta_l P_l(\cos\theta) \right|^2$$

J-PARC E40
YN scattering



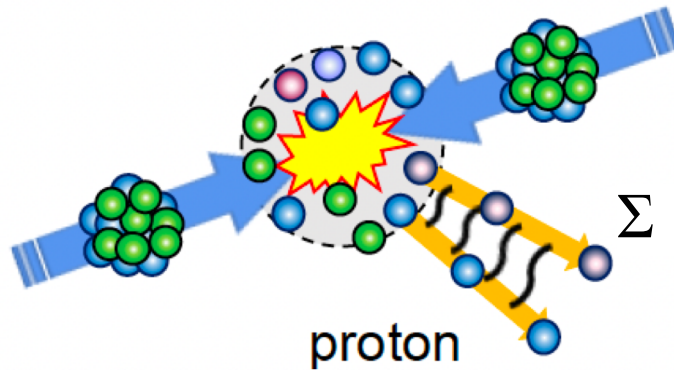
T. Nanamura et al., Prog. Theor. Exp. Phys. **2022** 093D01



FEMTOSCOPY: NEW TOOL TO STUDY THE HADRON INTERACTION

Open new era for the hadron-hadron interaction!!

Experimental progress



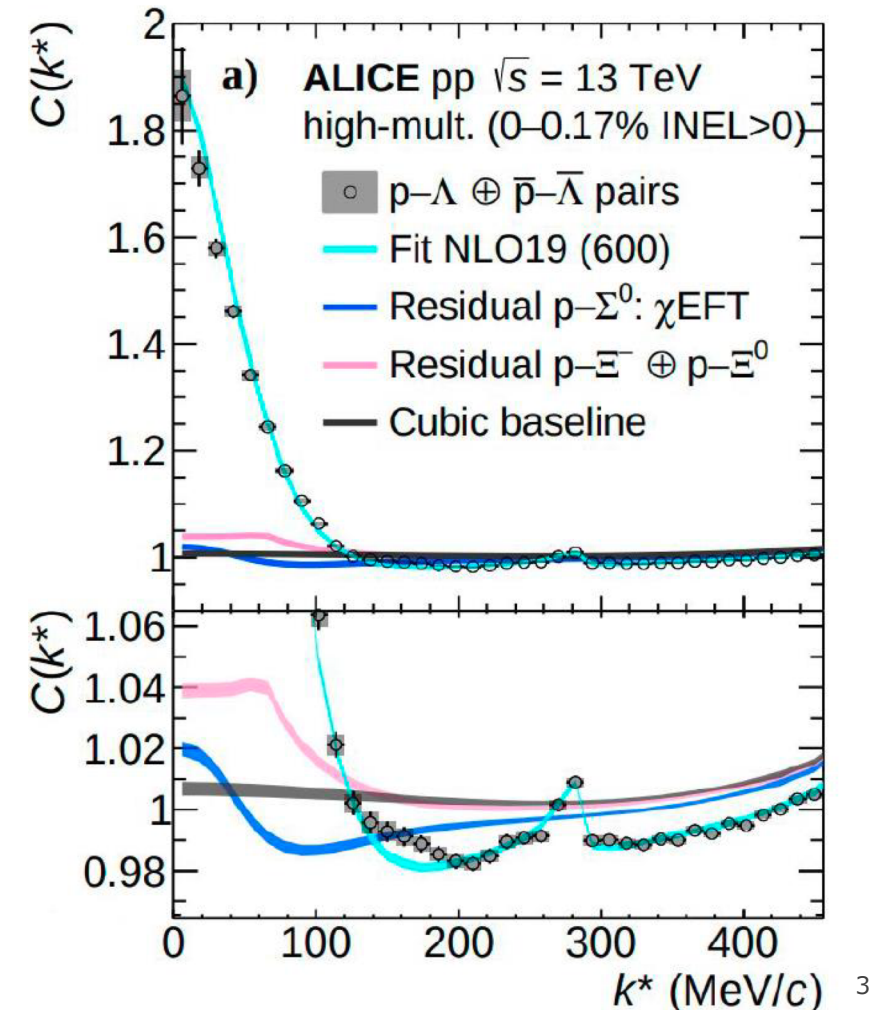
BB interaction by femtoscopy

$$c(k^*) = \int S(\vec{r}^*) |\Psi(\vec{k}^*, \vec{r}^*)|^2 d^3r^*$$

Fix **source size**($S(\vec{r}^*)$) $\rightarrow$

Study interaction from **wave function** ($\Psi(\vec{k}^*, \vec{r}^*)$)

ALICE Collaboration,
Phys. Lett. B 833
(2022) 137272



CURRENT ISSUES OF FEMTOSCOPY

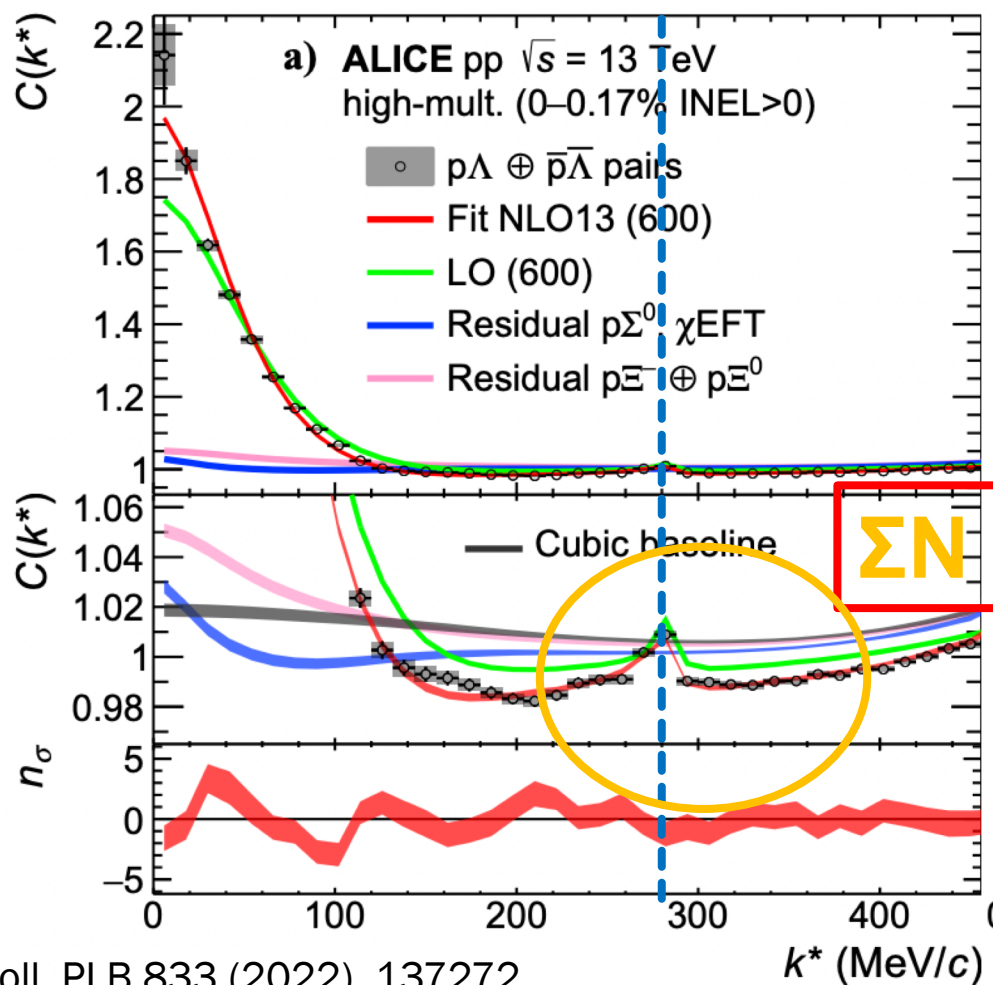
- Femtoscopy → Isospin and **spin averaged** interaction (f_0 , d_0)
- CF corresponds to **total cross section (σ)** in the scattering experiment.
 - Scattering experiment:
PWA to decompose them using $d\sigma/d\Omega$ and spin observables
 - Can CF(Femtoscopy) be decomposed with spin observables?
→ Y. Ichikawa *et al.*, J. Subatomic Part. Cosmol. 3, 100040 (2025).
- **Neutral hadrons** (ex: η , η') are difficult to study (measure).

→ Cusp spectroscopy is complementary to compensate these issues.

CUSP STRUCTURE IN FEMTOSCOPY

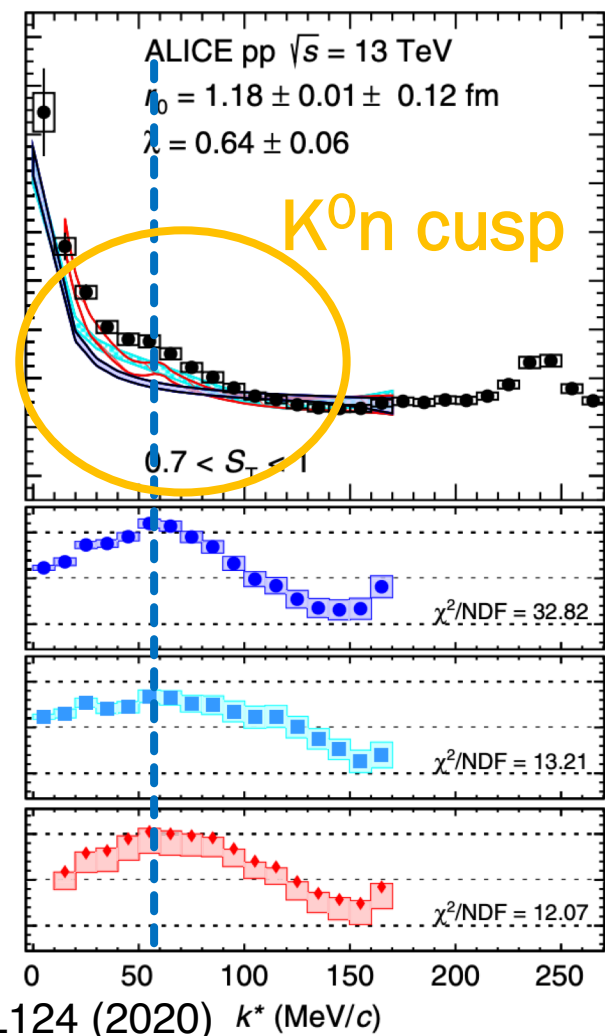
Λp

ΣN threshold



$K^- p$

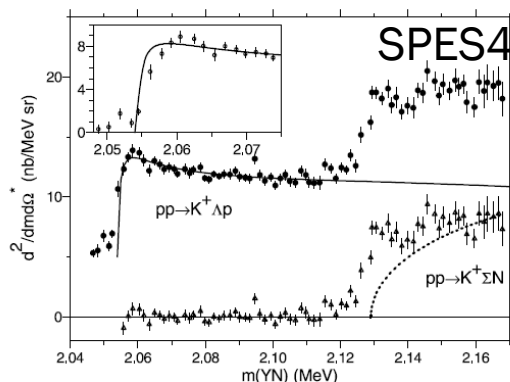
$K^0 n$ threshold



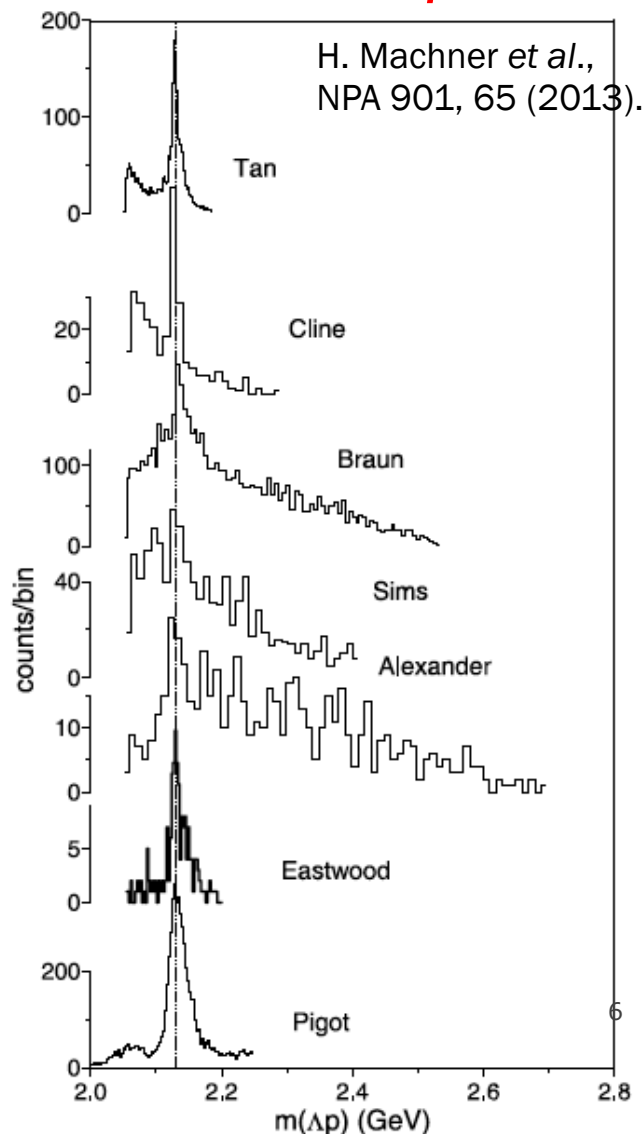
"ΣN CUSP"

Clear enhancement
around ΣN threshold
(~2.13 GeV/c²)

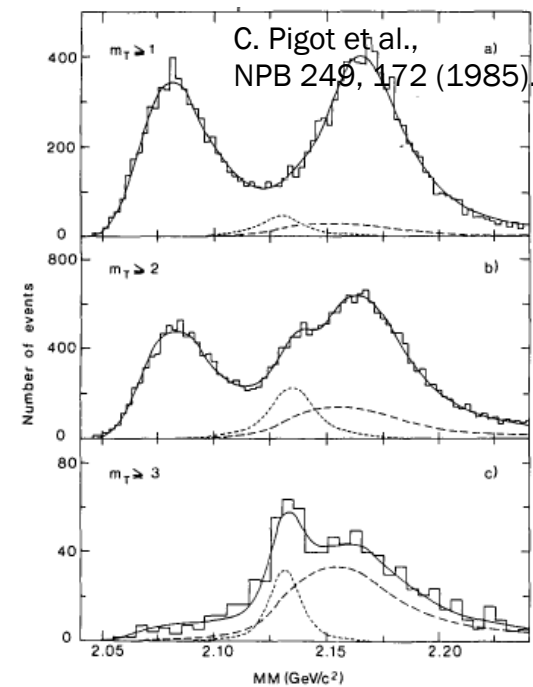
$pp \rightarrow \Lambda p K^+$



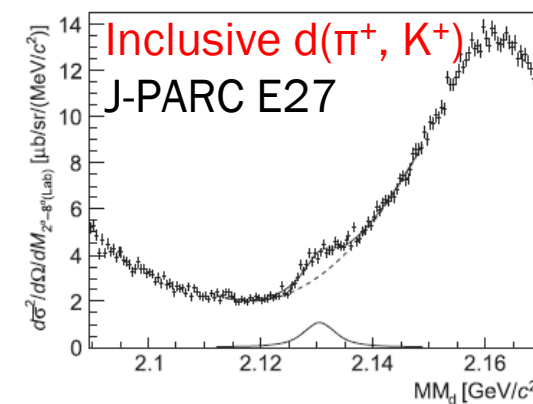
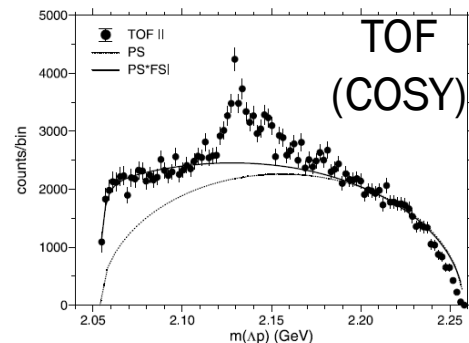
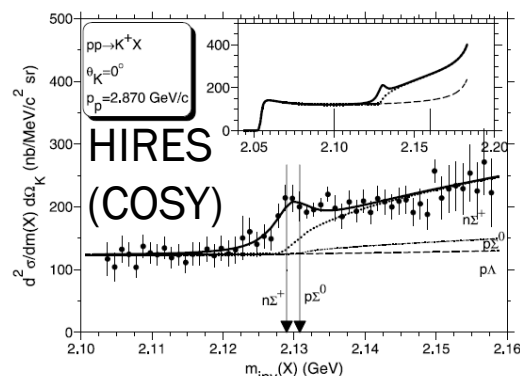
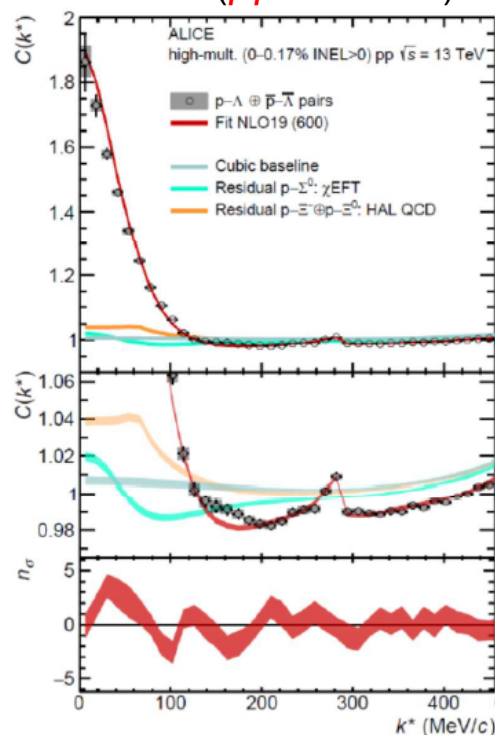
$K^-d \rightarrow \Lambda p \pi$



$\pi^+d \rightarrow \Lambda p K^+$



ALICE(pp@13TeV)



J-PARC E90 EXPERIMENT: “ Σ N CUSP SPECTROSCOPY”

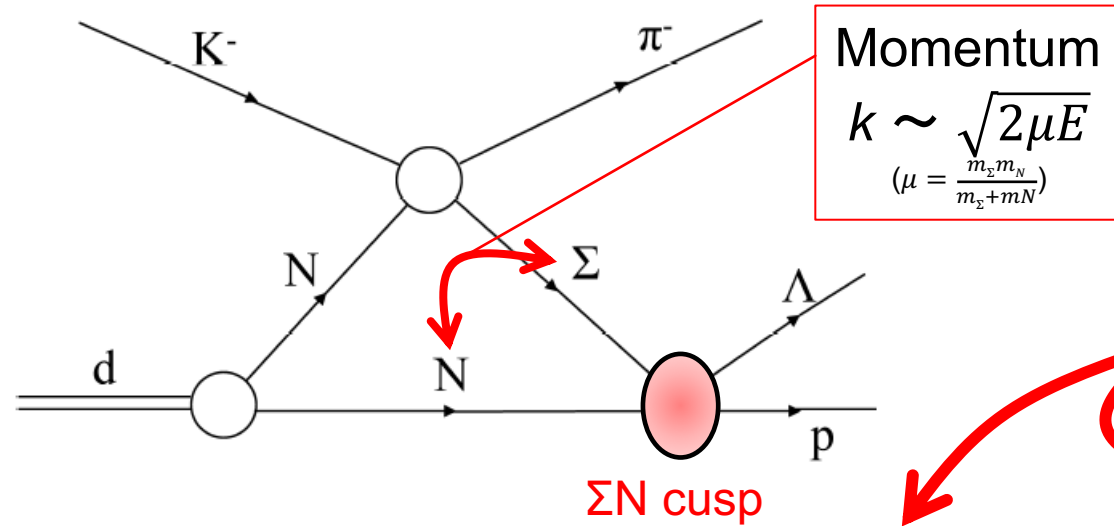
- Deduce the Σ N scattering length of $(T, S) = (1/2, 1)$ channel by fitting “ Σ N cusp” spectrum shape observed in the missing mass of the $d(K^-, \pi^-)$ reaction.
 - Unstable bound state (Σ N dibaryon)? or Virtual state?

The key of this experiment is the excellent missing-mass resolution thanks to the S-2S spectrometer (used in E70) and high statistics.

We will be able to achieve the best resolution of 0.4 MeV in σ , which is two times better than the past experiment (HIRES at COSY).

ΣN CUSP AND ΣN SCATTERING LENGTH

$$A_\Sigma = \underline{a} + i\underline{b}$$



ΣN interaction strength
Attractive, repulsive

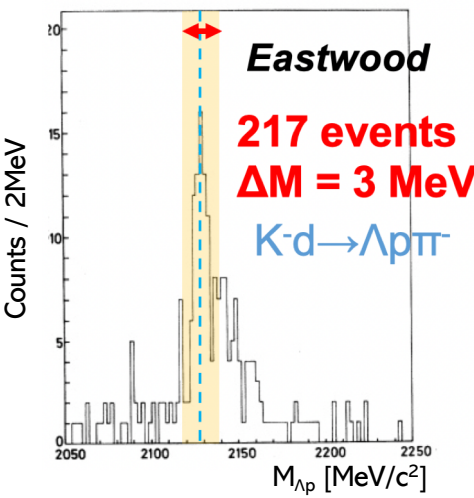
ΣN ↔ ΛN
coupling strength

dσ/dE can be described by A_Σ

$$\left[\begin{array}{l} [E < \text{Threshold}] \, d\sigma/dE \propto \frac{4\pi b}{\{(1+k\underline{b})^2 + (k\underline{a})^2\}} \\ [E > \text{Threshold}] \, d\sigma/dE \propto \frac{4\pi b}{\{(1+|ik\underline{a}|)^2 + (-ik\underline{b})^2\}} \end{array} \right]$$

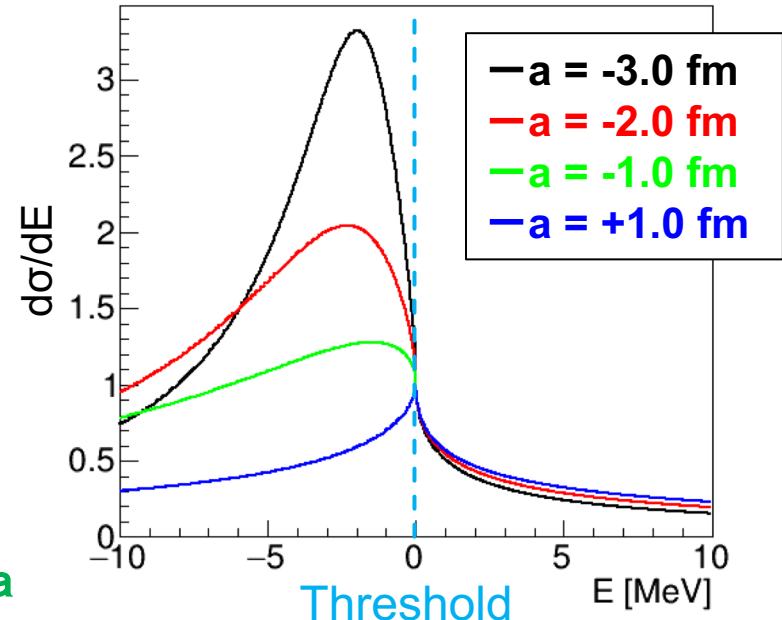
Cusp formula by Dalitz

ΣN threshold

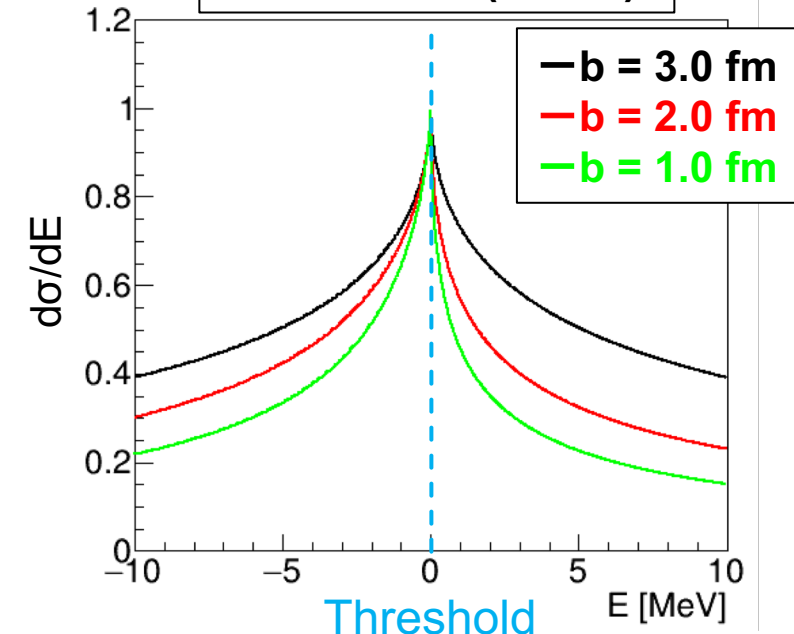


Bubble chamber data

b = 2.0 fm (Fixed)

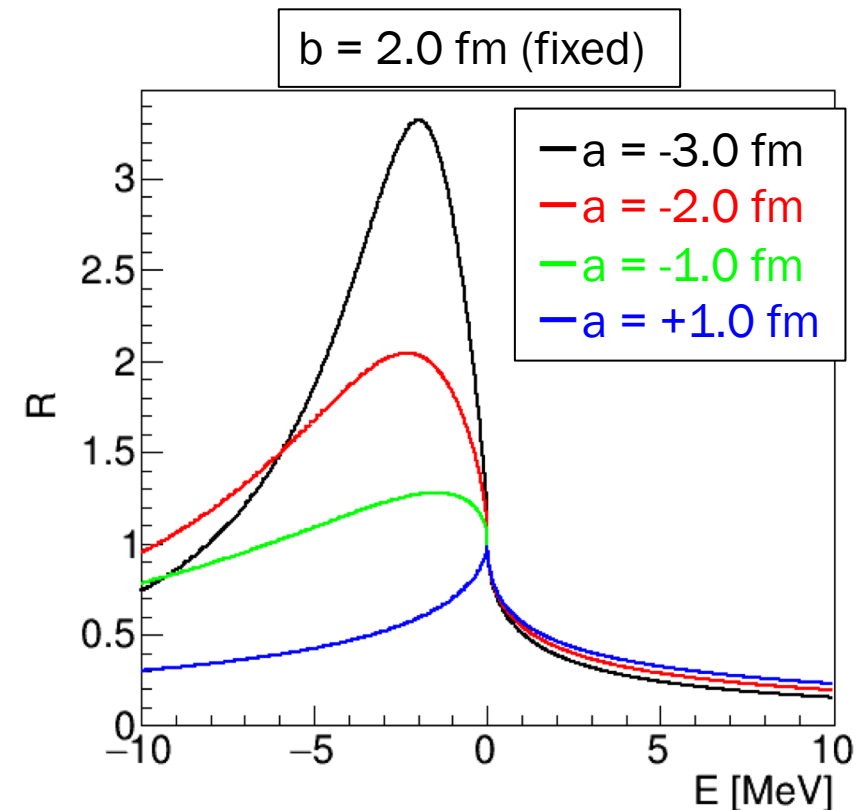


a = +1.0 fm (Fixed)



“ ΣN CUSP”

- “ ΣN cusp” is measured by $K^-d \rightarrow \pi^- \Lambda p$ reaction etc..
 - T: $T = 1/2$ (Λp final state)
 - S: 3S_1 is favored, D-target; observed in forward angles



Above threshold: $R = \frac{4\pi b}{\{(1+kb)^2 + (ka)^2\}} \sim 1 - 2kb + O(k^2)$

Below threshold: $R = \frac{4\pi b}{\{(1+\kappa a)^2 + (\kappa b)^2\}} \sim 1 - 2\kappa a + O(\kappa^2)$, $k = i\kappa$
($k \sim \sqrt{2\mu E}$)

“ ΣN Cusp” can be expressed by the ΣN scattering length ($A_\Sigma = a + ib$) of the $(T, S) = (1/2, ^3S_1)$ channel!!

$\Sigma N(T=1/2, {}^3S_1)$ SCATTERING LENGTH (THEORY)

Model	J04	J04c	J-A	NSC 97f	NSC 89	ND	NF	NB
a [fm]	3.83	3.63	-2.37	-1.03	2.54	2.06	-1.29	-3.0
b [fm]	3.01	3.09	3.74	2.41	0.26	4.64	3.02	1.8

Model	chiral EFT (NLO13)				chiral EFT (NLO19)			
Λ [MeV]	500	550	600	650	500	550	600	650
a [fm]	-2.61	-2.44	-2.27	-2.06	-0.95	-0.98	-2.29	-1.95
b [fm]	2.89	3.11	3.29	3.59	4.77	4.59	3.39	3.38

$$A_\Sigma = a + ib$$

$a > 0$: Attractive

$a < 0$: Bound state

Large ambiguity!!

THRESHOLD CUSP

Cusp structure can be expressed by the **scattering length** (for B'C'), $A = a + ib$

■ B'C' → BC amplitude

$$f_{B'C',BC} \sim \frac{\sqrt{b}}{1 - ikA}, \text{ Pole position: } k \sim -\frac{i}{A}$$

■ (Two body scattering amplitude)

$$f = \frac{1}{k} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin \delta_l P_l(\cos \theta)$$

$$\rightarrow (\text{s-wave}) \quad f = \frac{1}{k} e^{i\delta_0} \sin \delta_0 = \frac{1}{k \cot \delta - ik} \rightarrow \frac{a}{1 - ika}$$

$$k \cot \delta = \frac{1}{a} - \left(\frac{r_{eff}}{2} \right) k^2 + \dots$$

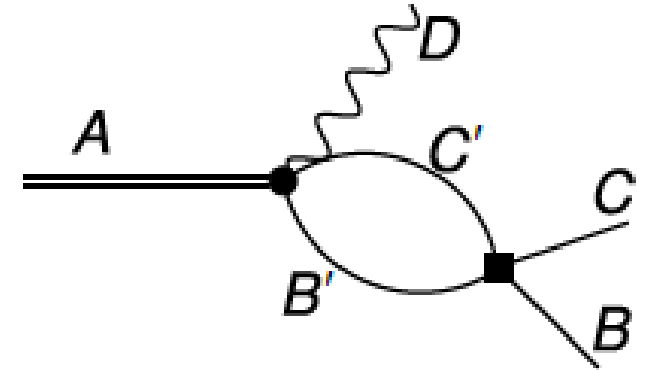
■ Reaction rate (R) : $\frac{d\sigma^2}{d\Omega dE} \propto |f_{B'C',BC}|^2$

$$\text{Above threshold: } R = \frac{4\pi b}{\{(1 + kb)^2 + (ka)^2\}}$$

$$\text{Below threshold: } R = \frac{4\pi b}{\{(1 + \kappa a)^2 + (\kappa b)^2\}}, \quad k = i\kappa \text{ (due to analytic continuation)}$$

$$\text{Reduced mass} \\ \mu = m_{B'} m_{C'} / (m_{B'} + m_{C'})$$

$$k(\text{relative momentum for B'C'}) \sim \sqrt{2\mu E}$$



For the “ ΣN cusp”,
B' = Σ , C' = N, B = Λ , C = N

Scattering amplitude with Square-well potential

(Square-well potential w complex term)

$$V(r) = \begin{cases} -V_0, & r < R \\ 0, & r > R \end{cases} \quad (V_0 = \text{Re } V_0 + i \text{Im } V_0) \quad (\text{Re } V_0 > 0, \text{Im } V_0 > 0)$$

S-wave Schrödinger equation

$$\left[-\frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial r^2} + V(r) \right] u = Eu \quad \therefore u(r) = \begin{cases} A \sin \kappa r & (r < R) \\ B \sin(\kappa r + \delta) & (r > R) \end{cases}$$

Boundary conditions at $r = R$

$$\begin{cases} A \sin \kappa R = B \sin(\kappa R + \delta) \\ A \kappa \cos \kappa R = B k \cos(\kappa R + \delta) \end{cases}$$

$$f_0 = R \left(\frac{\tan x}{x} - 1 \right) \quad , \quad d_0 = R - \frac{R^3}{3f_0^2} + \frac{R^2}{x^2 f_0} \quad (x = qR = \frac{\sqrt{2mV_0}}{\hbar} R)$$

Scattering amplitude

$$f(k) = \frac{1}{k \cot \delta - ik} = \frac{1}{\frac{1}{f_0} + \frac{1}{2} d_0 k^2 - ik} \quad \rightarrow \rightarrow \rightarrow \quad \psi_k(r) \simeq e^{ikr} + f(k) \frac{e^{ikr}}{r}$$

Scattering length and effective range have complex term

$$(f_0 = \text{Re } f_0 + i \text{Im } f_0, \quad d_0 = \text{Re } d_0 + i \text{Im } d_0)$$

S-Matrix: amplitude ratio

Incident wave / Scattered wave

$$u(r) = B \sin(\kappa r + \delta) = \frac{B}{2i} (e^{i(\kappa r + \delta)} - e^{-i(\kappa r + \delta)}) = \frac{B}{2i} (e^{i\delta} e^{i\kappa r} - e^{-i\delta} e^{-i\kappa r})$$

$$S = \frac{e^{i\delta}}{e^{-i\delta}} = e^{2i\delta} = e^{2i\text{Re}\delta} e^{-2\text{Im}\delta} \quad \therefore |S| < 1$$

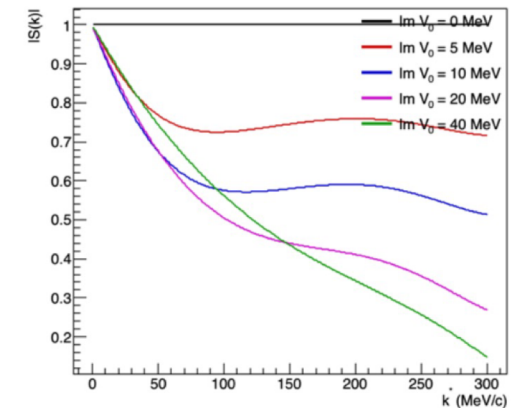
attenuation

$$u(r) = B \sin(\kappa r + \delta) = u_k(r) = \frac{B}{2i} [S(k) e^{+i\kappa r} - e^{-i\kappa r}]$$

Boundary conditions at $r = R$

$$\begin{cases} A \sin \kappa R = \frac{B}{2i} [S(k) e^{+i\kappa R} - e^{-i\kappa R}] \\ A \kappa \cos \kappa R = \frac{B k}{2} [S(k) e^{+i\kappa R} + e^{-i\kappa R}] \end{cases}$$

$$\therefore S(k) = e^{-2i\kappa R} \frac{\kappa \cot \kappa R + ik}{\kappa \cot \kappa R - ik}$$



$\text{Re } V_0 = 20 \text{ MeV}, R = 2.5 \text{ fm}$ の時の虚部による $|S|$ の変化

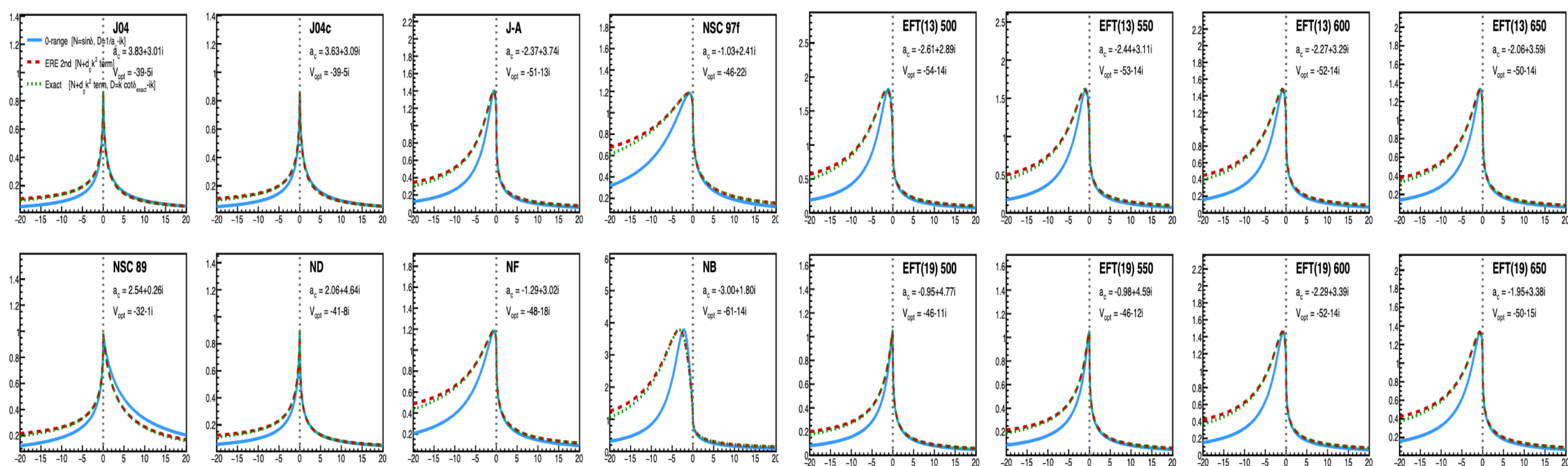
Scattering parameters
↔ Square-well potential

We can also calculate cusp from $k \cot \delta$ directly.

Results with square-well potential with fixed $R = 1.4$ fm (π exchange)

Model	J04	J04c	J-A	NSC 97f	NSC 89	ND	NF	NB
a [fm]	3.83	3.63	-2.37	-1.03	2.54	2.06	-1.29	-3.00
b [fm]	3.01	3.09	3.74	2.41	0.26	4.64	3.02	1.80
Re(V_opt) [MeV]	-38.9	-38.9	-51.0	-45.9	-32.3	-41.4	-47.7	-61.4
Im(V_opt) [MeV]	-4.8	-5.1	-12.7	-22.0	-1.0	-7.8	-17.5	-14.0
Re(d0) [fm]	1.53	1.53	1.30	1.28	1.30	1.46	1.31	1.20
Im(d0) [fm]	-0.10	-0.11	-0.15	-0.28	0.02	-0.14	-0.22	-0.12

Model	EFT(13) 500	EFT(13) 550	EFT(13) 600	EFT(13) 650	EFT(19) 500	EFT(19) 550	EFT(19) 600	EFT(19) 650
a [fm]	-2.61	-2.44	-2.27	-2.06	-0.95	-0.98	-2.29	-1.95
b [fm]	2.89	3.11	3.29	3.59	4.77	4.59	3.39	3.38
Re(V_opt) [MeV]	-54.2	-52.9	-51.7	-50.3	-46.3	-46.3	-51.5	-50.4
Im(V_opt) [MeV]	-14.5	-14.4	-14.4	-13.8	-11.1	-11.5	-14.0	-14.8
Re(d0) [fm]	1.42	1.42	1.42	1.42	1.46	1.46	1.46	1.46
Im(d0) [fm]	-0.16	-0.16	-0.16	-0.16	-0.18	-0.18	-0.18	-0.18

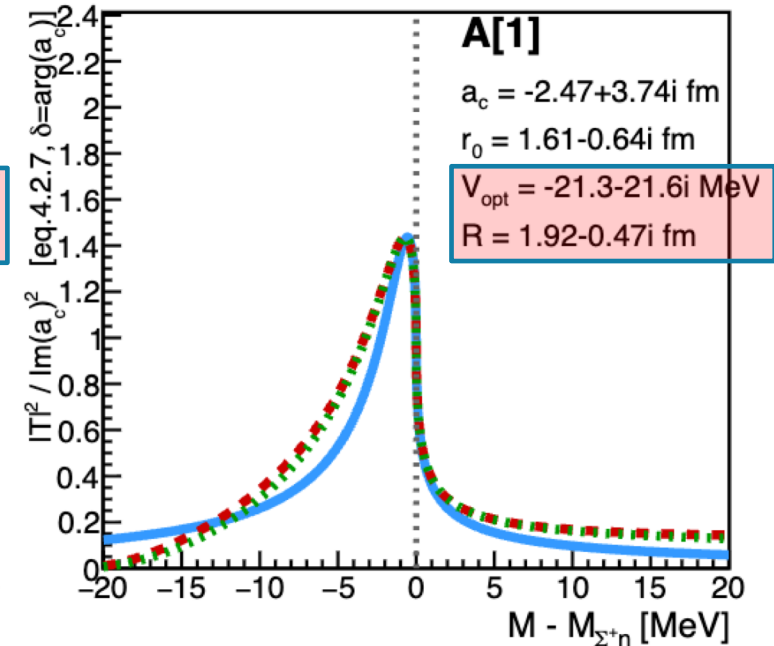
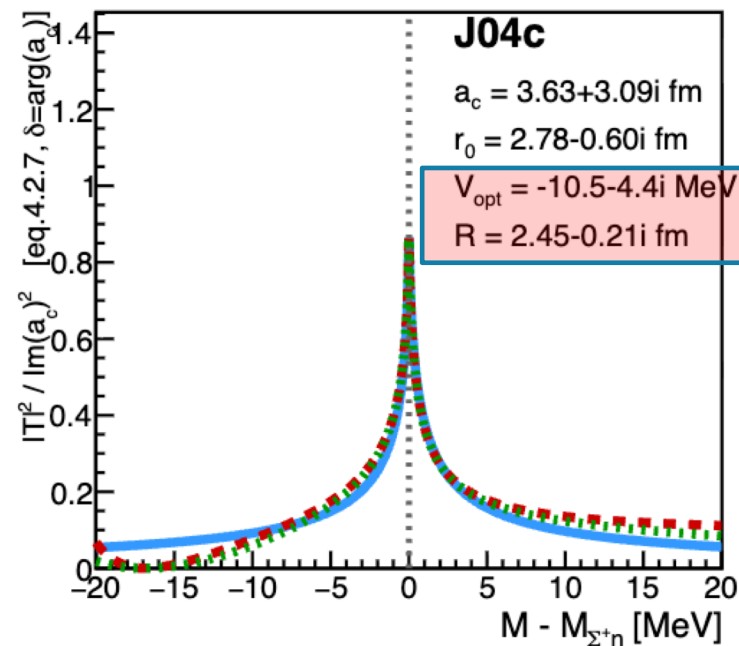
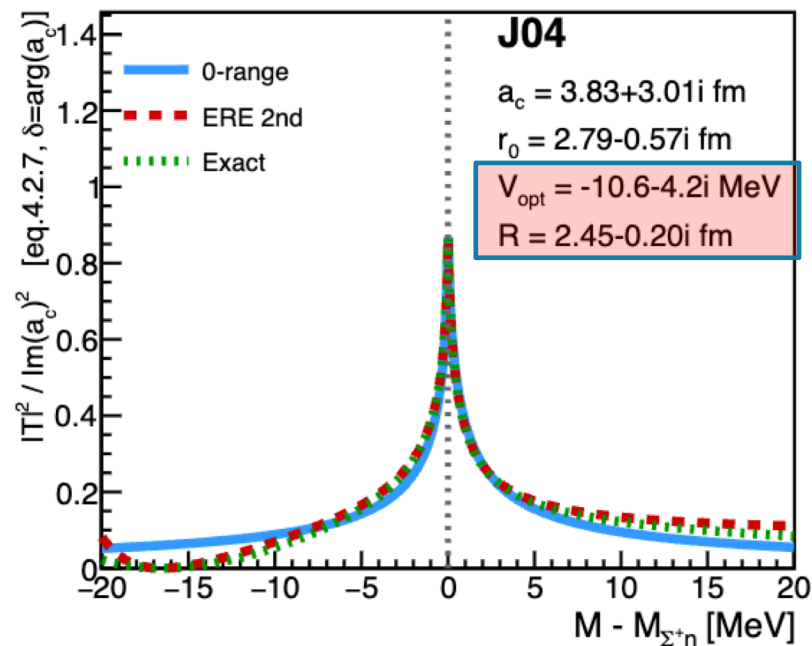


Results using Squared-well Potentials with Effective-range input

Channel	Model	$a_t(\text{fm})$	$r_t(\text{fm})$
ΛN	J04	-1.66	2.93
	J04c	-1.57	3.08
	A [1]	-1.59	3.16

$\Sigma N(I = 1/2)$	J04	$-3.83 - i3.01$	$2.79 - i0.57$
	J04c	$-3.63 - i3.09$	$2.78 - i0.60$
	A [1]	$2.47 - i3.74$	$1.61 - i0.64$

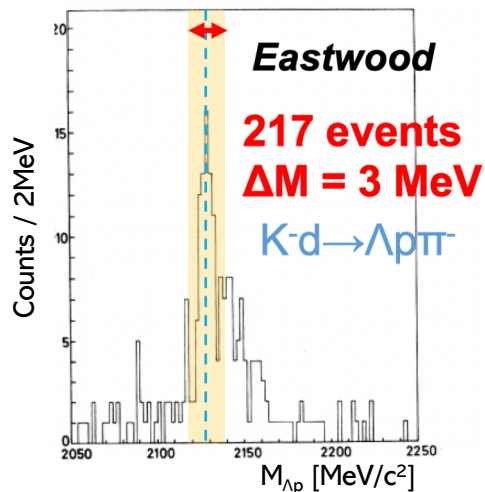
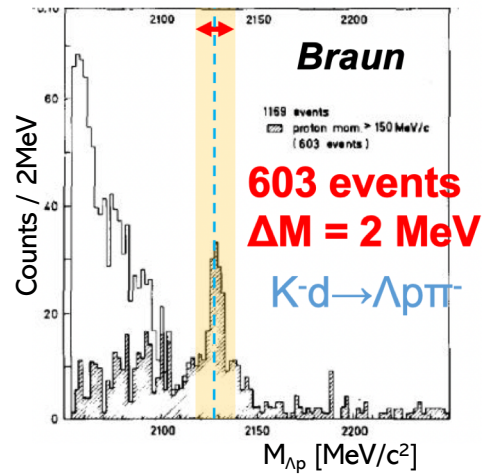
Effect is limited in this case



J-PARC E90 EXPERIMENT

Bubble chamber data

- low resolution ($\Delta M > 1 \text{ MeV}$)
- small statistics ($< 1000 \text{ events}$)



K^- : K1.8 beamline spectrometer

$\Delta p/p = 3.3 \times 10^{-4}$ in FWHM (Design)

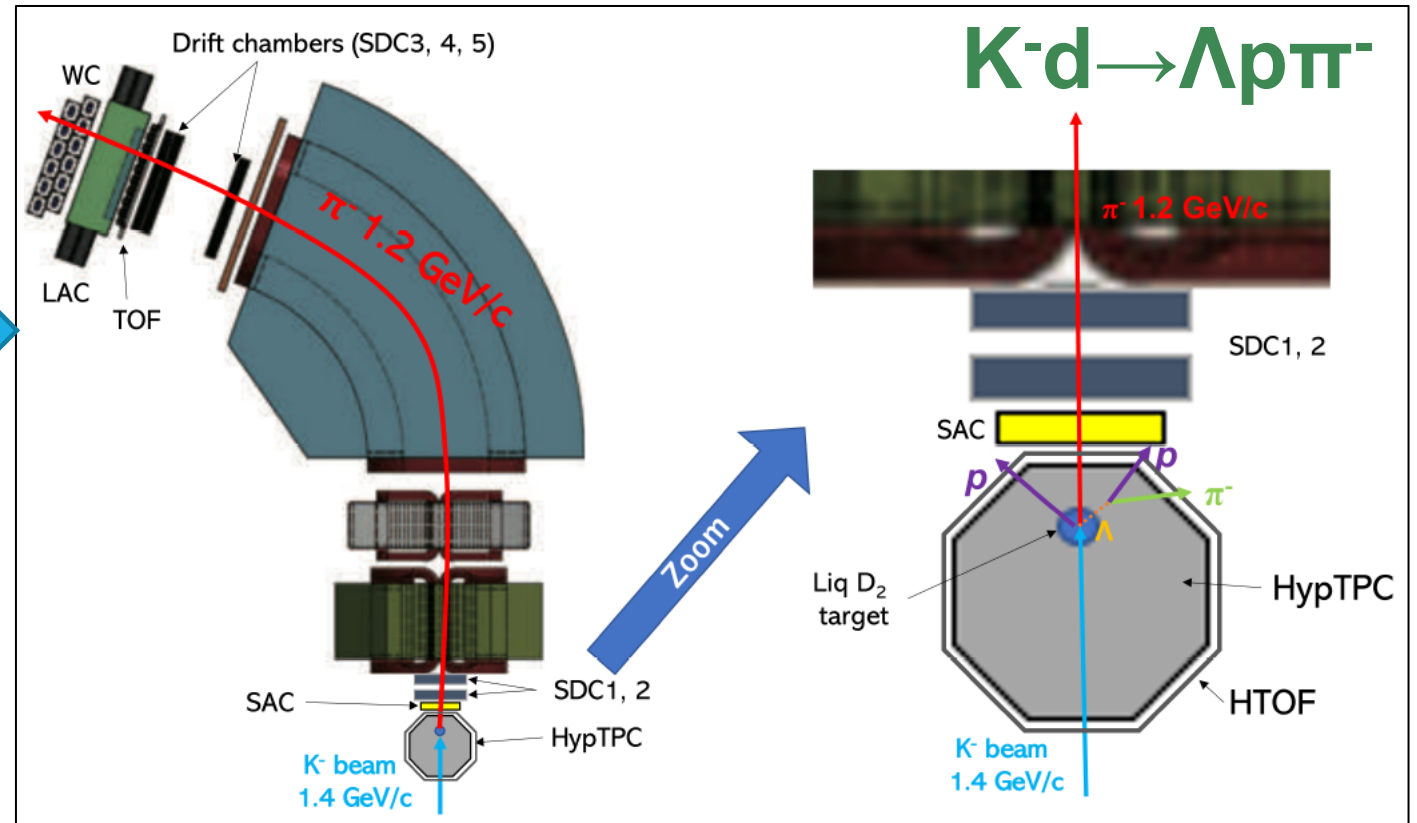
π^- : S-2S spectrometer

$\Delta p/p = 6.0 \times 10^{-4}$ in FWHM (Design)

(K^-, π^-) : (K18 + S2S) Spectrometer

Decay particles: HypTPC

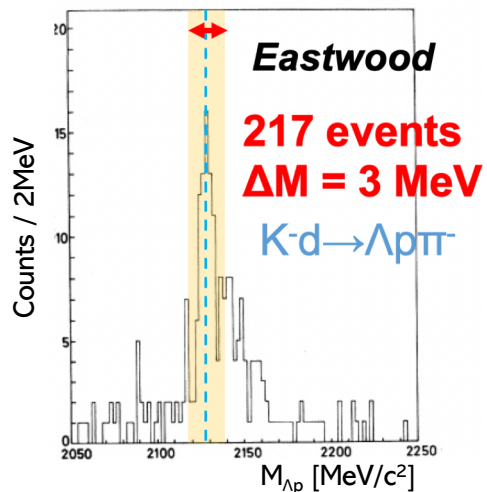
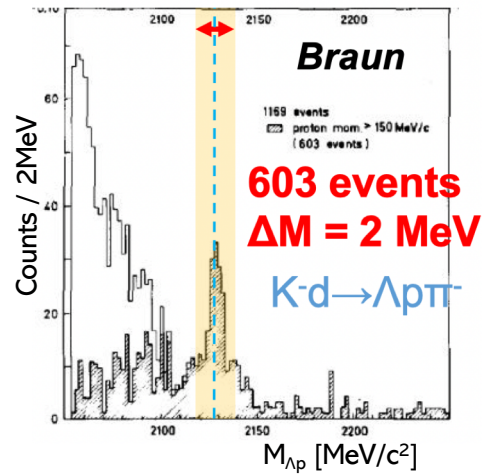
High resolution !



J-PARC E90 EXPERIMENT

Bubble chamber data

- low resolution ($\Delta M > 1 \text{ MeV}$)
- small statistics (< 1000 events)



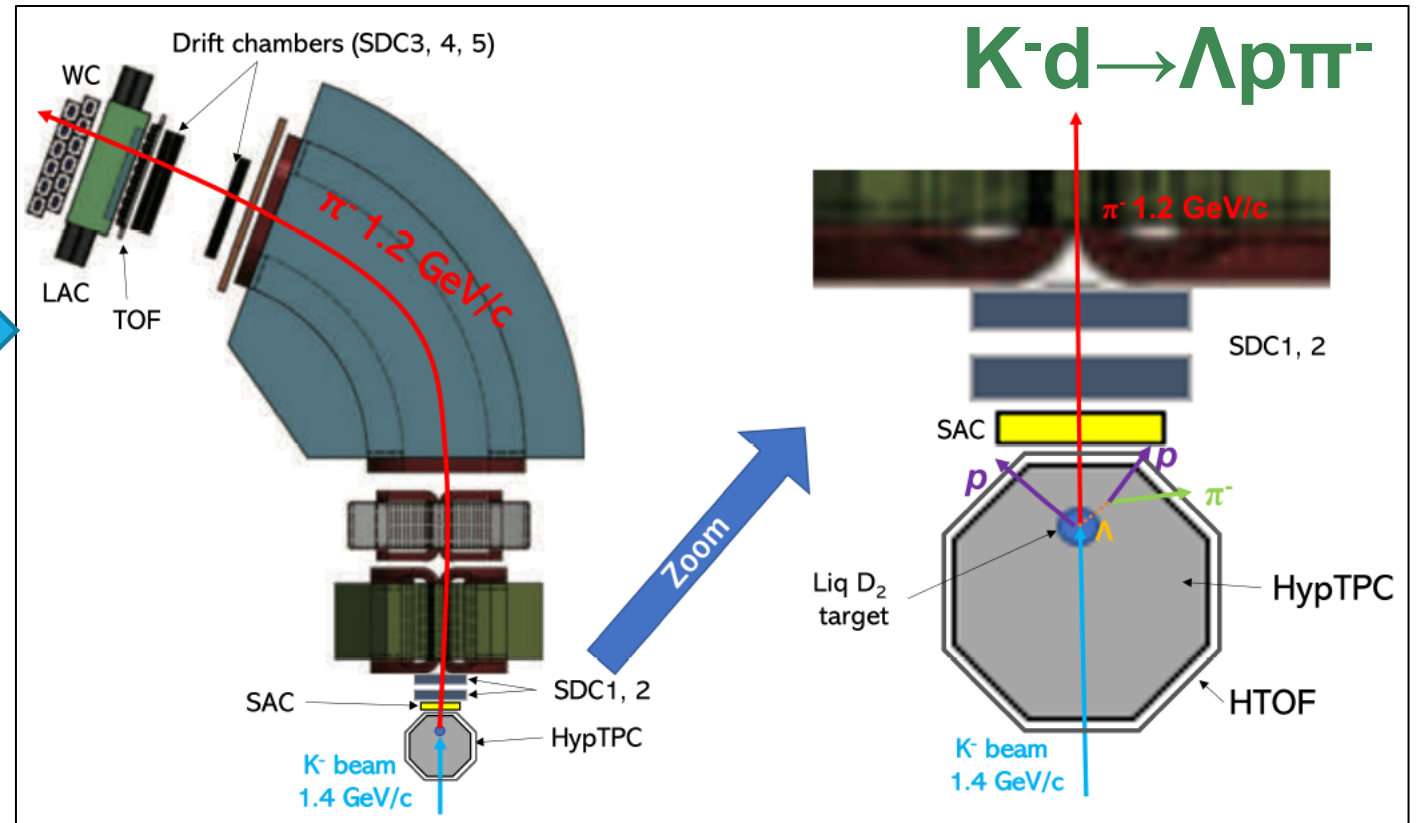
E90

- High resolution: $\Delta M = 0.4 \text{ MeV}/c^2(\sigma)$
- High statistics: 14,000 events

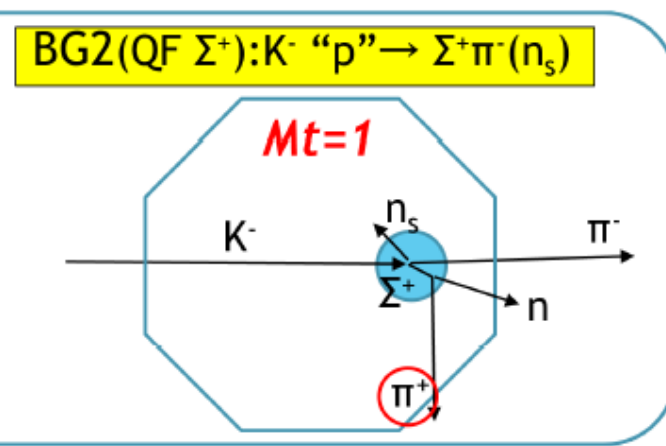
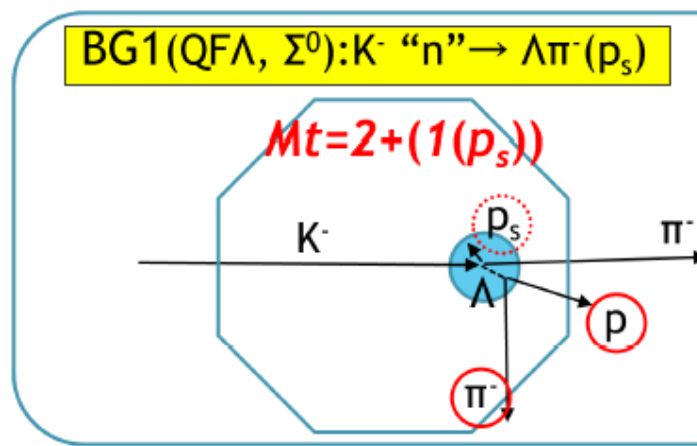
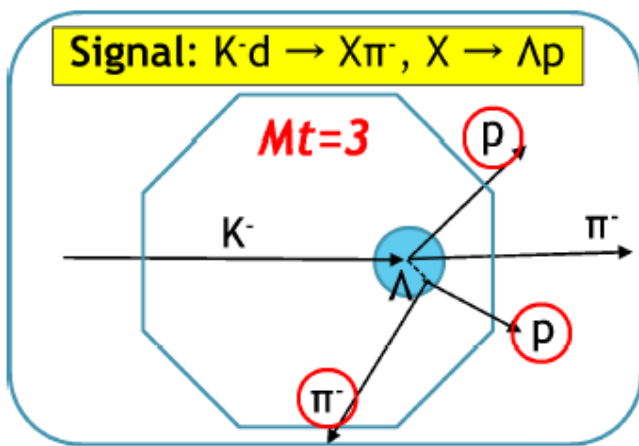
(K^-, π^-) : (K18 + S2S) Spectrometer

Decay particles: HypTPC

High resolution !

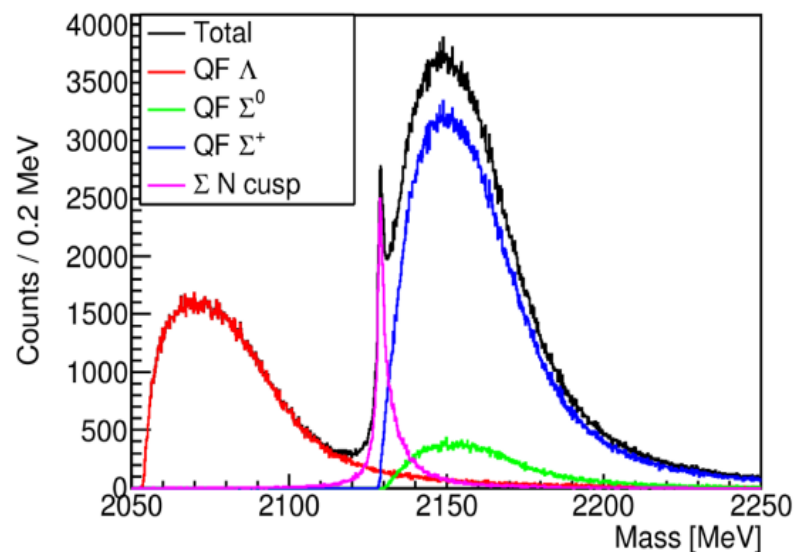


QF BACKGROUND SUPPRESSION BY HYPTPC



Simulated inclusive spectrum $d(K^-, \pi)$

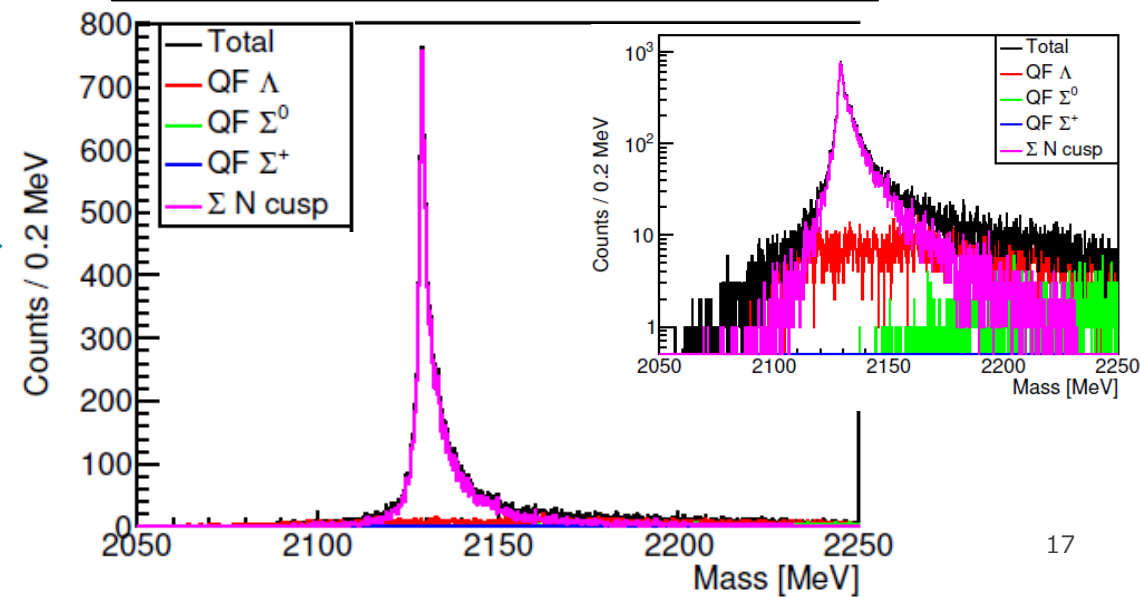
Expected spectrum
for the 15 days beam time



$\beta > 0.24$ for p
 $\beta > 0.4$ for π

Multiplicity = 3

Multiplicity = 3 without (K^-, π)



BACK GROUND REDUCTION METHOD SUBMITTED TO PTEP

[arXiv:2606.22750](https://arxiv.org/abs/2606.22750)

PTEP

Preprint number: XXXX-XXXX

Development of a Neural Network-Based Background Suppression Technique for ΣN Cusp Spectroscopy at J-PARC

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S. H. Kim³, F. Oura^{2,1}, H. I. Lee⁴, R. J. Saito¹, K. Shimazaki¹,
R. Sasaki¹, and Y. Nakayama¹

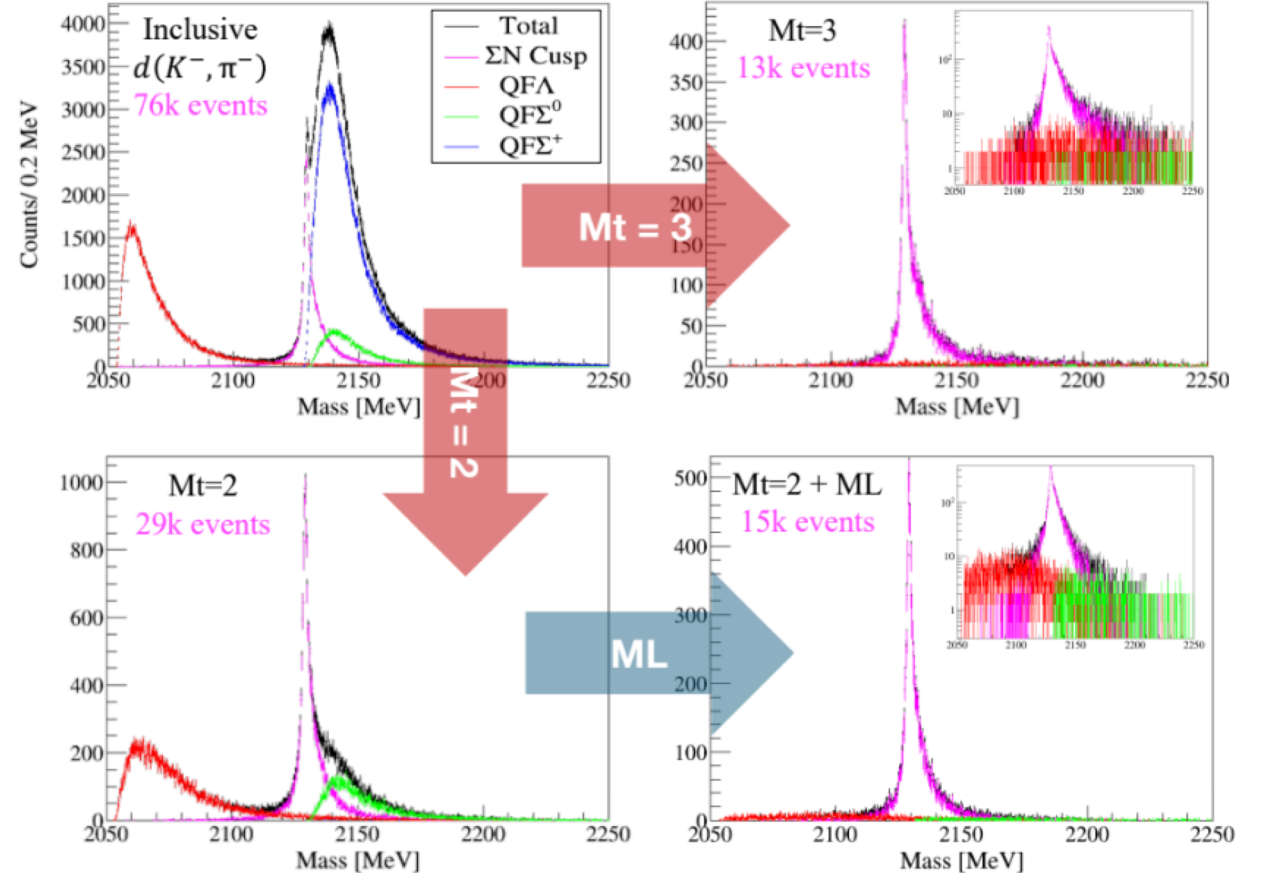
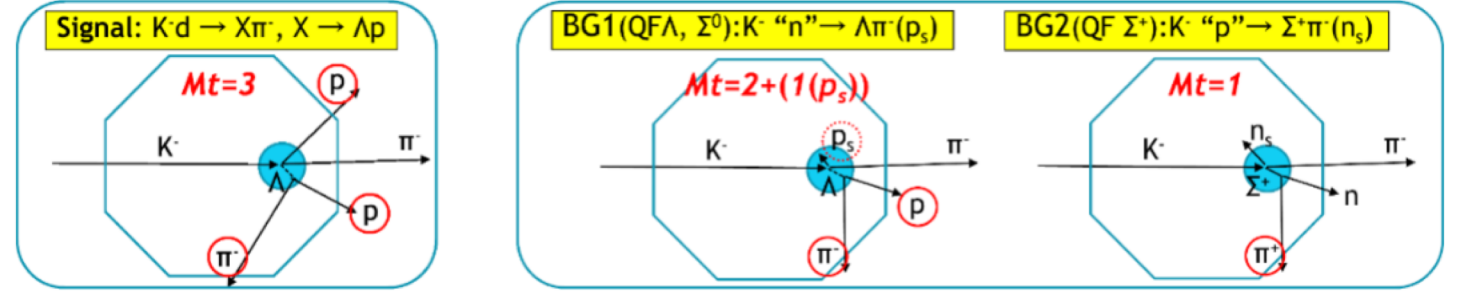
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A clear spectral enhancement, known as the “ ΣN cusp”, has been observed near the ΣN threshold in the $d(K^-, \pi^-)$ reaction. To understand the dynamical origin of this enhancement, the J-PARC E90 experiment aims to investigate the missing-mass spectrum with an unprecedented resolution of 0.4 MeV (σ). In this experiment, a Hyperon Time Projection Chamber (HypTPC) is utilized to detect charged decay products and suppress severe contamination from quasi-free (QF) background processes. While a conventional track multiplicity condition of three ($Mt=3$) effectively suppresses these QF events, it restricts the signal statistics to approximately 17% and introduces a mass-dependent acceptance bias that distorts the spectrum. In contrast, events with a track multiplicity of two ($Mt=2$) offer roughly double the statistical power ($\sim 39\%$) with minimal mass dependence, but they suffer from heavy background contamination. To fully exploit the $Mt=2$ events, we developed an innovative background suppression technique based on a neural network. By constructing a binary classification model using the HypTPC track topology and energy loss (dE/dx) as input features, we successfully discriminated the signal from QF backgrounds. This machine learning approach achieves a signal-to-noise ratio comparable to the strict $Mt=3$ condition while preserving the integrity of the spectral shape. By combining this independent ML-selected $Mt=2$ sample with the conventional $Mt=3$ sample, the total usable statistics are effectively doubled compared to traditional methods, significantly enhancing the sensitivity for determining the ΣN cusp parameters.

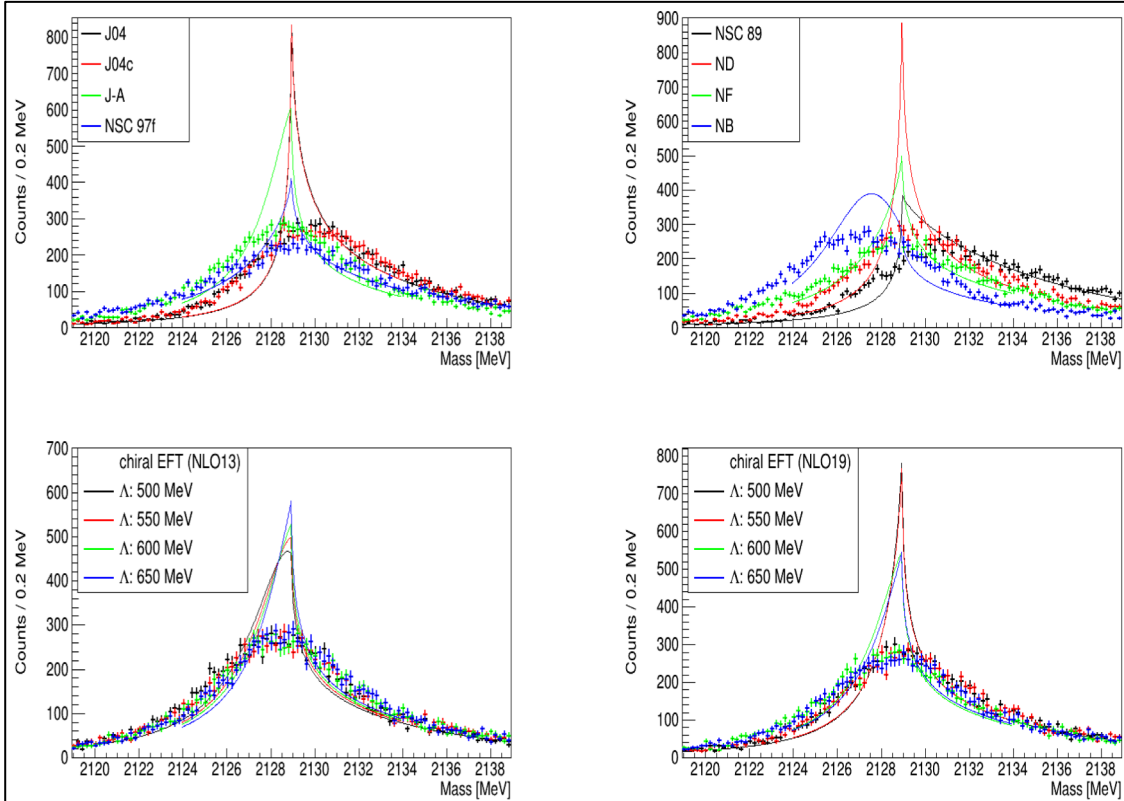
Key of E90: High resolution $\Delta M = 0.4 \text{ MeV}$

Theoretical
Value of
 ΣN scattering
length
($T=1/2, {}^3S_1$)

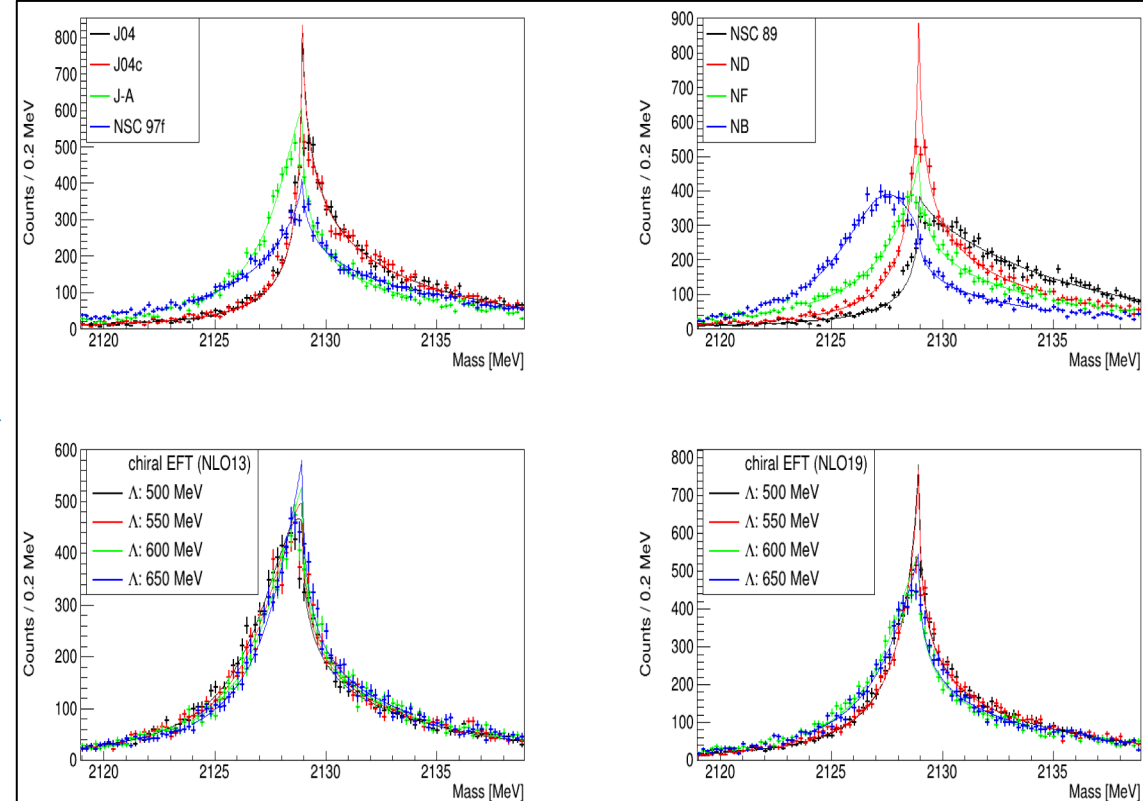
Model	J04	J04c	J-A	NSC 97f	NSC 89	ND	NF	NB
a [fm]	3.83	3.63	-2.37	-1.03	2.54	2.06	-1.29	-3.0
b [fm]	3.01	3.09	3.74	2.41	0.26	4.64	3.02	1.8
Model	chiral EFT (NLO13)				chiral EFT (NLO19)			
Λ [MeV]	500	550	600	650	500	550	600	650
a [fm]	-2.61	-2.44	-2.27	-2.06	-0.95	-0.98	-2.29	-1.95
b [fm]	2.89	3.11	3.29	3.59	4.77	4.59	3.39	3.38

ΣN -dibaryon !?

Sensitivity of past experiment ($\Delta M = 2 \text{ MeV}$)

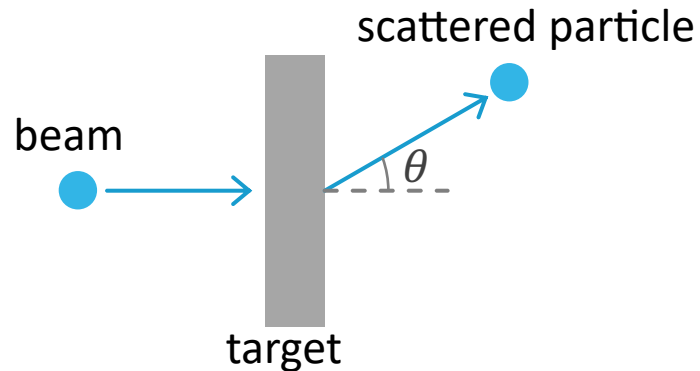


Sensitivity of E90 ($\Delta M = 0.4 \text{ MeV}$)



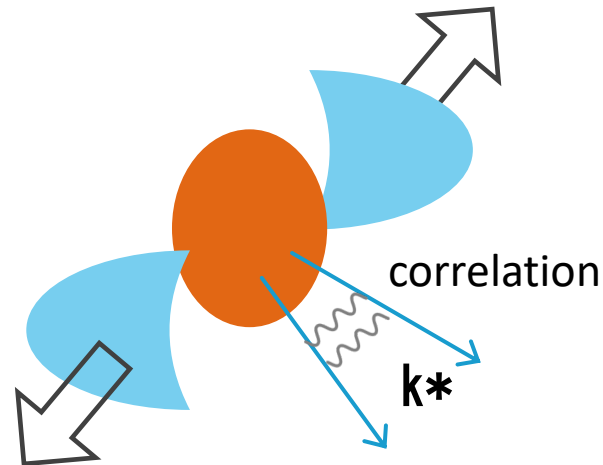
EXPERIMENTAL TECHNIQUE TO STUDY THE HADRON-HADRON INTERACTION

Scattering Experiment



The beam and target are limited

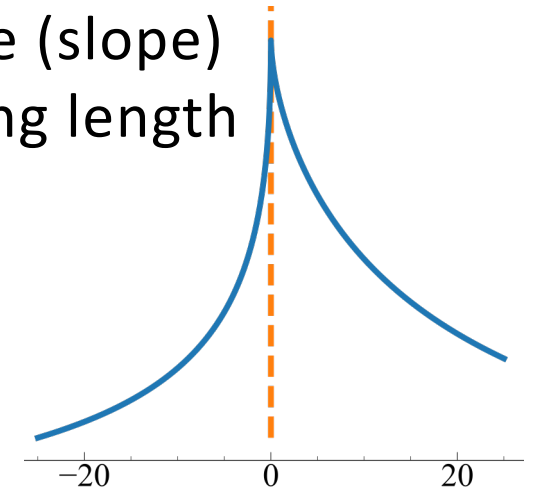
Femtoscscopy



Neutral particles are difficult to measure.
Spin decomposition.

Cusp Spectroscopy

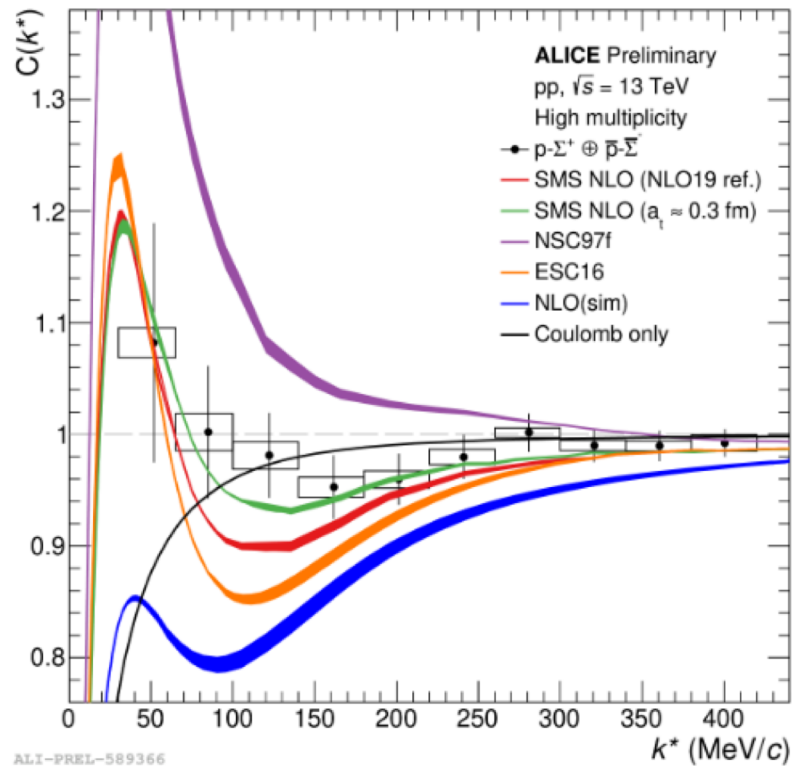
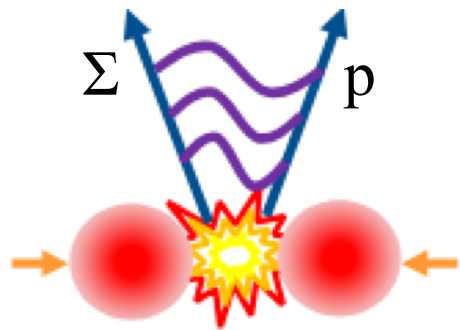
Cusp shape (slope)
→ scattering length



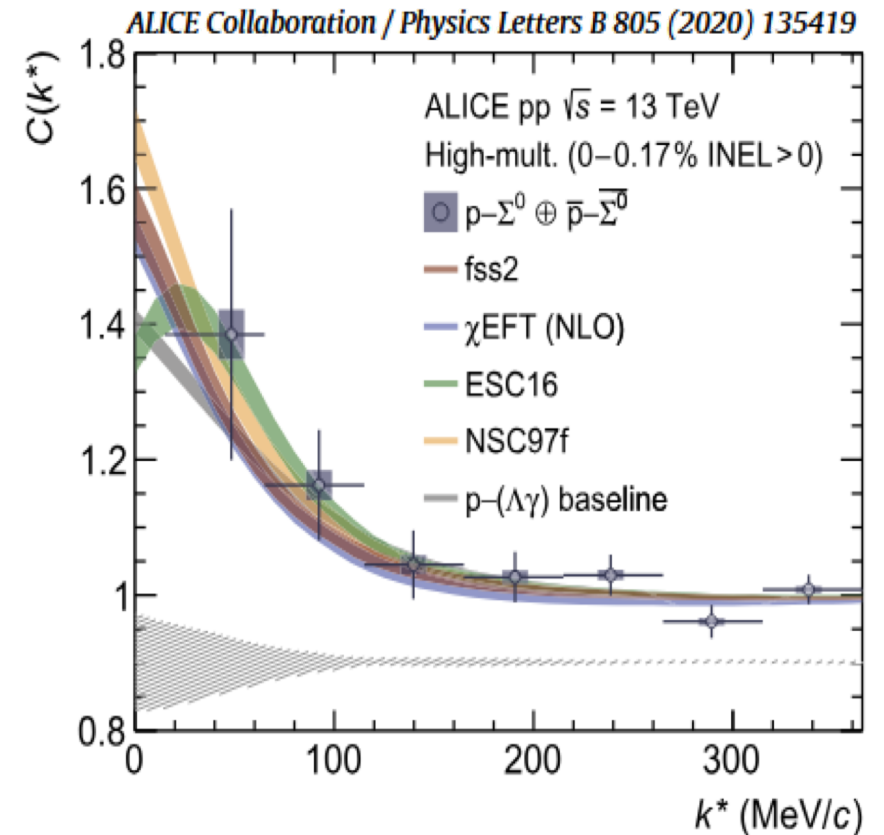
No necessary to measure the neutral particles such as $\Lambda\eta$
→ Next talk

Σp Femtoscopy at ALICE

ALICE(Run2) $\Sigma^+ p$



ALICE(Run2) $\Sigma^0 p$



SYNERGY WITH NEW TOOLS [YN SYSTEM] (FEMTO, CUSP, YN-SCATTERING)

- ΛN interaction ($T=1/2, S=(0, 1)$)

- Λp scattering experiment including spin observable

ΛN - ΣN interaction will
be completely determined

$S=0, 1$

ΛN - ΣN coupling (imaginary part)

- ΣN interaction ($T=(1/2, 3/2), S=(0, 1)$)

- J-PARC E90 (ΣN cusp) : $T=1/2, S=1$

- $\Sigma^0 p$ Femto: $T=(1/2, 3/2), S=(0, 1)$

- $\Sigma^+ p$ Femto: $T=(3/2), S=(0, 1)$

$T=1/2, S=(0,1)$

$T=1/2, S=0$

Spin decomposition!

Isospin decomposition!

SUMMARY

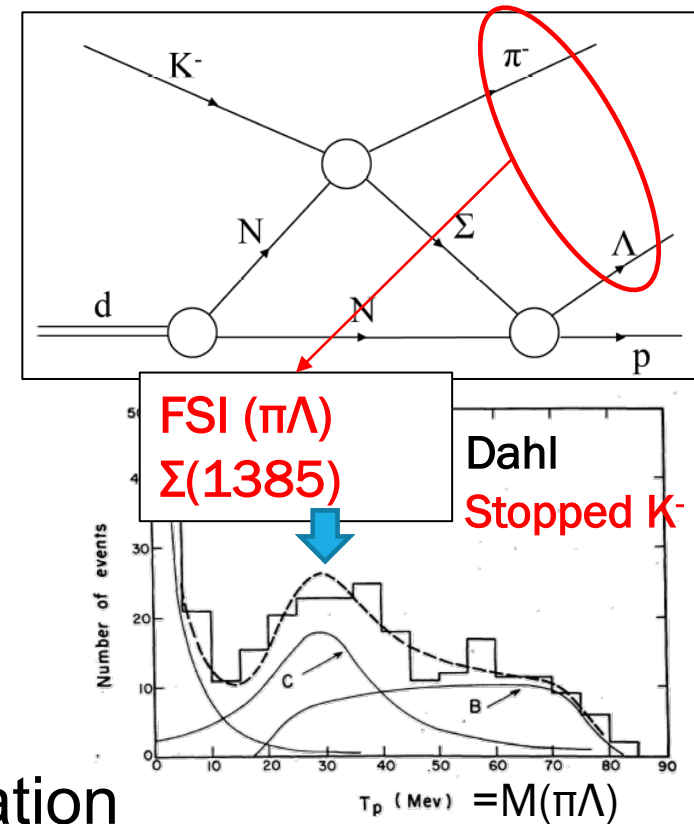
- Femtoscopy is powerful tool to open new era of hadron interactions.
- Issues:
 - Isospin and spin averaged interaction is given.
 - Neutral particles (ex. η , η') are difficult to measure.
- Cusp spectroscopy will be also good tools to study hadron interaction.
- J-PARC E90 will determine the ΣN scattering length with $(T, S) = (1/2, 1)$ channel.
- Synergy with new tools (Femto, Cusp, YN-scattering) will reveal the YN interaction for each isospin and spin combination!



BACK UP

REQUIREMENTS FOR THE “ ΣN CUSP” EXPERIMENT

- Good energy resolution ($\sigma < 1$ MeV)
- High statistics ($> 10^4$ events)
- Good Signal / Noise (S/N) ratio
- Avoid FSI **except for the ΣN**
 - **× Stopped** $K^-d \rightarrow \Lambda p \pi^-$ reaction ($p_\pi \sim p_\Lambda \sim p_p$)
FSI: $\pi\Lambda$, πp , YN (YN FSI = ΣN cusp signal)
 - **○ In-flight** $K^-d \rightarrow \Lambda p \pi^-$ reaction ($p_K \sim p_\pi \gg p_\Lambda \sim p_p$)
FSI: YN (YN FSI = ΣN cusp signal), ○ impulse approximation
- Decompose 1S_0 and 3S_1 contribution
($K^-d \rightarrow \Lambda p \pi^-$ reaction: extract only 3S_1 contribution by D-target property)



There was no experiment to satisfy these requirements!!

$\bar{K}N$ scattering length

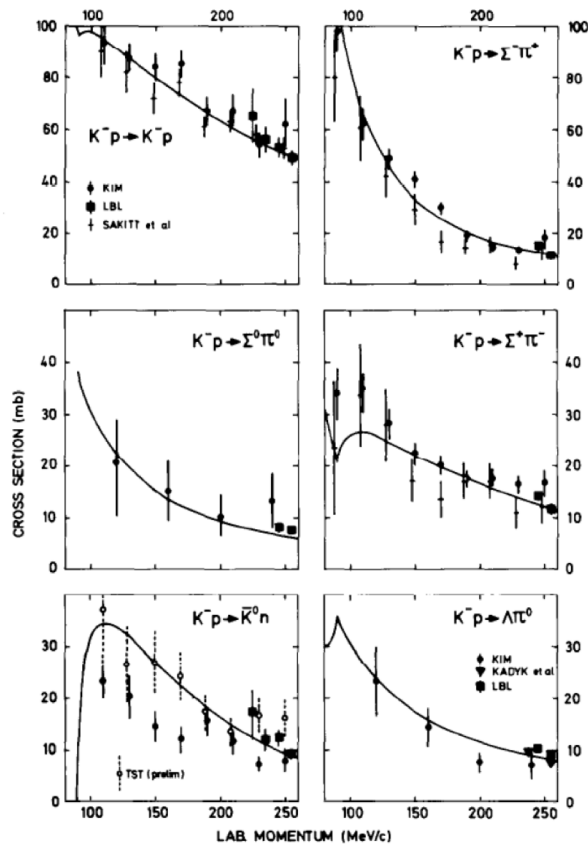


Figure 1.2: The low energy K^-p scattering data and the curves calculated from the M -matrix parameters. This figure is taken from Ref. [5].

① A.D. Martin, Nucl. Phys. B179, 33 (1981).

$$\begin{aligned} a^{I=0} &= -1.70 + i0.65 \text{ fm}, \\ a^{I=1} &= 0.37 + i0.60 \text{ fm}, \end{aligned}$$

② B. Borasoy et al., Phys. Rev. C74, 055201 (2006).
Chiral SU(3) before SIDDHARTA KpX.

$$\begin{aligned} \text{"WT"}: \quad a_0 &= (-1.45 + i0.85) \text{ fm}, \quad a_1 = (0.65 + i0.76) \text{ fm}, \\ \text{"WTB"}: \quad a_0 &= (-1.72 + i0.77) \text{ fm}, \quad a_1 = (0.09 + i0.76) \text{ fm}, \\ \text{"full"}: \quad a_0 &= (-1.64 + i0.75) \text{ fm}, \quad a_1 = (-0.06 + i0.57) \text{ fm}. \end{aligned} \quad (11)$$

③ Y. Ikeda, T. Hyodo and W. Weise, Nucl. Phys. A 881, 98 (2012).
Chiral SU(3) after SIDDHARTA KpX.

$$\text{SIDDHARTA:} \quad \text{Re } a(K^-p) = -0.65 \pm 0.10 \text{ fm}, \quad \text{Im } a(K^-p) = 0.81 \pm 0.15 \text{ fm}, \quad (25)$$

$$a(K^-p) = -0.70 + i0.89 \text{ fm (NLO)} \rightarrow \mathbf{a(K^-p) = \frac{1}{2}(a(I=0)+a(I=1))}$$

$$a(K^-n) = 0.57 + i0.73 \text{ fm (NLO)} \rightarrow \mathbf{a(K^-n) = a(I=1)}$$

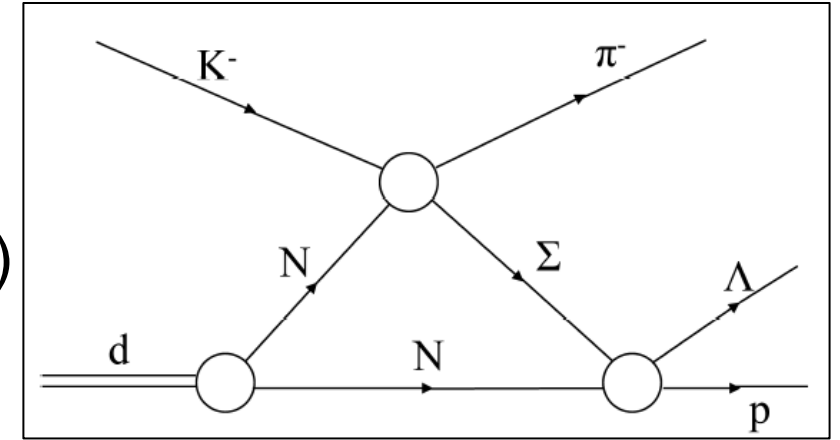
$$\rightarrow \mathbf{a(I=0) = -1.97 + i1.05 \text{ fm}}$$

“ΣN cusp” $K^-d \rightarrow \pi^- \Lambda p$ reaction

R.H. Dalitz, Nucl. Phys. A354, 101 (1981).

Amplitude of the elementary reaction

$$f(\theta) \propto T(\bar{K}d \rightarrow \Lambda N \pi) \sim \underbrace{T(\bar{K}N \rightarrow \pi \Sigma)}_{\text{Deuteron factor}} \underbrace{F_d(\vec{Q}_\Sigma, k_\Sigma)}_{\text{Amplitude of } \Sigma N \rightarrow \Lambda N \text{ reaction}} \underbrace{T(\Sigma N \rightarrow \Lambda N)}_{\text{Important term}}$$



Deuteron factor

$$F_d(Q_\Sigma, k_\Sigma) = \int \frac{e^{ik_\Sigma r}}{r} e^{i\vec{Q}_\Sigma \cdot \vec{r}} \psi_d(r) d^3r$$

$$\vec{Q}_\Sigma = \vec{q} m_N / (m_N + m_\Sigma)$$

$\psi_d(r)$: deuteron wave function

Amplitude of $\Sigma N \rightarrow \Lambda N$ reaction (*Important term*)

$$T(\Sigma N \rightarrow \Lambda N) \propto \frac{\sqrt{b}}{1 - ikA}$$

$$\text{Above threshold: } R = \frac{4\pi b}{\{(1+kb)^2 + (ka)^2\}} \sim 1 - 2kb + O(k^2)$$

$$\text{Below threshold: } R = \frac{4\pi b}{\{(1+\kappa a)^2 + (\kappa b)^2\}} \sim 1 - 2\kappa a + O(\kappa^2), \quad k = i\kappa$$

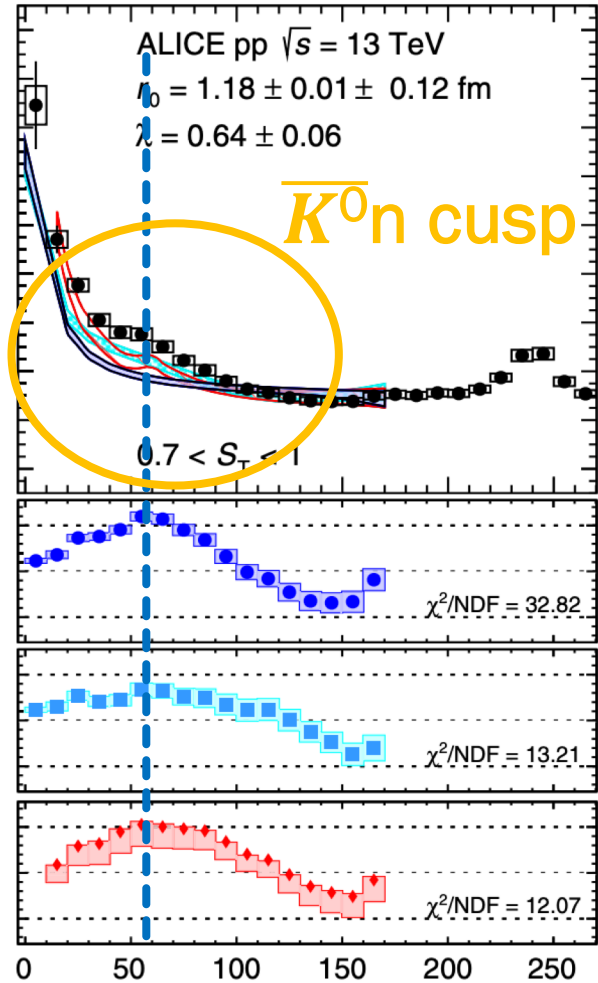
$(k \sim \sqrt{2\mu E})$

“ΣN Cusp” can be expressed by the ΣN scattering length ($A_\Sigma = a + ib$) of the **(T, S) = (1/2, 3S_1)** channel!!

$\overline{K}^0 n$ cusp structure (calculation)

$K^- p$

$\overline{K}^0 n$ threshold



—①(Martin) $A = 0.37 + 0.60$ fm

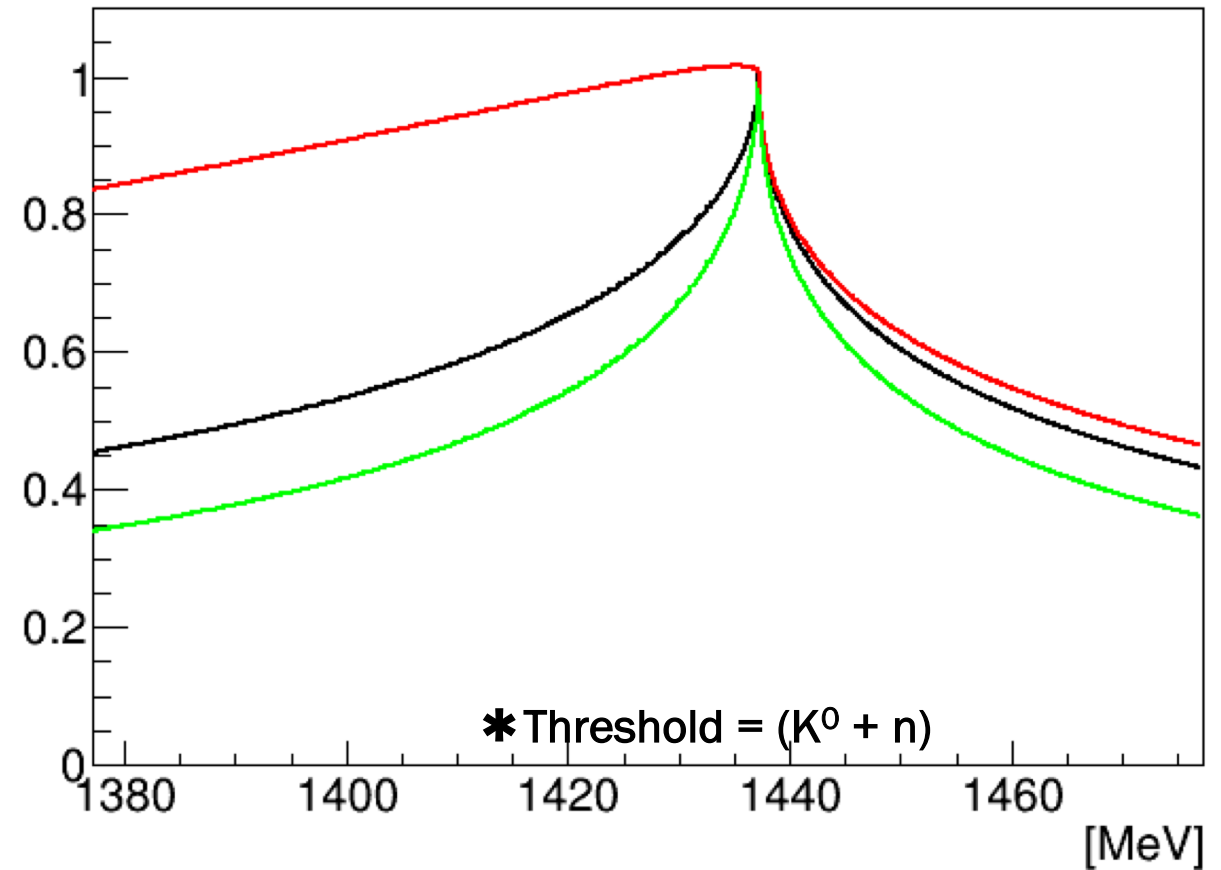
—②(Chiral Old) $A = -0.06 + 0.57$ fm

—③(Chiral IHW) $A = 0.57 + 0.73$ fm

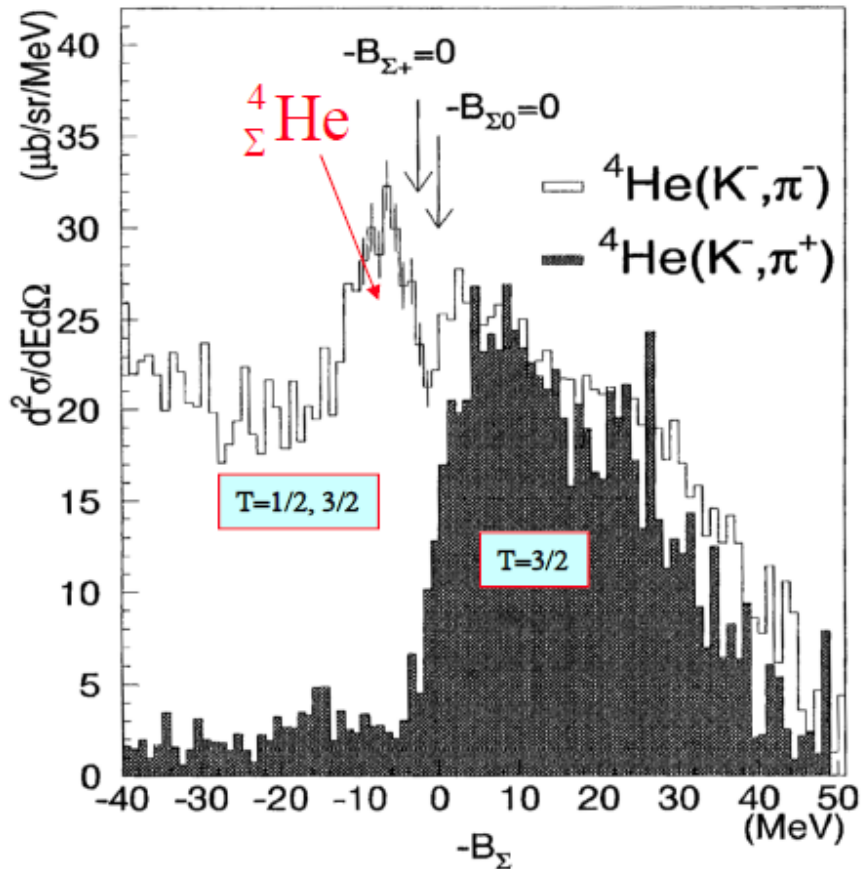
$E = (1491.3, 107.6)$ MeV

$E = (1615.4, -37.9)$ MeV

$E = (1454.1, 67.7)$ MeV



IMPORTANCE OF $(T, S) = (1/2, 1)$ CHANNEL IN $^4_\Sigma\text{He}$



ΣN interaction has strong T and S dependence.
 $V_{T=3/2, s=1}$ and $V_{T=1/2, s=0}$ are expected to be repulsive due to quark Pauli-blocking effect.
 $\rightarrow$ No Σ -hypernuclei in large A system.

ΣN cusp channel: $V_{T=1/2, s=1}$
 (expected to be attractive potential and origin for the $^4_\Sigma\text{He}$ bound state)

$\Sigma N N N$ (4body) system

ΣN (2body) system

$^4\text{He}(K^-, \pi^+):$

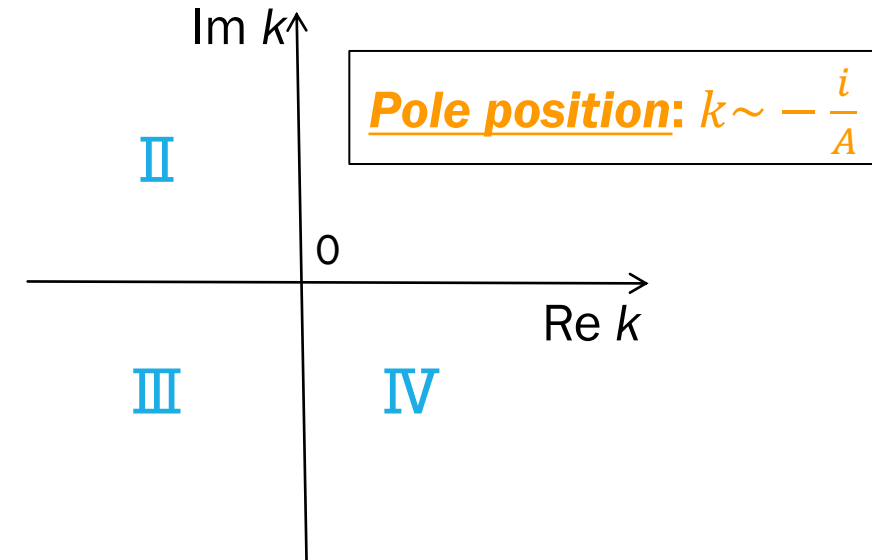
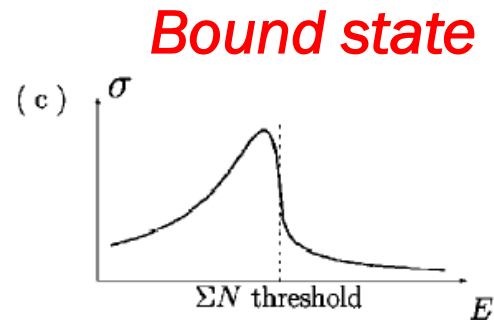
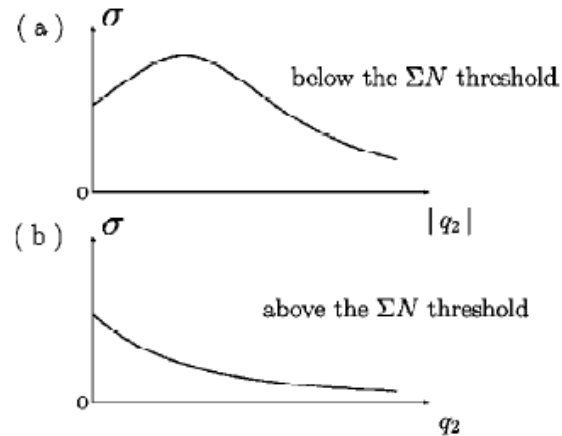
$^4\text{He}(K^-, \pi^-):$

$\Sigma N N N$ (4body) system		ΣN (2body) system	
T	S	$\bar{V}_{\Sigma N} (V_{TS})$	
$\frac{3}{2}$	0	$\frac{5}{18} V_{\frac{3}{2}0}$	$+\frac{1}{2} V_{\frac{3}{2}1}$
$\frac{1}{2}$	0	$\frac{4}{9} V_{\frac{3}{2}0}$	$+\frac{2}{9} V_{\frac{1}{2}0}$
			$+\frac{1}{18} V_{\frac{1}{2}0}$
			$+\frac{1}{2} V_{\frac{1}{2}1}$

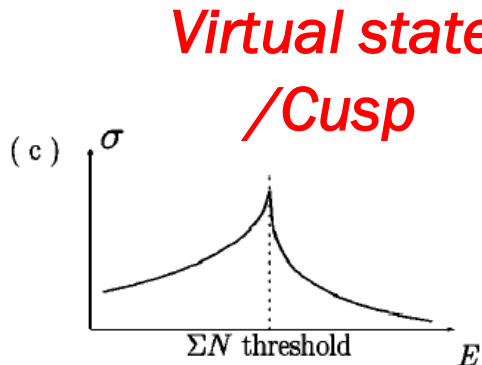
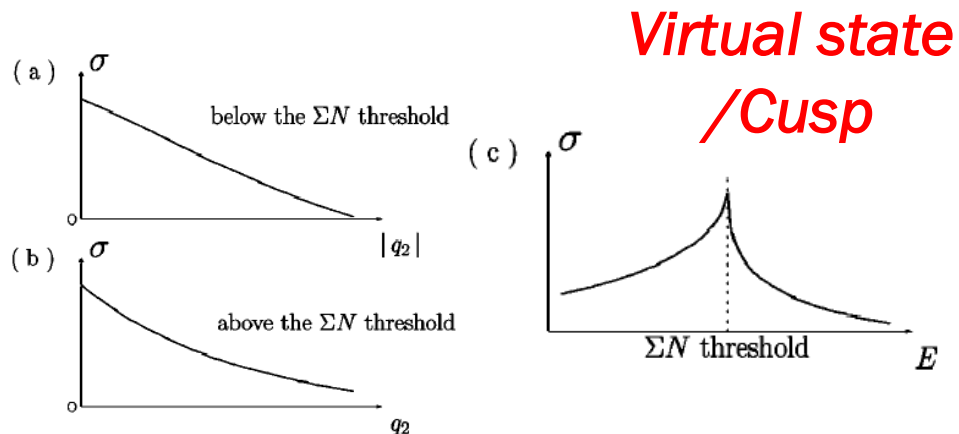
POLE POSITION vs CROSS SECTION ($d\sigma/dE$)

K. Miyagawa and H. Yamamura, PRC 60, 024003 (1999).

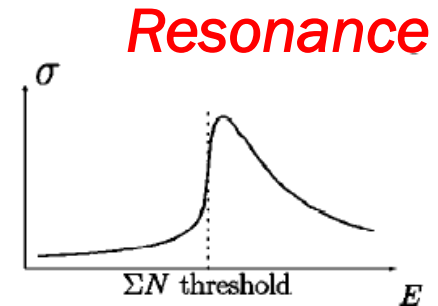
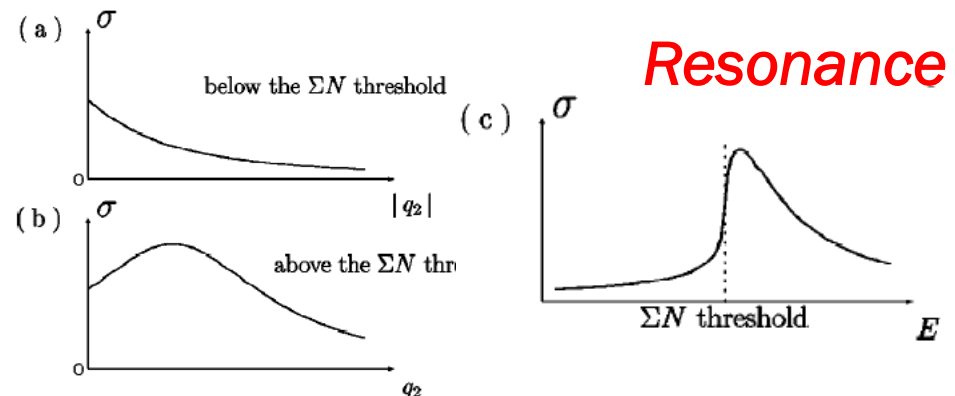
- Pole in (II) quad



- Pole in (III) quad



- Pole in (IV) quad

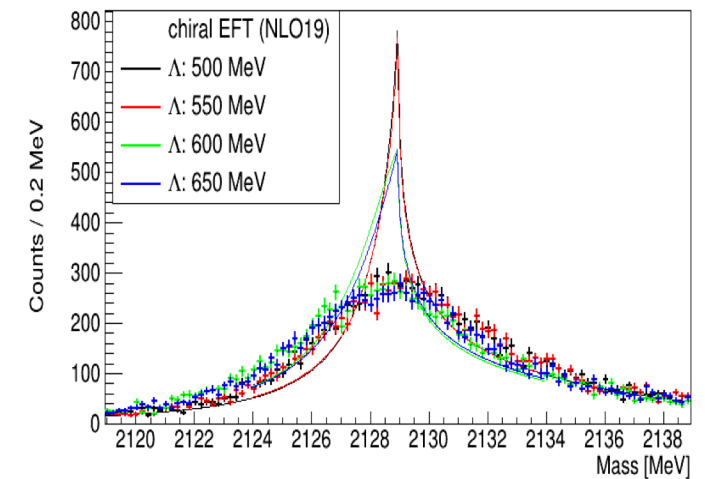
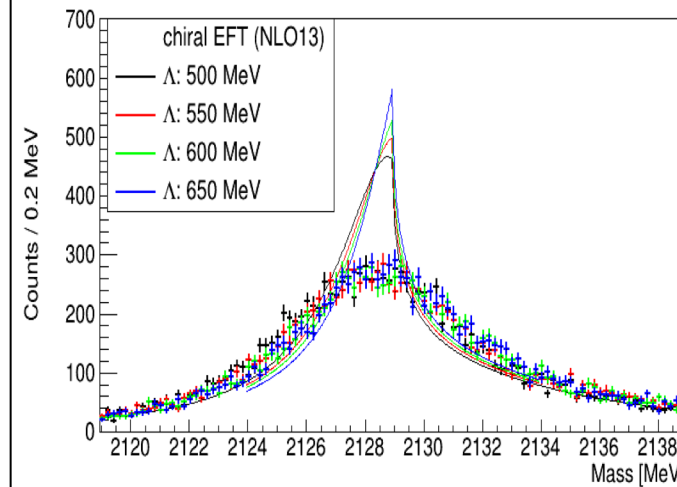
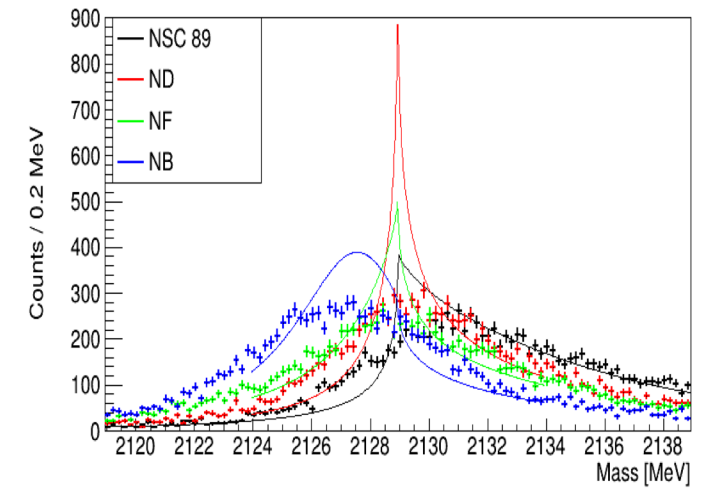
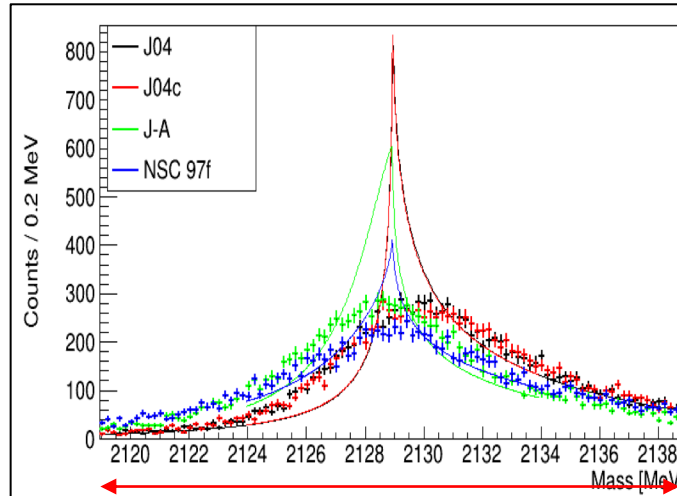
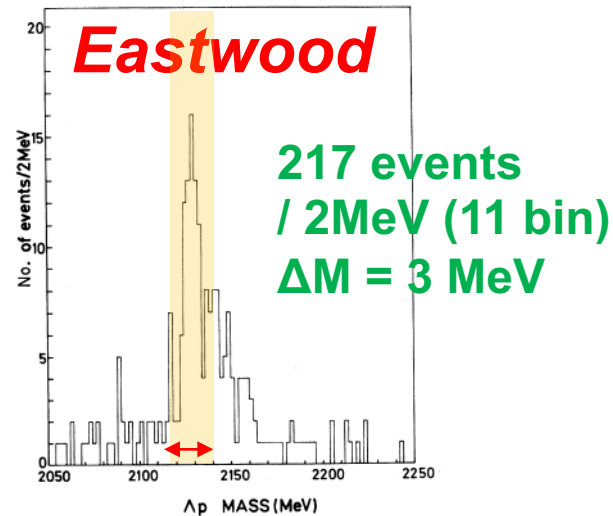
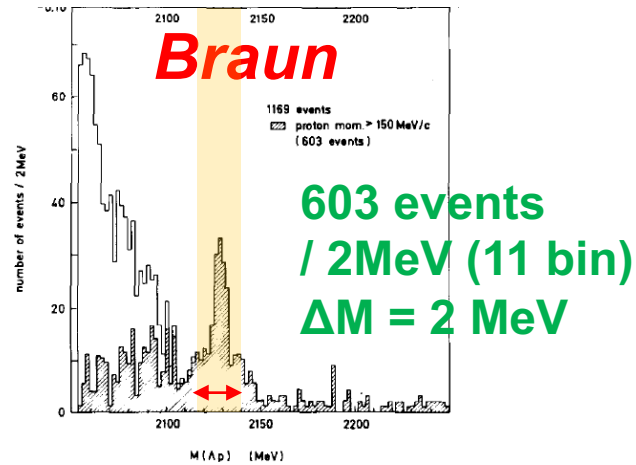


Sensitivity of **past experiment's resolution** ($\Delta M = 2 \text{ MeV}$)

with E90 statistics (1.4×10^4 events) / 0.2 MeV
Old bubble chamber: $< 10^3$ events

Model	J04	J04c	J-A	NSC 97f	NSC 89	ND	NF	NB
a [fm]	3.83	3.63	-2.37	-1.03	2.54	2.06	-1.29	-3.0
b [fm]	3.01	3.09	3.74	2.41	0.26	4.64	3.02	1.8

Model	chiral EFT (NLO13)				chiral EFT (NLO19)			
Λ [MeV]	500	550	600	650	500	550	600	650
a [fm]	-2.61	-2.44	-2.27	-2.06	-0.95	-0.98	-2.29	-1.95
b [fm]	2.89	3.11	3.29	3.59	4.77	4.59	3.39	3.38

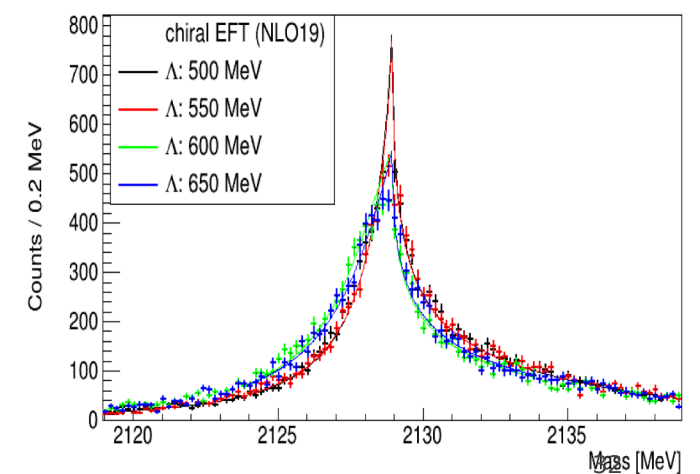
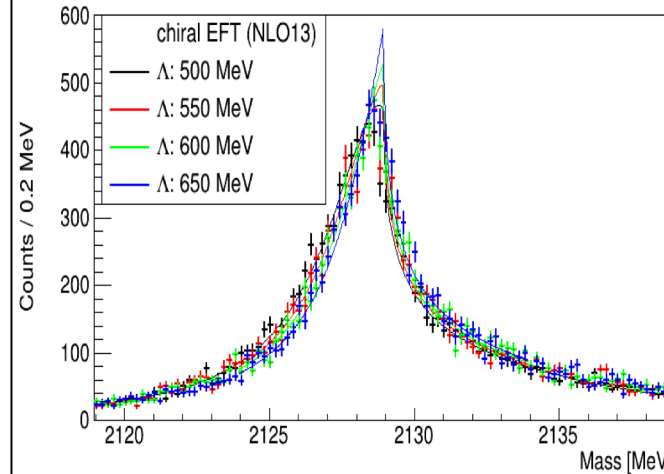
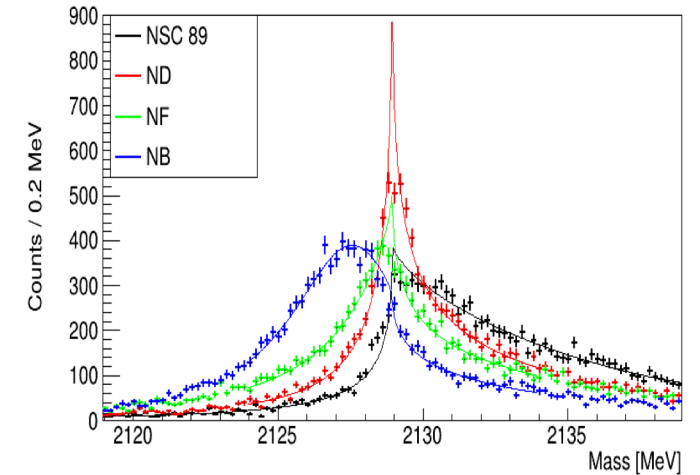
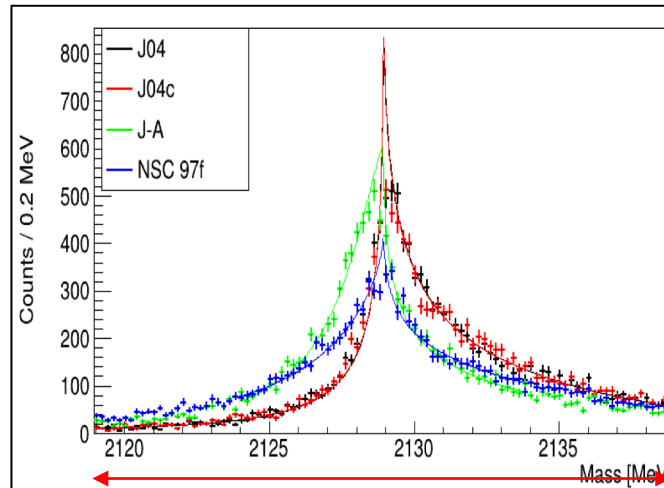
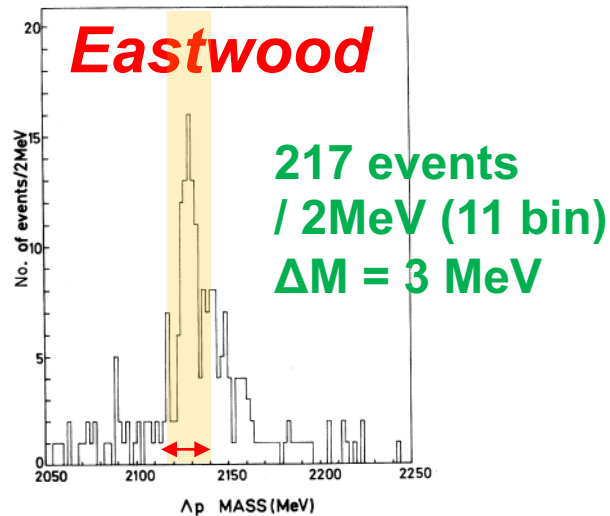
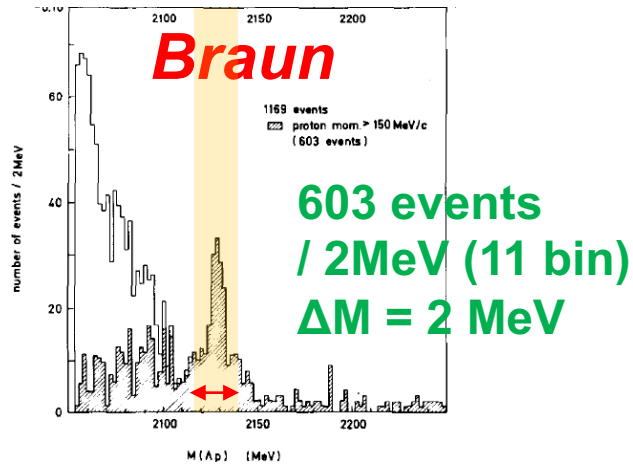


Sensitivity of E90's resolution ($\Delta M = 0.4 \text{ MeV}$)

with E90 statistics (1.4×10^4 events) / 0.2 MeV
Old bubble chamber: $< 10^3$ events

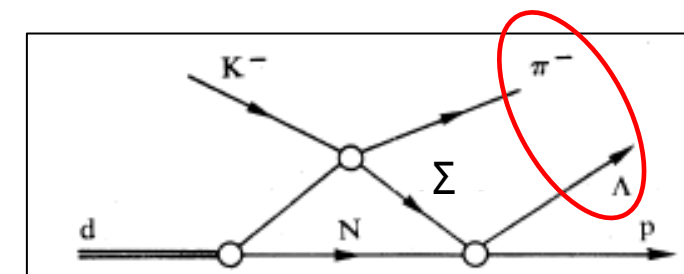
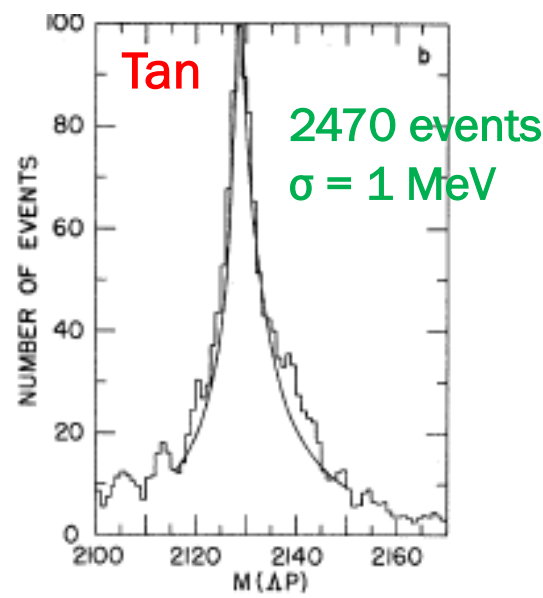
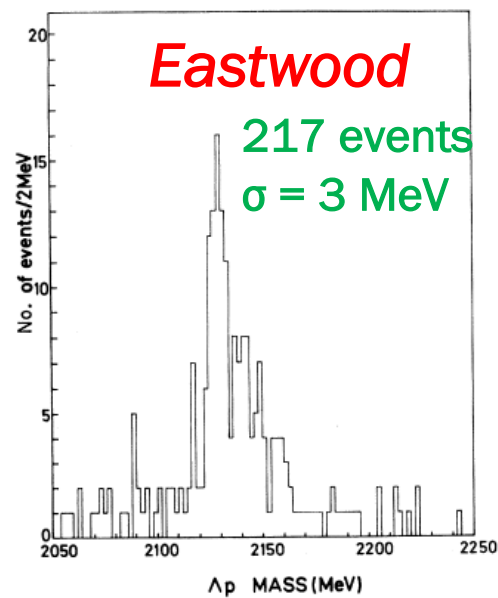
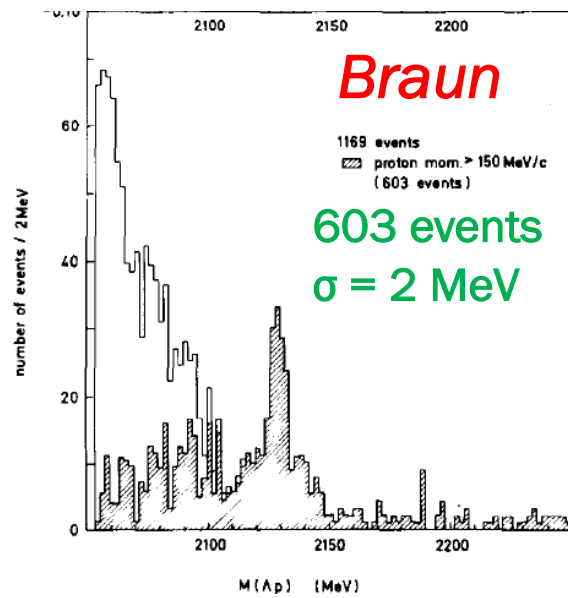
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Model	chiral EFT (NLO13)				chiral EFT (NLO19)			
Λ [MeV]	500	550	600	650	500	550	600	650
a [fm]	-2.61	-2.44	-2.27	-2.06	-0.95	-0.98	-2.29	-1.95
b [fm]	2.89	3.11	3.29	3.59	4.77	4.59	3.39	3.38



Bubble chamber

	Reaction	Comments	Statistics	Resolution
Braun	Inflight $d(K^-, \pi^-) \Lambda p$ 680 – 840 MeV/c	Low statistic, worse resolution	603 events ($\cos \theta > 0.9$, momcut)	2 MeV
Eastwood	Inflight $d(K^-, \pi^-) \Lambda p$ 1450, 1650 MeV/c	Low statistic worse resolution	217 events ($\cos \theta > 0.9$, momcut)	3 MeV
Tan	stopped $d(K^-, \pi^-) \Lambda p$	Large FSI	2470 events	1 MeV
Pigot	Inflight $d(K^-, \pi^-)$, $d(\pi^+, K^+)$	Poor resolution	Uncertain	9.1 MeV ($d(K^-, \pi^-)$ 1.4 GeV/c)
$pp \rightarrow \Lambda p K^+$ (COSY etc)	$pp \rightarrow \Lambda p K^+$	$^1S_0 + ^3S_1$ admixture Worse SN	High	0.8 MeV
ALICE	pp (Femtoscropy)	$^1S_0 + ^3S_1$ admixture	High	No description
J-PARC E27	$d(\pi^+, K^+)$ (Inclusive)	Worse SN (inclusive)	High	1.4 MeV



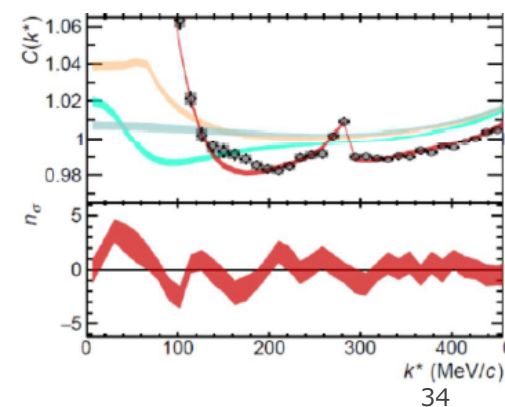
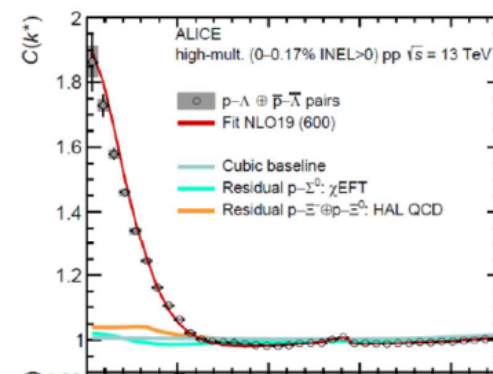
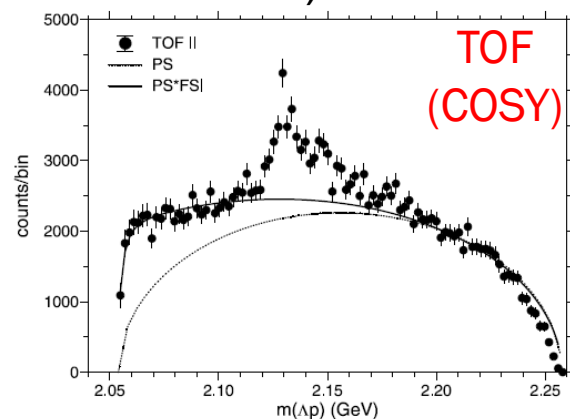
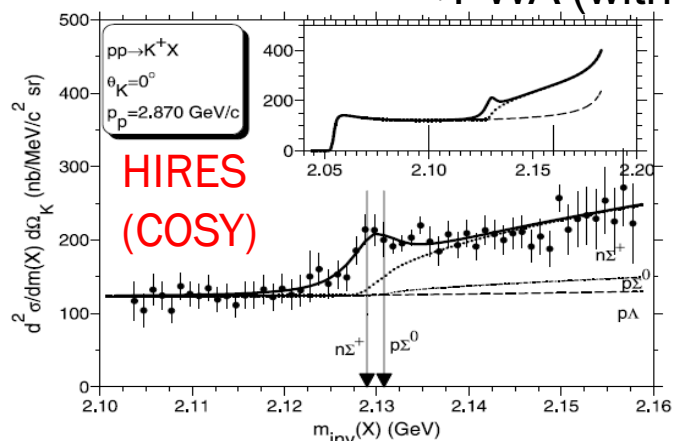
Stopped K^- reaction

- Multiple K^- scattering
- FSI: $\pi\Lambda$, πp ($p_\pi \sim p_\Lambda \sim p_p$)**
[YN FSI = Signal]

	Reaction	Comments	Statistics	Resolution
Braun	Inflight $d(K^-, \pi^-) \Lambda p$ 680 – 840 MeV/c	Low statistic, worse resolution	603 events ($\cos \theta > 0.9$, momcut)	2 MeV
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$pp \rightarrow \Lambda p K^+$ (COSY etc)	$pp \rightarrow \Lambda p K^+$	$^1S_0 + ^3S_1$ admixture Worse SN	High	0.8 MeV
ALICE	pp (Femtoscropy)	$^1S_0 + ^3S_1$ admixture	High	No description
J-PARC E27	$d(\pi^+, K^+)$ (Inclusive)	Worse SN (inclusive)	High	1.4 MeV

$pp \rightarrow \Lambda p K^+$: Good resolution, Worse SN, $^1S_0 + ^3S_1$ mixed,
Complicated reaction mechanism (via N^* , Δ^*)
→ PWA (with Spin observable)

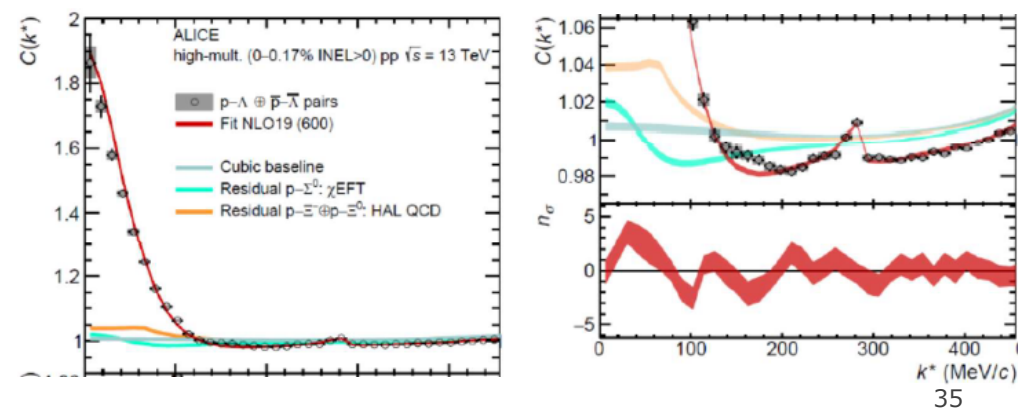
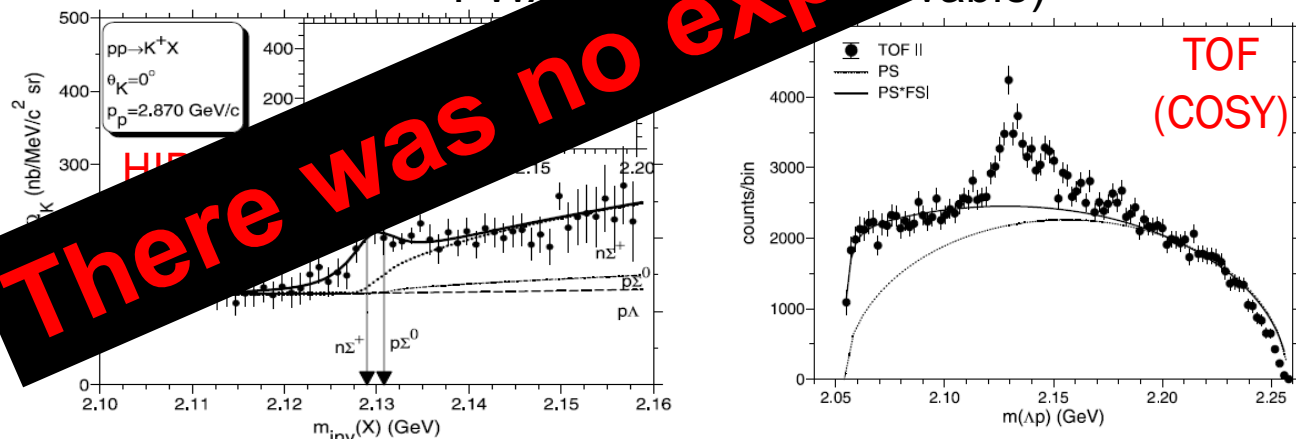
ALICE ($pp@13\text{TeV}$, Femtoscopy)
 $^1S_0 + ^3S_1$ mixed → Spin observable



	Reaction	Comments	Statistics	Resolution
Braun	Inflight $d(K^-, \pi^-) \Lambda p$ 680 – 840 MeV/c	Low statistic, worse resolution	603 events ($\cos \theta > 0.9$, momcut)	2 MeV
Eastwood	Inflight $d(K^-, \pi^-) \Lambda p$ 1450, 1650 MeV/c	Low statistic worse resolution	217 events ($\cos \theta > 0.9$, momcut)	3 MeV
Tan	stopped $d(K^-, \pi^-) \Lambda p$	Large FSI	2470 events	
Pigot	Inflight $d(K^-, \pi^-)$, $d(\pi^+, K^+)$	Poor resolution	Unclear	9.1 MeV ($d(K^-, \pi^-)$ 1.4 GeV/c)
$pp \rightarrow \Lambda p K^+$ (COSY etc)	$pp \rightarrow \Lambda p K^+$	$^1S_0 + ^3S_1$ admixture Worse resolution		0.8 MeV
ALICE	pp (Femtoscscopy)	$^1S_0 + ^3S_1$ mixed	High	No description
J-PARC E27	$d(\pi^+, K^+)$ (Inclusive)	(Inclusive)	High	1.4 MeV

$pp \rightarrow \Lambda p K^+$: Good resolution, Worse FSI, $^1S_0 + ^3S_1$ mixed,
Complicated reaction mechanism (via N^* , Δ^*)
 $\rightarrow$ PWA (Spin observable)

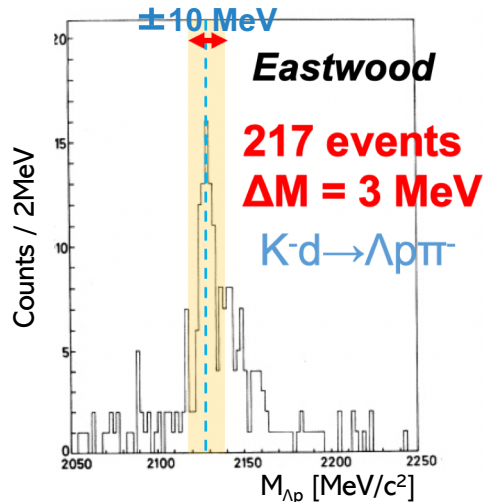
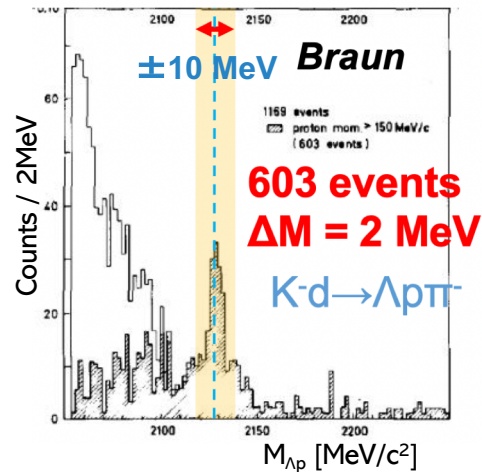
ALICE ($pp@13\text{TeV}$, Femtoscscopy)
 $^1S_0 + ^3S_1$ mixed $\rightarrow$ Spin observable



SENSITIVITY OF OLD BUBBLE CHAMBER

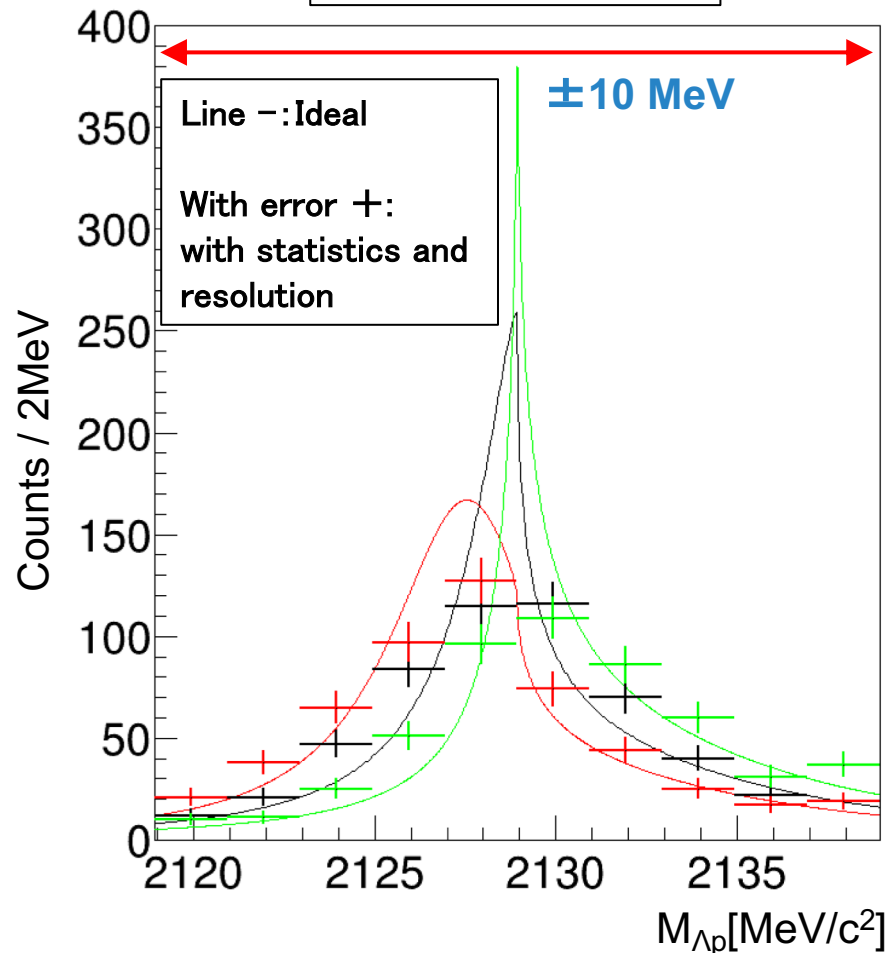
Bubble chamber data

- low resolution ($\Delta M > 1 \text{ MeV}$)
- small statistics ($< 1000 \text{ events}$)

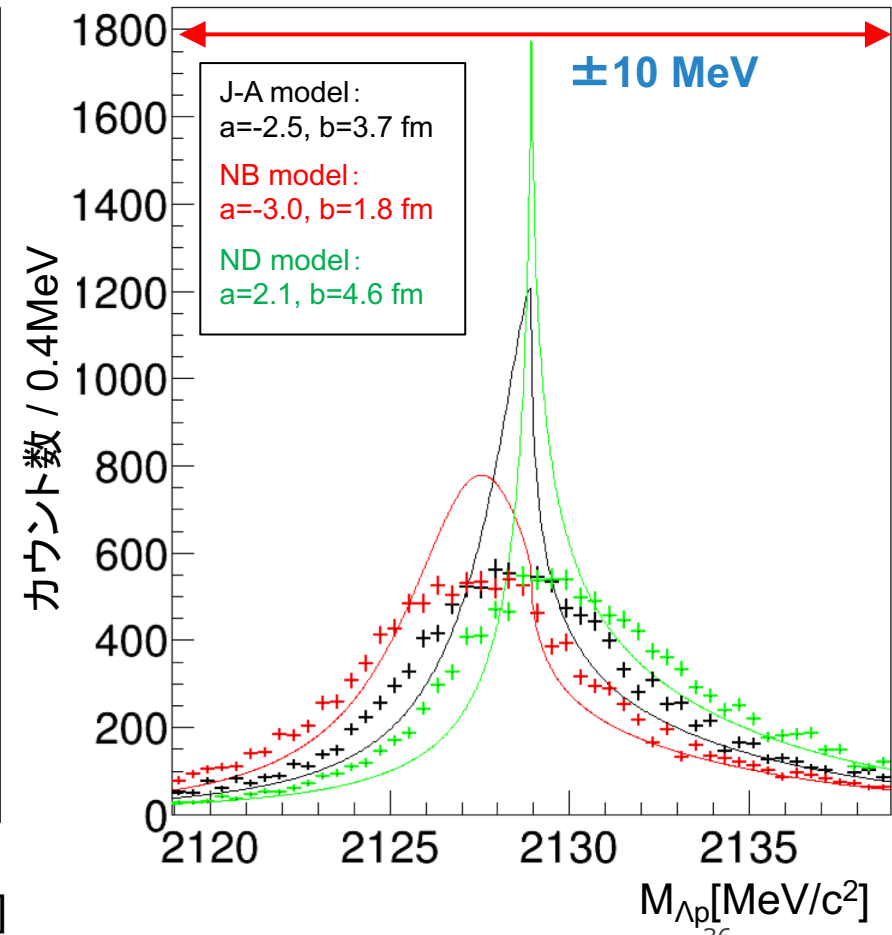


Resolution is the most important ! !

$\Delta M = 2 \text{ MeV}$
600 events



$\Delta M = 2 \text{ MeV}$
14,000 events



CROSS SECTION OF ELEMENTARY PROCESS

